AEA FUS 253

AEA Technology
Fusion
(UKAEA/Euratom Fusion Association)

Fokker-Planck Study of Fast Ion Losses in JET

C A Gardner, M R O'Brien and M Cox

November 1993

AEA Technology
Fusion

Culham, Abingdon
Oxfordshire OX14 3DB
United Kingdom
Telephone 0235 463595
Facsimile 0235 463770

Fokker-Planck Study of Fast Ion Losses in JET

C A Gardner, M R O'Brien and M Cox

Microwave and Interpretation Department
AEA Fusion, Culham

1 Introduction

A recent study of direct losses of fast ions near the JET separatrix using a particle following code to trace the drift orbits has shown that the radial electric field can greatly modify the loss cone in velocity space [1]. These calculations also suggest that a small negative potential within the plasma might lead to greater losses than with $E_r = 0$. The enhancement in the losses could lead to a still more negative plasma potential, accompanied by a further increase in losses, which could provide a mechanism to explain the L $\rightarrow$ H transition.

A quantitative investigation of this effect is required. To this end the 3D Fokker-Planck code BANDIT-3D has been modified to include the loss cones. The calculation of the ion distributions has been made on several, unconnected, circular flux surfaces clustered close to the separatrix with each flux surface having its own loss cone. The loss cones are calculated from the conserved quantities assuming a parabolic potential profile. The first section describes this calculation and explains the qualitative features of the loss cones, the second section briefly outlines the Fokker-Planck equation solved, the third section examines the results obtained, and conclusions are given in the final section.

2 Ion Loss Cone

In this section we describe, for convenience, the loss cone characteristics already discussed in reference [1]. In the drift orbit approximation the conserved quantities for non-relativistic ions are: the total energy, $\frac{1}{2}mv^2 + q\Phi$, the magnetic moment, $mv_\perp^2/2B$, and the generalised toroidal angular momentum, $Rmv_\varphi + q\Psi$. Here Φ is the electric potential and Ψ the poloidal flux. Consider a fast ion starting a distance Δr within the separatrix at the outside of the plasma. If, on its drift orbit, the ion just reaches the separatrix at the X-point, then:

$$
\begin{aligned}
1/2\, m\, v_o^2 \; + \; q\, \Phi_o &= 1/2\, m\, v_X^2 \; + \; q\, \Phi_X \\
v_{\parallel X}^2 &= v_X^2 \; - \; v_{\perp o}^2\, B_X/B_o \\
R_o\, m\, v_{\varphi o} &= R_X\, m\, v_{\varphi X} \; + \; q\, (\Psi_X \; - \; \Psi_o).
\end{aligned}
$$

The o, X subscripts denote outside and X-point (R_o, R_X are the major radii of the outboard side of the flux surface, and the X-point respectively). In this study we take $R_o/R_X = 1.6$ to model JET.

Now, $\Delta\Psi = \Psi_X - \Psi_o \simeq R_o B_{po} \Delta r$ where B_{po} is the poloidal field on the outboard side of the flux surface, and using the approximation $v_\varphi \simeq v_\parallel$ (although this might not be a very good approximation near the edge of JET), the above equations give

$$
\begin{aligned}
q/m(R_o B_{po} \Delta r) &= R_o v_o \cos\theta_o - R_X \sigma [v_o^2 - 2q/m(\Phi_X - \Phi_o) \\
&\quad - v_o^2 \sin^2\theta_o R_o/R_X]^{1/2}
\end{aligned}
\tag{1}
$$

Here we have neglected the contribution of B_p to B and therefore taken $B \propto 1/R$. θ_o is the ion pitch-angle on the outboard side of the flux surface and σ is the sign of $v_\parallel$ at the X-point. σ must have the same sign as $\cos\theta_o$ for passing particles whereas for trapped particles we may have $\sigma = +1$ or $\sigma = -1$ depending upon whether the particle is on the 'co' or 'counter' leg of its orbit. Equation (1) gives a boundary in velocity space for those ions that just reach the X-point, whence the loss cone is defined by the criterion: if in equation (1) the RHS $\geq$ LHS then the ion is lost through the separatrix otherwise the ions remain within the main body of the plasma.

The potential profile is taken to be parabolic;

$$
\Phi_X - \Phi_o = \alpha(\Delta r/\rho_\theta)^2 \times T_i
$$

Here, ρ_θ is the ion poloidal Larmor radius at the edge of the plasma, T_i is the ion temperature in eV, and the value of α facilitates changes in the size of the electric field. This choice is convenient since the magnitude of Φ is typically $\sim$ the local ion temperature one poloidal Larmor radius within the separatrix. A parabolic profile reflects the fact that Φ is observed to peak at the separatrix, dropping again in the sheath.

Figure 1 shows an ion loss cone in velocity space, in units of the thermal velocity $(2T_i/m)^{1/2}$, for a flux surface which is $\Delta r \simeq 1.1$cm within the separatrix with $R_{maj} = 2.96$m, $a = 1.2$m, $B_\varphi(R_{maj}) = 2.3$T, $B_{po} = 0.52$T, $T_i = 1.0$keV and zero electric field. The main features are a narrow nose, angular extent $\Delta\theta_o \simeq 10^o$, and a broad region $\Delta\theta_o \simeq 55^o$ for E_i(keV) > 4.5keV. Also note that the loss cone is symmetric in velocity space, in the trapped region, within the trapped/passing boundary TPBD (the dotted lines), but only exists for $v_\| > 0$ in the passing region, outside the TPBD. Loss cones for other surfaces are obtained simply by scaling the energies ($\sim\ (\Delta r)^2$) [1].

