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Abstract

The effect of a non-uniform resistive wall on the stability of plasma MHD modes is examined. For the case of a tokamak plasma interacting with a wall possessing toroidally varying resistivity the kink mode dispersion relation is found to reduce to a surprisingly simple form. The influence of a wall with toroidal gaps on tokamak plasma stability is investigated in some detail. Under some circumstances kink modes are found to 'explode' through the gaps with ideal growth-rates. A similar investigation is made for a modular wall constructed of alternate thick and thin sections.

I Introduction

The interaction of magnetohydrodynamical (MHD) plasma instabilities with a resistive wall in toroidal pinches has been extensively studied in the literature.[1–22] It is generally accepted that eddy currents induced in the wall can moderate the growth of an otherwise ideally unstable kink mode, so that it evolves on some characteristic resistive time-scale of the wall. Such modes are usually referred to as 'wall modes'. The interaction of a rotating tearing mode island with self-induced wall eddy currents is thought to generate a non-linear slowing torque which effectively brakes the rotation once a critical island width is exceeded.[19,22] This effect is important because a non-rotating (or 'locked') tearing mode is generally more unstable than a rapidly rotating one, since the non-rotating mode is able to penetrate through the wall.[11]

This paper is concerned with the stability of MHD modes in tokamaks which possess close fitting walls with toroidally varying electrical resistance. In fact, most modern tokamaks are of this type since their vacuum vessels are of modular construction, with thick low-resistance sections (containing the diagnostic ports) separated by thin high-resistance bellows sections.[23–25] It is clearly of interest to establish whether wall modes grow on the relatively slow resistive time-scale of the thick sections, the much faster resistive time-scale of the thin sections, or some appropriate average of the two. For obvious engineering reasons magnetic pick-up coils tend to be attached to the thick sections of the vacuum vessel. Distortions induced in the structure of MHD modes by the non-uniform eddy currents flowing in the vacuum vessel (e.g. 'ballooning' of modes through the thin sections of the vessel) need to be taken into account during the interpretation of pick-up coil data, otherwise spurious results may be obtained. Some tokamaks possess thick conducting walls with insulating toroidal breaks.[26,27] In such devices it may be possible for an ideally unstable kink mode to defeat the moderating effect of wall eddy currents by 'ballooning' through the insulating breaks where no eddy currents can flow.

In next-generation tokamaks, such as ITER,[28] the interaction of MHD instabilities with the wall is likely to be of particular significance because of the large dimensions envisaged for such devices. This follows since the critical island width for the 'locking' of rotating tearing modes to the wall is a rapidly decreasing function of machine dimensions, due to the comparatively feeble mode rotation found in large devices.[22] Thus, 'locked modes', which interact strongly with the wall, may be a common occurrence in next-generation tokamaks.

The above discussion highlights the importance of gaining as complete an understanding as possible of the interaction of MHD instabilities with *realistic* walls, including the effects of modularity, insulating breaks, gaps, diagnostic ports, etc. In Section II of this paper a general formalism is developed for analysing the influence of a wall with toroidally varying resistance on the stability of kink modes. In conventional tokamaks the kink mode dispersion relation is found to reduce to a surprisingly simple form. Section III investigates the effect of a wall with toroidal gaps on the stability of both kink modes and tearing modes. In Section IV a similar investigation is made for a wall of modular construction. Finally, this paper is summarised in Section V.

II Analysis

A Introduction

In the following, the standard cylindrical tokamak limit is adopted, and the usual right-handed cylindrical polar coordinates r, θ, z are employed. The perturbed magnetic field $\delta\mathbf{B}$ is written in terms of the perturbed poloidal flux ψ, so that $\delta\mathbf{B} = \nabla\psi \wedge \hat{\mathbf{z}}$. In general, the eddy currents flowing in a wall with toroidally varying resistance couple MHD modes with different toroidal mode-numbers n. However, modes with different poloidal mode-numbers m remain independent (in the cylindrical limit). The perturbed flux in the wall due to a general mode with poloidal mode-number m is written

$$\psi(r_{\rm w},\theta,\phi) \equiv \psi_{\rm w}(\theta,\phi) = \sum_n \Psi_{\rm w}^n \exp[i(m\theta - n\phi)], \tag{1}$$

where $r_{\rm w}$ is the wall minor radius, $\phi = z/R_0$, and R_0 is the simulated major radius. The wall is assumed to lie in the 'thin-shell' limit, where the skin depth is much larger than the actual wall thickness, so that the flux is approximately constant across the wall.[10]

B The Wall Eddy Currents

The radially integrated helical eddy currents induced in a *uniform* wall are given by

$$\mathbf{I}_{\rm w}(\theta,\phi) = -\gamma\sigma_{\rm w}\delta_{\rm w}\sum_n \Psi_{\rm w}^n \exp[i(m\theta - n\phi)]\,\hat{\boldsymbol{\imath}}^n, \tag{2}$$

where $\sigma_{\rm w}$ is the wall conductivity, $\delta_{\rm w}$ is the wall thickness, $\gamma \equiv d\ln\Psi_{\rm w}^n/dt$ is the growth-rate (all harmonics of the wall flux are assumed to have the same time dependence), and

$$\hat{\boldsymbol{\imath}}^n = \left(\hat{\mathbf{z}} + \frac{n}{m}\frac{r_{\rm w}}{R_0}\,\hat{\boldsymbol{\theta}}\right) \Big/ \left(1 + \frac{n^2}{m^2}\frac{r_{\rm w}^2}{R_0^2}\right)^{1/2} \tag{3}$$

is a unit vector with m/n helical pitch.[22] The eddy currents induced in a wall with toroidally varying resistance take the form

$$\mathbf{I}_{\rm w}(\theta,\phi) = -\gamma\sigma_{\rm w}\delta_{\rm w}\sum_n \Psi_{\rm w}^n \exp[i(m\theta - n\phi)] \times$$

$$\left\{\hat{\boldsymbol{\imath}}^n + i\frac{r_{\rm w}}{mR_0}\frac{d\ln(\sigma_{\rm w}\delta_{\rm w})}{d\phi}\,\hat{\boldsymbol{\theta}} \bigg/ \left(1+\frac{n^2}{m^2}\frac{r_{\rm w}^2}{R_0^2}\right)^{1/2}\right\}, \tag{4}$$

where poloidal 'return currents' have been added in order to maintain the divergence free nature of the current pattern. The above analysis assumes that the eddy currents have negligible radial components, so that the current pattern in the wall is essentially two dimensional. This is a reasonable assumption in the 'thin-shell' limit.

C The 'Jump' Conditions at the Wall

The 'jump' in the radial derivative of the m/n harmonic of poloidal flux induced by wall eddy currents is[22]

$$\Delta\Psi_{\rm w}^n = -\mu_0 r_{\rm w}\oint\oint \mathbf{I}_{\rm w}\cdot\hat{\boldsymbol{\imath}}^n\,\exp[-i(m\theta - n\phi)]\,\frac{d\theta}{2\pi}\frac{d\phi}{2\pi}, \tag{5}$$

which yields

$$\Delta\Psi_{\rm w}^n \simeq \sum_k \oint \gamma\tau_{\rm w}(\phi)\exp(-ik\phi)\,\frac{d\phi}{2\pi}\,\Psi_{\rm w}^{n+k}\,\hat{\boldsymbol{\imath}}^n.\,\hat{\boldsymbol{\imath}}^{n+k} + \mathcal{O}(\epsilon_{\rm w}), \tag{6}$$

where $\epsilon_{\rm w} = r_{\rm w}/R_0$ is the inverse aspect-ratio of the wall, and

$$\tau_{\rm w}(\phi) = \mu_0 r_{\rm w}\sigma_{\rm w}(\phi)\delta_{\rm w}(\phi) \tag{7}$$

is the toroidally varying wall time-constant. Later on in this section it is demonstrated that the smallest angular scale of the 'effective wall' time-constant is $\mathcal{O}(\epsilon_{\rm w})$. It follows that the maximum error involved in the neglect of the return currents in Eqn. (6) is $\mathcal{O}(\epsilon_{\rm w})$. For $n \sim \mathcal{O}(1)$ modes

$$\hat{\boldsymbol{\imath}}^n.\,\hat{\boldsymbol{\imath}}^{n+k} \simeq \frac{1}{(1+k^2\epsilon_{\rm w}^2/m^2)^{1/2}} \tag{8}$$

to a good approximation. Thus, Eqn. (6) reduces to

$$\Delta\Psi_{\rm w}^n \simeq \sum_k \oint \gamma\tau_{\rm w}(\phi)\exp(-ik\phi)\,\frac{d\phi}{2\pi}\,\frac{\Psi_{\rm w}^{n+k}}{(1+k^2\epsilon_{\rm w}^2/m^2)^{1/2}}. \tag{9}$$

Note that according to Eqn. (9) low-n toroidal harmonics (i.e. $|n| \ll m/\epsilon_{\rm w}$), whose stability is affected by wall eddy currents which are predominantly *toroidal* in nature, cannot couple effectively to very high-n harmonics (i.e. $|n| \gg m/\epsilon_{\rm w}$), whose stability is affected by eddy currents which are predominantly *poloidal* in nature. In fact, coupling is only effective for toroidal mode-numbers in the range $-m/\epsilon_{\rm w} \lesssim n \lesssim m/\epsilon_{\rm w}$.

