

SOME COMMENTS ON MAGNETIC FIELD SHAPING IN THETA-PINCH COILS

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A B S T R A C T

Recent studies of the Theta-pinch have shown that small irregularities in the magnetic field of the single turn coil influence the equilibrium of the plasma. One factor determining the field shape in the coil is the current flow pattern in the transmission line (collector - tab system) that feeds the current from the condenser bank output cables. This flow pattern can be analysed into harmonics which are found to decay as they propagate along a parallel sided transmission line. The amplitude of any non-uniformity is decreased by at least three orders of magnitude in traversing a square tab.

It may be possible to reduce the asymmetry in the coil magnetic field with a dummy tab, whose length can be estimated with the harmonic analysis. Experimental results obtained with a resistive analogue demonstrate the properties of dummy tabs.

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INTRODUCTION

Recent developments in studies of plasma in the Theta-pinch have led to an investigation of drift of plasma away from the axis of the discharge tube^(1,2). In one experiment⁽²⁾ this drift was so strong as to cause the plasma to hit the walls of the tube.

Reynolds, et al⁽¹⁾ have shown that this drift is driven by a gradient of magnetic field across the coil diameter. This gradient arises from the fact that the rapidly rising magnetic field is generated in a single turn coil by current fed to the coil from a condenser bank via a parallel plate transmission line (called a collector plate)(Fig.1). Several factors determine the distribution of magnetic field in the coil, two are very important: first, non-uniformity in the current feed to the collector plate and secondly non-uniformities in the current required by the coil. Some of the latter have been analysed in detail by Aldridge⁽³⁾.

An important contribution made by Reynolds et al⁽¹⁾ was an attempt to match the feed current to the coil current using an inductance in the collector plate which varied in value across the width of the collector (inductive lens). The particular problem of designing a lens for changing a non-uniform current distribution into a uniform one has been analysed by Whipple⁽⁴⁾. He considered the propagation of the Fourier components of the current distribution along a parallel sided collector (sometimes called a tab) and the refraction of current flow lines at a boundary separating two regions of collector with different spacing between the plates.

There are several points which arise from this type of analysis which we wish to present in this note. First, the analysis shows that each Fourier component decays exponentially as it propagates along a long tab. Consequently a long tab can be used to feed a coil with nearly uniform current distribution. A comparison of the inductance in a long tab with that in a lens which has an equivalent effect, shows that there is little difference between them.

One can also consider the problem of providing the coil with a non-uniform current, such as is needed for a coil with mirrors. In principle a uniform current distribution can be converted into the required non-uniform distribution using a lens, but due to the decay of non-uniformities propagating along the tab, this lens has to be very strong. Furthermore, there are two sources of non-uniformity which no lens can correct for. One is the presence of currents which flow from the inside of the collector to the outside of the coil to support the magnetic field strength of the 'return' flux⁽³⁾. The other is the

time dependence of the current distribution caused by axial motion of the plasma⁽¹⁾. The presence of plasma changes the tube inductance; if the amount of plasma per unit length is a function of axial distance and time, then the current distribution along the axis will change with time.

As a result of these considerations an improved form of construction of the collector coil and tab assembly is suggested. It contains no features which have not been used in one form or another by other workers. We are concerned only with trying to show, at least quantitatively, what effect each feature will have. As we show, one can study some of these effects on a scale model.

BASIC EQUATIONS

The basic equations which describe the variation of voltage on the collector plate have been formulated by Whipple⁽⁴⁾.

The co-ordinate system to be used is shown in Fig.1: y is the distance along the collector plates measured from the feed cables; x is the distance across the plates measured from the centre line; z is normal to the plates which are separated by a gap h . Assuming the voltage V , magnetic field H , and current density j all vary as $\exp(i\omega t)$ then

$$\frac{\partial V}{\partial x} = -\omega h H y = 4\pi h \omega j x, \quad \dots (1a)$$

$$\frac{\partial V}{\partial y} = \omega h H x = -4\pi h \omega j y. \quad \dots (1b)$$

Since

$$\frac{\partial H x}{\partial y} = \frac{\partial H y}{\partial x},$$

$$\frac{\partial}{\partial x} \frac{1}{h} \frac{\partial V}{\partial x} + \frac{\partial}{\partial y} \frac{1}{h} \frac{\partial V}{\partial y} = 0 \quad \dots (2)$$

Essentially the neglect of H_z implies that h is uniform with x or y , or varies slowly over a scale length. For the sake of the present note we will say that h is constant except at discontinuities. Equation 2 now reduces to

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \quad \dots (3)$$

This equation can be solved analytically for simple cases as Whipple⁽⁴⁾ has shown. In more complex cases it can be solved using a computer technique or an analogue.

