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Memorandum

ETNA
A FUEL-COOLANT INTERACTION CODE

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Abingdon Oxfordshire

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ABSTRACT

ETNA is a computer code for solving the equations of a recently proposed model for fuel-coolant interactions.

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Title of Program : ETNA

Computer : ICL 4/70 Installation: Culham Laboratory

Operating System : ICL Multijob

Programming Languages used : Standard FORTRAN

High Speed store required : 19000 words

No. of bits in a word : 32

Overlay Structure : None

No. of Magnetic Tapes required : None

Other Peripherals used : Line printer

No. of cards in program deck : 2463

Card Punching Code : EBCDIC

Typical Running Time : On the ICL 4/70 at Culham Laboratory 5 cycles for one value of the external pressure takes 10 seconds

Unusual features of the program : Standard FORTRAN is used except for the NAMELIST facility and the abnormal entry statement, ENTRY, into the subroutine ELIPSE ⟨2.4⟩

1. INTRODUCTION

The computer code ETNA is a program for fuel-coolant interactions, FCI, calculations. The basic version of the code described here simulates the FCI model equations given by Buchanan (1974); however the structure of the code is sufficiently flexible to allow for substantial modifications and improvements in the basic equations. The structure of ETNA is based on the OLYMPUS package of Christiansen and Roberts (1974). MKS units are used throughout.

§ 2 contains a brief description of the physical model; § 3 is on numerical methods; and § 4 describes the structure of the code.

2. THE PHYSICAL MODEL

In this section we describe briefly the physics which the code ETNA simulates. A complete description has been given elsewhere (Buchanan 1974). The interaction is divided into five stages, the last four of which occur cyclically.

Stage 1. As a result of some triggering mechanism the liquids come into intimate contact and a vapour bubble is formed. This stage is solely a means of supplying the initial perturbation which precipitates the interaction. All our calculations start with the assumption that the initial bubble has been formed and thus the initial condition for the code is a bubble adjacent to the fuel surface.

Stage 2. The vapour bubble expands and then collapses as a result of condensation in the subcooled coolant. The collapse is asymmetric and a high velocity jet of liquid coolant, directed towards the fuel, is formed. The expansion phase is governed by the Rayleigh equation:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = (P-P_o)/\rho_c \qquad \ldots (2.1)$$

R and P are the radius and pressure of the bubble at time t, measured from the start of the expansion, and P_o and ρ_c are the ambient pressure and coolant liquid density. If it is assumed that the expansion is adiabatic and that the value of γ is $4/3$ (the value for water vapour), then the maximum radius of the bubble is

$$R_m = R_i(1 + Z_r) \qquad \ldots (2.2)$$

where Z_r is the real solution of the cubic equation

$$Z^3 + 4Z^2 + 6Z - 3(P_i/P_o - 1) = 0 \qquad \ldots (2.3)$$

There is only one real solution of Eq (2.3). R_i and P_i are the initial radius and pressure of the bubble for the i^{th} cycle. Note the double role of the subscript i to denote 'initial' and 'i^{th}'. The time taken to expand to R_m is

$$t_g = R_i\left(\frac{3\rho_c}{2P_o}\right)^{\frac{1}{2}} \int_0^{Z_r} \frac{(1 + z)^2\, dZ}{[3(P_i/P_o - 1)Z - Z^4-4Z^3-6Z^2]^{\frac{1}{2}}} \qquad \ldots (2.4)$$

When the maximum radius is reached the vapour condenses and the bubble collapses under the external pressure P_o. The equation of motion is:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = -P_o/\rho_c \qquad \ldots (2.5)$$

and the time for collapse is

$$t_c = R_m\left(\frac{\rho_c}{3P_o}\right)^{\frac{1}{2}} \frac{\Gamma(5/6)\Gamma(1/2)}{\Gamma(4/3)} = 0.915\ R_m\left(\frac{\rho_c}{P_o}\right)^{\frac{1}{2}} \qquad \ldots (2.6)$$

where Γ is the gamma function. This collapse results in the formation of a jet of liquid coolant. The results of Plesset and Chapman (1971) indicate that the velocity, length and diameter of the jet when it strikes the fuel surface are

$$\left.\begin{aligned} V_o &= V_c(P_o/\rho_c)^{\frac{1}{2}} \\ L_o &= L_c\, R_m \\ d_o &= d_c\, R_m \end{aligned}\right\} . \qquad \ldots (2.7)$$

The constants V_c , L_c and d_c are determined by the degree of departure from spherical symmetry. For bubble collapse adjacent to a solid wall Plesset and Chapman's results indicate that:

$$\left.\begin{aligned} V_c &= 13 \\ L_c &= 0.493 \\ d_c &= 0.237 \end{aligned}\right\} \quad . \qquad \ldots (2.8)$$

and these values have been used in ETNA.

Stage 3. The jet of coolant (diameter d_o and length L_o) enters the fuel with velocity V_o . As the jet penetrates it disintegrates and mixes with the surrounding fuel. The surface area of contact between the fuel and the jet is given by:

$$A = A_o \exp(t/\tau) \qquad \ldots (2.9)$$

$$\tau = \frac{11}{4}\left(\frac{\rho_f}{\rho_c}\right)^{\frac{1}{2}} \frac{d_o}{V_o} \qquad \ldots (2.10)$$

A_o is the initial surface area of the jet and t is measured from the start of penetration. Whilst the jet remains liquid its volume is almost constant. Consequently, since the area increases exponentially the average distance S between material surfaces must decrease exponentially.

$$S = S_o \exp(-t/\tau) \qquad \ldots (2.11)$$

S of course cannot decrease indefinitely and we denote the minimum value that is physically attainable by S_m .