D The 'Effective Wall'

Let

$$\Psi_{\rm w}^n = \int_{-\infty}^{\infty} \bar{\Psi}_{\rm w}(\eta)\exp(in\eta)\,\frac{d\eta}{2\pi}, \tag{10}$$

which can be inverted to give

$$\bar{\Psi}_{\rm w}(\eta) = \sum_n \Psi_{\rm w}^n \frac{2\sin\eta}{\eta}\exp(-in\eta), \tag{11}$$

using the identity

$$\int_{-\infty}^{\infty} \frac{2\sin\eta}{\eta}\exp[i(n-n')\eta]\,\frac{d\eta}{2\pi} = \delta_{nn'}. \tag{12}$$

It follows from Eqn. (1) that

$$\psi_{\rm w}(\theta,\phi) = \frac{\phi}{2\sin\phi}\bar{\Psi}_{\rm w}(\phi)\exp(im\theta). \tag{13}$$

Equation (9) can be rewritten

$$\Delta\Psi_{\rm w}^n \simeq \oint\int_{-\infty}^{\infty} \gamma\tau_{\rm w}(\phi)\bar{\Psi}_{\rm w}(\eta)\exp(in\eta)\sum_k \frac{\exp[ik(\eta-\phi)]}{(1+k^2\epsilon_{\rm w}^2/m^2)^{1/2}}\,\frac{d\phi}{2\pi}\frac{d\eta}{2\pi}, \tag{14}$$

which reduces to

$$\Delta\Psi_{\rm w}^n \simeq \int_{-\infty}^{\infty} \gamma\bar{\tau}_{\rm w}(\eta)\bar{\Psi}_{\rm w}(\eta)\exp(in\eta)\,\frac{d\eta}{2\pi}. \tag{15}$$

where

$$\bar{\tau}_{\rm w}(\phi) = \oint \tau_{\rm w}(\phi')h(\phi-\phi')\,\frac{d\phi'}{2\pi}, \tag{16}$$

and

$$h(\phi) \equiv \sum_k \frac{\exp(ik\phi)}{(1+k^2\Delta\phi_{\rm w}^2)^{1/2}}. \tag{17}$$

Here, $\Delta\phi_{\rm w} \equiv \epsilon_{\rm w}/m$. Note that according to Eqs. (16) and (17):

$$\bar{\tau}_{\rm w}(\phi+2\pi) = \bar{\tau}_{\rm w}(\phi), \tag{18(a)}$$

$$\oint h(\phi)\,\frac{d\phi}{2\pi} = 1, \tag{18(b)}$$

$$\oint \bar{\tau}_{\rm w}(\phi)\,\frac{d\phi}{2\pi} = \oint \tau_{\rm w}(\phi)\,\frac{d\phi}{2\pi}, \tag{18(c)}$$

$$h(\phi) \simeq \frac{2}{\Delta\phi_{\rm w}}K_0(|\phi|/\Delta\phi_{\rm w}) \qquad \text{for } \Delta\phi_{\rm w} \ll 1, \tag{18(d)}$$

$$\mathrm{lt}_{\,\Delta\phi_{\rm w}\to 0}\, h(\phi) \to 2\pi\delta(\phi), \tag{18(e)}$$

where K_0 is a standard Bessel function.

It is clear from Eqs. (15) and (16) that, as a consequence of the lack of coupling to high-n harmonics discussed in Section II.C, MHD modes experience an 'effective wall' whose toroidally varying time-constant is the convolution of the actual time-constant with a smoothing function, $h(\phi)$, whose characteristic (toroidal) angular spread is $\Delta\phi_{\rm w}$. According to Eqn. (18)(c), the 'effective wall' possesses the same average time-constant as the real wall.

E The Stability of Wall Modes

Consider the stability of wall modes, which behave ideally (i.e. with no reconnection) at all rational surfaces within the plasma. The perturbed poloidal flux can be written

$$\psi(r,\theta,\phi) = \sum_n \psi^n(r)\exp[i(m\theta - n\phi)], \tag{19}$$

where $\psi^n(r)$ satisfies the m/n cylindrical tearing mode equation[22] and the physical boundary conditions at $r = 0$ and $r \to \infty$. In addition, $\psi^n(r)$ is zero at the m/n rational surface, provided it lies within the plasma. There is a wall stability index associated with each of the coupled toroidal harmonics:

$$E^n_{\rm ww} \equiv \left[r\frac{d\psi^n}{dr}\right]_{r_{\rm w-}}^{r_{\rm w+}} \Big/ \psi^n(r_{\rm w}). \tag{20}$$

Asymptotic matching across the wall yields

$$\Delta\Psi^n_{\rm w} = E^n_{\rm ww}\Psi^n_{\rm w} \tag{21}$$

for each harmonic. It follows from Eqn. (15) that

$$E^n_{\rm ww}\Psi^n_{\rm w} = \int_{-\infty}^{\infty}\gamma\bar{\tau}_{\rm w}(\eta)\bar{\Psi}_{\rm w}(\eta)\exp(in\eta)\,\frac{d\eta}{2\pi} \tag{22}$$

for wall modes.

It is easily demonstrated that for large-n (i.e. $1 \ll |n| \ll m/\epsilon_{\rm w}$) the $\psi^n(r)$ become vacuum-like (i.e. they are essentially unaffected by the equilibrium plasma current) and $E^n_{\rm ww} \simeq -2m$. Note that the standard tokamak orderings (and the cylindrical tearing mode equation) break down at very high-n (i.e. $|n| \sim m/\epsilon_{\rm w}$), so $E^n_{\rm ww} \neq -2m$ in this limit. However, this effect can be neglected given the ineffective coupling to such high-n modes in tokamaks (see Section II.C).

F The Wall Mode Dispersion Relation

Suppose that

$$E^n_{\rm ww} \neq -2m \qquad \text{for } n_1 \leq n \leq n_2,$$

$$E^n_{\rm ww} = -2m \qquad \text{otherwise}, \tag{23}$$

which, according to the discussion in Section II.E, is a reasonable assumption for tokamaks, but not for reversed-field pinches (RFPs) where kink modes typically have $n \sim m/\epsilon_{\rm w}$.[16] Let

$$\bar{\Psi}_{\rm w}(\eta) = \sum_{k=n_1}^{n_2} \left\{ \bar{f}_k(\eta) + \alpha_k \frac{2\sin\eta}{\eta} \exp(-ik\eta) \right\}, \tag{24}$$

so that

$$\Psi_{\rm w}^n = \sum_{k=n_1}^{n_2} \int_{-\infty}^{\infty} \bar{f}_k(\eta) \exp(in\eta) \frac{d\eta}{2\pi} + \alpha_n. \tag{25}$$

Here, the α_n are arbitrary complex constants satisfying $\alpha_n = 0$ for all n not in the range $n_1 \rightarrow n_2$. It follows from Eqn. (22) that for all toroidal mode-numbers not in this range

$$\sum_{k=n_1}^{n_2} \int_{-\infty}^{\infty} \left[(2m + \gamma\bar{\tau}_{\rm w})\bar{f}_k + \alpha_k \gamma\bar{\tau}_{\rm w} \frac{2\sin\eta}{\eta} \exp(-ik\eta) \right] \exp(in\eta) \frac{d\eta}{2\pi} = 0. \tag{26}$$

