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CLM L2

LECTURES ON THE HYDROMAGNETIC STABILITY OF A CYLINDRICAL PLASMA

VIII THE ROLE OF ANISOTROPIC PRESSURE IN THE THEORY OF THE STABILIZED PINCH

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ABSTRACT

In the simple theory of the stabilized pinch it is assumed that the plasma has isotropic pressure. In practice this may well not be true and it is known that new instabilities may arise when the pressure is anisotropic. In particular, in the simple theory, the uniform magnetic field in the plasma always exerts a stabilizing influence whereas plane waves propagating in a uniform magnetic field may be unstable if the pressure is anisotropic.

This lecture is divided into two main parts. In the first the qualitative effect of anisotropic pressure is demonstrated by supposing the plasma to be governed by the double adiabatic hydromagnetic equations of Chew, Goldberger and Low. These equations neglect heat flow along field lines and they underestimate the instability.

In the second part a more accurate dispersion relation, given by Chandrasekhar, Kaufman and Watson, is solved for several values of the ratio of parallel and perpendicular components of pressure. In this case both ion and electron distribution functions are assumed to be Gaussian in both parallel and perpendicular components of velocity and the ratio of parallel to perpendicular pressure is taken to be the same for each species. It is shown that the domain of instability increases if the ratio of pressures is either very large or very small but that the greatest stability occurs when the parallel pressure slightly exceeds the perpendicular pressure.

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C O N T E N T S

1. INTRODUCTION

Recently the stability of a cylindrical plasma obeying the idealised hydromagnetic equations has become rather well understood. In particular, following the papers of Newcomb(9) and Suydam(11), complete necessary and sufficient conditions for the stability of such a system are available. In practice the situation is not quite so clear; application of these criteria requires a discussion of the presence or absence of zeros of solutions of a second order differential equation. Calculations based on these criteria have recently been performed by Whiteman and Copley(14)

Parallel with the progress in the theory of plasma stability there has been the experimental occurrence of instabilities in situations which ideal hydromagnetics would describe as stable. The most serious discrepancy is in the case of the hard core pinch; this configuration, which has been mentioned in passing in (12), is one in which the plasma column is hollow and an axial current in a conducting rod through its centre exceeds the axial current flowing in the opposite direction through the plasma. Such a configuration is definitely stable according to hydromagnetics but instabilities have been observed by both Aitken, Burcham and Reynolds(1) and Birdsall, Colgate and Furth(2). One possibility is that end effects in a finite cylindrical tube introduce perturbations leading to instability but another more serious possibility is that the idealised hydromagnetic equations are not an adequate representation of the true equations.

The possibility to be considered in this lecture is that the particle distribution functions and hence the plasma pressure tensor are anisotropic. This is likely in a low density plasma in a strong magnetic field. In a high density plasma, frequent collisions keep the distribution functions isotropic and ensure a truly localised hydrodynamics. When collisions are infrequent this is no longer the case. If the magnetic field is strong, the particles gyrate in tight orbits around the lines of force but they are more or less free to move along the field lines; this situation is discussed in the present lecture.

A first attempt to obtain a set of equations in this case was made by Chew, Goldberger and Low(6). They obtained, instead of the single adiabatic law for pressure and density variations, two equations governing the behaviour of the components of the pressure tensor along and across the field lines. These double adiabatic equations take account of the anisotropy caused by the introduction of a strong magnetic field but they are still local equations and they do not take full account of the possibility of heat flow along the magnetic field.

A more complete system of equations was considered by Chandrasekhar, Kaufman and Watson(3,4) and they also studied the stability problem discussed in this lecture in (5). Rosenbluth and Rostoker(10) and Kruskal and Oberman(7) subsequently considered the stability of arbitrary equilibrium configurations in the case in which the heat flow along field lines is included and they showed that an energy principle could be formulated to deal with this problem. As usual the system is stable if and only if a certain integral, δW, is positive for all perturbations. Energy principles also apply for the idealised hydromagnetic and double adiabatic equations; the corresponding integrals are δW_H and δW_{DA}.

When the equilibrium configuration is isotropic it can be shown that

$$\delta W_H \leq \delta W \leq \delta W_{DA}. \tag{1.1}$$

The second inequality also applies when the equilibrium is anisotropic. The inequalities (1.1) show that, for problems in which the initial configuration is isotropic, the idealised hydromagnetic equations give the worst result; thus if they predict stability there is no need to study the more refined equations. However the more refined equations enable equilibria with anisotropic distributions to be studied and this can lead to the prediction of further instabilities.

That this does lead to new instabilities can be demonstrated by consideration of a very simple problem. In the case of idealised hydromagnetics it is well known that, if there is only an axial magnetic field, the worst that can happen is that the system is marginally stable. However in the case of either the double adiabatic equations or the Chandrasekhar, Kaufman, Watson equations*, it is easy to show that plane hydromagnetic waves can become unstable if the anisotropy is large enough.

*These will be referred to as C.K.W. equations in what follows.

Thus Lüst(8) has shown, using the double adiabatic equations, that instability of plane waves in infinite space with a uniform magnetic field occurs if either

$$p_{\parallel} > p_{\perp} + B^2/4\pi \tag{1.2}$$

or

$$p_{\perp}^2/p_{\parallel} > 6(p_{\perp} + B^2/8\pi), \tag{1.3}$$

where $B, p_{\parallel}$ and $p_{\perp}$ are the equilibrium values of the magnetic field and the components of the pressure tensor along and across the field lines. Similarly, using the C.K.W. equations and making some further assumptions about the symmetry of the problem, it has been shown (5) that plane waves are unstable if either inequality (1.2) is satisfied or

$$p_{\perp}^2/p_{\parallel} > p_{\perp} + B^2/8\pi. \tag{1.4}$$

It can be seen that, in agreement with what has been stated above, the C.K.W. criterion is violated more readily than the double adiabatic criterion.

It is immediately apparent that this result can be expected to have an adverse effect on the gross stability of the stabilized pinch; its possible influence on localised instabilities is not discussed here*. The gross stability of the stabilized pinch is largely due to the inherent stability of an axial magnetic field. It has been seen above that this property may be lost when the equilibrium is anisotropic.

Both the double adiabatic equations and the C.K.W. equations predict instabilities of the stabilized pinch caused by the anisotropy of the equilibria. Because the C.K.W. equations are non-local and are much more complicated than the double adiabatic equations, detailed derivation of stability criteria is given for the double adiabatic case. This enables the qualitative effects of anisotropic pressure to be seen without unduly complicated algebra. Numerical results are however given

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*Strictly speaking the structure of the plasma surface should be considered because the equations are only valid if the surface region is not too narrow. However this criticism applies equally to the original discussion of the stabilized pinch and the difference between the gross properties should be found by the present treatment.

for the case discussed by Chandrasekhar, Kaufman and Watson(5). It seems likely that these results are more valid than the corresponding double adiabatic results. For example, in the case of an isotropic equilibrium, the C.K.W. equations predict the same results as idealised hydromagnetics but the double adiabatic equations predict much greater stability.

2. DERIVATION OF STABILITY CRITERIA.

The double adiabatic equations for a plasma are:

$$\rho \frac{d\underline{v}}{dt} = - \operatorname{div} \underline{\underline{p}} + \operatorname{curl} \underline{B} \times \underline{B}/4\pi, \tag{2.1}$$

$$\frac{\partial \rho}{\partial t} = - \operatorname{div} \rho \underline{v}, \tag{2.2}$$

$$\frac{\partial \underline{B}}{\partial t} = \operatorname{curl}(\underline{v} \times \underline{B}), \tag{2.3}$$

$$\frac{d}{dt} (p_{\parallel} B^2/\rho^3) = 0 \tag{2.4}$$

and

$$\frac{d}{dt} (p_{\perp}/B\rho) = 0, \tag{2.5}$$

where the plasma has density ρ, pressure tensor $\underline{\underline{p}}$ and velocity $\underline{v}$ and carries a magnetic field $\underline{B}$. The pressure tensor has the form

$$\underline{\underline{p}} = \begin{vmatrix} p_{\parallel} & 0 & 0 \\ 0 & p_{\perp} & 0 \\ 0 & 0 & p_{\perp} \end{vmatrix} \tag{2.6}$$

referred to local cartesian coordinates at any point in the plasma with the 1 axis along the field line.

An equilibrium configuration is considered in which a cylinder of plasma of radius r_o has density ρ_o, pressure $p_{o\parallel}$ and $p_{o\perp}$ and contains a magnetic field $(o, o, B_o b_i)$. The plasma is surrounded by a vacuum containing a magnetic field $(o, B_o r_o/r, B_o b_e)$ and this in turn is surrounded by a cylindrical conducting wall of radius Λr_o. Pressure balance across the plasma–vacuum interface requires that

$$1 + b_e^2 - b_i^2 = 8\pi p_{o\perp}/B_o^2 . \tag{2.7}$$

The stability of this system is to be studied.

In a cylindrical polar coordinate system the pressure tensor can be written

$$\left.\begin{aligned}
p_{rr} &= p_{\parallel}B_r^2/B^2 + p_{\perp}(B_\theta^2 + B_z^2)/B^2, \\
p_{\theta\theta} &= p_{\parallel}B_\theta^2/B^2 + p_{\perp}(B_z^2 + B_r^2)/B^2, \\
p_{zz} &= p_{\parallel}B_z^2/B^2 + p_{\perp}(B_r^2 + B_\theta^2)/B^2, \\
p_{r\theta} &= p_{\theta r} = (p_{\parallel} - p_{\perp})B_rB_\theta/B^2, \\
p_{\theta z} &= p_{z\theta} = (p_{\parallel} - p_{\perp})\, B_\theta B_z/B^2, \\
p_{zr} &= p_{rz} = (p_{\parallel} - p_{\perp})\, B_z B_r/B^2.
\end{aligned}\right\} \qquad (2.8)$$

Also the divergence of the pressure tensor has components

$$\left.\begin{aligned}
(\mathrm{div}\underline{\underline{p}})_r &= \frac{\partial p_{rr}}{\partial r} + \frac{1}{r}\frac{\partial p_{r\theta}}{\partial\theta} + \frac{\partial p_{rz}}{\partial z} + \frac{p_{rr}-p_{\theta\theta}}{r}, \\
(\mathrm{div}\underline{\underline{p}})_\theta &= \frac{\partial p_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial p_{\theta\theta}}{\partial\theta} + \frac{\partial p_{\theta z}}{\partial z} + \frac{2p_{r\theta}}{r}, \\
(\mathrm{div}\underline{\underline{p}})_z &= \frac{\partial p_{zr}}{\partial r} + \frac{1}{r}\frac{\partial p_{z\theta}}{\partial\theta} + \frac{\partial p_{zz}}{\partial z} + \frac{p_{zr}}{r}.
\end{aligned}\right\} \qquad (2.9)$$

Perturbations about the equilibrium are considered in which any variable q has the form

$$q = q_0 + q_1(r)\; e^{i(m\theta+kz)+\omega t}. \qquad (2.10)$$

The linearised forms of equations (2.1) to (2.5) are

$$\rho_0\omega\underline{v}_1 = -\,\mathrm{div}\,\underline{\underline{p}}_1 + \mathrm{curl}\,\underline{B}_1 \times \underline{B}_0/4\pi, \qquad (2.11)$$

$$\omega\rho_1 = -\,\rho_0\mathrm{div}\,\underline{v}_1, \qquad (2.12)$$

$$\omega\underline{B}_1 = \mathrm{curl}(\underline{v}_1 \times \underline{B}_0), \qquad (2.13)$$

$$p_{1\parallel}/p_{0\parallel} + 2B_{1z}/B_0b_i - 3\rho_1/\rho_0 = 0 \qquad (2.14)$$

and

$$p_{1\perp}/p_{0\perp} - B_{1z}/B_0b_i - \rho_1/\rho_0 = 0. \qquad (2.15)$$

The linearised forms of equations (2.8) are

$$\left.\begin{aligned} p_{1rr} &= p_{1\theta\theta} = p_{1\perp}, \\ p_{1zz} &= p_{1\parallel}, \\ p_{1\theta r} &= 0, \\ p_{1\theta z} &= (p_{0\parallel} - p_{0\perp})B_{1\theta}/B_0 b_i, \\ p_{1zr} &= (p_{0\parallel} - p_{0\perp})B_{1r}/B_0 b_i, \end{aligned}\right\} \qquad (2.16)$$

while equations (2.9) hold with all components of the pressure tensor replaced by first order quantities and $\partial/\partial\theta$ and $\partial/\partial z$ replaced by im and ik.