The symmetry in the trapped region occurs because we assume that the co-trapped ($v_\| > 0$) and the counter-trapped ($v_\| < 0$) ions are equivalent since in the Fokker-Planck calculations we will neglect finite orbit width effects in describing the ion distribution on a flux surface. There are two distinct ways in which the trapped ions can be lost to the X-point: those that are lost directly to the X-point without bouncing and those that are lost after bouncing [1]. These correspond to σ having the same sign and the opposite sign to $\cos\theta_o$ respectively. Finite orbit width effects, while outside the scope of the Fokker-Planck treatment, of course determine the loss cone and mean that for passing ions all those with $v_\| < 0$ are confined.

The effects of the electric field on the shape of the loss cone, previously discussed by Chankin et al. [1], are now described. Two distinct effects occur as the electric field is increased: (a) a reduction in the threshold energy for $\theta_o = 0$ for small electric fields followed by an increase for fields above a critical value (see figure 2), and (b) a gradual narrowing in the angular extent of the nose accompanied by an arching and shortening of the nose until the nose finally disappears altogether for $\alpha \gtrsim 1.75$. The behaviour of the threshold energy can be explained in the following terms. For small electric fields the ions are only marginally slowed down so that their velocities in the vicinity of the X-point are small compared to those ions in the absence of an electric field, thus they have more time to drift vertically and are more easily lost. However, eventually the electric field reaches a critical value, $\alpha \simeq 1.75$, above which the ions are slowed to such an extent that those that were previously lost do not even reach the X-point.

To determine whether the electric field leads to an increase or reduction in the ion losses we solve a Fokker-Planck equation to see whether the shrinking of the nose, or the reduction in the threshold energy for lost passing ions is the dominant effect.

3 Fokker-Planck Equation

BANDIT-3D solves the time dependent three dimensional Fokker-Planck equation for the ion distribution function $f(\theta_o, v, r, t)$ [2]. Here θ_o is the pitch-angle the ion makes with B at the outboard (low field) side of the plasma, and r is the radius of the flux surface which is assumed for simplicity to be circular. The ion speed v is approximated to be independent of position on the flux surface; this is a requirement of the Fokker-Planck code even though the approximation is invalid for non-zero radial electric fields. Once the loss cone has been determined then the loss term is incorporated into the Fokker-Planck equation as below:

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}} + \left(\frac{\partial f}{\partial t}\right)_{\text{loss}}$$

$$\left(\frac{\partial f}{\partial t}\right)_{\text{loss}} = - \frac{f}{\tau_{\text{loss}}}$$

$$\tau_{loss} = \begin{cases} 2\pi R/v_{\|o} & \text{if ion lost} \\ \infty & \text{if ion contained} \end{cases}$$

The radial grid used is non-uniform with a cluster of uniformly spaced points close to the separatrix. Radial transport connecting different flux surfaces, which can be included in BANDIT-3D, has been neglected for simplicity. We repeat that finite orbit width effects only determine the loss cone and do not otherwise affect the Fokker-Planck calculation.

4 Results

The ion distribution function in velocity space after twenty time steps $\simeq$ 3.4ms is shown in figure 3 for a flux surface 2.1cm inside the separatrix. For this calculation and all subsequent calculations, unless stated otherwise, the majority deuterium ions underwent self collisions, and collisions with background carbon ions (to give $Z_{eff} = 2.0$) and background electrons, and $R_{maj} = 2.96$m, $a = 1.2$m, $R_X = 2.6$m, $T_D = 1.0$keV, $T_e = 1.0$keV, $T_C = 0.5$keV, $n_D = 0.9 \times 10^{19}\text{m}^{-3}$, $n_e = 1.0 \times 10^{19}\text{m}^{-3}$, $B_{po} = 0.52$T and $B_\varphi(R_{maj}) = 2.3$T. Both the initial temperature and density profiles were flat and the numerical mesh consisted of twenty flux surfaces, eighty speed grid points, sixty pitch-angle grid points and the time step was $\Delta t = 0.172$ms.

The distribution function reflects the structure of the loss cone but with an additional, initially surprising, spike at the 'co' TPBD. This spike is due to those counter-passing ions close to the 'counter' TPBD becoming trapped, due to collisional effects, and appearing at the 'co' TPBD because of the equivalence between the co-trapped and counter-trapped regions in velocity space. The equivalence previously mentioned between the 'co' and 'counter' trapped ions is broken if finite banana orbit widths are considered. In this case the two legs of the trapped ion can belong to different flux surfaces at the inboard side of the plasma due to the significant drift in its orbit. Consequently the spike in the distribution function about the co-trapped/passing boundary would be smoothed out over a number of flux surfaces. In addition, the symmetry observed in the trapped region of the loss cone would not be exact.

Figure 4 shows the number of deuterium ions within 4cm of the separatrix over twenty time steps, total duration $\simeq$ 3.4ms, for a number of different values of α. What is obvious from figure 4 is that the number of ions lost decreases as the electric field is increased. This result is counter to that proposed by Chankin et al. [1]: that the number of ions lost should initially increase as the electric field rises and then decrease above a critical value for the electric field. Chankin et al. concluded this from the behaviour of the broad region in the loss cone as a function of the electric field.

However, it is clear from our results that the broad region only comes into effect for velocities a few times greater than the thermal velocity and has little effect on the total number of ions if the initial distribution is Maxwellian. It is therefore the nose, which extends to much lower velocities, that dominates even though this has a smaller angular extent: it shrinks as the electric field is increased thereby reducing the number of ions lost to the separatrix. Calculation of the ion current created from the approximate gradients of the curves in figure 4 reveals that the current drops from $\simeq$ 600A, where $\alpha = 0.0$, to $\simeq$ 328A, where $\alpha = 1.5$. Thus although the currents calculated are similar to those calculated by Hugill [3] to explain the potential changes observed during H modes, it appears that a negative plasma potential does indeed act to retard the ion loss.