The most general solution is

$$\bar{f}_k(\eta) = -\alpha_k \frac{2\sin\eta}{\eta} \frac{\beta_k + \gamma\bar{\tau}_{\rm w}(\eta)}{2m + \gamma\bar{\tau}_{\rm w}(\eta)} \exp(-ik\eta), \tag{27}$$

where the β_n are arbitrary complex constants satisfying $\beta_n = 0$ for n all not in the range $n_1 \rightarrow n_2$. Equations (24) and (27) give

$$\bar{\Psi}_{\rm w}(\eta) = \sum_{k=n_1}^{n_2} \alpha_k \frac{2\sin\eta}{\eta} \frac{2m - \beta_k}{2m + \gamma\bar{\tau}_{\rm w}(\eta)} \exp(-ik\eta). \tag{28}$$

For each toroidal mode-number in the range n_1 to n_2 Eqn. (22) yields

$$\alpha_n E_{\rm ww}^n = \sum_{k=n_1}^{n_2} \alpha_k \int_{-\infty}^{\infty} \frac{2\sin\eta}{\eta} \frac{(E_{\rm ww}^n + 2m - \beta_k)\gamma\bar{\tau}_{\rm w}(\eta) + E_{\rm ww}^n\beta_k}{2m + \gamma\bar{\tau}_{\rm w}(\eta)} \exp[i(n-k)\eta] \frac{d\eta}{2\pi}, \tag{29}$$

which can be rearranged to give

$$\sum_{k=n_1}^{n_2} \hat{\alpha}_k \int_{-\infty}^{\infty} \frac{2\sin\eta}{\eta} \frac{2m + E_{\rm ww}^n}{2m + \gamma\bar{\tau}_{\rm w}(\eta)} \exp[i(n-k)\eta] \frac{d\eta}{2\pi} = \hat{\alpha}_n, \tag{30}$$

where $\hat{\alpha}_n = \alpha_n \times (2m - \beta_n)$.

Now, $2m + \gamma\bar{\tau}_{\rm w}(\eta)$ is a periodic function of η with period 2π [see Eqn. (18)(a)]. It follows that

$$\frac{1}{2m + \gamma\bar{\tau}_{\rm w}(\eta)} = \sum_k \oint \frac{\exp(ik\phi)}{2m + \gamma\bar{\tau}_{\rm w}(\phi)} \frac{d\phi}{2\pi} \exp(-ik\eta), \tag{31}$$

so using Eqn. (12)

$$\int_{-\infty}^{\infty} \frac{2\sin\eta}{\eta} \frac{\exp(in\eta)}{2m + \gamma\bar{\tau}_{\rm w}(\eta)} \frac{d\eta}{2\pi} \equiv \oint \frac{\exp(in\phi)}{2m + \gamma\bar{\tau}_{\rm w}(\phi)} \frac{d\phi}{2\pi}. \tag{32}$$

Thus, the wall mode dispersion relation can be written

$$\sum_k \hat{\alpha}_k \oint \frac{2m + E_{\rm ww}^n}{2m + \gamma\bar{\tau}_{\rm w}(\phi)} \exp[i(n-k)\phi] \frac{d\phi}{2\pi} = \hat{\alpha}_n \tag{33}$$

for all n. Note that $\hat{\alpha}_n = 0$ if $E_{\rm ww}^n = -2m$, as was initially assumed in Eqn. (24).

The wall mode dispersion relation takes the form of a matrix equation:

$$\mathbf{A}\cdot\boldsymbol{\alpha} = \mathbf{0}, \tag{34}$$

where $\boldsymbol{\alpha}$ is the complex vector of the $\hat{\alpha}_n$ values, and

$$A_{ab} = \delta_{ab} - \oint \frac{2m + E^a_{\rm ww}}{2m + \gamma\bar{\tau}_{\rm w}(\phi)}\exp[i(a-b)\phi]\frac{d\phi}{2\pi}. \tag{35}$$

Here, $\mathbf{A}$ is a complex $N\times N$ matrix, where N is the number of modes for which $E^n_{\rm ww} \neq -2m$.

Finally, it is easily demonstrated from Eqs. (4), (12), (13), (17), and (28), that

$$\Psi^n_{\rm w} = \sum_k \hat{\alpha}_k \oint \frac{\exp[i(n-k)\phi]}{2m + \gamma\bar{\tau}_{\rm w}(\phi)}\frac{d\phi}{2\pi}, \tag{36}$$

and

$$\psi_{\rm w}(\theta,\phi) = \sum_n \hat{\alpha}_n \frac{\exp[i(m\theta - n\phi)]}{2m + \gamma\bar{\tau}_{\rm w}(\phi)}, \tag{37}$$

with

$$\mu_0 r_{\rm w}\mathbf{I}_{\rm w} = -\left(\hat{\mathbf{z}} + i\frac{\epsilon_{\rm w}}{m}\,\hat{\boldsymbol{\theta}}\,\frac{d}{d\phi}\right)\gamma\tau_{\rm w}(\phi)\oint \psi_{\rm w}(\theta,\phi')h(\phi'-\phi)\frac{d\phi'}{2\pi}. \tag{38}$$

G Discussion

After some analysis, the wall mode dispersion relation is found to take the surprisingly simple form (34), in which only those coupled toroidal harmonics whose stability indices differ appreciably [i.e. by $\mathcal{O}(1)$] from the vacuum value $-2m$ are *explicitly* included. The remaining toroidal harmonics are, in fact, *implicitly* included in the calculation without any approximation. This great simplification is possible because most of the coupled toroidal harmonics in tokamaks possess the same stability index, $-2m$. No corresponding simplification occurs for RFPs, or for coupled poloidal harmonics in tokamaks, because in both cases the coupled modes generally have widely different stability indices.

III The Effect of a Wall with Toroidal Gaps

A The Stability of Wall Modes

1 The Single-Mode Approximation

Consider, first, the limit $\Delta\phi_{\rm w}\rightarrow 0$ in which, according to Section II.D, the 'effective wall' experienced by MHD modes becomes identical with the real wall. Suppose that only a single mode has a wall stability index which differs significantly from the vacuum value $-2m$. This is not an unusual situation, especially if a low mode-number rational surface lies just outside the edge of the plasma current channel. Table 1 shows values of $E^n_{\rm ww}$ (for $m = 3$ poloidal harmonics) calculated for a Wesson-like equilibrium current profile $j_z(r) \propto (1 - r^2/a^2)^\nu$, with $\nu = 1.46$, $q(0) = 1.2$, $q(a) = 2.95$, and $r_{\rm w}/a = 1.0$. Here, $q(r)$ is the conventional tokamak safety-factor profile.[22] Table 1 indicates that only the 3/1 mode has a wall stability index which differs appreciably from the vacuum value -6. In this situation, the wall mode dispersion relation (34) takes the particularly simple form

n	$E^n_{\rm ww}$
-4	-5.831
-3	-5.790
-2	-5.723
-1	-5.595
0	-5.244
1	+2.167
2	-7.077
3	-6.479
4	-6.309
5	-6.228
6	-6.181

Table I: Values of the resistive wall mode stability index for $m = 3$ modes calculated for various different toroidal mode-numbers n. The equilibrium current profile is $j_z(r) \propto (1-r^2/a^2)^\nu$, with $\nu = 1.46$, $q(0) = 1.2$, $q(a) = 2.95$, and $r_{\rm w}/a = 1.0$.

$$\oint \frac{2m + E^n_{\rm ww}}{2m + \gamma\tau_{\rm w}(\phi)}\,\frac{d\phi}{2\pi} = 1, \tag{39}$$

where $E^n_{\rm ww} \neq -2m$ is the 'special' stability index.

Suppose that the wall is made of metal with an intrinsic time-constant $\tau_{\rm w}$, but that one or more toroidal sections of total fractional angular extent f are missing (so $f = 0$ corresponds to no gaps, and $f = 1$ to no metal). According to Eqn. (39), the wall mode dispersion relation is written

$$\gamma\tau_{\rm w} = \frac{E^n_{\rm ww}}{1 - f\,(1 + E^n_{\rm ww}/2m)}, \tag{40}$$

and using Eqn. (37),

$$\left|\frac{\psi_{\rm w\,gap}}{\psi_{\rm w\,mtl}}\right| = 1 + \frac{\gamma\tau_{\rm w}}{2m}. \tag{41}$$

Here, $\psi_{\rm w\,gap}$ is the perturbed poloidal flux in the gaps, and $\psi_{\rm w\,mtl}$ is the flux in the metal.