Stage 4. As the jet penetrates, heat transfer occurs. We assume that heat transfer takes place one-dimensionally across each element of fuel-coolant (see Buchanan 1974, Fig 4.). The thickness of the element of coolant that is effectively heated, x , is given initially by

$$x = 4\sigma\sqrt{t} \qquad \ldots (2.12)$$

When $4\sigma\sqrt{t}$ is equal to the thickness of the element, S , the coolant is heated throughout and the value of x is then given by Eqn (2.11). The temperature of the heated region satisfies the equation (Buchanan 1973)

$$c\rho_c \, x \, \frac{dT}{dt} = r_1 - r_2 T \qquad \ldots (2.13)$$

where r_1 is a parameter specifying the rate of heating of the jet per unit contact area due to heat transfer from the surrounding hot fluid; $r_2 T$ is inserted to take account of the fact that the heat transfer rate per unit contact area decreases as the temperature of the jet increases; and c is the specific heat of the coolant. Thus the temperature of the jet is given by:

i) When Eqn (2.12) applies

$$T = \frac{r_1}{r_2} - \left(\frac{r_1}{r_2} - T_o\right) \exp\left(- \frac{r_2}{2\rho_c c \sigma} \sqrt{t}\right) \qquad \ldots (2.14)$$

ii) When Eqn (2.11) applies

$$T = \frac{r_1}{r_2} + F \exp\left\{ - \frac{r_2 \tau}{\rho_c c \, S_o} \exp(t/\tau) \right\} \qquad \ldots (2.15)$$

where F is a constant such that at time t' Eqns (2.14) and (2.15) give the same value for T. t' is the solution of

$$4\sigma\sqrt{t} = S_o \exp(-t/\tau) \qquad \ldots (2.16)$$

iii) When S_m is attained

$$T = \frac{r_1}{r_2} - \left(\frac{r_1}{r_2} - T_1\right) \exp\left(- \frac{r_2}{\rho_c c \, S_m} t\right) \qquad \ldots (2.17)$$

T_1 is the temperature given by Eqn (2.16) with $t = t''$ where

$$S_m = S_o \exp(-t''/\tau) \qquad \ldots (2.18)$$

T_o is the initial temperature of the coolant.

Stage 5. The jet continues to be heated until it is vaporized, which happens at either the saturation temperature, T_{sat}, if nucleation sites are available (heterogeneous nucleation), or at the homogeneous nucleation temperature, T_{hn}, if no nucleation sites are available. The fraction of the jet that is vaporized, β, depends on the process, being unity for heterogeneous nucleation and 0.33 for homogeneous nucleation at 1 bar.

Since a finite mass of liquid is vaporized simultaneously and since the density cannot change instantaneously from ρ_ℓ , the saturated liquid density, to ρ_v , the saturated vapour density, we assume that the vapour is initially formed at ρ_ℓ and then expands adiabatically. The initial high pressure P_i is given by:

$$P_i \rho_\ell^{-\gamma} = P_v \rho_v^{-\gamma} \qquad \text{... (2.19)}$$

where P_v is the vapour pressure. We also assume that the vapour bubble is spherical with radius R_i given by:

$$\frac{4}{3}\pi R_i^3 \rho_\ell = \beta M_j \qquad \text{... (2.20)}$$

M_j is the mass of the jet. The pressure P_i in Eqn (2.19) is the same as the pressure P_i in Eqn (2.4).

The pressure in the liquid coolant at time t from vaporization and at a point r, measured from the centre of the bubble but outside the bubble, is given by:

$$P(r,t) - P_o = [P(a, \tau') - P_o] a/r \qquad \text{... (2.21)}$$

where

$$\tau' = t - \frac{r-a}{c_o} \qquad \text{... (2.22)}$$

c_o is the speed of sound in the liquid coolant while a is arbitrary $(a < r)$ but is conveniently chosen to be the bubble's radius at time τ'.

From this model a number of results can be derived analytically which provide useful checks on the computer code. The ratio of the peak pressures at r due to successive cycles is:

$$\frac{P_i(r) - P_o}{P_{i-1}(r) - P_o} = \left(\frac{3}{16} \beta d_c^2 L_c \frac{\rho_c}{\rho_\ell}\right)^{1/3} (1 + Z_r) \qquad \text{... (2.23)}$$

which for $P_o = 1$ bar gives

$$\frac{P_i(r) - P_o}{P_{i-1}(r) - P_o} = \begin{cases} 6.673 \text{ (heterogeneous)} \\ 2.899 \text{ (homogeneous)} \end{cases} \qquad \ldots (2.24)$$

As P_o is increased the ratio (2.23) decreases and becomes equal to unity when

$$P_o = \begin{cases} 67.5 \text{ bar} \quad \text{(heterogeneous)} \\ 13.0 \quad '' \quad \text{(homogeneous)} \end{cases} \qquad \ldots (2.25)$$

After N cycles the total mass of coolant heated during jet penetration is:

$$M_c = \frac{\pi}{4} \rho_c d_c^2 L_c \frac{\xi^N - 1}{\xi - 1} R_{m_1}^3 \qquad \ldots (2.26)$$

where R_{m_1} is the radius just prior to collapse of the initial perturbing bubble and

$$\xi = \frac{3}{16} \beta d_c^2 L_c \frac{\rho_c}{\rho_\ell} (1 + Z_r)^3 \ . \qquad \ldots (2.27)$$

The ratio of the kinetic energy of the jet E_j just as it starts to penetrate, to the energy of the bubble E_b calculated as the work done by the bubble expanding to its maximum radius, is independent of cycle number and to order $O(^1/P_i)$ is given by

$$\frac{E_j}{E_b} = \frac{3}{32} d_c^2 L_c V_c^2 = 0.439 \qquad \ldots (2.28)$$

3. NUMERICAL ANALYSIS

Most of the numerical work in the code is straight-forward function evaluation. The only parts that merit attention here are; 1) evaluation of Z_r, the real root of the cubic equation; 2) evaluation of the integral (2.4), and 3) the solution of Eqn (2.16).

1. Evaluation of Z_r: It is a trivial matter to show that Eqn (2.3) has only one real root. Cubic equations may be solved exactly by Cardan's method (Turnbull 1952) and this method has been used here. After the root is found the solution is checked for accuracy.