The disparity between Chankin's proposition and the results calculated from the Fokker-Planck calculations could be due to the rather simplistic model adopted for the loss time. When the loss times calculated by Chankin for zero electric field were examined it was evident that our model was inadequate. The major discrepancy between the two was the total absence in the former of a sharp transition in the loss times as a function of $1/v_{\|o}$. This transition represents a change in the ion orbit from one that is lost directly through the separatrix to one that bounces, hence a longer transition time, before it is lost through the separatrix. Figure 5 shows the loss times calculated on a flux surface $\Delta r = 2$cm within the separatrix by Chankin. Therefore the loss time model was altered to incorporate this transition in an approximate manner: below the transition point a linear dependence in the loss time with $1/v_{\|o}$ was assumed, the approximate gradient estimated from the figure, while above the same linear dependence was assumed but with the gradient $1.5 - 2.5$ times steeper, as measured from the figure. However even with this improved model a reduction in the ion losses with increasing negative potential was still observed.

An interesting feature of figure 4 is the apparent convergence of the ion loss curves as α is increased towards $\alpha = 1.50$ with the $\alpha = 0.874$ and $\alpha = 1.50$ curves barely distinguishable. This suggests that there may be a local maximum in the ion loss current between these two values of α. Therefore an improved parameter scan in α was undertaken: the conditions were identical to the previous case except that the time resolution was improved by doubling the number of time steps while halving their duration. Also we changed the potential profile to have a more realistic spatial dependence, namely parabolic for $\Delta r < \rho_\theta$ and linear for $\Delta r > \rho_\theta$.

Figure 6 shows that there is indeed a local maximum in the ion loss current at $\alpha \simeq 1.25$.

This upturn which is only a few Amps can be attributed to the behaviour of the loss cone as α is increased. As commented before there is an initial shrinkage in the extent of the nose accompanied by a shift in the broad region of the loss cone to lower $v_{||}$ as α is increased from zero thus reducing the ions lost. But eventually the reduction in the nose and its final disappearance is compensated by the shift of the broad region and so no further decrease in the ions lost occurs; this situation is represented by the minimum in figure 6. Beyond this minimum the broad region shifts still further, to lower energies, as α is increased; the nose is no longer significant and so the ion losses are greater when compared to the stationary situation mentioned before. Finally when α reaches a critical value the broad region starts to shift back to higher $v_{||}$ and the ion losses once again drop with increasing α. In figure 7 the ion loss currents are shown for a number of flux surfaces; each flux surface has $\sim$ the same area assigned to it and k is the flux surface label. There is a gradual increase in current as the last closed flux surface is approached as expected: this is simply because for larger Δr the loss cone shifts to higher energies resulting in a much smaller loss term due to the approximate Maxwellian nature of the ion distribution. Additionally a rapid fall in the current with increasing α for those flux surfaces furthest away from the separatrix ($\Delta r > \rho_\theta$) is observed: this is because the plasma potential grows more quickly with increasing α the further the flux surface is from the separatrix.

Lastly trapping effects, while still determining the loss cone, were switched off in the Fokker-Planck code to remove the spike at the co-TPBD in figure 3. The total current due to ion losses is shown in figure 8 for a number of different values for α. The current appears to have dropped by a sizeable factor when compared to the previous case (see figure 6), the factor depending on the value of α. Taking the comparison at $\alpha = 0.0$ we see that the factor is $\simeq 3.3$ which seems surprising in that we would only expect a factor of two drop (at this α value the nose, half of which has been removed, is dominant). This discrepancy is due to geometrical effects arising from differences in the way the Fokker-Planck code treats the trapping and no-trapping cases. In the trapping case the factor is approximately 1.65, compared to 1.0 in the no-trapping case, and once this is accounted for the ratio $I_{trap}/I_{no-trap}$ equals two as expected. As α grows the ratio $I_{trap}/I_{no-trap}$ falls: this is due to the increasing importance of the broad region of the loss cone, which is primarily in the passing region of velocity space.

5 Conclusions

On the basis of our simplified kinetic model, we conclude that giving the plasma a negative potential with respect to the separatrix reduces the ion losses due to vertical drift. This is contrary to the suggestion in [1] in which it was proposed that the behaviour of the ion losses as a function of radial electric field might be responsible for the L $\rightarrow$ H transition. Even though a closer investigation of the effect of the negative plasma potential on the ion losses revealed a slight upturn in the losses with increasing potential for $\alpha \sim 1$ this upturn is very small and does not explain the potential change observed in the L $\rightarrow$ H transition. However our Fokker-Planck calculation required several approximations: a) finite orbit width effects were neglected in the Fokker-Planck calculation, b) the ion speed v was assumed to be independent of poloidal angle, c) a simple and probably inaccurate model for the ion loss time was adopted, and d) radial transport between adjacent flux surfaces was ignored. Further calculations in which some of these approximations are not made may be of value to test the robustness of these conclusions.

Acknowledgements

The work described was undertaken for an Article 14 contract with the JET Joint Undertaking (contract JT7/9005(7)) and was funded by the UK Department of Trade and Industry and Euratom.

References

[1] *The effect of radial electric field on fast ion losses in divertor Tokamaks*, A V Chankin et al., Proc. of 19th Int. Conf. on Plasma Physics, Innsbruck 1992, part II, p 779.
[2] *Solution of Three Dimensional Fokker-Planck Equations for Tokamak Plasmas using an Operator Splitting Technique*, J S McKenzie, M R O'Brien and M Cox, Computer Phys. Comm. 66 (1991), p. 194.
[3] *The Energy Required to Establish Flow in an Axisymmetric Tokamak Plasma*, J. Hugill, TGN(92) 01.