Equations (40) and (41) imply that as $E^n_{\rm ww}$ approaches the vacuum limit $-2m$, the flux at the wall radius is mostly concentrated in the metal sections (i.e. $\psi_{\rm w\,gap} \rightarrow 0$), and the wall mode decays on the characteristic time-constant of the metal (i.e. $\gamma \simeq E^n_{\rm ww}/\tau_{\rm w}$). As the mode approaches marginal stability (i.e. $E^n_{\rm ww} \rightarrow 0$), the fluxes in the gap and metal sections of the wall gradually even out, and the flux becomes uniform (i.e. $|\psi_{\rm w\,gap}/\psi_{\rm w\,mtl}| = 1$) at the marginal stability point $E^n_{\rm ww} = 0$. For weakly unstable/stable modes (i.e. $|E^n_{\rm ww}| \ll 1$), the typical growth/decay time-scale is the average time-constant of the metal and gap sections of the wall [i.e. $\gamma \simeq E^n_{\rm ww}/\oint \tau_{\rm w}(\phi)\,d\phi/2\pi = E^n_{\rm ww}/\tau_{\rm w}(1-f)$]. As the mode becomes significantly unstable [i.e. $E^n_{\rm ww} \sim {\cal O}(1)$], the flux at the wall radius starts to concentrate in the gap sections of the wall (i.e. $|\psi_{\rm w\,gap}/\psi_{\rm w\,mtl}| > 1$), and the characteristic growth-time decreases. Eventually, at a critical wall mode stability index,

$$(E^n_{\rm ww})_{\rm crit} = 2m\left(\frac{1}{f} - 1\right), \tag{42}$$

the flux is entirely concentrated in the gap sections of the wall (i.e. $|\psi_{\rm w\,mtl}| = 0$), and the mode becomes ideal in nature (i.e. the resistive growth-rate tends to infinity). For $E^n_{\rm ww} > (E^n_{\rm ww})_{\rm crit}$, the wall eddy currents are insufficient to moderate the growth-rate, and the mode 'explodes' through the gaps on a typical ideal external-kink time-scale (i.e. the growth-rate is moderated by plasma inertia).

According to Eqn. (40), the wall mode dispersion relation depends only on the total (angular) fraction of gaps in the wall, and not on their distribution (i.e. the dispersion relation is the same for one large gap as for two half-sized gaps, etc.). Equation (41) implies that the poloidal flux at the wall radius changes discontinuously at the metal/gap boundaries, corresponding to the situation shown schematically in Fig 1. It can be seen that the eddy current vortices in the metal sections of the wall have 'square' ends, giving rise to infinite poloidal return currents flowing along the metal/gap boundaries [see Eqn. (38)]. Such behaviour is clearly unphysical, and can be avoided by taking into account the decoupling of very high-n toroidal harmonics from low-n harmonics discussed in Section II.C. As a consequence of this decoupling, wall modes experience an 'effective wall' whose toroidally varying time-constant is the convolution of the actual time-constant with a smoothing function of (toroidal) angular width $\Delta\phi_{\rm w} = \epsilon_{\rm w}/m$ [see Eqn. (16)].

For a wall made up of N metal sections of intrinsic time-constant $\tau_{\rm w}$, separated by N evenly spaced gaps, the variation of the effective wall time-constant with toroidal angle is given by

$$\frac{\bar{\tau}_{\rm w}(\phi)}{\tau_{\rm w}} = \frac{1}{2} + \frac{2}{\pi}\sum_{j=1}^{\infty} \frac{(-1)^{j-1}\cos[(2j-1)N\phi]}{(2j-1)\,[1+\{(2j-1)N\Delta\phi_{\rm w}\}^2]^{1/2}}. \tag{43}$$

Figure 2 shows the effective time-constant calculated for $N = 6$ (corresponding to gaps of angular width $\Delta\phi_{\rm g} = \pi/6$), with various different values of the smoothing angle $\Delta\phi_{\rm w}$. It can be seen that as $\Delta\phi_{\rm w}$ increases, the gaps start to merge with the metal sections until eventually modes experience a uniform effective wall whose time-constant is half that of the metal sections of the real wall. The maximum/minimum values of the effective time-constant are given by

$$\frac{\bar{\tau}_{\rm w\ max/min}}{\tau_{\rm w}} = \frac{1}{2} \pm g(\Delta\phi_{\rm w}/\Delta\phi_{\rm g}), \tag{44}$$

where $g(\lambda)$ is plotted in Fig. 3.

The above analysis suggests that wall modes cannot 'see' gaps whose angular extent is much less than the smoothing angle $\Delta\phi_{\rm w}$. Thus, the previous conclusion that the wall mode dispersion relation depends only on the total angular extent of gaps, and not their distribution, needs to be qualified. This conclusion is, in fact, only true for gaps whose angular extent is much greater than $\Delta\phi_{\rm w}$. In particular, if a large gap (i.e. $\Delta\phi_{\rm g} \gg \Delta\phi_{\rm w}$) is split up into many small gaps (i.e. $\Delta\phi_{\rm g} \ll \Delta\phi_{\rm w}$) of the same total angular extent, then wall modes experience a uniform effective wall with no gaps and a time-constant which is the average time-constant of the real wall.

The decoupling of very high-n from low-n harmonics also ensures that poloidal return currents are spread over a region of toroidal angular extent $\Delta\phi_{\rm w}$, or toroidal length $2\pi\, r_{\rm w}/m$, instead of flowing right on the metal/gap boundaries [see Eqn. (38)]. This effect gives rise to eddy current vortices of the form shown schematically in Fig. 4. Note that the toroidal extent of the return current region is the same as the poloidal spacing of the vortices. This ensures that the eddy current vortices have rounded ends at the metal/gap boundaries, instead of the unphysical 'square' ends shown in Fig. 1.

2 The Coupling of Toroidal Harmonics

Suppose that two toroidal harmonics (n_1 and n_2, say) have wall stability indices which differ significantly from the vacuum value $-2m$. In the limit where the angular extents of the metal and gap sections of the wall are much larger than the smoothing angle, $\Delta\phi_w$, the analysis of Section II.F yields:

$$\psi_{\text{w gap}}(\theta,\phi) \propto \alpha_1 \exp[i(m\theta - n_1\phi)] + \alpha_2 \exp[i(m\theta - n_2\phi)], \tag{45(a)}$$

$$\left|\frac{\psi_{\text{w gap}}}{\psi_{\text{w mtl}}}\right| = 1 + \frac{\gamma\tau_w}{2m}, \tag{45(b)}$$

$$\frac{\alpha_2}{\alpha_1} = \frac{\gamma\tau_w(1 + E_{ww}^{n_2}/2m)c_{12}^*}{\gamma\tau_w[1 - f(1 + E_{ww}^{n_2}/2m)] - E_{ww}^{n_2}}$$

$$= \frac{\gamma\tau_w[1 - f(1 + E_{ww}^{n_1}/2m)] - E_{ww}^{n_1}}{\gamma\tau_w(1 + E_{ww}^{n_1}/2m)c_{12}}, \tag{45(c)}$$

where

$$c_{12} = \int_{\text{gaps}} \exp[i(n_1 - n_2)\phi]\,\frac{d\phi}{2\pi}. \tag{46}$$

According to Eqn. (45)(c), the mode growth-rate is given by

$$\gamma\tau_w \times 2a = E_{ww}^{n_1}[1 - f(1 + E_{ww}^{n_2}/2m)] + E_{ww}^{n_2}[1 - f(1 + E_{ww}^{n_1}/2m)]$$

$$\pm\sqrt{(E_{ww}^{n_1} - E_{ww}^{n_2})^2(1-f)^2 + 4|c_{12}|^2 E_{ww}^{n_1}E_{ww}^{n_2}(1 + E_{ww}^{n_1}/2m)(1 + E_{ww}^{n_2}/2m)}, \tag{47}$$

with

$$a = [1 - f(1 + E_{ww}^{n_1}/2m)][1 - f(1 + E_{ww}^{n_2}/2m)]$$

$$-|c_{12}|^2(1 + E_{ww}^{n_1}/2m)(1 + E_{ww}^{n_2}/2m). \tag{48}$$

In most respects, the behaviour of the coupled modes is analogous to that of the corresponding uncoupled modes (see Section III.A.1). For instance, as $E_{ww}^{n_j}$ (where j is 1 or 2) approaches the vacuum value $-2m$, the associated root of Eqn. (47) approaches $\gamma\tau_w = -2m$, and the magnetic flux becomes entirely concentrated in the metal sections of the wall. Furthermore, as $E_{ww}^{n_j}$ approaches zero, the corresponding root of (47) approaches the marginal value $\gamma\tau_w = 0$, and the magnetic flux becomes evenly distributed in the wall. However, mode coupling does affect the onset of ideal growth through the gaps, which now occurs when $a = 0$.