All the perturbed quantities except v_{1z} can now be eliminated from equations (2.11) to (2.16) and (2.9). It is shown in Appendix 1 that an equation for v_{1z} is obtained in the form

$$rDrD\, v_{1z} = (m^2 + \alpha^2 r^2)v_{1z}, \qquad (2.17)$$

where D means d/dr and

$$\alpha^2 = \frac{\{\rho_0\omega^2 + k^2[B_0^2 b_i^2/4\pi + p_{0\perp} - p_{0\parallel}]\}\ \{\rho_0\omega^2 + 3k^2 p_{0\parallel}\}}{\{\rho_0\omega^2[B_0^2 b_i^2/4\pi + 2p_{0\perp}] + k^2[3B_0^2 b_i^2 p_{0\parallel}/4\pi + 6p_{0\perp}p_{0\parallel} - p_{0\perp}^2]\}}. \qquad (2.18)$$

Equation (2.17) is a modified Bessel Equation and it can be solved immediately to give

$$v_{1z} = \mathcal{A}\, I_m(\alpha r). \qquad (2.19)$$

Expressions can then be found for the other perturbed quantities. Thus

$$v_{1r} = -ik\lambda\alpha\mathcal{A}I_m'(\alpha r)/[\rho_0\omega^2 + k^2(B_0^2 b_i^2/4\pi + p_{0\perp} - p_{0\parallel})], \qquad (2.20)$$

$$v_{1\theta} = mk\lambda\mathcal{A}I_m(\alpha r)/r[\rho_0\omega^2 + k^2(B_0^2 b_i^2/4\pi + p_{0\perp} - p_{0\parallel})], \qquad (2.21)$$

$$B_{1r} = k^2 B_0 b_i \lambda\alpha\mathcal{A}I_m'(\alpha r)/\omega[\rho_0\omega^2 + k^2(B_0^2 b_i^2/4\pi + p_{0\perp} - p_{0\parallel})], \qquad (2.22)$$

$$B_{1\theta} = imk^2\lambda B_0 b_i\mathcal{A}I_m(\alpha r)/\omega[\rho_0\omega^2 + k^2(B_0^2 b_i^2/4\pi + p_{0\perp} - p_{0\parallel})], \qquad (2.23)$$

$$B_{1z} = -B_0 b_i(\rho_0\omega^2 + 3p_{0\parallel}k^2)\mathcal{A}I_m(\alpha r)/ikp_{0\perp}\omega, \qquad (2.24)$$

$$\rho_1 = -\rho_0[\rho_0\omega^2+(3p_{0\parallel}-p_{0\perp})k^2]\mathfrak{a}I_m(\alpha r)/ikp_{0\perp}\omega, \quad (2.25)$$

$$p_{1\perp} = -[2\rho_0\omega^2+(6p_{0\parallel}-p_{0\perp})k^2]\mathfrak{a}I_m(\alpha r)/ik\omega \quad (2.26)$$

and

$$p_{1\parallel} = -p_{0\parallel}[\rho_0\omega^2+3(p_{0\parallel}-p_{0\perp})k^2]\mathfrak{a}I_m(\alpha r)/ikp_{0\perp}\omega, \quad (2.27)$$

where

$$\lambda = \{\rho_0\omega^2(B_0^2b_i^2/4\pi+2p_{0\perp})+k^2(3B_0^2b_i^2p_{0\parallel}/4\pi+6p_{0\perp}p_{0\parallel}-p_{0\perp}^2)\}/p_{0\perp}k^2. \quad (2.28)$$

The perturbed magnetic field in the vacuum has the well known form

$$B_{1z}^v = \mathfrak{B}K_m(kr) + CI_m(kr), \quad (2.29)$$

$$B_{1r}^v = -i\mathfrak{B}K_m'(kr) - iCI_m'(kr) \quad (2.30)$$

and

$$B_{1\theta}^v = (m/kr)[\mathfrak{B}K_m(kr)+CI_m(kr)]. \quad (2.31)$$

Boundary conditions have now to be applied on the perturbed plasma surface and on the conducting wall. The perturbed plasma surface has the equation

$$r = r_0+(v_{1r}/\omega)\, e^{i(m\theta+kz) + \omega t}. \quad (2.32)$$

The boundary conditions are that the normal component of the magnetic field and the total pressure $p_\perp + B^2/8\pi$ are continuous across the interface and the normal component of the magnetic field vanishes at the conducting wall. The unit normal to the plasma-vacuum interface is

$$\underline{n} = (-1,0,0) + (0, imv_{1r}/\omega r_0, ikv_{1r}/\omega)e^{i(m\theta+kz)+\omega t}. \quad (2.33)$$

When these boundary conditions are applied an equation is obtained for ω in terms of m,k and the equilibrium quantities. This dispersion relation is derived in Appendix 2 and it has the form

$$(m+b_e kr_0)^2\,\phi\,(kr_0,\Lambda) + kr_0$$
$$= \alpha kr_0^2\,[b_i^2 + \frac{4\pi p_{0\perp}}{B_0^2}\;\frac{2\rho_0\omega^2+(6p_{0\parallel}-p_{0\perp})k^2}{\rho_0\omega^2 + 3p_{0\parallel}k^2}]\frac{I_m(\alpha r_0)}{I_m'(\alpha r_0)}, \quad (2.34)$$

where

$$\phi\,(kr_0,\Lambda) = \frac{K_m(kr_0)I_m'(\Lambda kr_0)-I_m(kr_0)K_m'(\Lambda kr_0)}{K_m'(kr_0)I_m'(\Lambda kr_0)-I_m'(kr_0)K_m'(\Lambda kr_0)}. \quad (2.35)$$

In the derivation of this equation m and k have been supposed to be positive; because of this positive and negative values of b_e must be considered simultaneously.

It can be seen that there is a general similarity between equation (2.34) and the dispersion relation for the stabilized pinch with isotropic pressure governed by the idealised hydromagnetic equations. Marginal stability occurs for values of m and k such that equation (2.34) is satisfied with $\omega = 0$; that is

$$(m+b_e kr_o)^2\phi + kr_o = \alpha_o kr_o^2 \left[b_i^2 + \frac{4\pi p_{o\perp}}{B_o^2}\frac{6p_{o\parallel}-p_{o\perp}}{3p_{o\parallel}}\right]\frac{I_m(\alpha_o r_o)}{I_m'(\alpha_o r_o)}, \quad (2.36)$$

where α_o is the value of α with ω put equal to zero. The behaviour of equation (2.36) depends on whether α_o^2 is positive or negative. If α_o^2 is negative, the argument of the Bessel functions on the right hand side of equation (2.36) is imaginary and, as k varies, $\alpha_o I_m(\alpha_o r_o)/I_m'(\alpha_o r_o)$ takes all real values infinitely many times. Thus there are certainly wavenumbers of marginal stability and the system is definitely unstable. The condition that α_o is imaginary is just that one of equations (1.2) and (1.3) is satisfied. Thus the stabilized pinch is certainly unstable whenever the plane waves are unstable.

If α_o is real and m is not equal to zero, the right hand side of equation (2.36) exceeds the left hand side both for large values and small values of k. The condition for stability is that the right hand side always exceeds the left hand side. The same condition can be shown to be true when m is zero; in this case if the right hand side is larger for small k it can be shown to be larger for all k. Thus for real α_o the stability criterion is

$$\alpha_o kr_o^2 \left[b_i^2 + \frac{4\pi p_{o\perp}}{B_o^2}\left(2 - \frac{p_{o\perp}}{3p_{o\parallel}}\right)\right]\frac{I_m(\alpha_o r_o)}{I_m'(\alpha_o r_o)} > (m+b_e kr_o)^2\phi + kr_o. \quad (2.37)$$

Although the above criterion is generally similar to the well known stabilized pinch criterion (13), it does not reduce to it if the pressure is isotropic. Thus if $p_{o\parallel} = p_{o\perp}$, equation (2.37) becomes

$$\alpha_o kr_o^2 \left[b_i^2 + \frac{20\pi p_{o\perp}}{3B_o^2}\right]\frac{I_m(\alpha_o r_o)}{I_m'(\alpha_o r_o)} > (m+b_e kr_o)^2\phi + kr_o, \quad (2.38)$$

where in this case $\alpha_0^2 = k^2/(1+20\pi p_{o\perp}/3B_0^2 b_i^2)$.

Criterion (2.38) can thus be written

$$\frac{b_i^2 k^3 r_0^2}{\alpha_0} \frac{I_m(\alpha_0 r_0)}{I_m'(\alpha_0 r_0)} > (m+b_e k r_0)^2 \phi + k r_0, \tag{2.39}$$

where $\alpha_0 < k$. The corresponding stabilized pinch criterion is

$$b_i^2 k^2 r_0^2 I_m(kr_0)/I_m'(kr_0) > (m+b_e k r_0)^2 \phi + k r_0. \tag{2.40}$$

Since $I_m(X)/XI_m'(X)$ is a monotonically decreasing function of X the double adiabatic criterion is more easily satisfied than criterion (2.40). This is in agreement with the result expressed in inequality(1.1)

Chandrasekhar, Kaufman and Watson(5) have obtained an alternative stability criterion which is valid when the C.K.W. equations are satisfied. With the additional assumptions that the distributions of parallel and perpendicular velocities are both Gaussian but that they have different dispersions ('temperatures') and that the ions and electrons have the same 'temperatures', their criterion is

$$\alpha_0 k r_0^2 \left[b_i^2 + \frac{8\pi p_{o\perp}}{B_0^2}\left(1 - \frac{p_{o\perp}}{p_{o\parallel}}\right)\right] \frac{I_m(\alpha_0 r_0)}{I_m'(\alpha_0 r_0)} > (m+b_e k r_0)^2 \phi + k r_0, \tag{2.41}$$

where here

$$\alpha_0^2 = k^2 \left[b_i^2 + \frac{4\pi p_{o\perp}}{B_0^2}\left(1 - \frac{p_{o\parallel}}{p_{o\perp}}\right)\right] \Big/ \left[b_i^2 + \frac{8\pi p_{o\perp}}{B_0^2}\left(1 - \frac{p_{o\perp}}{p_{o\parallel}}\right)\right]. \tag{2.42}$$

This criterion holds when α_0 is real; when α_0 is imaginary the system is unstable and this corresponds to criteria (1.2) and (1.4). Criterion (2.41) does reduce to (2.40) when $p_{o\perp}$ is put equal to $p_{o\parallel}$.