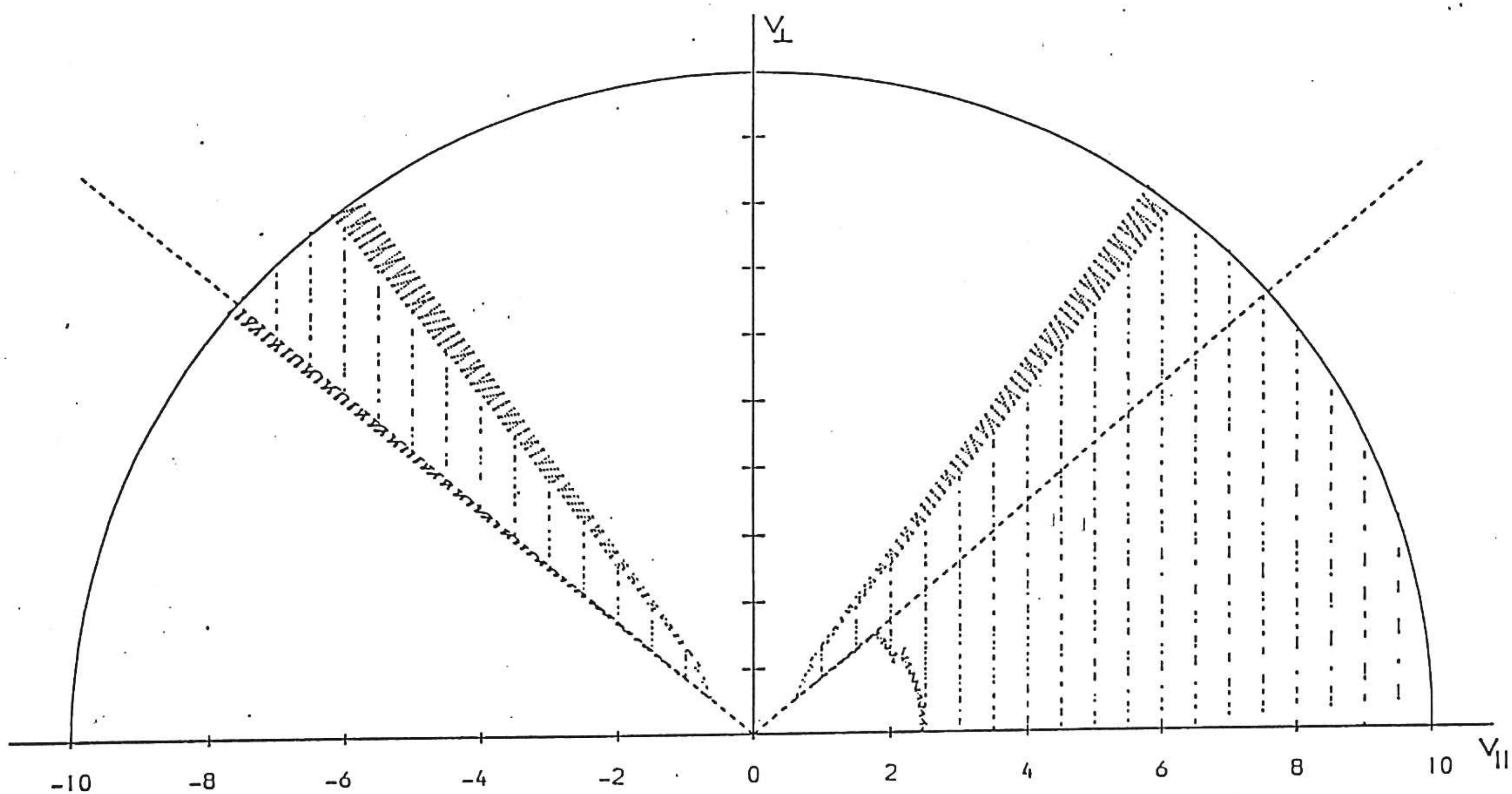

Figure 1: Ion loss cone in velocity space. ($\alpha = 0$)