The simple case which will now be considered is that of a parallel plate collector.

CURRENT IN A PARALLEL-PLATE COLLECTOR

The final stage of the collector is usually a parallel-sided plate, or tab, of width equal to the coil (Fig.2). The solution of equation 3 in this geometry is:

$$V = V_0 (1 - \alpha y) + \sum_{n=1}^{\infty} V_n \cos nkx (e^{-nky} + \beta_n e^{nky}) \quad \dots (4)$$

where the constants are determined by the boundary conditions, and k equals π/a , where a is the half width of the tab.

The corresponding solutions for j_x, j_y are

$$j_x = j_n \sin nkx (e^{-nky} + \beta_n e^{nky}) , \quad \dots (5a)$$

$$j_y = j_0 + \sum j_n \cos nkx (e^{-nky} - \beta_n e^{nky}) . \quad \dots (5b)$$

An important implication of this is that if j_y is a function of x then it is also a function of y , so that j_x can only be zero in one plane. That is, one cannot have a set of parallel current flow lines if the current density is non-uniform, i.e. is a function of x . Since the current density is usually non-uniform, the current flow lines are usually non-parallel.

CORRECTION OF CURRENT NON-UNIFORMITIES IN THE TAB

SOURCE OF NON-UNIFORMITIES

The collector design used in the Culham Megajoule condenser bank⁽⁵⁾ is typical of most; it is shown in Fig.1. The current from the condensers is fed by cables to the back of an 8 metre wide plate, which extends for 1 metre in the direction of current flow. To this is joined the tab 2 metres wide and 1 metre long which feeds the coil.

The effect of this shaping is to curve the current feed lines to bring them together. Consequently, even if the current distribution is uniform at the feed point and at the coil, it is non-uniform at the start of the tab.

The amplitude of this non-uniformity of the current distribution has been estimated from a computer calculation and a resistance mat analogue. For the parameters listed above j_1 is 10% of j_0 and j_2 is 3%. They do not change significantly as the tab length is increased beyond a length equal to its width.

CORRECTION OF CURRENT FEED VARIATIONS WITH A LONG TAB

Suppose that a long tab is closed at a plane $y = y_0$ with a coil and j_y is required to be uniform at that point. Then

$$\beta_n = e^{-2nky_0} ,$$

and j_n depends on the boundary condition at the feed to the tab described above. Consequently the amplitude of the harmonics in j_x will vary as e^{-nky_0} . It seems reasonable that the largest component will be the first harmonic. If the tab length is increased so varying y_0 , the amplitude will decrease as $\exp(\frac{-\pi y_0}{a})$ where a is the half-width of the

tab. Typically for a tab 200 cms wide the e-fold length will be 32 cms, the ten-fold length 73 cms, the hundred-fold length 150 cms.

Higher harmonics will decay more rapidly.

CORRECTION OF CURRENT FEED WITH A LENS

An alternative method of correcting current feed non-uniformities, was suggested by Reynolds et al⁽¹⁾ and was analysed by Whipple⁽⁴⁾. Their suggestion was to insert, across the collector, a region of increased gap width one of whose boundaries is shaped. The transition across a discontinuity in gap width is given by the equation:-

$$h_1 H_{n1} = h_2 H_{n2}$$

$$H_{t1} = H_{t2}$$

where the suffixes n and t denote values normal to and tangential to the boundary. (These equations are only correct at such a distance from the discontinuity that the local H_z can be neglected).

Whipple has shown that choosing the correct shape for the boundary can make the emergent current lines parallel. Closing the lens with a straight boundary then leads to no further change in field shape. If the first harmonic non-uniformity which one wishes to smooth out has a fractional amplitude ϵ , then the shape of the boundary required to remove this is

$$y = 2 \epsilon a \frac{h_1}{h_1 - h_2} \cos \frac{2 \pi x}{a} .$$

(There will be residual non-uniformities of order ϵ^2 .) The inductance of such a lens will be of order

$$4 \frac{\pi y h_2}{a} \times 10^{-9} \text{ henries ,}$$

or, for

$$L_2 \gg L_1 ,$$

$$8 \pi \epsilon h_1 \times 10^{-9} \text{ henries .}$$

It is interesting to compare this with the inductance of a tab needed to reduce the non-uniformity of amplitude ϵ by a factor $1/\epsilon$. Suppose ϵ is 10% then the tab length will be $\sim \frac{a}{\pi}$, and the inductance will be $2 h_1$ nanohenries, this compares with πh_1 nanohenries for a lens. (There may be more waste inductance on a lens.)