Solutions of criterion (2.41) will be discussed in the next section. The full derivation of the criterion can be found in (5) and its form when distributions are not Gaussian or when ion and electron 'temperatures' are different is given there.

3. RESULTS

First consider the restrictions placed on possible values of the equilibrium magnetic fields by inequalities (1.2) and (1.4) and the condition of pressure balance. Define, following Chandrasekhar, Kaufman and Watson

$$\beta = 4\pi p_{o\perp}/B_o^2 \tag{3.1}$$

and

$$\eta = p_{o\perp}/p_{o\parallel}. \tag{3.2}$$

Equation (2.7) shows that

$$2\beta = 1 + b_e^2 - b_i^2 \tag{3.3}$$

and, as β cannot be negative, b_e and b_i must satisfy

$$b_i^2 \leqslant 1 + b_e^2. \tag{3.4}$$

Inequality (1.2) must never be satisfied. Thus if η is less than unity

$$b_i^2 \geqslant \beta[\eta^{-1} - 1].$$

When equation (3.3) is used this becomes

$$b_i^2 \geqslant \{(1 - \eta)/(1 + \eta)\} (1 + b_e^2). \tag{3.5}$$

Similarly if η is greater than unity

$$b_i^2 \geqslant \{(\eta - 1)/\eta\} (1 + b_e^2). \tag{3.6}$$

For any given value of η, (3.4) and one of (3.5) and (3.6) define a region in the (b_e, b_i) plane outside which there can be no stability. The critical regions are shown for a set of values of η in Figure 1. Also shown is the actual region of stability for one value of the ratio of wall radius to plasma radius for $\eta = 1$; as mentioned in Section 2, the results in this case are exactly the well known stabilized pinch results.

Criterion (2.41) must now be applied in the region of the (b_e, b_i) plane which is not ruled out by (3.4), (3,5) and (3.6). As usual the cases m = o and m $\geqslant$ 1 must be considered separately and from the results of Chandrasekhar, Kaufman and Watson it appears that m = 1 perturbations are less stable than those with m > 1. When m = o the criterion is even in b_e but when m = 1 negative values of b_e are less stable than positive values. Thus results are obtained for m = o and m = 1 and negative b_e.

The m = o criterion can be solved quite simply. It can be seen that, if it is satisfied for k close to zero, it is satisfied for all larger k. When k is small the criterion has the limiting form

$$2[\eta b_i^2 + (1-\eta)(1+b_e^2)] > 1 - 2b_e^2/(\Lambda^2-1)$$

$$\text{or} \quad 2\eta b_i^2 > (2\eta-1) - 2b_e^2[(1-\eta) + 1/(\Lambda^2-1)]. \qquad (3.7)$$

Criterion (3.7) is the equation of another curve in the (b_e, b_i) plane below which there can be no stability. This curve depends on Λ as well as on η. It can be seen that it places no restriction on possible values of b_i when $\eta \leq 0.5$; in this case the right hand side of criterion (3.7) can never be positive.

The solution of the m = 1 criterion is more difficult. Values of η and Λ are first chosen; then a curve of marginal stability is obtained as follows. If a value of b_e is chosen, the right hand side of criterion (2.41) can be plotted as a function of kr_o. The left hand side of the criterion is now a function of b_i^2 and kr_o. For one value of b_i^2 the curve of the left hand side as a function of kr_o touches the curve of the right hand side. This is the value of b_i^2 for marginal stability and no stability can occur for b_i^2 less than this value. The procedure can then be followed for further values of b_e.

The Mercury computer has been used to obtain the solution of the m = 1 criterion. The method of solution is briefly as follows. Figure 2 shows the general behaviour of the right hand side of criterion (2.41). For small values of kr_o the function is negative and it has the maximum value of + 1 at $C(kr_o = 1/|b_e|)$. The function first becomes positive at A and the tangent from the origin touches it at B. Since the left hand side of the criterion is everywhere concave upwards, the touching point of the two curves must be between A and B. The right hand side is first calculated and the range (A,B) is identified. The left hand side now need only be calculated for values of kr_o between the points A and B. The left hand side is first calculated for $b_i^2 = 1 + b_e^2$; if this curve cuts the curve AB, the system is unstable for all allowable values of b_i^2. Otherwise the left hand side is next calculated for b_i^2 half way between the limits given by (3.4) and either (3.5) or (3.6). An iteration procedure is now used in which the alteration in the value of b_i^2 is halved in each step, the values being increased if the curves cut and decreased if they do not. The iteration stops when the difference between successive values of b_i^2 is less than some prearranged amount. The actual results of the computation for one set of values of (η, Λ, b_e) are shown in figure 3.

Results have been obtained for eight values of η(5.0, 2.0, 1.5, 1.0, 0.75, 0.5, 0.2, 0.1) and three values of Λ(3.0, 2.0, 1.5). The full results obtained for $m = o$, $m = 1$ and from (3.4) to (3.6) are shown in figures 4 – 11; one figure for each each value of η.

A quantity which is of interest experimentally is the proportion of the energy, which is within the conducting walls, which is in the form of plasma thermal energy. This has been calculated for one value of the ratio of wall radius to discharge radius for values of (b_e, b_i) for which the system is marginally stable. The results are shown in the table. The fraction of energy in the plasma is given by the formula

$$\varepsilon_p = (1+b_e^2-b_i^2)(1+1/2\eta)/\{(1+b_e^2-b_i^2)(1+1/2\eta)+b_i^2+(\Lambda^2-1)b_e^2+2\ln\Lambda\}. \quad (3.8)$$

It can be seen that both the region of stability and the plasma thermal energy have a maximum value when the parallel pressure is somewhat larger than the perpendicular pressure.

4. DISCUSSION

The aim of this lecture has now been achieved. It has been shown that the introduction of anisotropic equilibrium distribution functions can lead to new instabilities and, for one particular form of the zero order distribution function, the révised stabilized pinch stability diagrams have been calculated. However these numerical results do depend on the assumptions made and it is perhaps worthwhile to demonstrate that there are certainly equilibrium distribution functions for which the stability requirements are more stringent.

The simplest thing to do is to consider criteria (1.2) and (1.4). Criterion (1.2) is not affected by different assumptions about the zero order distribution function but Chandrasekhar, Kaufman and Watson(5) show that the general form of criterion (1.4) is

$$(\overline{S} + \overline{R})p_\perp + B^2/4\pi < o. \quad (4.1)$$

In this inequality

$$\bar{S}p_{\perp} = S_i p_{\perp i} + S_e p_{\perp e} \qquad (4.2)$$

and

$$\bar{R}p_{\perp} = R_i p_{\perp i} + R_e p_{\perp e},$$

where the suffices i and e refer to ions and electrons. For either species

$$Sp_{\perp} = \frac{m}{4n} \iint s^4 \left(\frac{\partial f}{\partial q^2} - \frac{\partial f}{\partial s^2} \right) dq ds^2 \qquad (4.3)$$

and

$$Rp_{\perp} = \frac{-\frac{e}{2} \iint s^2 \frac{\partial f}{\partial q^2} dq ds^2 \sum_{i,e} \frac{e}{2} \iint s^2 \frac{\partial f}{\partial q^2} dq ds^2,}{\sum_{ie} \frac{e^2}{m} \iint \frac{\partial f}{\partial q^2} dq ds^2} \qquad (4.4)$$

where the distribution functions f are functions of q^2 and s^2 where q and s are the parallel and perpendicular components of the velocity. The sums are over the two species of particles.

When the distribution functions are Gaussian in both velocity components so that

$$f = e^{-\alpha q^2 - \beta s^2}, \qquad (4.6)$$

the expressions (4.4) and (4.5) simplify considerably. In fact it is possible to show that

$$Sp_{\perp} = 2p_{\perp}(1-\eta) \qquad (4.7)$$

and

$$Rp_{\perp} = e\eta \sum_{i,e} e\eta / \sum_{i,e} (e^2\eta/p_{\perp}), \qquad (4.8)$$

where now there is a separate value of $\eta, (p_{\perp}/p_{\parallel})$, for each type of particle. If $\eta_i = \eta_e$, then R = o since $e_i = -e_e$; in addition $\bar{S}$ satisfies equation (4.7) with $p_{\perp}$ being the total pressure. This is the case for which calculations have been done.

If the values of η_i and η_e are different, then

$$(\bar{S}+\bar{R})p_{\perp} = 2p_{\perp i}(1-\eta_i) + 2p_{\perp e}(1-\eta_e)$$

$$+ (\eta_i - \eta_e)^2 / [\eta_i/p_{\perp i} + \eta_e/p_{\perp e}]. \qquad (4.9)$$

This expression can be rearranged to give

$$(\bar{S}+\bar{R})p_{\perp} = 2p_{\perp}(1-\eta) - (p_{\perp i}p_{\parallel e}-p_{\perp e}p_{\parallel i})^2/p_{\parallel i}p_{\parallel e}(p_{\parallel i}+p_{\parallel e}). \qquad (4.10)$$

The criterion for instability (4.1) then becomes

$$\eta > 1 + \frac{B^2}{8\pi p_{\perp}} - \frac{(p_{\perp i}p_{\parallel e}-p_{\perp e}p_{\parallel i})^2}{2(p_{\perp i}+p_{\perp e})(p_{\parallel i}+p_{\parallel e})p_{\parallel i}p_{\parallel e}} . \qquad (4.11)$$

In this form it can be seen that the critical value of η, at which instability first occurs, is reduced if the energy is rearranged so that $\eta_i \neq \eta_e$. The complete criterion clearly imposes restrictions on the individual η's for the two species of particles as well as on the total η. Thus if $p_{\parallel e} = 0$ so that $\eta_e = \infty$, the last term in (4.11) becomes infinite so that criterion (4.11) is always satisfied.

If the distribution functions are not Gaussian it is not so easy to give general results and expressions (4.4) and (4.5) would have to be evaluated for each species of particle. It should also be mentioned that the Chandrasekhar, Kaufman, Watson treatment rests on a solution of the Boltzmann equation which assumes that the particle Larmor radii are small; if this is not valid there may be other instabilities not derived here.

ACKNOWLEDGEMENT

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APPENDIX I

REDUCTION OF THE PERTURBED PLASMA EQUATIONS

Equations (2.12) to (2.15) enable ρ_1, $\underline{\underline{p}}_1$ and $\underline{B}_1$ to be expressed in terms of $\underline{v}_1$. Thus

$$\omega\rho_1 = -\rho_0 \text{div } \underline{v}_1, \tag{A1.1}$$

$$\omega B_{1r} = ikB_0 b_i v_{1r}, \tag{A1.2}$$

$$\omega B_{1\theta} = ikB_0 b_i v_{1\theta}, \tag{A1.3}$$

$$\omega B_{1z} = ikB_0 b_i v_{1z} - B_0 b_i \text{div } \underline{v}_1, \tag{A1.4}$$

$$\omega p_{1\parallel}/p_{0\parallel} = -\text{ div } \underline{v}_1 - 2ikv_{1z} \tag{A1.5}$$

and

$$\omega p_{1\perp}/p_{0\perp} = -2 \text{ div } \underline{v}_1 + ikv_{1z}. \tag{A1.6}$$

Equations (2.9), (2.16), (A1.5) and (A1.6) then combine to give

$$\omega(\text{div}\underline{\underline{p}}_1)_r = -2p_{0\perp}D\text{div } \underline{v}_1 + ikp_{0\perp}Dv_{1z} - k^2(p_{0\parallel}-p_{0\perp})v_{1r}, \tag{A1.7}$$

$$\omega(\text{div}\underline{\underline{p}}_1)_\theta = -(2im/r)p_{0\perp}\text{div}\underline{v}_1 - (mk/r)p_{0\perp}v_{1z} - k^2(p_{0\parallel}-p_{0\perp})v_{1\theta} \tag{A1.8}$$

and

$$\omega(\text{div}\underline{\underline{p}}_1)_z = -ikp_{0\perp}\text{div } \underline{v}_1 + k^2(3p_{0\parallel}-p_{0\perp})v_{1z}, \tag{A1.9}$$

where D means d/dr.