2. Evaluation of the integral (2.4): This is an elliptic integral and can, after some tedious algebra, be reduced to a combination of the three canonical forms; the elliptic integrals of the first, second and third kinds respectively (Whittaker and Watson 1927). However, these basic forms must then be evaluated numerically. In this case, it is just as adequate and easier to evaluate the integral numerically, the only minor problem being that the integrand diverges at both limits. This problem is easily overcome (Buchanan 1974) by observing that

$$\text{Integral (2.4)} = \int_0^{Z_r} F(Z)\,dZ + 2\left(\frac{Z_r}{a}\right)^{\frac{1}{2}} + \frac{2(1+Z_r)^2}{(3Z_r^2 + 8Z_r + 6)^{\frac{1}{2}}} \qquad \ldots (3.1)$$

where

$$F(Z) = \frac{(1+Z)^2}{(aZ - Z^4 - 4Z^3 - 6Z^2)^{\frac{1}{2}}} - \frac{1}{(aZ)^{\frac{1}{2}}} - \frac{(1+Z_r)^2 (Z_r - Z)^{-\frac{1}{2}}}{(3Z_r^3 + 8Z_r^2 + 6Z_r)^{\frac{1}{2}}} \qquad \ldots (3.2)$$

$$a = 3(P_i/P_o - 1)$$

$F(Z)$ is a perfectly well-behaved analytic function in the region $[0, Z_r]$. At $Z=0$ and $Z=Z_r$, $F(Z)$ has the values

$$\left.\begin{aligned} F(0) &= \frac{-(1+Z_r)^2}{Z_r(3Z_r^2 + 8Z_r + 6)^{\frac{1}{2}}} \\ F(Z_r) &= -(aZ_r)^{-\frac{1}{2}} \end{aligned}\right\} . \qquad \ldots (3.3)$$

To effect the numerical integration of $F(Z)$ a Gaussian ten point formula is used (Abramowitz and Stegun 1965). Initially ten points are used over the whole range. The range is then divided into two equal parts and ten point integration used within each range. If the answers to the whole range and divided range integrations are not equal to within some specified limit, each range is further subdivided and ten point integration used again in each range. This process is continued until the required accuracy is achieved. Gaussian integration is performed over the interval $[-1, 1]$ and thus each integral must be between these limits. This is effected each time by a simple linear change of variable. Near the limits 0 and Z_r the function F(Z) is evaluated by a series expansion, otherwise its evaluation is straight forward.

To determine the pressure history at a point r the incomplete form of the integral (2.4) is required with some $Z \neq Z_r$ for the upper limit. This is easily done using the same routines as before with the upper limit changed.

3. Solution of Eqn (2.16). This is equivalent to finding the zeros of the function

$$G(Z) = 4\sigma\sqrt{Z} - S_o\, e^{-Z/\tau} \qquad \ldots (3.4)$$

In fact it is obvious that this function has only one real zero for positive Z . The rule of false position is used (Hochstrasser 1962).

4. STRUCTURE OF THE COMPUTER CODE

The code ETNA consists of a main program plus subprograms. Of the subprograms, some are part of the ICL 4-70 Multijob Operating System and hence need not concern us, some are part of the OLYMPUS package (Roberts 1974, Christiansen and Roberts 1974) and the remainder are divided into five classes as described by Christiansen and Roberts (1974). The classes 1, 3, 4 and 5 almost identical to those of Christiansen and Roberts (1974). Table 1 contains a list of all subprograms in the classes 1-5 together with their purpose; Table 2 contains a list of variables in the labelled common blocks together with their purpose; and Fig 4.1 is a flow diagram of class 2 subprograms. Those subprograms marked with an asterisk are described in more detail in the text. Each of the subprograms of ETNA is decimally numbered according to OLYMPIAN conventions. References of the type ⟨2.5⟩ can be correlated with the list in Table 1. The source deck also contains this index. On the ICL 4-70 the corresponding file name is denoted by C2S5 (class 2, subprogram 5) followed by a type code defining the file eg FORTRAN source code F, object module Y.

CUBERT ⟨2.3⟩: This is a double precision subroutine which calculates the real root of the cubic equation. It calls the function DSIGN(X,Y)⟨2.16⟩. DSIGN is a double precision function which transfers the sign of Y to the absolute value of X. The only reason for supplying this subprogram is that the system-supplied version does not work.

ELIPSE ⟨2.4⟩: This routine evaluates the elliptic integral. Normally the complete elliptic integral is evaluated; however the routine contains the statement ENTRY MOVE(TYM,MOT) which allows abnormal entry and the calculation of the incomplete elliptic integral.

HEAT ⟨2.8⟩: This is a rather lengthy routine whose control is as follows:

1. Decide which of the PDE and ODE methods is used to solve the heat conduction problem (Buchanan 1973). At present the ODE method is always used.

2. Solve equation (2.16) for t'.

3. Determine if Eqn (2.11) is always operable or if a specified minimum particle size S_m is to be included. If the latter is the case then Eqn (2.18) is solved for t''.

4. Compare t' and t''. If $t' > t''$ then Eqn (2.15) is never used. Equations (2.14) and (2.18) (possibly) are used to calculate the time required to heat to the vaporization temperature, TBRAK. The fraction of the jet vaporized is then calculated. If $t' < t''$ the solution may involve Eqn (2.15). If the temperature using Eqn (2.14) and t' is greater than TBRAK then Eqn (2.15) need not be used. If this is not the case Eqn (2.15) must be used and the constant F found. If Eqn (2.15) with $t=t''$ does not give a temperature greater than TBRAK then Eqn (2.18) must also be used. The fraction of the jet that is vaporized is found.

5. The pressure and radius of the new bubble is calculated.

SOLRTT ⟨2.17⟩: This routine solves Eqn (2.14) for t with the temperature T set equal to TBRAK.