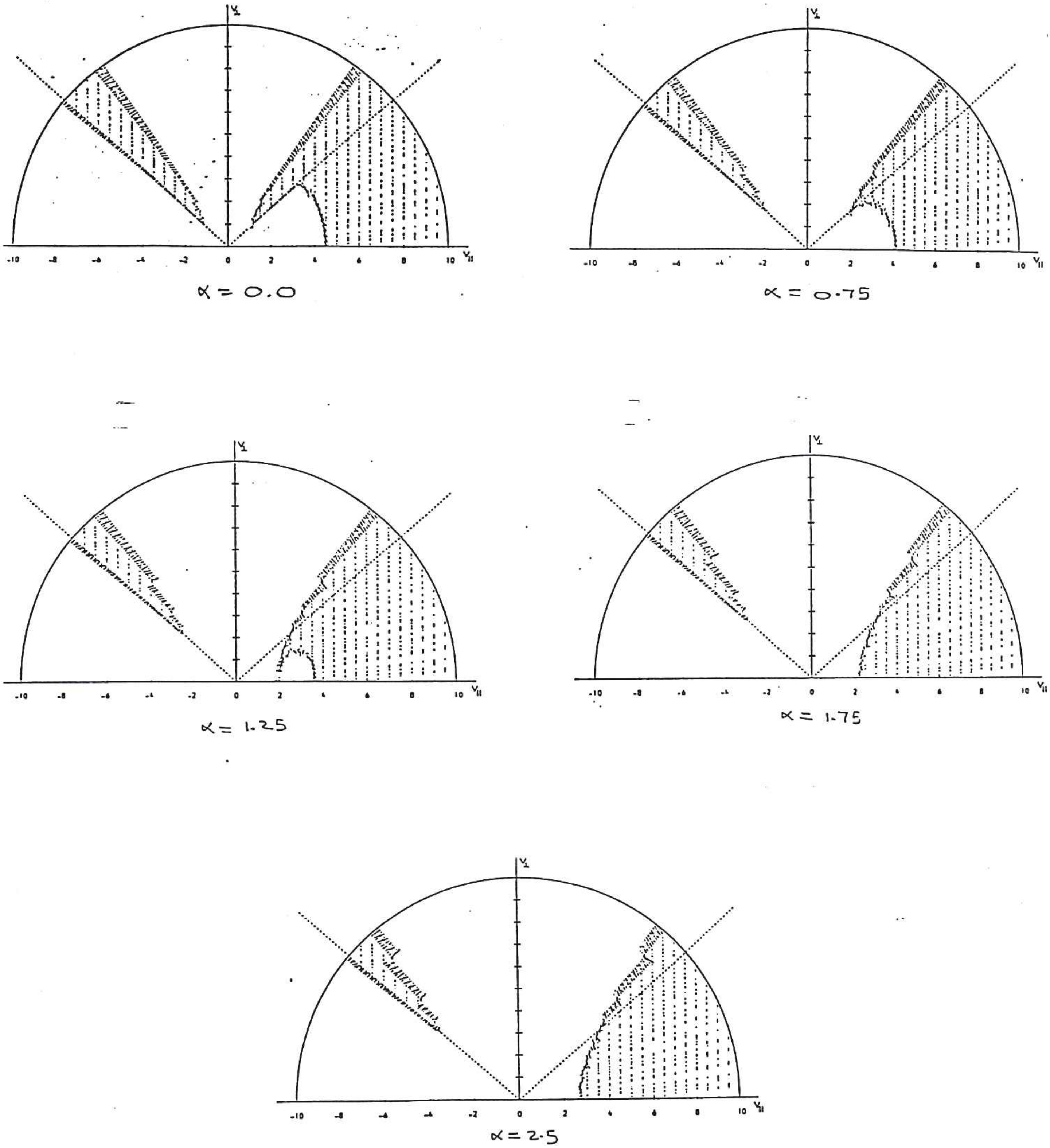

Figure 2: Loss cones for different negative potentials.

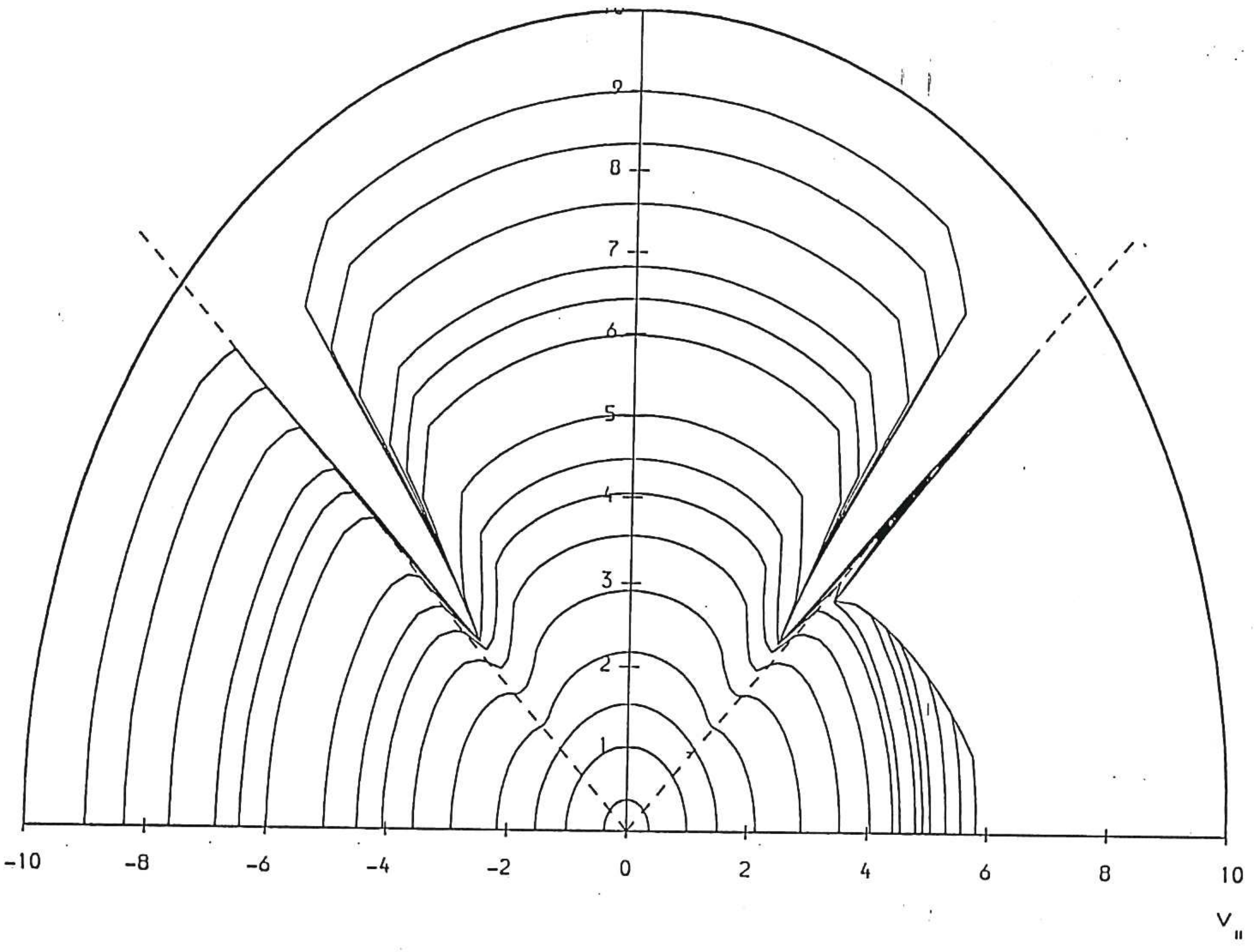

Figure 3: Ion distribution function in velocity space. ($\Delta r = 2.1$ cm)