Mode coupling is most effective when the two stability indices are equal; i.e. when $E_{ww}^{n_1} = E_{ww}^{n_2} = E_{ww}^{n}$. For this special case, Eqn. (47) reduces to

$$\gamma\tau_w = \frac{E^n_{ww}}{1-(f\pm|c_{12}|)(1+E^n_{ww}/2m)}, \tag{49}$$

so the critical stability index for the 'explosion' of the mode through the gaps is

$$(E^n_{ww})_{crit} = 2m\left(\frac{1}{f+|c_{12}|}-1\right). \tag{50}$$

It is clear, by comparison with Eqn. (42), that mode coupling tends to *reduce* the critical stability index needed for ideal growth. Note that $f+|c_{12}|\leq 1$, so $(E^n_{ww})_{crit}$ is never negative.

B The Interaction with Rotating Tearing Modes

Suppose that a single toroidal harmonic (mode number n, say) has a rational surface lying inside the plasma at minor radius r_s. In this situation, the stability of the m/n mode is governed by the following coupled equations:[22,29,30]

$$\Delta\Psi_s - E_{ss}\Psi_s - E_{sw}\Psi^n_w = 0, \tag{51(a)}$$

$$\Delta\Psi^n_w - E^n_{ww}\Psi^n_w - E_{sw}\Psi_s = 0, \tag{51(b)}$$

where Ψ_s is the m/n reconnected flux at the rational surface, $\Delta\Psi_s$ is the 'jump' in the radial derivative of the m/n flux across the rational surface, E_{ss} is the fixed-boundary m/n tearing stability index (calculated assuming zero flux in the wall), and $E_{ss}+(E_{sw})^2/(-E^n_{ww})$ is the corresponding free-boundary stability index (calculated assuming zero eddy currents in the wall). As before, Ψ^n_w is the m/n flux in the wall, $\Delta\Psi^n_w$ is the 'jump' in the radial derivative of the m/n flux across the wall, and E^n_{ww} is the m/n wall stability index (calculated assuming zero reconnection at the rational surface). The stability of non-resonant modes (i.e. $n'\neq n$) is again governed by

$$\Delta\Psi^{n'}_w - E^{n'}_{ww}\Psi^{n'}_w = 0. \tag{52}$$

According to standard Rutherford island theory,[31] the non-linear evolution of the m/n tearing mode satisfies

$$\tau_R\frac{d}{dt}\left(\frac{W}{r_s}\right) = r_s\Delta'_s \equiv E_{ss}+E_{sw}\,\mathrm{Re}\left(\frac{\Psi^n_w}{\Psi_s}\right), \tag{53}$$

where W is the island width, $\tau_R = 0.8227\,\mu_0 r_s^2/\eta_\parallel(r_s)$ is the resistive diffusion time-scale, and $\eta_\parallel$ is the parallel plasma resistivity. The non-linear toroidal electromagnetic torque acting at the rational surface due to eddy currents flowing in the wall is given by[22,29,30]

$$\delta T_\phi(r_s) = \frac{2n\pi^2 R_0}{\mu_0}\times E_{sw}\,\mathrm{Im}(\Psi^n_w\Psi^*_s). \tag{54}$$

Consider the simplest possible case, where the wall stability indices of the non-resonant modes do not differ appreciably from the vacuum value $-2m$. This situation is analogous to that studied in Section III.A.1 provided $E^n_{ww}\to E^n_{ww}+E_{sw}\Psi_s/\Psi^n_w$ and $\gamma\to -i\omega$. Here, ω is the angular rotation frequency of the m/n magnetic island. It is assumed that the m/n wall mode is intrinsically stable, so that $E^n_{ww}<0$. If follows from Eqn. (40) (with the usual proviso that the angular scales of the wall are much larger than the smoothing angle $\Delta\phi_w$) that

$$\frac{\Psi_{\rm w}^n}{\Psi_{\rm s}} = \frac{(1 - i\omega\tau_{\rm w} f/2m)\, E_{\rm sw}}{-i\omega\tau_{\rm w}[1 - f(1 + E_{\rm ww}^n/2m)] - E_{\rm ww}^n}. \tag{55}$$

Equations (36) and (37) yield

$$|\psi_{\rm w\,gap}| = \frac{(1 - i\omega\tau_{\rm w}/2m)\,\Psi_{\rm w}^n}{(1 - i\omega\tau_{\rm w}/2m)f + (1 - f)}, \qquad (56)(a)$$

$$|\psi_{\rm w\,mtl}| = \frac{\Psi_{\rm w}^n}{(1 - i\omega\tau_{\rm w}/2m)f + (1 - f)}, \qquad (56)(b)$$

while Eqs. (53) and (54) imply that

$$r_{\rm s}\Delta_{\rm s}' = E_{\rm ss} + (E_{\rm sw})^2 \times \frac{(\omega\tau_{\rm w})^2[1 - f(1 + E_{\rm ww}^n/2m]f/2m - E_{\rm ww}^n}{(\omega\tau_{\rm w})^2[1 - f(1 + E_{\rm ww}^n/2m)]^2 + (E_{\rm ww}^n)^2}, \qquad (57)(a)$$

$$\delta T_\phi(r_{\rm s}) = \frac{2n\pi^2 R_0}{\mu_0} \times |\Psi_{\rm s}|^2 (E_{\rm sw})^2 \times \frac{\omega\tau_{\rm w}(1 - f)}{(\omega\tau_{\rm w})^2[1 - f(1 + E_{\rm ww}^n/2m)]^2 + (E_{\rm ww}^n)^2}. \qquad (57)(b)$$

In the high frequency limit, $\omega\tau_{\rm w} \gg 1$, Eqs. (55)–(57) reduce to:

$$\frac{\Psi_{\rm w}^n}{\Psi_{\rm s}} \simeq \frac{E_{\rm sw}\, f/2m}{[1 - f(1 + E_{\rm ww}^n/2m)]}, \qquad (58)(a)$$

$$\frac{|\psi_{\rm w\,gap}|}{\Psi_{\rm s}} \simeq \frac{E_{\rm sw}/2m}{[1 - f(1 + E_{\rm ww}^n/2m)]}, \qquad (58)(b)$$

$$\frac{|\psi_{\rm w\,mtl}|}{|\psi_{\rm w\,gap}|} \simeq i\frac{2m}{\omega\tau_{\rm w}}, \qquad (58)(c)$$

$$r_{\rm s}\Delta_{\rm s}' \simeq E_{\rm ss} + \frac{(E_{\rm sw})^2 f/2m}{[1 - f(1 + E_{\rm ww}^n/2m)]}, \qquad (58)(d)$$

$$\delta T_\phi(r_{\rm s}) \simeq \frac{2n\pi^2 R_0}{\mu_0} \frac{|\Psi_{\rm s}|^2 (E_{\rm sw})^2 (1 - f)}{(\omega\tau_{\rm w})[1 - f(1 + E_{\rm ww}^n/2m)]^2}. \qquad (58)(e)$$

Equation (58)(a) indicates that a wall with gaps is unable to completely shield the perturbed flux due to a rapidly rotating tearing mode island from the region outside the wall. According to Eqs. (58)(b) and (58)(c) the rotating flux is able to penetrate through the wall by 'squeezing' through the gaps. Of course, in the limit where the gaps become very narrow (i.e. $f \to 0$) the amount of flux which gets through the wall becomes negligible. Equation (58)(d) shows that the tearing mode stability index asymptotes to the fixed-boundary value as the gaps become very narrow (i.e. $f \to 0$), and asymptotes to the free-boundary value as the gaps become very wide (i.e. $f \to 1$). Finally, Eqn (58)(e) shows that the torque exerted on the rotating tearing mode island by eddy currents induced in the wall asymptotes to zero as the gaps become very wide (i.e. $f \to 1$).

The above results suggest that the interaction of a rapidly rotating tearing mode island with a wall possessing thin toroidal gaps is very similar to the corresponding interaction with a uniform wall, except that in the former case a small amount of rotating magnetic flux gets through the wall, and the slowing down torque exerted on the island is slightly reduced. A thin toroidal limiter (radius r_L, say) can be modelled as a wall with a very large gap (i.e. f just less than unity). According to the above analysis, such a limiter is ineffective at shielding magnetic flux from the region $r > r_L$, and only exerts a comparatively weak slowing down torque on any rotating islands inside the plasma.