The third component of equation (2.11) can then be written

$$\rho_0\omega^2 v_{1z} = ikp_{0\perp}\text{div } \underline{v}_1 + (p_{0\perp} - 3p_{0\parallel})k^2 v_{1z}$$

or

$$\text{div } \underline{v}_1 = [\rho_0\omega^2 + (3p_{0\parallel}-p_{0\perp})k^2]v_{1z}/ikp_{0\perp}. \tag{A1.10}$$

The first component of equation (2.12) becomes

$$\rho_0\omega^2 v_{1r} = 2p_{0\perp}D \text{ div } \underline{v}_1 - ikp_{0\perp}Dv_{1z} + k^2(p_{0\parallel}-p_{0\perp})v_{1r}$$

$$- k^2B_0^2b_i^2v_{1r}/4\pi - ikB_0^2b_i^2 Dv_{1z}/4\pi + B_0^2b_i^2 D\text{div } \underline{v}_1/4\pi,$$

or

$$[\rho_0\omega^2 + k^2(B_0^2b_i^2/4\pi + p_{0\perp}-p_{0\parallel})]v_{1r} = -ik\lambda Dv_{1z}, \tag{A1.11}$$

where $\lambda = [\rho_0\omega^2(B_0^2 b_i^2/4\pi + 2p_{0\perp}) + k^2(3B_0^2 b_i^2 p_{0\parallel}/4\pi + 6p_{0\perp}p_{0\parallel} - p_{0\perp}^2)]/p_{0\perp}\,k^2$. (A1.12)

Similarly the second component of equation (2.12) yields

$$[\rho_0\omega^2 + k^2(B_0^2 b_i^2/4\pi + p_{0\perp} - p_{0\parallel})]\, v_{1\theta} = mk\lambda v_{1z}/r. \qquad (A1.13)$$

Equations (A1.10), (A1.11) and (A1.13) can now be combined to give an equation for v_{1z}. Thus

$$-(ik\lambda/r)DrDv_{1z} + (imk/r^2)\lambda v_{1z} + ik[\rho_0\omega^2 + k^2(B_0^2 b_i^2/4\pi + p_{0\perp} - p_{0\parallel})]v_{1z}$$

$$= [\rho_0\omega^2 + (3p_{\parallel 0} - p_{\perp 0})k^2][\rho_0\omega^2 + k^2(B_0^2 b_i^2/4\pi + p_{0\perp} - p_{0\parallel})]v_{1z}/ikp_{0\perp}.$$

This equation can be rearranged in the form

$$rDrD\, v_{1z} = (m^2 + \alpha^2 r^2)v_{1z}, \qquad (A1.14)$$

where

$$\alpha^2 = \frac{\{\rho_0\omega^2 + k^2(B_0^2 b_i^2/4\pi + p_{0\perp} - p_{0\parallel})\}\{\rho_0\omega^2 + 3k^2 p_{0\parallel}\}}{\{\rho_0\omega^2(B_0^2 b_i^2/4\pi + 2p_{0\perp}) + k^2(3B_0^2 b_i^2 p_{0\parallel}/4\pi + 6p_{0\perp}p_{0\parallel} - p_{0\perp}^2)\}}. \qquad (A1.15)$$

APPENDIX II

DERIVATION OF DISPERSION RELATION

The continuity of the normal component of the magnetic field across the plasma-vacuum interface implies that

$$B^v_{1r} - imB_o v_{1r}/\omega r_o - ikB_o b_e v_{1r}/\omega = B_{1r} - ikB_o b_i v_{1r}/\omega \equiv o, \qquad (A2.1)$$

where this equation now has to be applied on the surface $r = r_o$. The total pressure is continuous provided that

$$p_{1\perp} + B_o b_i B_{1z}/4\pi = B_o b_e B^v_{1z}/4\pi + B_o B^v_{1\theta}/4\pi - B_o^2 v_{1r}/4\pi r_o \omega, \qquad (A2.2)$$

also on $r = r_o$. In addition

$$B^v_{1r} = o \qquad (A2.3)$$

on $r = \Lambda r_o$.

These three equations can be rewritten

$$-i\mathcal{B}K_m'(kr_o) - iCI_m'(kr_o) = k\lambda\alpha B_o(m+b_e kr_o)\mathcal{A}I_m'(\alpha r_o)/\omega r_o[\rho_o\omega^2 + k^2(B_o^2 b_i^2/4\pi + p_{o\perp} - p_{o\parallel})], \qquad (A2.4)$$

$$\begin{aligned} & B_o(m+b_e kr_o)[\mathcal{B}K_m(kr_o) + CI_m(kr_o)]/4\pi kr_o \\ &= -ik\lambda\alpha B_o^2\mathcal{A}I_m'(\alpha r_o)/4\pi\omega r_o[\rho_o\omega^2 + k^2(B_o^2 b_i^2/4\pi + p_{o\perp} - p_{o\parallel})] \\ &- [2\rho_o\omega^2 + (6p_{o\parallel} - p_{o\perp})k^2]\mathcal{A}I_m(\alpha r_o)/ik\omega \\ &- B_o^2 b_i^2(\rho_o\omega^2 + 3p_{o\parallel}k^2)\mathcal{A}I_m(\alpha r_o)/4\pi ikp_{o\perp}\omega \end{aligned} \qquad (A2.5)$$

$$\text{and} \qquad \mathcal{B}K_m'(\Lambda kr_o) + CI_m'(\Lambda kr_o) = 0. \qquad (A2.6)$$

Equation (A2.6) can now be used to eliminate C from equations (A2.4) and (A2.5). Thus

$$\begin{aligned} & -i\mathcal{B}[K_m'(kr_o)I_m'(\Lambda kr_o) - I_m'(kr_o)K_m'(\Lambda kr_o)]/I_m'(\Lambda kr_o) \\ &= k\lambda\alpha B_o(m+b_e kr_o)\mathcal{A}I_m'(\alpha r_o)/\omega r_o[\rho_o\omega^2 + k^2(B_o^2 b_i^2/4\pi + p_{o\perp} - p_{o\parallel})] \end{aligned} \qquad (A2.7)$$

and

$$\begin{aligned} & B_o(m+b_e kr_o)\mathcal{B}[K_m(kr_o)I_m'(\Lambda kr_o) - I_m(kr_o)K_m'(\Lambda kr_o)]/4\pi kr_o I_m'(\Lambda kr_o) \\ &= -ik\lambda\alpha B_o^2\mathcal{A}I_m'(\alpha r_o)/4\pi\omega r_o[\rho_o\omega^2 + k^2(B_o^2 b_i^2/4\pi + p_{o\perp} - p_{o\parallel})] \end{aligned}$$

$$- [2\rho_0\omega^2+k^2(6p_{0\parallel}-p_{0\perp})]\, \mathcal{A} I_m(\alpha r_0)/ik\omega$$

$$- B_0^2 b_i^2\, (\rho_0\omega^2+3p_{0\parallel}k^2)\mathcal{A} I_m(\alpha r_0)/4\pi i k p_{0\perp}\omega. \qquad \text{(A2.8)}$$

$\mathcal{A}$ and $\mathcal{B}$ can now be eliminated between equations (A2.7) and (A2.8) to give the dispersion relation. After some rearrangement this has the form

$$(m+b_e k r_0)^2 \left[\frac{K_m(kr_0)I_m'(\Lambda kr_0)-I_m(kr_0)K_m'(\Lambda kr_0)}{K_m'(kr_0)I_m'(\Lambda kr_0)-I_m'(kr_0)K_m'(\Lambda kr_0)}\right] + kr_0$$

$$= \alpha k r_0^2 \left[b_i^2 + \frac{4\pi p_{0\perp}}{B_0^2}\, \frac{2\rho_0\omega^2 + (6p_{0\parallel}-p_{0\perp})k^2}{\rho_0\omega^2 + 3p_{0\parallel} k^2}\right] \frac{I_m(\alpha r_0)}{I_m'(\alpha r_0)}. \qquad \text{(A2.9)}$$

TABLE

Fraction of energy contained in plasma in marginally stable configurations.

$\Lambda = 2.0$

$\eta \backslash b_e$	0.0	0.1	0.2	0.3
0.1	0.25	0.20	0.13	0.07
0.2	0.28	0.22	0.15	0.07
0.5	0.32	0.25	0.17	0.08
0.75	0.35	0.27	0.18	0.09
1.0	0.28	0.27	0.18	0.09
1.5	0.18	0.18	0.17	0.09
2.0	0.13	0.13	0.13	0.08
5.0	0.05	0.05	0.05	0.05