THE PROBLEM OF NON-UNIFORM CURRENTS IN THE COIL

ORIGINS OF NON-UNIFORMITY

In the previous section we have considered methods of minimising non-uniformities arising from the feed side of the collector system in order to provide a uniform current

distribution in a coil. However, in practice, the current distribution at the coil is non-uniform. There are several reasons for this, amongst which the following are important: (cf Reynolds et al⁽¹⁾).

- (i) End-effect. As shown by Aldridge⁽³⁾ the curvature of field at the ends of a coil produces a peak in current density at the ends.
- (ii) Mirror current. Increasing the field strength at the ends to form a mirror by restricting the area through which the flux passes, requires a higher current density in these regions.
- (iii) Plasma effect. The existence of plasma in the tube, which is non-uniform along the length, causes a variation in the coil inductance along the coil length and so alters the current distribution.
- (iv) External currents. As shown by Aldridge⁽³⁾ the return flux on the outside of the coil requires a larger current than does the return flux on the outside of the collector.

PRODUCTION OF NON-UNIFORM CURRENT DISTRIBUTIONS

Production of the non-uniform current distribution required by effects (i) and (ii) should in principle be possible using a lens to distort a uniform distribution. Here we think it is important to draw a distinction between a lens which straightens the non-uniform feed current distribution and one which distorts the uniform distribution which is so produced. Of course since only one boundary of a lens, as described above, is actually used for current shaping, one can use both boundaries of one lens, shaping them independently to perform the separate functions.

The shape of a boundary needed to distort the current flow, can be calculated by repeating Whipple's calculations with a reversed direction of current flow. There is an important difference from Whipple's calculation in that the non-uniform current distribution produced by the boundary changes as it propagates away from the boundary. Consequently the 'strength' of the lens must be greater the farther it is from the coil.

Taking this into account, it is possible to calculate the shape of lens required to provide the current and voltage distribution needed at the coil, and so produce in the coil an axi-symmetric magnetic field.

This would be true if only effects (i) and (ii) were important. However, effect (iv) requires the transfer of current from the tab inside to the coil outside. The analysis of such a situation is very difficult but some points can be derived by the analogy of a rectangular section coil which is longer than the tab (Fig.2). The essential point about such an analogue is that it brings out the importance of the boundary condition at the coil wall opposite the slot in the coil where one connects the tab. At this wall V must be

zero, and J_y is specified. Together these specify each β_n in the coil region and so determine the variation with y of j_x, j_y in the coil, together with their amplitudes.

Each harmonic will vary with y monotonically as $\cosh nky$ so that there cannot be a magnetic well in y at one given y value. The transfer currents can be represented by these harmonics; since they vary as $\cosh nky$ they will flow near the slot.

The obvious limitation of this model is that it neglects H_z whilst one knows that it is the three dimensional effect which gives rise to the existence of the wells. However it would be surprising if inclusion of this effect would allow one adequate freedom to specify j_y at the slot to be the same as that at the opposite wall when it is known that current must be transferred to the outside of the coil near this slot.

This lack of symmetry between the slot and the opposite wall of the coil is even more important when one considers that the plasma changes the coil inductance and may make the inductance vary along the length, so varying the current distribution with time. Again this is difficult to analyse, but the rectangular coil analogue can be used to show that there will be transfer currents (i.e. j_x) and magnetic field curvature.

PROPOSAL FOR MINIMISING FIELD ASYMMETRIES

It is clear that it is difficult to produce an axi-symmetric magnetic field in a single turn coil, instead it is necessary to consider how to minimise the non-uniformities which arise. We propose the following:

(i) Control of Feed Non-Uniformities

We suggest that it is preferable to use a long tab rather than a lens. If the 'crowbar' systems are applied to the Megajoule bank experiment they will produce time dependent feed current non-uniformities, which cannot be corrected with a lens. A long tab will minimise the effect of such variations.