HISTOR ⟨2.17⟩: This routine calculates the pressure as a function of time at the point r using Eqn (2.21). As this equation stands t is the independent variable. The pressure $P(r,t)$ is related to the pressure in the bubble at the retarded time τ' when the bubble radius is a. To avoid having to iterate to find consistent values of a and τ' it is much more convenient to regard t as the dependent variable. So an a is chosen and the time for the bubble to expand to radius a is found by evaluating the incomplete elliptic integral. This involves the abnormal entry into ELIPSE ⟨2.4⟩ . Since a and τ' are known t can be found from Eqn (2.22) and of course $P(a,\tau')$ is just the pressure of the bubble when it has a radius a .

SOLEXP ⟨2.19⟩: This routine solves Eqn (2.15) for t with T set equal to TBRAK. The constant F is such that Eqn (2.14) and (2.15) give the same value for T when $t = t'$.

INFORM ⟨5.1⟩: This is a diagnostic routine which prints out selected common variables if any errors occur in the calculation. A modified version of the system DEBUG also calls this routine so that if DEBUG is called, the current state of the common variables is available to the user.

The variable TBRAK is used solely to calculate the time required for vaporization. The saturated density values are read as data. The default option of TBRAK is 373.15 corresponding to heterogeneous nucleation at 1 bar. The time to heat to TBRAK takes no account of the additional time required to supply the latent heat; however, an estimate of the time to heat to the saturation temperature plus the time to supply the latent can be obtained by using the equivalent temperature approximation, <u>ie</u> TBRAK=373.15 + L/C where L is the latent heat and C the specific heat. This modified value of TBRAK must of course be read in as data.

The variable GASDEN is the density of the vapour when it is formed. By assumption this density is the saturated <u>liquid</u> density corresponding to the particular vaporization temperature used (cf Eqn 2.19). The variables RATE1 and RATE2 must satisfy the condition RATE1/RATE2 $<$ TBRAK.

As far as possible the program has been tested against analytical solutions. In this respect the expressions 2.23-2.28 are most useful. The only calculation that cannot be tested against an analytical formula is the total elapsed time. Obviously time solutions of Eqns (2.14)-(2.18) are easily checked; the only part that is not easily checked is the evaluation of the elliptic integral. The Gaussian integration scheme has been checked against a different integral which can be evaluated analytically.

The default option for all common variables is given in PRESET ⟨1.3⟩. These options can be overwritten by using the NAMELIST facility in DATA ⟨1.4⟩. All variables supplied by the user must be in MKS units. The default values will give an FCI calculation for tin and water with vaporization by heterogeneous nucleation. The calculation is started by an initial bubble collapsing and the radius of this bubble is specified as RINIT. A cycle is defined to be that period between successive formation of two bubbles, so that a cycle ends when a bubble is about to expand. The expansion time of the first cycle is of course zero since the calculation is started with bubble collapse.

Output from two test runs is shown in Appendices 1 and 2. The first run is the default run (Appendix 1) when no data is supplied. This corresponds to an FCI at an external pressure of 1 bar with vaporization by heterogeneous nucleation. The second run (Appendix 2) corresponds to an

FCI at 5 bar with homogeneous nucleation. The variables reset in DATA ⟨1.4⟩ are shown below:

```
PRESS=  5.0E 05
PINIT=  3.614E 08
DELP=   5.0E 05
GASDEN= 728.2
TBRAK=  565.
FRAC1=  0.2547
```

Some of the relevant thermodynamic properties for water can be obtained from UK Steam Tables (1970), others such as T_{hn} must be calculated by the user from one of the theories of homogeneous nucleation (Frenkel 1946 and Cooper 1952, provide useful reviews).

14 February 1975
dg

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TABLE 1 - INDEX OF SUBPROGRAMS

NB. Subprograms marked * are described in more detail in the text

Class 1 Prologue

Name	No	Dummy Arguments	Purpose
LABRUN	1.1		Label the run
CLEAR	1.2		Clear the common blocks
PRESET	1.3		Define default values
DATA	1.4		Define data specific to the run
AUXVAL	1.5		Define auxiliary variables
INITAL	1.6		Define initial conditions
RESUME	1.7		Restart the calculation
START	1.8		Start the calculation

Class 2 Calculation

Name	No	Dummy Arguments	Purpose
STEPON	2.1		Advance the calculation by one cycle
BUBBLE	2.2		Solve the bubble dynamics problem
*CUBERT	2.3		Find the real root of the cubic equation
*ELIPSE	2.4		Evaluate the elliptic integral
GAUSS	2.5	ANS,RLIM1,RLIM2,D E,C,MOTION	10 Point Gaussian integration scheme
FUNC	2.6	Y1,D,E,C,MOTION	Evaluate the integrand
JET	2.7		Evaluate the jet characteristics
*HEAT	2.8		Solve the heat transfer problem
THICK	2.9		Solve the thickness equation
YTHICK	2.10	XX	Evaluate the thickness function
TSQRTT	2.11	T	Find temperature on the assumption that the square root approximation is valid.
*SQLRTT	2.12		Solution of square root approximation
ARIA	2.13	T	Evaluates the area at time T.
KONST	2.14		Integration constant in exponential mass approximation.
TEMEXP	2.15	T	Temperature at time T according to the exponential mass approximation.
DSIGN	2.16	XX,YY	Transfer of sign.
*HISTOR	2.17		Calculate pressure at point AR.
PTIME	2.18	AA,PTT	Calculate retarded time
*SOLEXP	2.19		Solution of exponential approximation.

Class 3 Output

OUTPUT	3.1	K	Output routine

Class 4 Epilogue

TESEND	4.1		Test for completion of run
ENDRUN	4.2		Terminate the run.

Class 5 Diagnostics

*INFORM	5.1		Diagnostic routine.

The terms 'square root' or 'square root mass' approximation and 'exponential' or 'exponential mass' approximation above and in the program listing refer to Eqns (2.14) and (2.15) respectively.