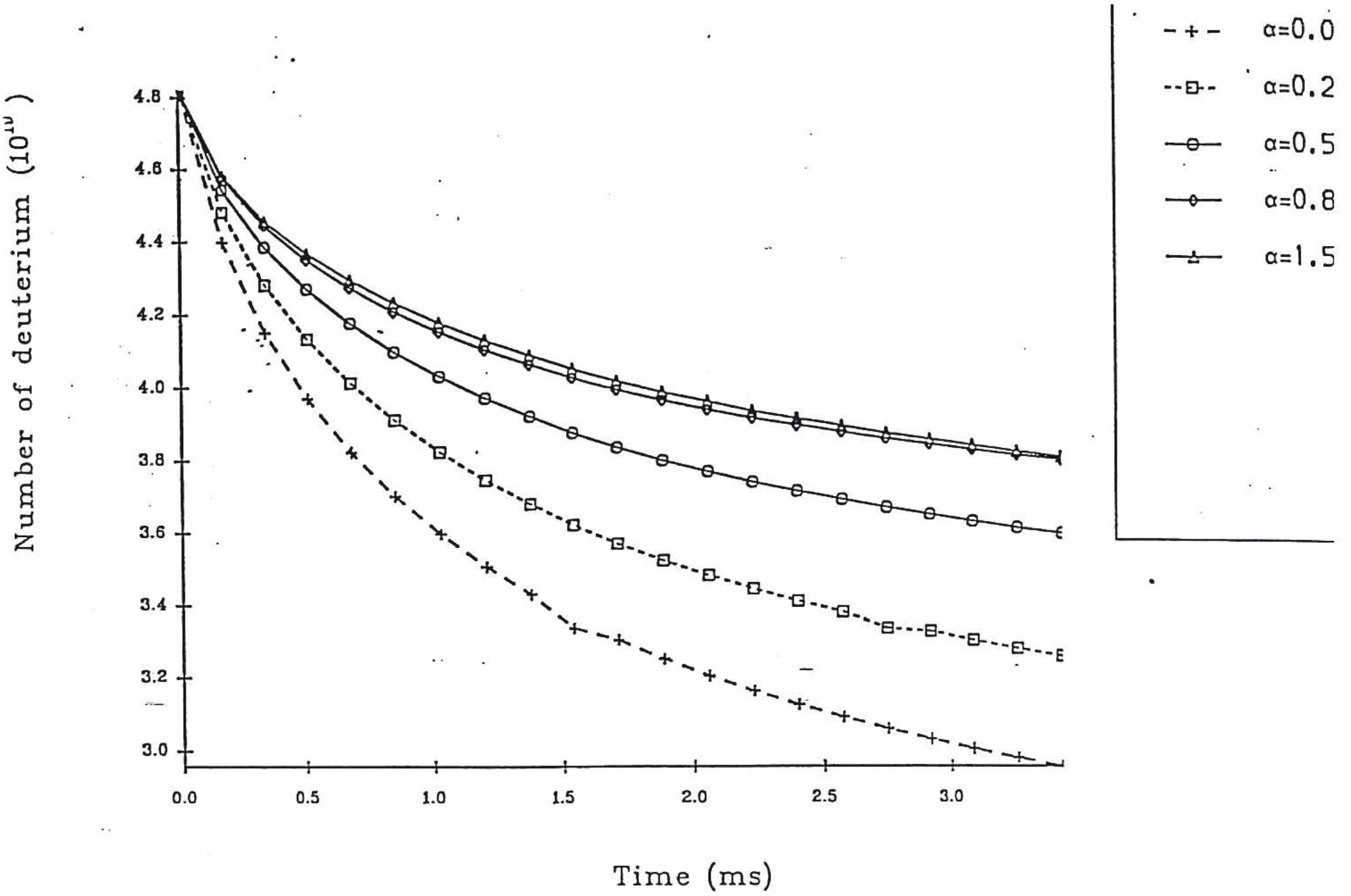

Figure 4: Number of deuterium ions within 4cm of separatrix.