IV The Effect of a Modular Wall

A The Stability of Wall Modes

Consider a wall made up of alternate thick and thin sections of time-constants τ_{w_1} and τ_{w_2}, respectively ($\tau_{w_1} > \tau_{w_2}$). It is initially assumed that the angular extent of these sections is much greater than the smoothing angle, $\Delta\phi_w$, so that the effective wall is almost identical with the real wall. For this simple case, the single-mode dispersion relation (39) reduces to

$$\gamma^2 \tau_{w_1}\tau_{w_2} + \gamma\left[2m\left\{f_1\tau_{w_1} + f_2\tau_{w_2}\right\} - E^n_{ww}\left\{f_2\tau_{w_1} + f_1\tau_{w_2}\right\}\right] - 2m\,E^n_{ww} = 0, \tag{59}$$

where f_1 is the total angular extent of the thick sections, and $f_2 \equiv 1 - f_1$ is the total extent of the thin sections. According to Eqn. (37), the ratio of the flux in the thin sections of the wall to that in the thick sections is

$$\left|\frac{\psi_{w_2}}{\psi_{w_1}}\right| = \frac{2m + \gamma\tau_{w_1}}{2m + \gamma\tau_{w_2}}. \tag{60}$$

In the limit $\tau_{w_2} \ll \tau_{w_1}$, Eqn. (59) possess the following asymptotic solutions:

$$\gamma\tau_{w_1} \simeq \frac{E^n_{ww}}{1 - f_2(1 - E^n_{ww}/2m)}, \tag{61)(a}$$

$$\left|\frac{\psi_{w_2}}{\psi_{w_1}}\right| \simeq 1 + \frac{\gamma\tau_{w_1}}{2m}, \tag{61)(b}$$

for $E^n_{ww} \ll 2m\,(1/f_2 - 1)$, and

$$\gamma\tau_{w_2} \simeq f_2 E^n_{ww}, \tag{62)(a}$$

$$\left|\frac{\psi_{w_2}}{\psi_{w_1}}\right| \simeq \frac{\tau_{w_1}}{\tau_{w_2}}, \tag{62)(b}$$

for $E^n_{ww} \gg 2m\,(1/f_2 - 1)$. It can be seen, by comparison with the results of Section III.A.1, that for a stable or moderately unstable mode the thin sections of the wall act rather like gaps. However, for a very unstable mode the finite conductivity of the thin sections limits the accumulation of magnetic flux there, which has the effect of limiting the mode growth-rate. In fact, the mode is unable to evolve on a faster time-scale than the time-constant of the thin sections of the wall. Of course, if the angular extent of the thick and thin sections is much less than the smoothing angle, $\Delta\phi_w$, then wall modes experience a uniform effective wall whose time-constant is the average time-constant of the real wall.

B The Interaction with Rotating Tearing Modes

Suppose that a single toroidal harmonic (mode number n, say) has a rational surface lying inside the plasma, and that the stability indices of the non-resonant harmonics do not differ appreciably from the vacuum value $-2m$. In the high frequency limit, $\omega\tau_{w_1} \gg 1$ and $\omega\tau_{w_2} \gg 1$, a similar analysis to that of Section III.B yields:

$$\frac{\Psi_w^n}{\Psi_s} \simeq \frac{i}{\omega}\left(\frac{f_2}{\tau_{w_2}} + \frac{f_1}{\tau_{w_1}}\right) E_{sw}, \tag{63)(a}$$

$$\frac{\psi_{w_1}}{\Psi_s} \simeq i\frac{E_{sw}}{\omega\tau_{w_1}}, \tag{63)(b}$$

$$\frac{\psi_{w_2}}{\Psi_s} \simeq i\frac{E_{sw}}{\omega\tau_{w_2}}, \tag{63)(c}$$

$$r_s\Delta_s' \simeq E_{ss}, \tag{63)(d}$$

$$\delta T_\phi(r_s) \simeq \frac{2n\pi^2 R_0}{\mu_0}\frac{|\Psi_s|^2(E_{sw})^2}{\omega}\left(\frac{f_2}{\tau_{w_2}} + \frac{f_1}{\tau_{w_1}}\right). \tag{63)(e}$$

The above results indicate that a modular wall is able to shield the flux due to a rapidly rotating tearing island from the region beyond the wall. Equations (63)(b) and (63)(c) show that the residual flux tends to concentrate in the thin sections of the wall. According to Eqn. (63)(d), the tearing mode stability index asymptotes to the fixed-boundary value. Finally, Eqn. (63)(c) shows that the slowing torque exerted on a rotating island is the same as that exerted by a uniform wall with the same average resistance as the modular wall.

V Summary

In principle, the effect of a non-uniform resistive wall on the stability of plasma MHD modes is calculated as follows. First, the spectrum of helical eddy currents induced in the wall by a general plasma mode is evaluated [see Eqn. (4)]. Next, the influence of a given helicity of eddy current on the stability of MHD modes is obtained [see Eqs. (5), (21), and (51)]. Finally, these two sets of information are combined together in a self-consistent manner to yield a dispersion relation for the various coupled modes in the problem. In general, this is a very difficult calculation to perform. However, there is a considerable simplification for the special case considered in this paper of a tokamak surrounded by a wall with toroidally varying resistance. This comes about because most of the different toroidal harmonics coupled by non-uniform wall eddy currents have the same MHD free-energy (see Sections II.E and II.F). No corresponding simplification takes place for RFPs, or for tokamaks surrounded by walls with poloidally varying resistance, because in both cases the modes coupled by non-uniform wall eddy currents have quite disparate MHD free-energies.

In a general tokamak, low mode-number toroidal harmonics (i.e. $|n| \ll m/\epsilon_w$), whose stability is influenced by wall eddy currents which are predominantly toroidal in nature, cannot couple effectively to very high mode-number harmonics (i.e. $|n| \gg m/\epsilon_w$), whose stability is influenced by eddy currents which are predominantly poloidal in nature (see Section II.C). As a direct consequence, MHD modes experience an 'effective wall' whose toroidally varying time-constant is the convolution of the actual time-constant with a smoothing function whose characteristic (toroidal) angular spread is $\Delta\phi_{\rm w} = \epsilon_{\rm w}/m$ (see Section II.D). Here, m is the common poloidal mode-number of the various modes coupled by wall eddy currents, and $\epsilon_{\rm w}$ is the inverse aspect-ratio of the wall.

Consider the stability of a set of coupled wall modes (i.e. modes which do not reconnect magnetic flux inside the plasma) with common poloidal mode-number m. The MHD free-energy associated with each mode is parameterised by a wall stability index $E^n_{\rm ww}$ (where n is the toroidal mode number–see Section II.E). In a general tokamak plasma, the stability indices of most modes lie close to the vacuum value $-2m$. Suppose that only one mode (toroidal mode-number n, say) has a stability index which differs appreciably from the vacuum value. In this situation, the wall mode dispersion relation reduces to the particularly simple form

$$\oint \frac{2m + E^n_{\rm ww}}{2m + \gamma\bar{\tau}_{\rm w}(\phi)}\frac{d\phi}{2\pi} = 1, \tag{64}$$

and the perturbed poloidal flux at the wall radius is given by

$$\psi_{\rm w}(\theta,\phi) \propto \frac{\exp[i(m\theta - n\phi)]}{2m + \gamma\bar{\tau}_{\rm w}(\phi)} \tag{65}$$

[see Eqs. (34)–(37)]. Here, γ is the growth-rate, $\bar{\tau}_{\rm w}(\phi)$ is the toroidally varying time-constant of the effective wall, and $E^n_{\rm ww}$ is the 'special' stability index. If there are N modes with 'special' stability indices, then the wall mode dispersion relation takes the form of an $N\times N$ matrix equation [see Eqn. (34)].

Toroidal gaps are incorporated into tokamak vacuum vessels in order to suppress eddy currents and thereby reduce the penetration time-scale for the vertical magnetic field. However, it is clear from Eqn. (4) that toroidal gaps only interfere strongly with the non-helical (i.e. $m/n = 0/0$) component of the wall eddy current. Helical eddy currents are not suppressed because, unlike the non-helical current, they are able to turn around before reaching the gaps (see Fig. 4). Of course, large gaps attenuate helical eddy currents to some extent because they reduce the effective area of the wall.