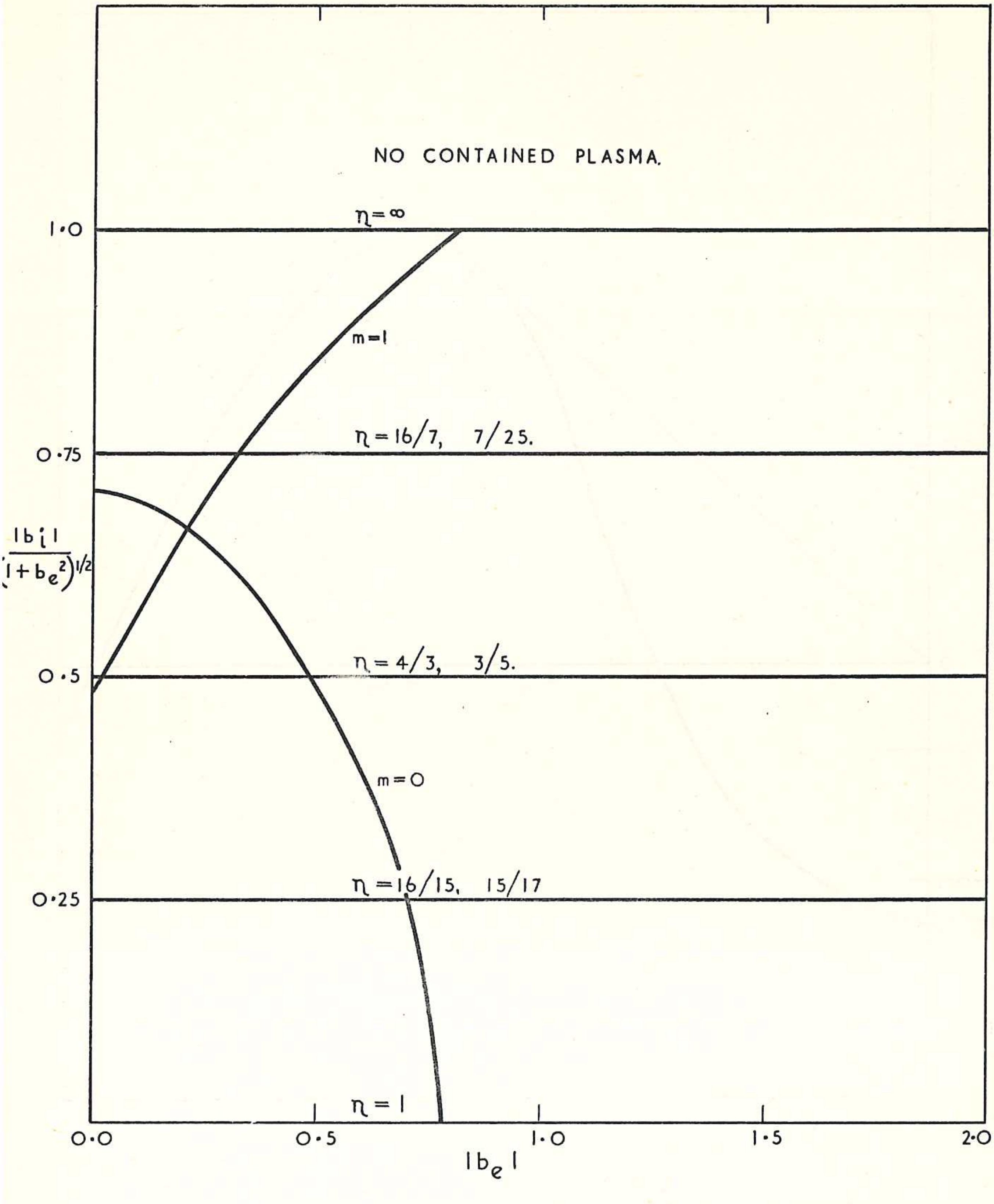

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FIG. 1. STABILITY DIAGRAM FOR PLANE WAVES

FOR A GIVEN VALUE OF η, PLANE WAVES ARE STABLE FOR VALUES OF $|b_i|/(1+b_e^2)^{1/2}$ ABOVE THE STRAIGHT LINE MARKED WITH THE VALUE OF η. ALSO SHOWN ARE THE ACTUAL CURVES OF MARGINAL STABILITY FOR THE STABILIZED PINCH FOR $\eta=1$, $\Lambda=1{\cdot}5$, $m=0$ AND $m=1$.

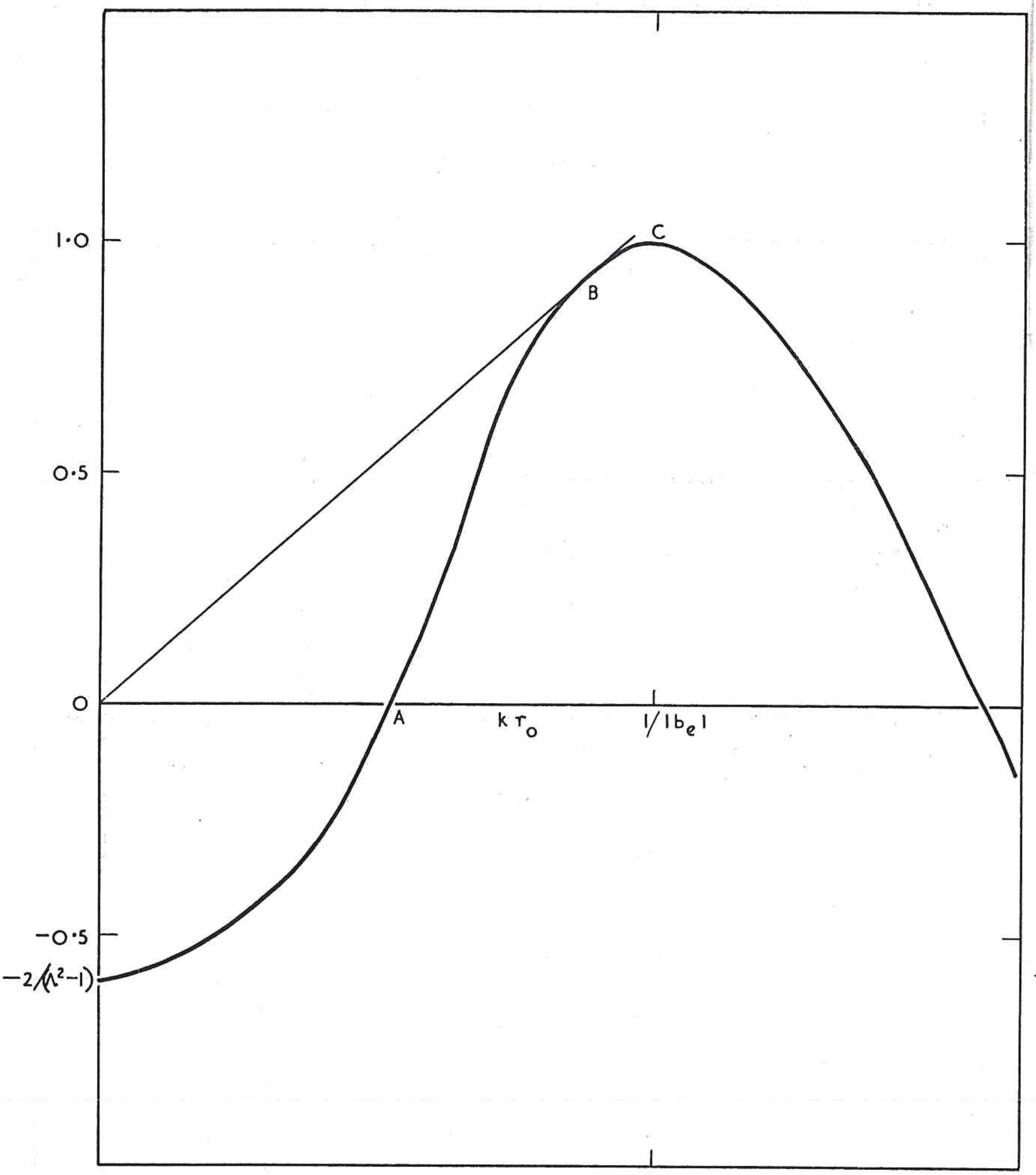

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FIG. 2. GRAPH OF RIGHT HAND SIDE OF STABILITY CRITERION.

THE RIGHT HAND SIDE OF THE CRITERION IS NEGATIVE FOR SMALL kr_o, FIRST BECOMES POSITIVE AT A AND REACHES ITS MAXIMUM VALUE OF +1 AT C. THE TANGENT FROM THE ORIGIN TOUCHES THE CURVE AT B.

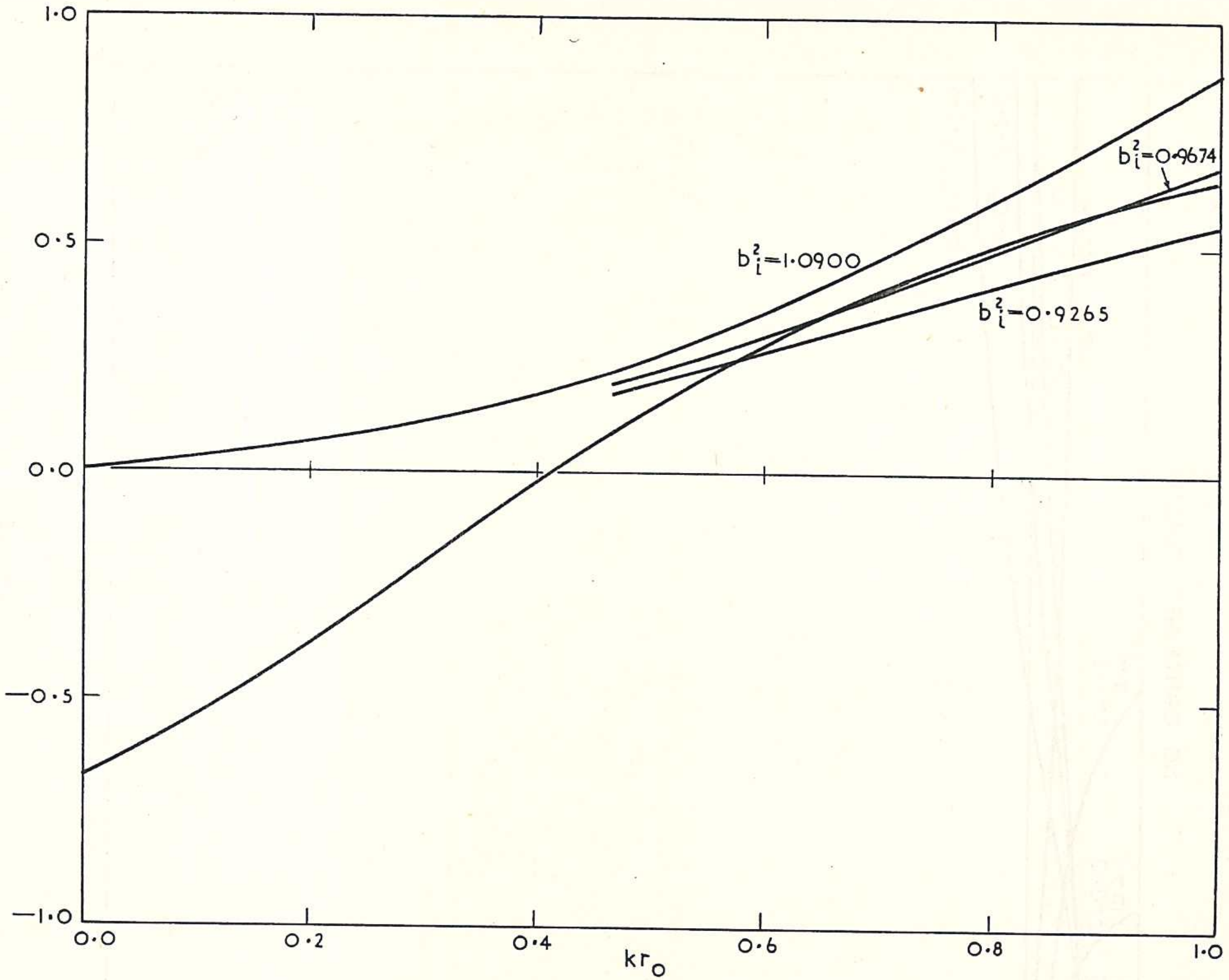

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FIG. 3. METHOD OF SOLUTION OF STABILITY CRITERION

THE RIGHT HAND SIDE OF THE STABILITY CRITERION IS SHOWN FOR ONE SET OF VALUES OF n, Λ AND b_e (5·0, 2·0, 0·3). THE LEFT HAND SIDE IS DRAWN FOR THREE VALUES OF b_i^2 SHOWING HOW THE CURVE WHICH TOUCHES THE RIGHT HAND SIDE IS APPROACHED.