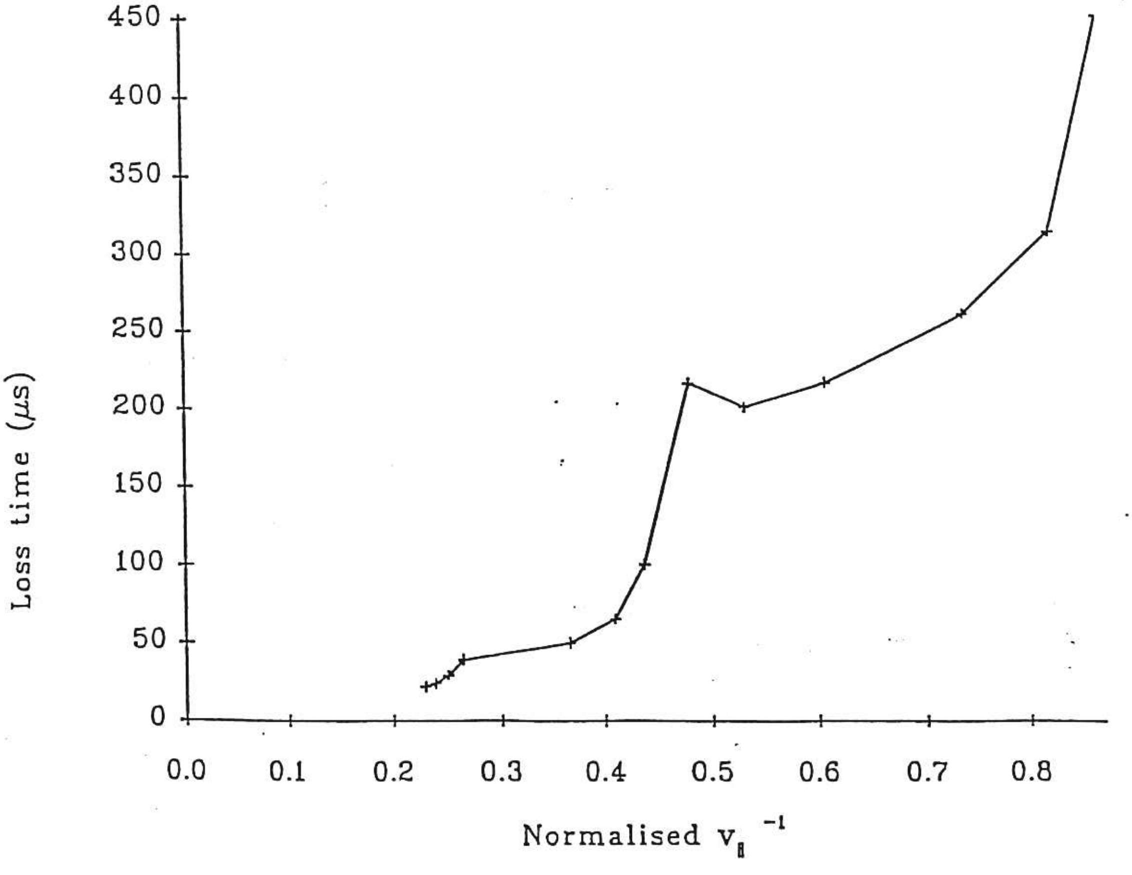

Figure 5: Loss times calculated by Chankin.

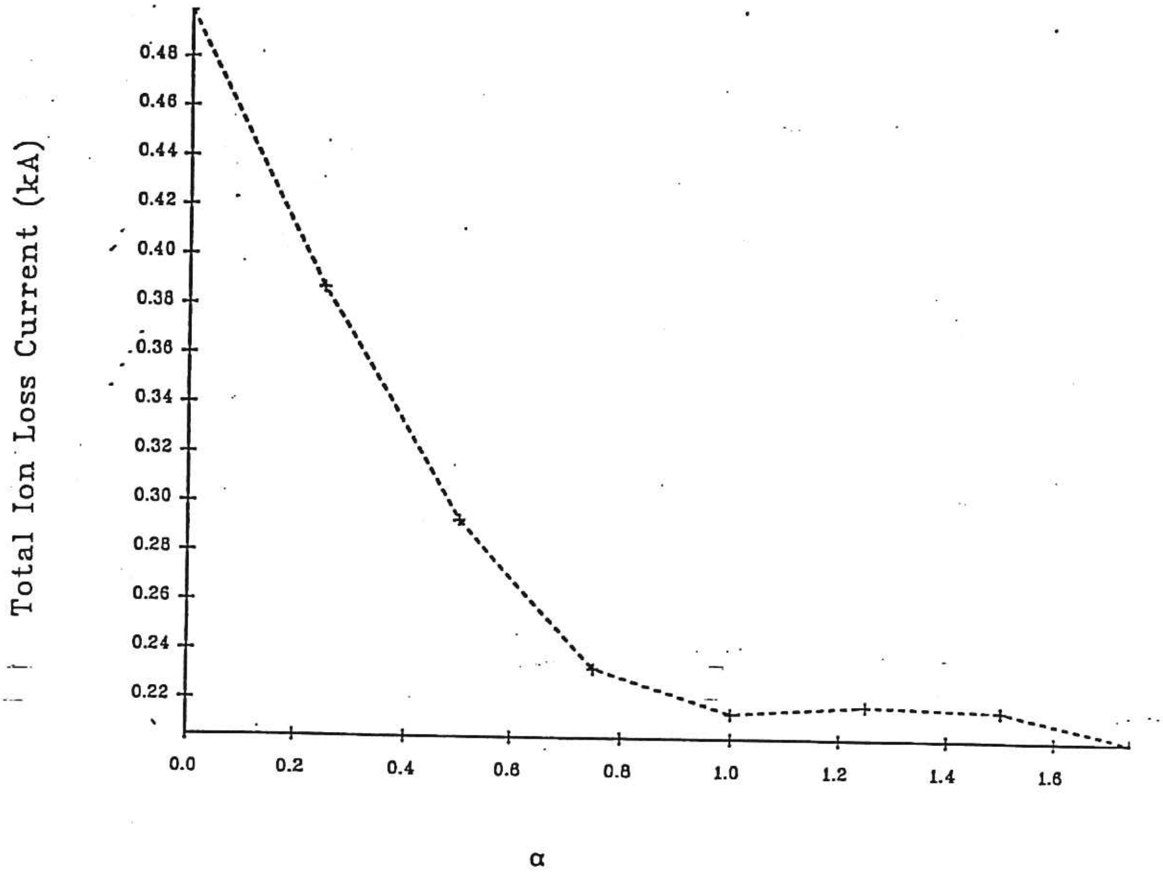

Figure 6: Total ion loss current.