The influence of toroidal gaps on wall mode stability is investigated in Section III.A. For the case where only one mode has a stability index which differs appreciably from the vacuum value, it is found that if the mode is stable the perturbed poloidal flux tends to concentrate in the metal sections of the wall, if the mode is marginally stable the flux becomes evenly distributed between the metal and gap sections, and if the mode is unstable the flux tends to concentrate in the toroidal gaps. In fact, if the stability index exceeds a critical value [see Eqn. (42)] the flux becomes entirely concentrated in the gap regions, and the mode can then 'explode' through the gaps with an ideal growth-rate. Of course, wall modes can only 'see' gaps whose angular extent is much greater than the smoothing angle $\Delta\phi_{\rm w}$. If more than one mode possesses a stability index which differs from the vacuum value, then the critical stability index needed for the ideal growth of a given mode is reduced (see Section III.A.2). Finally, a wall possessing toroidal gaps is unable to completely shield the flux of a rapidly rotating tearing island from the region beyond the wall because the flux is able to 'squeeze' through the gaps (see Section III.B).

In conclusion, the investigation of the influence of a wall with toroidal structure on tokamak stability has yielded a number of useful and interesting results. Hopefully, the techniques developed in this paper can be extended to deal with the intrinsically more difficult problem of the influence of a wall with poloidal structure on tokamak stability.

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Figure Captions

Fig. 1 Wall eddy current vortices with 'square' ends.

Fig. 2 Variation of the effective time-constant (normalised to the intrinsic time-constant of the metal sections of the wall) with toroidal angle (ϕ) for a wall possessing six evenly spaced toroidal gaps. Four different values of the smoothing angle $\Delta\phi_{w}$ are used: (a) $\Delta\phi_{w} = 1°$; (b) $\Delta\phi_{w} = 10°$; (c) $\Delta\phi_{w} = 30°$; (d) $\Delta\phi_{w} = 60°$.

Fig. 3 The function $g(\lambda)$, which specifies the maximum and minimum values of the effective wall time-constant as a function of smoothing angle [see Eqn. (44)].

Fig. 4 Realistic wall eddy current vortices.

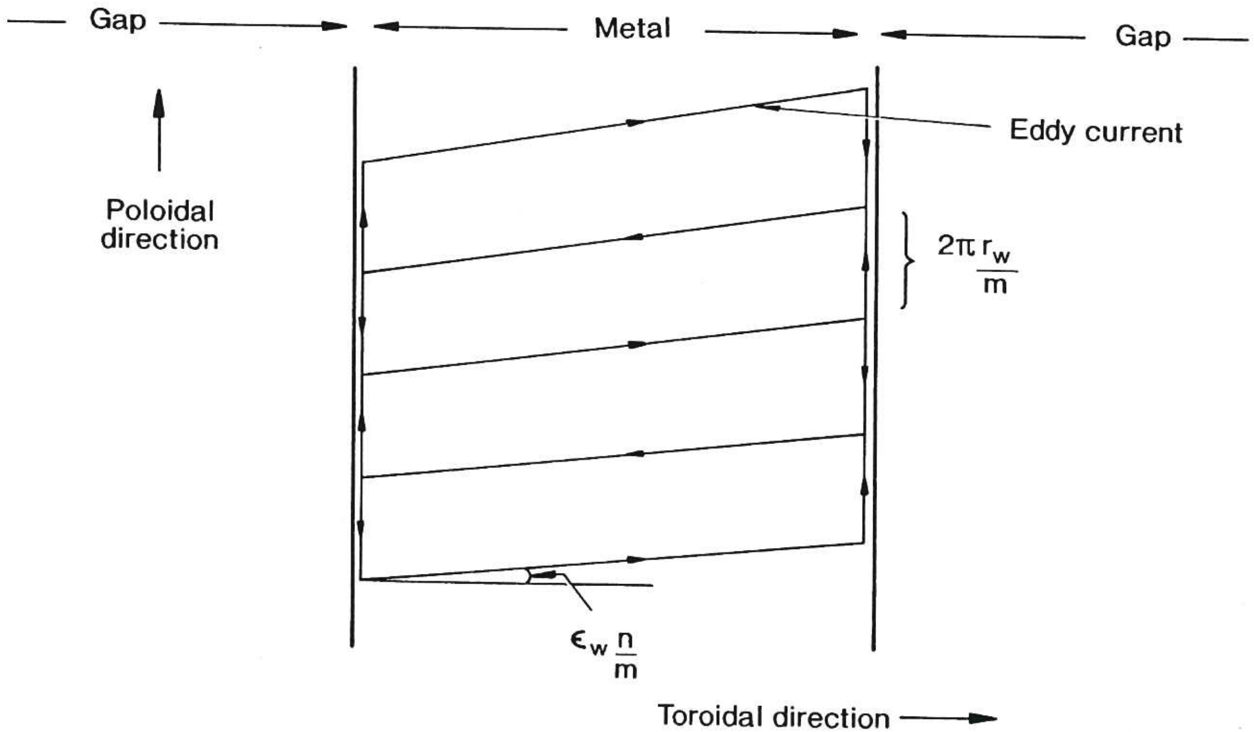
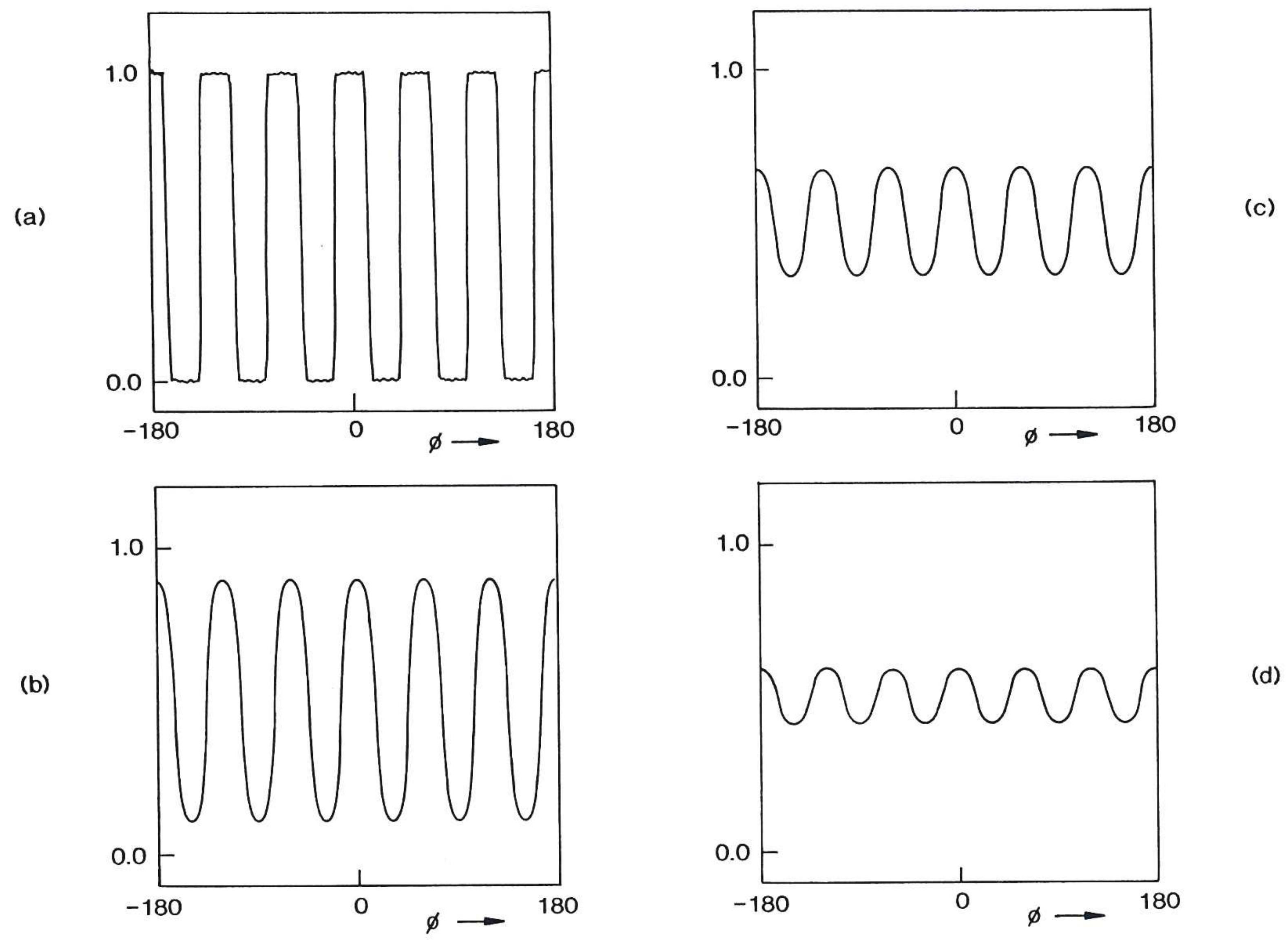

Gap
Metal
Gap
Poloidal direction
Eddy current
$2\pi r_w / m$
$\epsilon_w n / m$
Toroidal direction

Fig. 1.