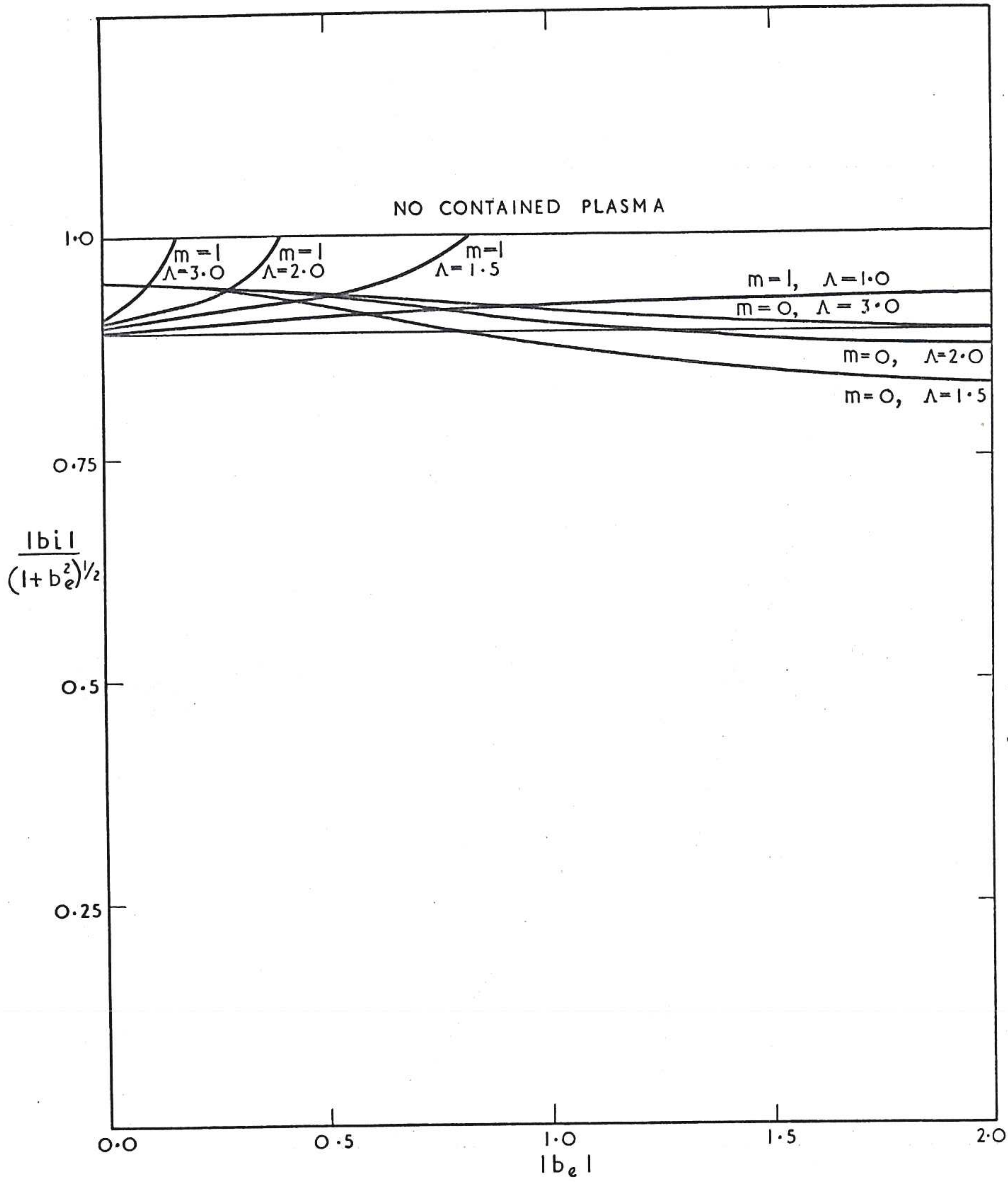

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FIG. 4. STABILITY DIAGRAM $\eta = 5\cdot0$

MARGINAL STABILITY CURVES ARE DRAWN FOR $m=0$ AND $m=1$ PERTURBATIONS. FOR GIVEN Λ, THE SYSTEM IS STABLE FOR VALUES OF $|b_e|$ AND $|b_i|/(1+b_e^2)^{1/2}$ ABOVE BOTH THE $m=0$ AND $m=1$ CURVES AND BELOW THE UPPER HORIZONTAL LINE.

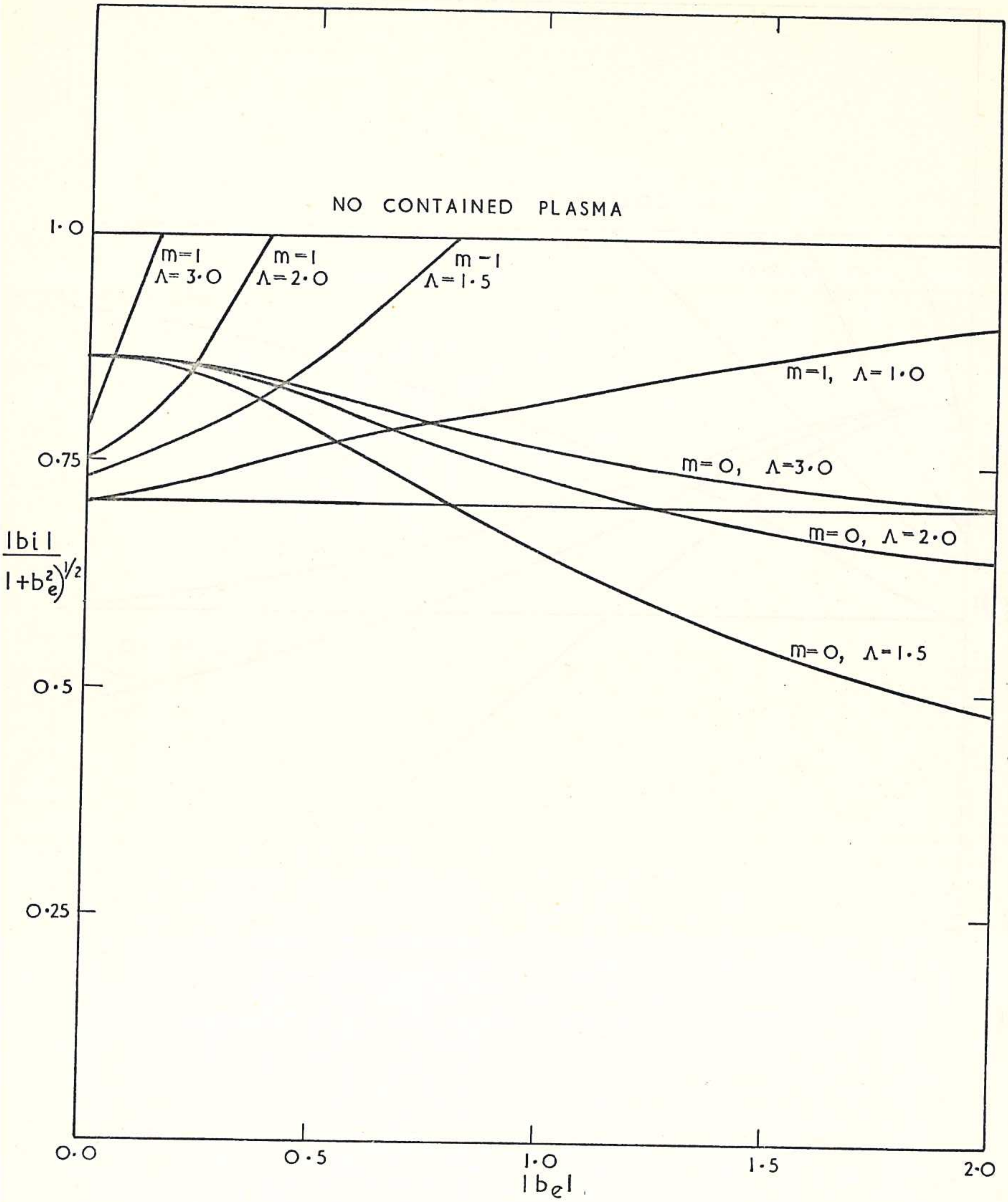

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FIG. 5. STABILITY DIAGRAM $\eta = 2\cdot 0$

MARGINAL STABILITY CURVES ARE DRAWN FOR m=0 AND m=1 PERTURBATIONS. FOR GIVEN Λ, THE SYSTEM IS STABLE FOR VALUES OF $|b_e|$ AND $|b_i|/(1+b_e^2)^{1/2}$ ABOVE BOTH THE m=0 AND m=1 CURVES AND BELOW THE UPPER HORIZONTAL LINE.

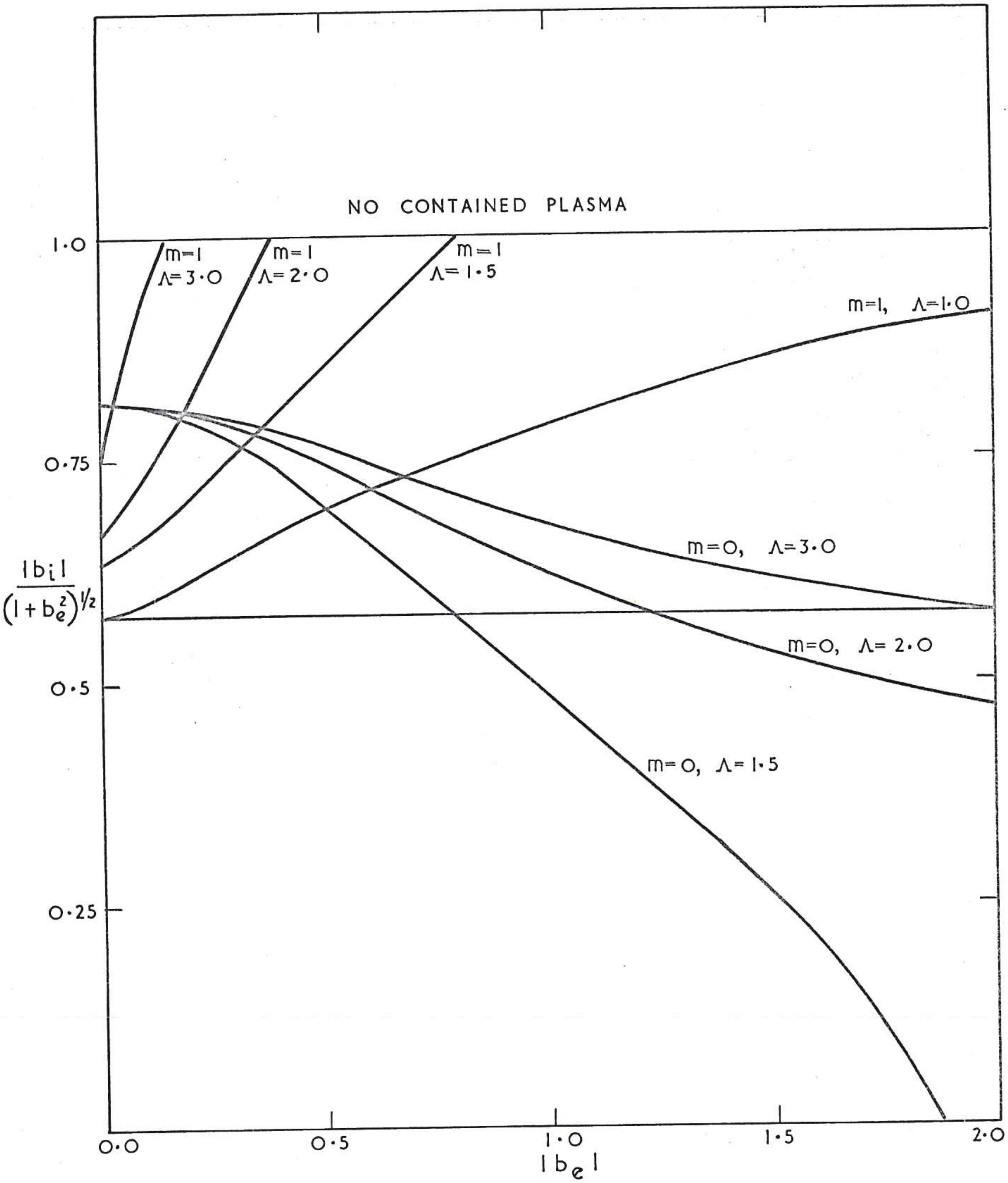

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FIG. 6. STABILITY DIAGRAM $\eta=1{\cdot}5$

MARGINAL STABILITY CURVES ARE DRAWN FOR m=0 AND m=1 PERTURBATIONS. FOR GIVEN Λ, THE SYSTEM IS STABLE FOR VALUES OF $|b_e|$ AND $|b_i|/(1+b_e^2)^{1/2}$ ABOVE BOTH THE m=0 AND m=1 CURVES AND BELOW THE UPPER HORIZONTAL LINE.