Variable	Default Value	Purpose
		Common Block FCI1.
RFINAL	0.	Final radius of bubble after expansion, R_m
RINIT	1.E-04	Initial radius of bubble at start of expansion, R_i This variable is used to start the calculation.
ZCUBE	0.	Real root of cubic equation, Z_r
TCOLL	0.	Collapse time of bubble, t_c
RHO	1000.	Density of bulk coolant, ρ_c
PRESS	1.01325E 05	Ambient pressure, P_o
TIME	0.	Total time for expansion and collapse of bubble, $t_c + t_g$
TXPAND	0.	Time for bubble to expand, t_g
PINIT	1.90083E 09	Pressure at which bubble is formed, P_i
X(I)I=1,5	0.9739065 0.8650634 0.6794096 0.4333954 0.1488743	Nodes for Gaussian ten point integration
W(I)I=1,5	0.0666713 0.1494513 0.2190864 0.2692667 0.2955242	Weights for Gaussian ten point integration
VJET	0.	Velocity of jet, V_o
RLJET	0.	Length of jet, L_o
DJET	0.	Diameter of jet, d_o
RJETV	13.	Proportionality constant for jet velocity, V_c
RJETL	4.78/9.70	Proportionality constant for jet length, L_c
RJETD	2.30/9.70	Proportionality constant for jet diameter, d_c
DELP	1.01325E 05	Pressure difference under which the bubble collapses, P_o
TAU	0.	Time constant for area increase, τ .
RJETTA	11./4.	Proportionality constant for area increase, $\frac{11}{4}(\rho_f/\rho_c)^{\frac{1}{2}}$
AREAO	0.	Initial area of jet, A_o
TCRIT	647.3	Critical temperature of coolant (not used)
TIMHET	0.	Time to heat to vaporization temperature
TWID	0.	Time solution of Eqn (2.16), t'
TEXP	0.	Temperature from exponential approximation, Eqn (2.15)
TEMP	0.	Temperature from square root approximation, Eqn (2.14)
GAMMA	21.E-05	Coefficient of thermal expansion of coolant (not used)
RKAPPA	4.45 E-10	Compressibility of coolant (not used)
TZERO	300.	Ambient coolant temperature, T_o .
GASDEN	958.13	Density of vapour at temperature and pressure at which vapour is formed (ie the saturated liquid density, see (Eqn 2.19))

SIGMA	3.879126E-04	Thermal diffusivity of coolant, σ
FACTOR	0.5	Proportionality constant for Eqn (2.16). This constant allows for the fact that S_o in Eqn (2.11) may not equal the diameter of the jet, d_o . For example, in Buchanan (1974) Fig 4, $S_o=d_o/2$ seems more appropriate.
RATE1	1.E 06	Rate of heating, r_1
RATE2	1.E 03	Rate of cooling, r_2
SPHT	4200.	Specific heat of coolant, c
AREA	0.	Area of jet at time t, A.
CONST	0.	Integration constant in Eqn (2.15), F.
TBRAK	373.15	Temperature at which the jet is vaporized. For example, T_{sat} or T_{hn}.
GAM	4./3.	Ratio of specific heats of coolant vapour, γ .
ENBUB	0.	Energy of bubble, E_b
ENJET	0.	Energy of jet, E_j
ELTIME	0.	Total elapsed time.
RLENTH	1.	Factor giving thickness in square root approximation, Eqn (2.14). The heated mass is defined somewhat arbitrarily to be $2\sigma\sqrt{t}$. This constant allows any multiple of $2\sigma\sqrt{t}$ to be used.
ACC1	1.E-05	Accuracy of solution of cubic equation
ACC2	1.E-05	Accuracy of evalution of integral
ACC3	1.E-08	Accuracy of solution of Eqn (2.16)
SRI	0.	Saves previous value of RINIT
SPARE(I)I=1,10		Spare vector used as follows:
SPARE(1)	0.	Controls number of runs
SPARE(2)	2.	If GE 0.5 then Eqns (2.17) and (2.18) are included in calculation.
SPARE(3)	3.E-07	Minimum particle size for cut-off, S_m .
SPARE(4)	0.	Thickness of jet at time of vaporization
SPARE(5)	0.	Not used
SPARE(6)	0.	Not used
SPARE(7)	0.	Stores ZCUBE
SPARE(8)	0.	Stores RINIT
SPARE(10)	0.	Total mass of coolant, M_c
NFREQ	1	Frequency of output
NMAX	5	Maximum number of cycles
NSCAN	4	Not used
METHOD	0	Switch to determine which of the ODE and PDE methods are used. Only the ODE method is used in the current version.

Common Block FCI2

PHIST(I)I=1,10	0.	Pressure at point r as a function of time, P(r,t)
PT(I)I=1,10	0.	Times at which P(r,t) is calculated
PTIME(I)I=1,10	0.	Total times at which P(r,t) is calculated
AR	0.1	Point where pressure is calculated, r .
ZMAX	0.	Upper limit of incomplete elliptic integral
SONCOL	1500.	Velocity of sound in coolant, c_o
FRAC1	1.	Maximum fraction of jet that can be vaporized, β
RIMP	0.	Impulse at point r .

Common Block FCI3

FREE(I)I=1,20		Spare vector used as follows
FREE(1)	1.	Factor modifying RJETV
FREE(2)	1.	Factor modifying RJETL
FREE(3)	1.	Factor modifying RJETD. These constants are intended for future use to take account of possible modifications to Eqn (2.8) as a result of non-condensible gas, etc.
FREE(4)	7000.	Density of fuel, ρ_f
FREE(I)I=5,20	0.	Not used.