(a)
1.0
0.0
-180
0
180
ø
(b)
1.0
0.0
-180
0
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(c)
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0.0
-180
0
180
ø

Fig. 2.

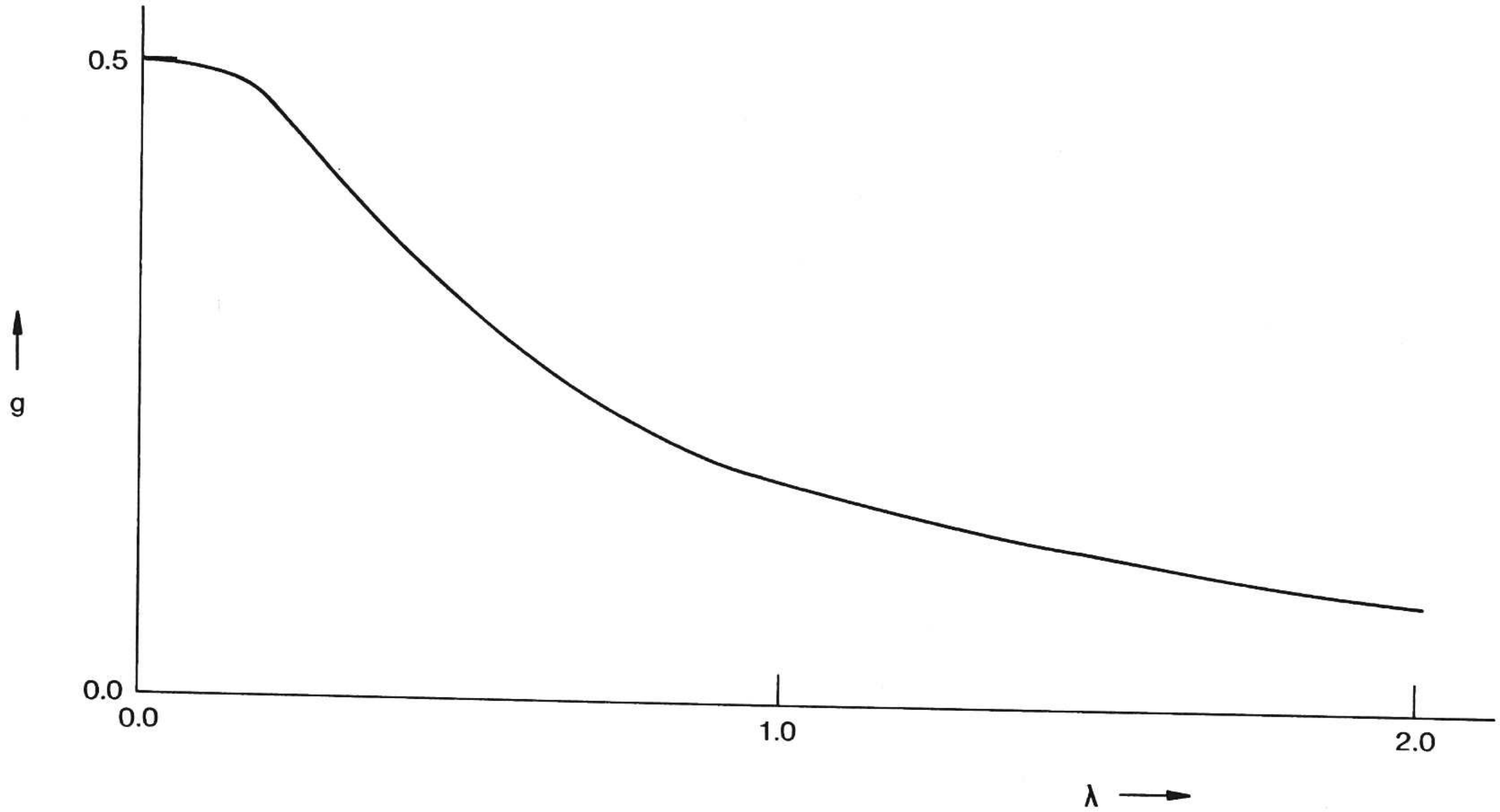

Fig. 3.

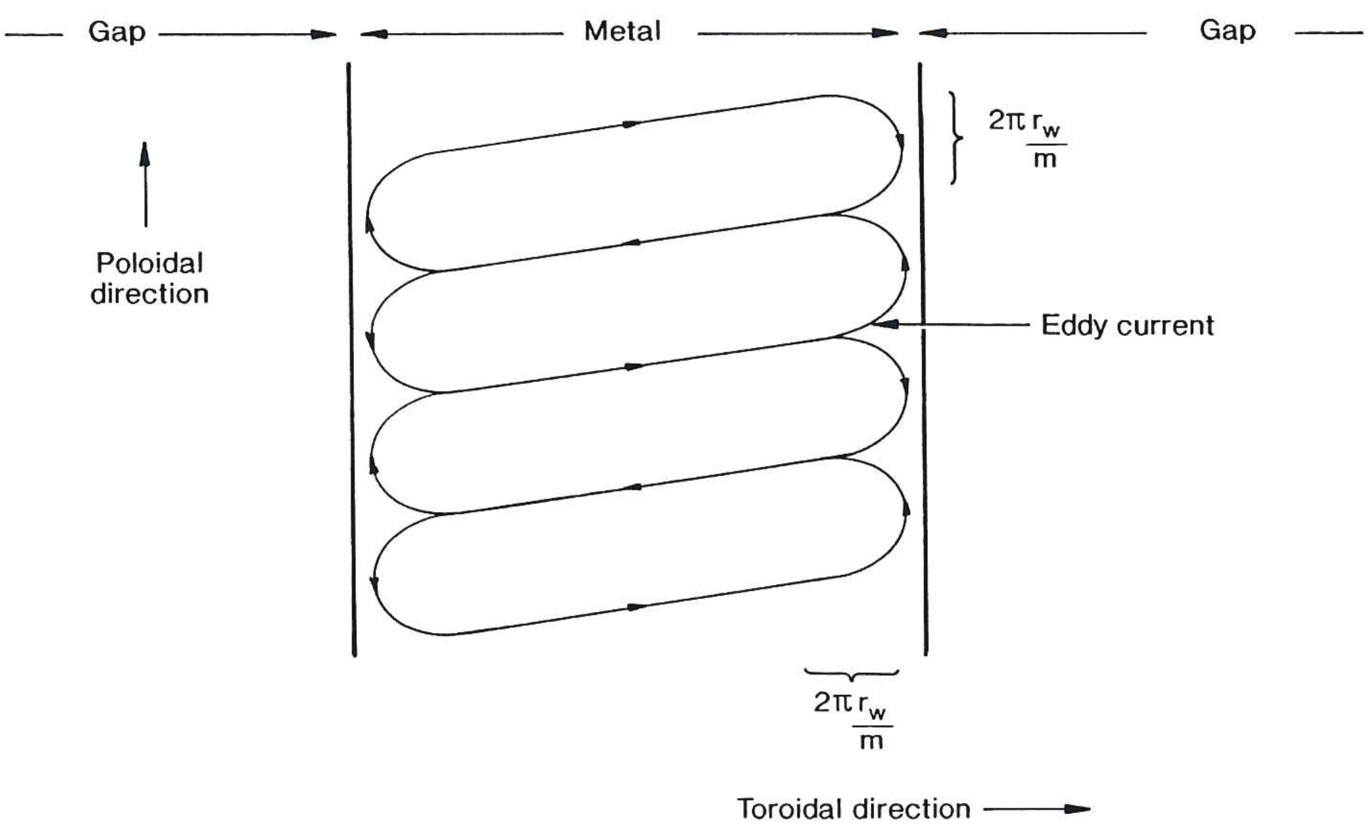
Gap
Metal
Gap
$2\pi r_w / m$
Poloidal direction
Eddy current
$2\pi r_w / m$
Toroidal direction

Fig. 4.