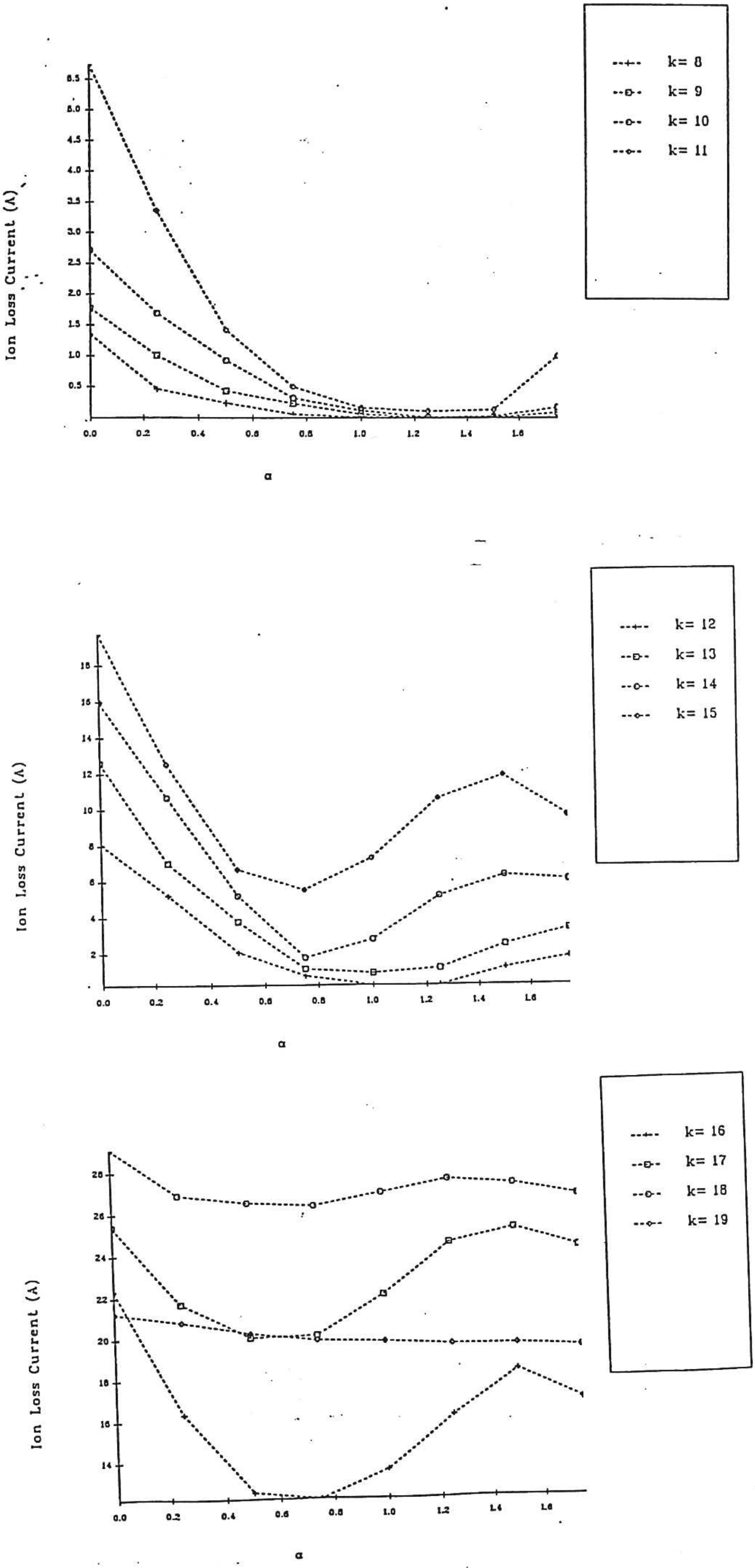

Figure 7: Ion loss currents for different flux surfaces.

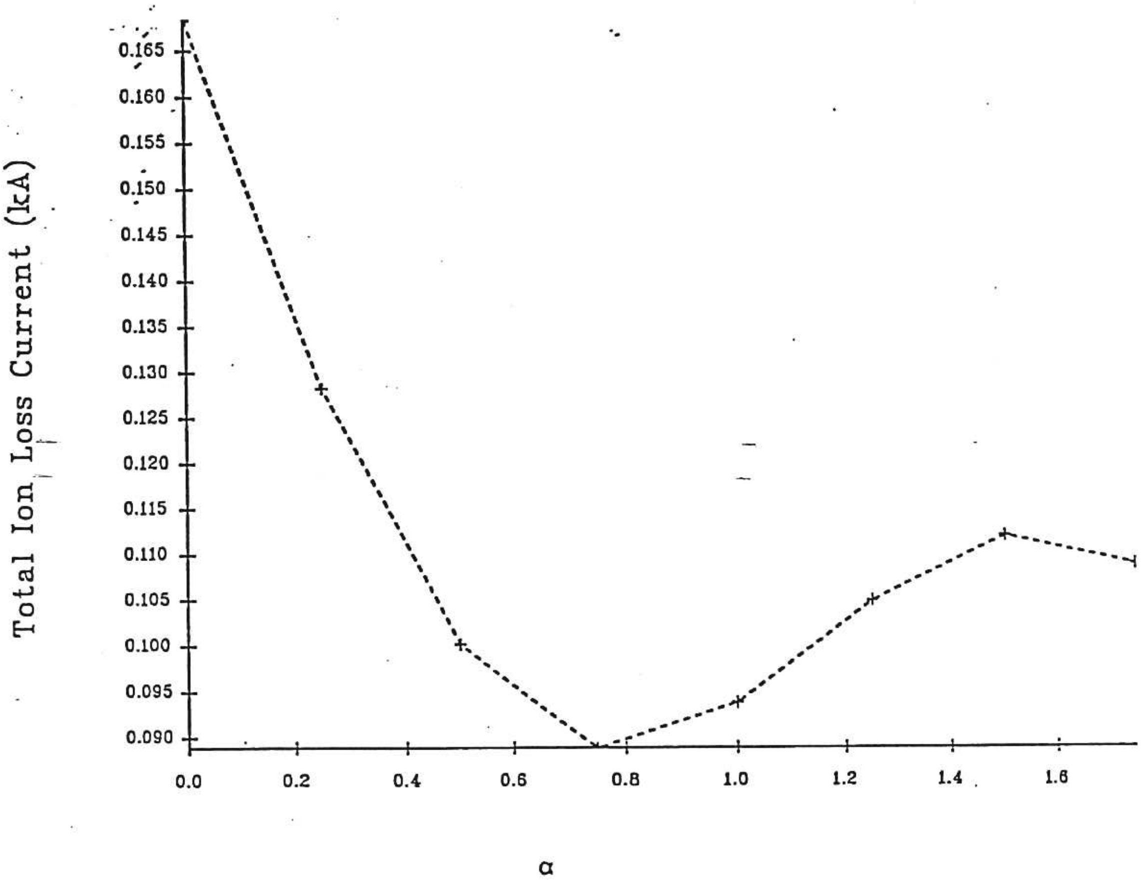

Figure 8: Total ion loss current (counter-trapped region removed).