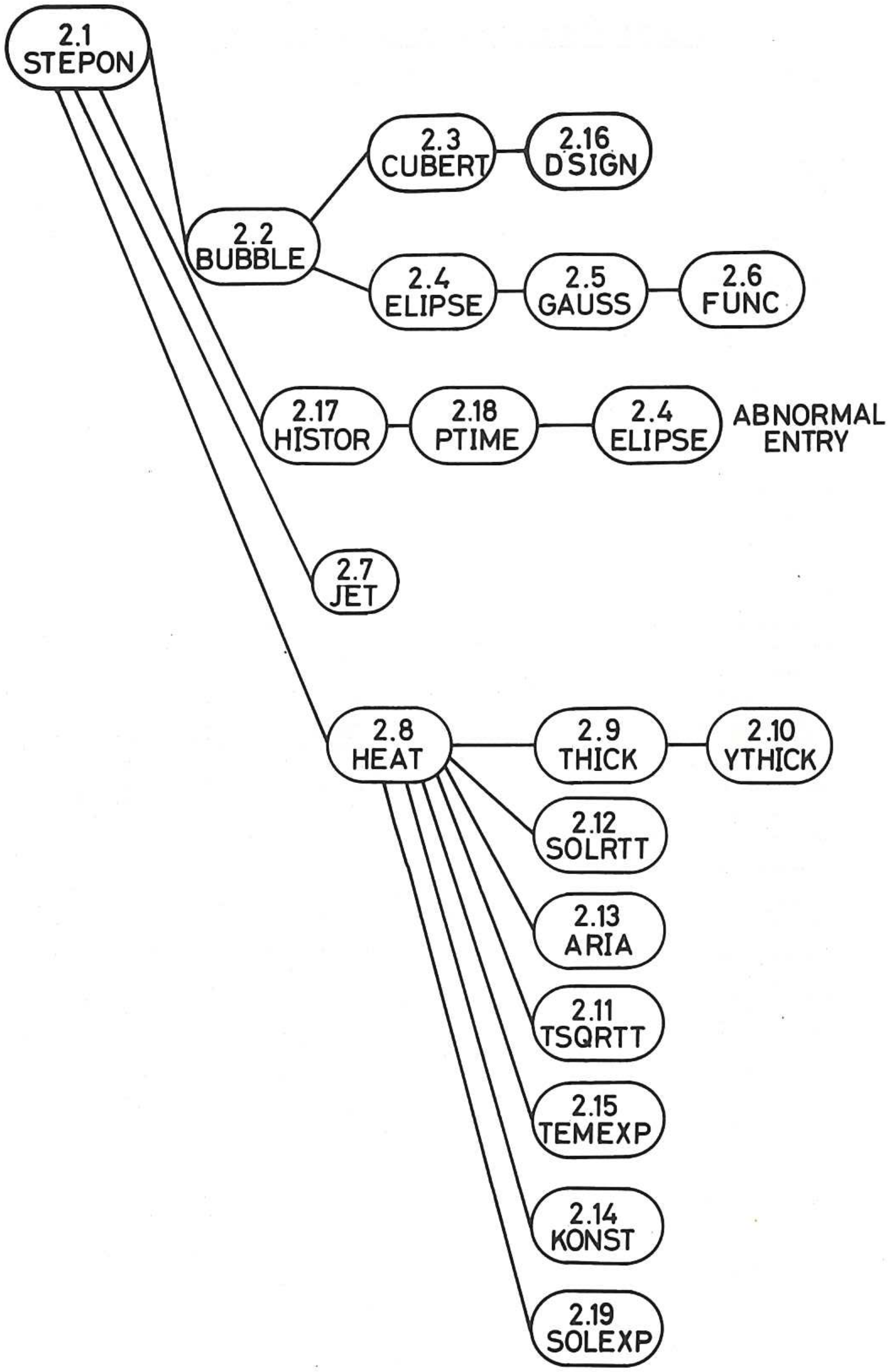

Fig.4.1 Flow Diagram of Class Two Subprograms

DEFAULT RUN WHEN NO DATA IS SUPPLIED

```
     INITIAL OUTPUT

     INITIAL RADIUS OF BUBBLE

RINIT    =  1.0000E-04

     DENSITY OF BULK COOLANT

RHO      =  1.0000E 03

     AMBIENT PRESSURE

PRESS    =  1.0133E 05

     INITIAL PRESSURE OF BUBBLE

PINIT    =  1.9008E 02

     PRESSURE DIFFERENCE COLLAPSING BUBBLE

DELP     =  1.0133E 05

     AMBIENT COOLANT TEMPERATURE

TZERO    =  3.0000E 02

     DENSITY OF VAPOUR AT FORMATION

GASDEN   =  9.5813E 02

     RATE OF HEATING

RATE1    =  1.0000E 06

     RATE OF COOLING

RATE2    =  1.0000E 05

     BREAKING TEMPERATURE

TBRAK    =  3.7315E 02

     MASS FRACTION OF VAPOUR

FRAC1    =  1.0000E 00

     POSITION OF TRANSDUCER

AR       =  1.0000E-01

     METHOD OF SOLUTION
     THE ODE METHOD IS USED

     FREQUENCY OF OUTPUT

NFREQ    =           1

     MAXIMUM NUMBER OF CYCLES

NMAX     =           5

     PERIODIC PRODUCTION OF OUTPUT

     CYCLE NUMBER

NSTEP    =           1

     BUBBLE DYNAMICS

SRI      =  1.0000E-04
RFINAL   =  1.0000E-04
TXPAND   =  0.0000E 00
TCOLL    =  9.9344E-06
TIME     =  9.9344E-06
ENBUB    =  8.5265E-13

     JET CHARACTERISTICS

VJET     =  1.3086E 02
RLJET    =  4.9278E-05
DJET     =  2.3711E-05
TAU      =  1.3184E-06
AREA0    =  3.6708E-09
ENJET    =  1.8631E-07
FRACTION =  1.0000E 00
MASS J   =  2.1760E-11
     CALCULATED TOTAL MASS
TOT MASS =  2.1760E-11
     TOTAL MASS BY DIRECT ADDITION
RTOTM    =  2.1760E-11

     HEAT TRANSFER

TIMHET   =  1.4220E-04
AREA     =  1.4507E-07
AI       =  3.9519E 01
S(T)     =  3.0000E-07
THID     =  2.1701E-06
TEXP     =  3.0095E 02
TEMP     =  3.0032E 02
RINIT    =  1.7568E-05

     TOTAL ELAPSED TIME

ELTIME   =  1.5214E-04

     PRESSURE HISTORY
```