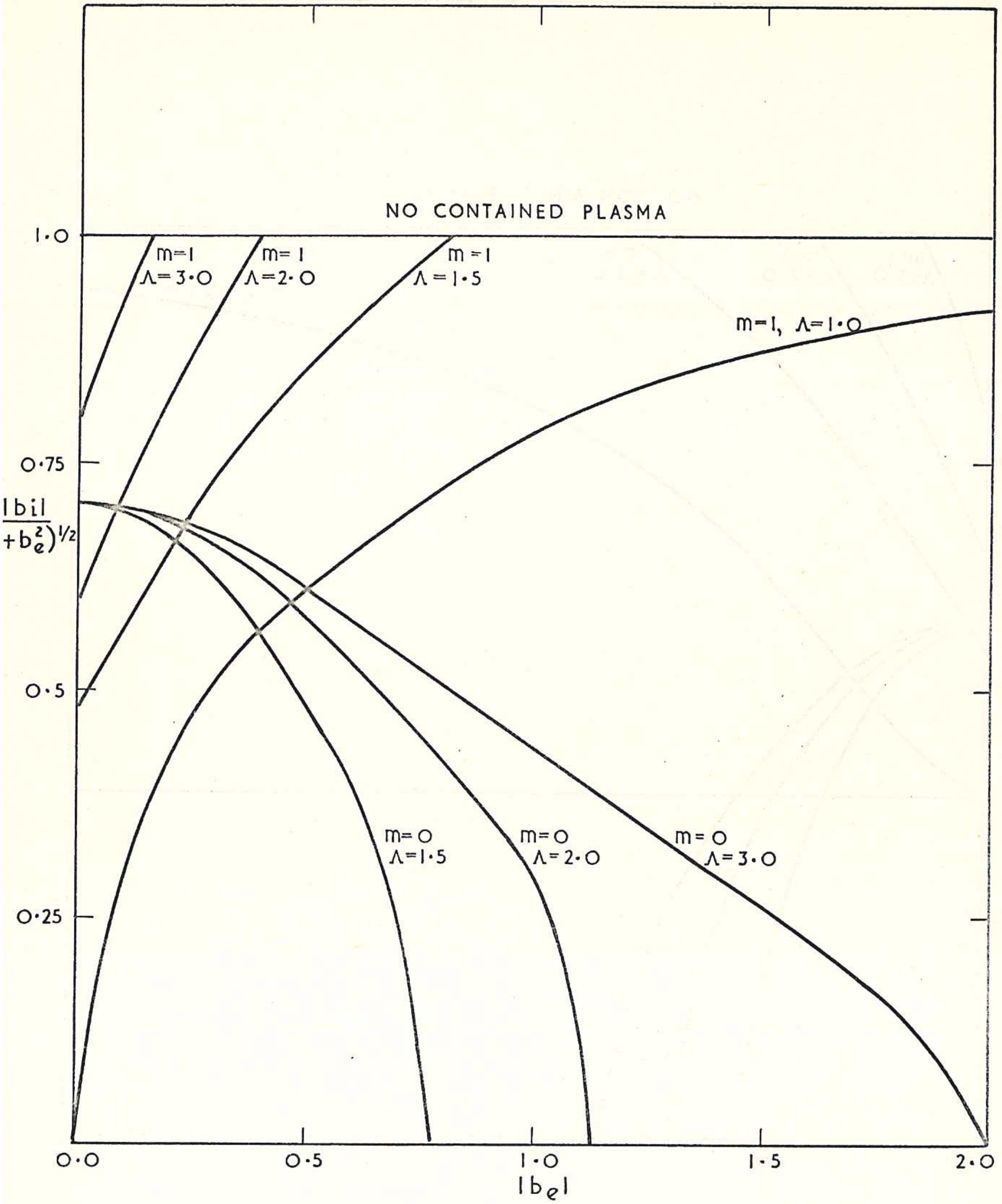

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FIG. 7. STABILITY DIAGRAM $\eta = 1{\cdot}0$

MARGINAL STABILITY CURVES ARE DRAWN FOR $m=0$ AND $m=1$ PERTURBATIONS FOR GIVEN Λ, THE SYSTEM IS STABLE FOR VALUES OF $|b_e|$ AND $|b_i|/(1+b_e^2)^{1/2}$ ABOVE BOTH THE $m=0$ AND $m=1$ CURVES AND BELOW THE UPPER HORIZONTAL LINE.

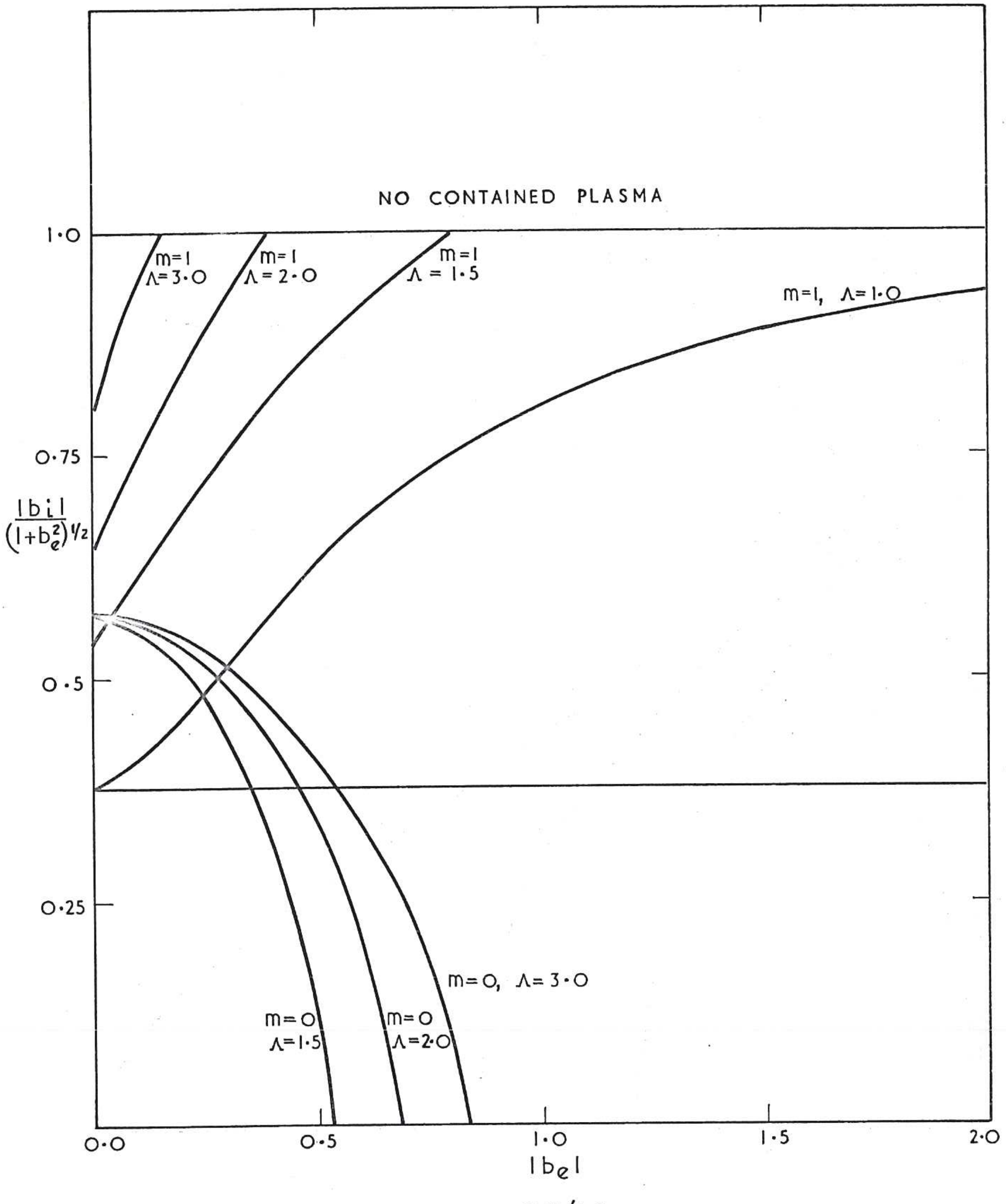

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FIG. 8. STABILITY DIAGRAM $\eta = 0\cdot75$

MARGINAL STABILITY CURVES ARE DRAWN FOR m=0 AND m=1 PERTURBATIONS. FOR GIVEN Λ, THE SYSTEM IS STABLE FOR VALUES OF $|b_e|$ AND $|b_i|/(1+b_e^2)^{1/2}$ ABOVE BOTH THE m=0 AND m=1 CURVES AND BELOW THE UPPER HORIZONTAL LINE.

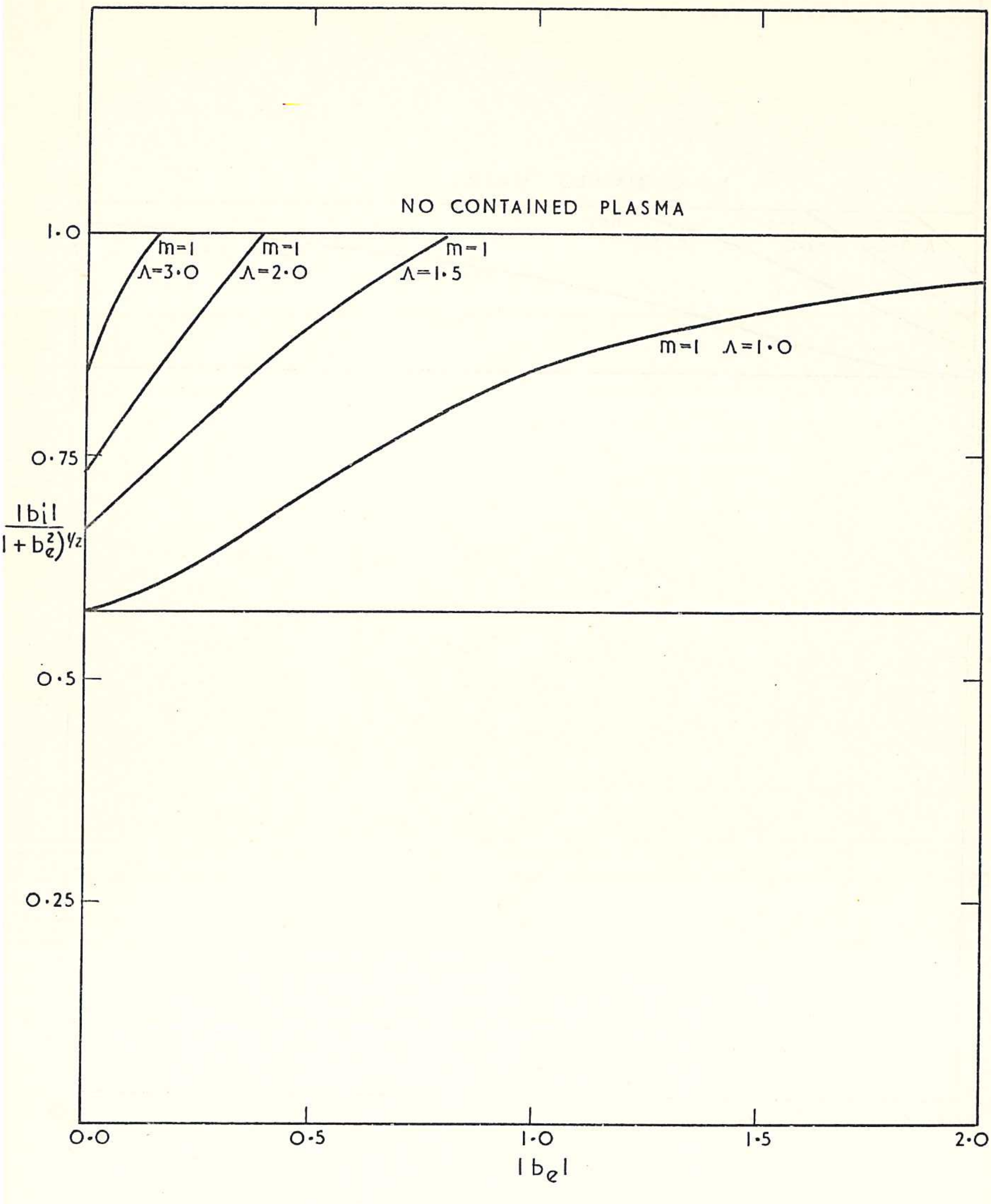

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FIG. 9. STABILITY DIAGRAM $\eta = 0{\cdot}5$

MARGINAL STABILITY CURVES ARE DRAWN FOR m=1 PERTURBATIONS. THERE ARE NO m=0 INSTABILITIES. FOR GIVEN Λ, THE SYSTEM IS STABLE FOR VALUES OF $|b_e|$ AND $|b_i|/(1+b_e^2)^{1/2}$ ABOVE THE m=1 CURVE AND BELOW THE UPPER HORIZONTAL LINE.

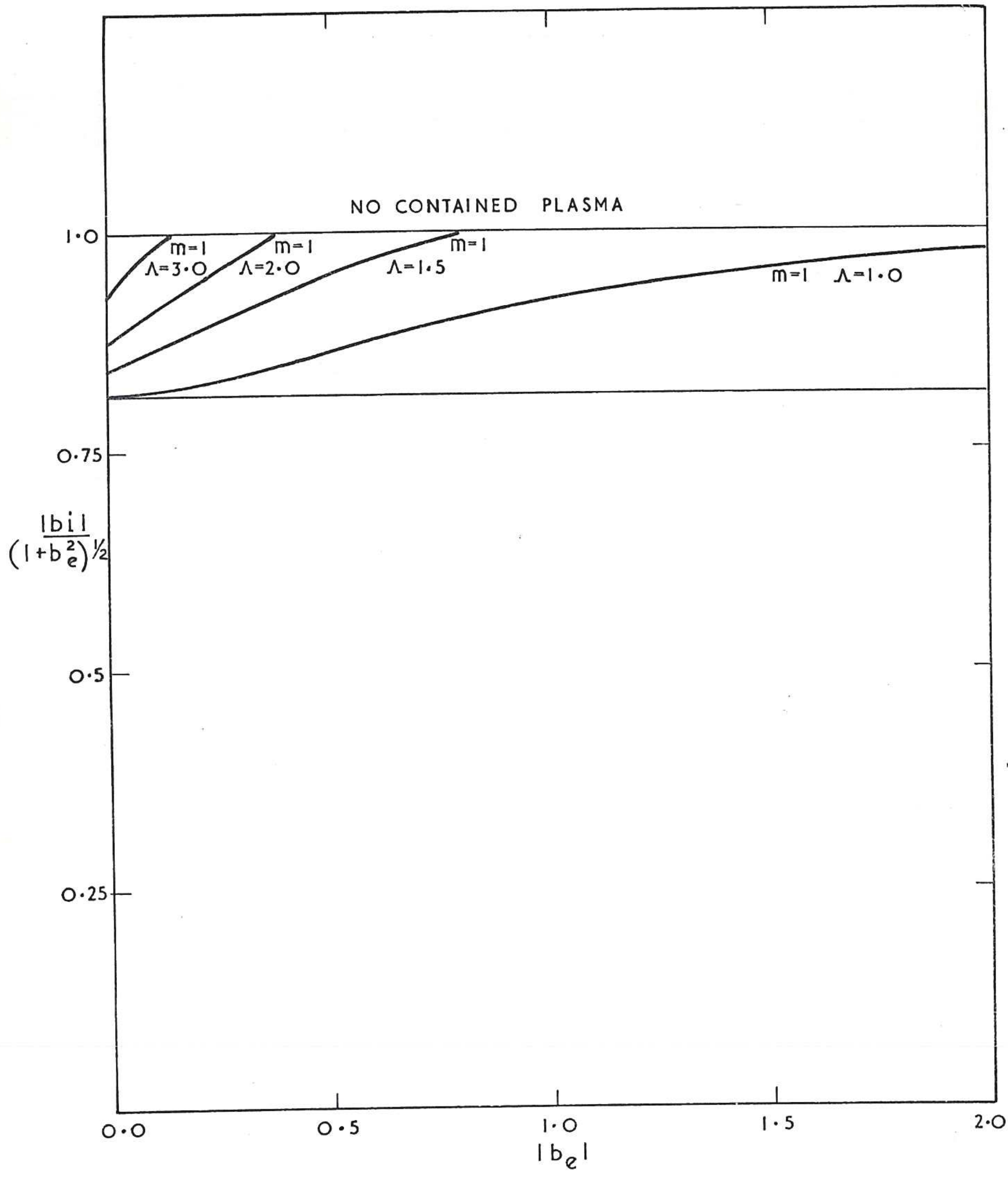

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FIG. 10. STABILITY DIAGRAM $\eta = 0 \cdot 2$

MARGINAL STABILITY CURVES ARE DRAWN FOR m=1 PERTURBATIONS. THERE ARE NO m=0 INSTABILITIES. FOR GIVEN Λ, THE SYSTEM IS STABLE FOR VALUES OF $|b_e|$ AND $|b_i|/(1+b_e^2)^{1/2}$ ABOVE THE m=1 CURVE AND BELOW THE UPPER HORIZONTAL LINE.

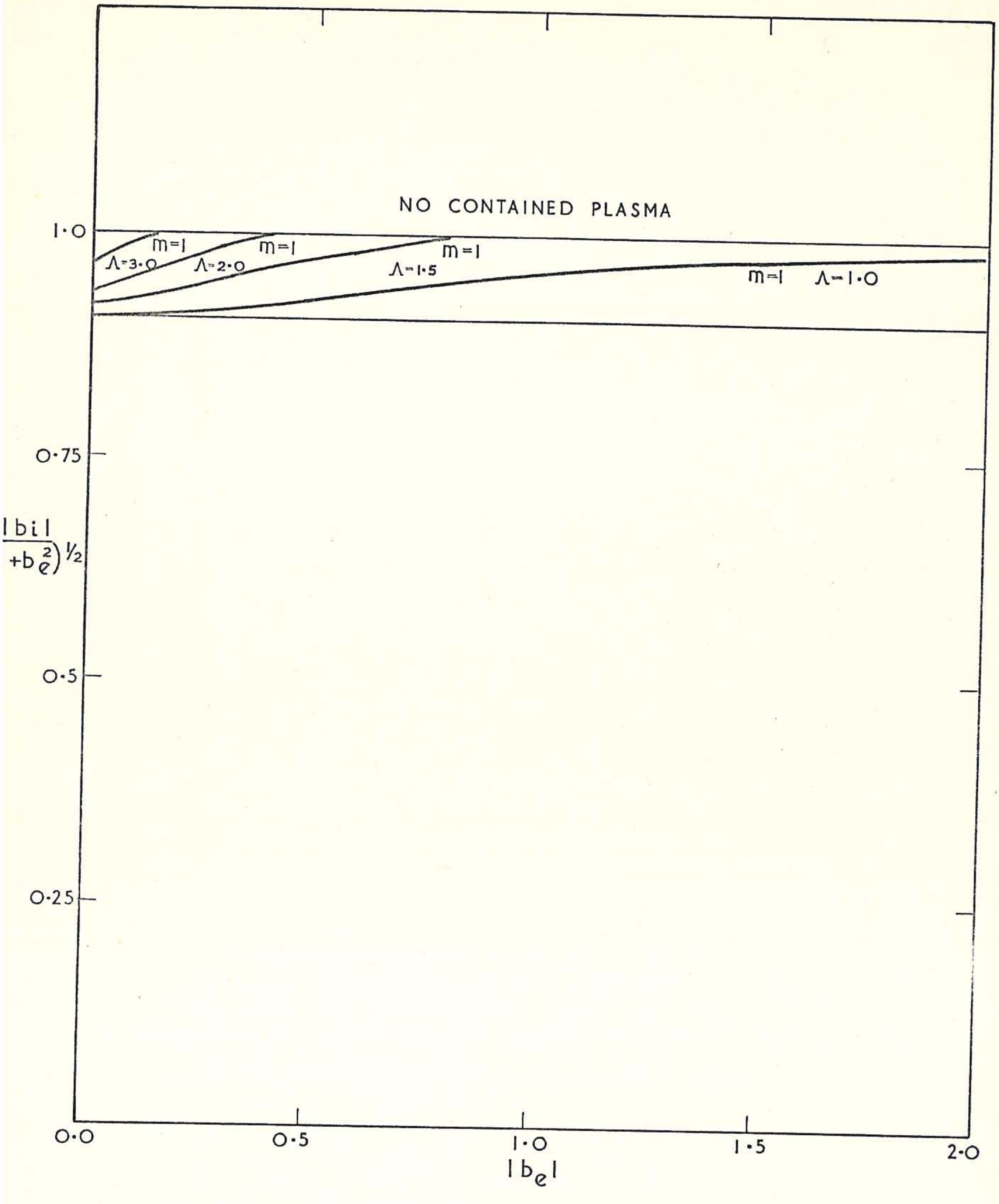

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FIG. 11. STABILITY DIAGRAM $\eta = 0 \cdot 1$

MARGINAL STABILITY CURVES ARE DRAWN FOR m=1 PERTURBATIONS. THERE ARE NO m=0 INSTABILITIES. FOR GIVEN Λ, THE SYSTEM IS STABLE FOR VALUES OF $|b_e|$ AND $|b_i|/(1+b_e^2)^{1/2}$ ABOVE THE m=1 CURVE AND BELOW THE UPPER HORIZONTAL LINE.