PRESSURE	TIME	TOTAL TIME
0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00
0.0000000E 00	0.0000000E 00	0.0000000E 00

```
RIMP     =  0.0000E 00

     CYCLE NUMBER

NSTEP    =           2

     BUBBLE DYNAMICS

SRI      =  1.7568E-05
RFINAL   =  6.6727E-04
TXPAND   =  6.1204E-05
TCOLL    =  6.6290E-05
TIME     =  1.2749E-04
ENBUB    =  1.2610E-04

     JET CHARACTERISTICS

VJET     =  1.3086E 02
RLJET    =  3.2882E-04
DJET     =  1.5822E-04
TAU      =  8.7971E-06
AREA0    =  1.6344E-07
ENJET    =  5.5353E-05
FRACTION =  1.0000E 00
MASS J   =  6.4650E-09
     CALCULATED TOTAL MASS
TOT MASS =  6.4868E-09
     TOTAL MASS BY DIRECT ADDITION
RTOTM    =  6.4868E-09

     HEAT TRANSFER

TIMHET   =  1.7790E-04
AREA     =  4.3100E-05
AI       =  2.6370E 02
S(T)     =  3.0000E-07
THID     =  2.1161E-05
TEXP     =  3.0565E 02
TEMP     =  3.0090E 02
RINIT    =  1.1723E-04

     TOTAL ELAPSED TIME

ELTIME   =  4.5753E-04

     PRESSURE HISTORY
```

PRESSURE	TIME	TOTAL TIME
0.4352394E 06	0.6665495E-04	0.2187905E-03
0.2171727E 06	0.6666547E-04	0.2188010E-03
0.1502843E 06	0.6667449E-04	0.2188100E-03
0.1244913E 06	0.6668924E-04	0.2188248E-03
0.1130268E 06	0.6671378E-04	0.2188493E-03
0.1074245E 06	0.6675487E-04	0.2188904E-03
0.1045010E 06	0.6682496E-04	0.2189605E-03
0.1029025E 06	0.6695125E-04	0.2190868E-03
0.1019957E 06	0.6720636E-04	0.2193419E-03
0.1006547E 06	0.1274257E-03	0.2795612E-03

```
RIMP     =  5.3470E-03

     CYCLE NUMBER

NSTEP    =           3

     BUBBLE DYNAMICS

SRI      =  1.1723E-04
RFINAL   =  4.4526E-03
TXPAND   =  4.0840E-04
TCOLL    =  4.4234E-04
TIME     =  8.5074E-04
ENBUB    =  3.7465E-02

     JET CHARACTERISTICS

VJET     =  1.3086E 02
RLJET    =  2.1942E-03
DJET     =  1.0558E-03
TAU      =  5.8701E-05
AREA0    =  7.2775E-06
ENJET    =  1.6446E-02
FRACTION =  1.0000E 00
MASS J   =  1.9208E-06
     CALCULATED TOTAL MASS
TOT MASS =  1.9273E-06
     TOTAL MASS BY DIRECT ADDITION
RTOTM    =  1.9273E-06

     HEAT TRANSFER

TIMHET   =  5.1455E-04
AREA     =  1.2806E-02
AI       =  1.7596E 03
S(T)     =  3.0000E-07
THID     =  1.8844E-04
TEXP     =  3.3424E 02
TEMP     =  3.0294E 02
RINIT    =  7.8222E-04

     TOTAL ELAPSED TIME

ELTIME   =  1.8228E-03

     PRESSURE HISTORY
```

PRESSURE	TIME	TOTAL TIME
0.2329461E 07	0.6658852E-04	0.5241216E-03
0.8743511E 06	0.6665837E-04	0.5241914E-03
0.4260223E 06	0.6671849E-04	0.5242515E-03
0.2559086E 06	0.6681675E-04	0.5243497E-03
0.1794084E 06	0.6698072E-04	0.5245137E-03
0.1420256E 06	0.6725482E-04	0.5247879E-03
0.1225181E 06	0.6772265E-04	0.5252557E-03
0.1118516E 06	0.6856532E-04	0.5260983E-03
0.1058004E 06	0.7026743E-04	0.5278005E-03
0.9685406E 05	0.4720974E-03	0.9296306E-03

```
RIMP     =  2.3749E-01
```

HOMOGENEOUS NUCLEATION WITH P_o = 5 bar

```
     INITIAL OUTPUT

     INITIAL RADIUS OF BUBBLE
 RINIT    =  1.0000E-04

     DENSITY OF BULK COOLANT
 RHO      =  1.0000E 03

     AMBIENT PRESSURE
 PRESS    =  5.0000E 05

     INITIAL PRESSURE OF BUBBLE
 PINIT    =  3.6140E 03

     PRESSURE DIFFERENCE COLLAPSING BUBBLE
 DELP     =  5.0000E 05

     AMBIENT COOLANT TEMPERATURE
 TZERO    =  3.0000E 02

     DENSITY OF VAPOUR AT FORMATION
 GASDEN   =  7.2820E 02

     RATE OF HEATING
 RATE1    =  1.0000E 06

     RATE OF COOLING
 RATE2    =  1.0000E 03

     BRAKING TEMPERATURE
 TBRAK    =  5.6500E 02

     MASS FRACTION OF VAPOUR
 FRAC1    =  2.5470E-01

     POSITION OF TRANSDUCER
 AR       =  1.0000E-01

     METHOD OF SOLUTION
     THE ODE METHOD IS USED

     FREQUENCY OF OUTPUT
 NFREQ    =              1

     MAXIMUM NUMBER OF CYCLES
 NMAX     =              5

     PERIODIC PRODUCTION OF OUTPUT

     CYCLE NUMBER
 NSTEP    =

     BUBBLE DYNAMICS

 SRI      =  1.0000E-04
 RFINAL   =  1.0000E-04
 TXPAND   =  0.0000E 00
 TCOLL    =  4.4721E-06
 TIME     =  4.4721E-06
 ENBUB    =  0.0000E 00

     JET CHARACTERISTICS

 VJET     =  2.9069E 02
 RLJET    =  4.9278E-05
 DJET     =  2.3711E-05
 TAU      =  2.2432E-07
 AREAO    =  3.6708E-07
 ENJET    =  9.1936E-07
 FRACTION =  1.0000E 00
 MASS J   =  2.1760E-11
     CALCULATED TOTAL MASS
 TOT MASS =  2.1760E-11
     TOTAL MASS BY DIRECT ADDITION
 RTOTM    =  2.1760E-11

     HEAT TRANSFER

 TIMHET   =  5.9980E-04
 AREA     =  1.4507E-07
 AI       =  3.9519E 01
 S(T)     =  3.0000E-07
 TWID     =  5.2781E-07
 TEXP     =  3.0025E 02
 TEMP     =  3.0016E 02
 RINIT    =  1.2203E-05

     TOTAL ELAPSED TIME
 ELTIME   =  6.0427E-04

     PRESSURE HISTORY

 PRESSURE            TIME                TOTAL TIME

 0.0000000E 00       0.0000000E 00       0.0000000E 00
 0.0000000E 00       0.0000000E 00       0.0000000E 00
 0.0000000E 00       0.0000000E 00       0.0000000E 00
 0.0000000E 00       0.0000000E 00       0.0000000E 00
 0.0000000E 00       0.0000000E 00       0.0000000E 00
 0.0000000E 00       0.0000000E 00       0.0000000E 00
 0.0000000E 00       0.0000000E 00       0.0000000E 00
 0.0000000E 00       0.0000000E 00       0.0000000E 00
 0.0000000E 00       0.0000000E 00       0.0000000E 00
 0.0000000E 00       0.0000000E 00       0.0000000E 00

 RIMP     =  0.0000E 00

     CYCLE NUMBER
 NSTEP    =              2

     BUBBLE DYNAMICS

 SRI      =  1.2203E-05
 RFINAL   =  1.5367E-04
 TXPAND   =  6.4866E-06
 TCOLL    =  6.8724E-06
 TIME     =  1.3359E-05
 ENBUB    =  7.5965E-05

     JET CHARACTERISTICS

 VJET     =  2.9069E 02
 RLJET    =  7.5726E-05
 DJET     =  3.6437E-05
 TAU      =  3.4471E-07
 AREAO    =  8.6685E-07
 ENJET    =  3.3362E-05
 FRACTION =  1.0000E 00
 MASS J   =  7.8964E-11
     CALCULATED TOTAL MASS
 TOT MASS =  1.0072E-10
     TOTAL MASS BY DIRECT ADDITION
 RTOTM    =  1.0072E-10

     HEAT TRANSFER

 TIMHET   =  6.0020E-04
 AREA     =  5.2643E-07
 AI       =  6.0729E 01
 S(T)     =  3.0000E-07
 TWID     =  8.7256E-07
 TEXP     =  3.0035E 02
 TEMP     =  3.0020E 02
 RINIT    =  1.8752E-05

     TOTAL ELAPSED TIME
 ELTIME   =  1.2178E-03

     PRESSURE HISTORY

 PRESSURE            TIME                TOTAL TIME

 0.5440388E 06       0.6665853E-04       0.6709325E-03
 0.5302826E 06       0.6666887E-04       0.6709427E-03
 0.5209071E 06       0.6667453E-04       0.6709483E-03
 0.5144083E 06       0.6668066E-04       0.6709546E-03
 0.5098430E 06       0.6668311E-04       0.6709620E-03
 0.5066023E 06       0.6669796E-04       0.6709718E-03
 0.5042852E 06       0.6671200E-04       0.6709858E-03
 0.5026171E 06       0.6673373E-04       0.6710077E-03
 0.5014072E 06       0.6677218E-04       0.6710461E-03
 0.4992537E 06       0.7305080E-04       0.6773246E-03

 RIMP     =  1.0377E-03

     CYCLE NUMBER
 NSTEP    =              3

     BUBBLE DYNAMICS

 SRI      =  1.8752E-05
 RFINAL   =  2.3615E-04
 TXPAND   =  9.9680E-06
 TCOLL    =  1.0561E-05
 TIME     =  2.0529E-05
 ENBUB    =  2.7567E-05

     JET CHARACTERISTICS

 VJET     =  2.9069E 02
 RLJET    =  1.1637E-04
 DJET     =  5.5994E-05
 TAU      =  5.2972E-07
 AREAO    =  2.0470E-01
 ENJET    =  1.2107E-05
 FRACTION =  1.0000E 00
 MASS J   =  2.8655E-10
     CALCULATED TOTAL MASS
 TOT MASS =  3.8727E-10
     TOTAL MASS BY DIRECT ADDITION
 RTOTM    =  3.8728E-10

     HEAT TRANSFER

 TIMHET   =  6.0092E-04
 AREA     =  1.9103E-06
 AI       =  9.3322E 01
 S(T)     =  3.0000E-07
 TWID     =  1.4364E-06
 TEXP     =  3.0050E 02
 TEMP     =  3.0024E 02
 RINIT    =  2.8816E-05

     TOTAL ELAPSED TIME
 ELTIME   =  1.8393E-03

     PRESSURE HISTORY

 PRESSURE            TIME                TOTAL TIME

 0.5676748E 06       0.6665416E-04       0.1284491E-02
 0.5465354E 06       0.6667004E-04       0.1284507E-02
 0.5321280E 06       0.6667881E-04       0.1284516E-02
 0.5221413E 06       0.6668818E-04       0.1284525E-02
 0.5151258E 06       0.6669965E-04       0.1284537E-02
 0.5101446E 06       0.6671478E-04       0.1284552E-02
 0.5065851E 06       0.6673633E-04       0.1284573E-02
 0.5040218E 06       0.6676969E-04       0.1284607E-02
 0.5021624E 06       0.6682883E-04       0.1284666E-02
 0.4998532E 06       0.7647726E-04       0.1294314E-02

 RIMP     =  2.4510E-03
```