For example, a square tab reduces the amplitude of the first harmonic of the current distribution by three orders of magnitude. From figures quoted for the Megajoule bank experiment, this would reduce the field gradient to less than 1/10th of a gauss/cm.

(ii) Minimising Effect of Current Non-Uniformities at the Coil

Since axi-symmetry cannot be obtained the next best is to aim for planar symmetry. This exists about a horizontal plane. It can also be produced about a vertical plane using a dummy tab connected to the coil directly opposite the feed tab (Fig.3). The presence of a long dummy tab could be a severe handicap to the arrangement of diagnostics so that it is important to determine its minimum useful length from experiments.

This tab permits one to have a magnetic axis which is coincident with the coil axis, and symmetry about the vertical and horizontal planes through that axis. However there will be an azimuthal variation in the strength of the magnetic field which may give rise to instabilities⁽⁵⁾.

One source of these asymmetries is the transfer current to the outer coil surface. This effect could be reduced by slotting the coil in the direction of current flow at intervals of about a coil diameter along the coil length (Fig.3(a)). Reynolds, et al⁽¹⁾ and Kolb⁽⁶⁾ report that such slotting can affect plasma drift.

Experiments with a resistive mat analogue (Appendix 1) and with a short single turn coil are in progress to investigate these problems. It is hoped that the data obtained will allow us to specify the required length of dummy tab and to evaluate the effect of slotting.

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APPENDIX I

DEMONSTRATION OF THE PROPERTIES OF A DUMMY TAB USING A RESISTIVE ANALOGUE

The equations which relate voltage and current in the two dimensional inductive system are also applicable, in principle, to a two dimensional resistive system. The inductive impedance per unit square, ωh , is replaced by the resistance per unit square, giving:

$$\begin{aligned} \frac{\partial V}{\partial x} &= R j_x \\ \frac{\partial V}{\partial y} &= R j_y \end{aligned} \quad \text{cf. equation 1} \quad \dots (6)$$

This permits some properties of inductive systems to be investigated with a resistive analogue. In particular it is possible to produce an analogue with which one can demonstrate the improvement of the magnetic field asymmetry in a coil using a dummy tab, as previously discussed in this report.

In the experiment we have performed, an analogue was made from a resistive fibre glass material and shaped as shown in Fig.4. There are three main sections, the normal tab which is long compared with its width and which produces a nearly uniform current feed, the dummy tab whose length can be varied by adjusting a shorting bar, and the coil. The coil is made wider than the tabs in order to produce a transfer current which simulates those, which flow from inside the collector to the outside of the coil as described by Aldridge⁽³⁾. This transfer current, a j_x current, produces non-uniformities in j_y and hence magnetic field asymmetries in a normal coil.

Two problems arise in making a precise analogue. In the coil the magnetic field is determined by three dimensional effects, and also the inductance per unit area varies, unlike the analogue whose resistance per unit area is constant. However these factors only alter the magnitudes of the asymmetries, the attenuating factor of the tab being unaffected.

Measurements were made of the voltage difference, ΔV , between two points, x_1 and x_2 in Fig.4, on the axis of the coil. This value ΔV is a measure of the asymmetry.

$$j_x = \frac{1}{R} \frac{\partial V}{\partial x} \approx \frac{1}{R} \frac{\Delta V}{x_2 - x_1}$$

Fig.5 shows a plot of ΔV against the length of the dummy tab. There are two interesting features of this result. If the shorting bar is moved from the back of the coil, position (a) Fig.4, to the start of the dummy tab, position (b) Fig.4, ΔV decreases by a factor of 5. As the length of the tab is increased, ΔV decreases exponentially as predicted by equation 5

in the main test. The experimental value of the exponent is 2.4 cms, as compared with the theoretical value of 2.43 cms, this being the half width of the coil divided by π .

The improvement in the current flow asymmetry is demonstrated clearly by plotting equipotential points. These were plotted using mapping pins, then photographed as in Fig.6. The shorting bar, which defines an equipotential line, was placed at positions (a) (b) and (c), as in Fig.4, the length of the tab in position (c) being equal to the coil diameter. Particularly apparent is the effect of moving the shorting bar from position (a) to position (b). This effect arises because of the change in boundary conditions. When the shorting bar is at (a), ΔV is zero from R to R', so that j_x is zero and j_y is everywhere normal to R R'. Consequently current must diverge continuously from TT' to RR'. However when the shorting bar is at (b) ΔV is zero only from S to S' and j_y is zero in the regions RS and R'S'. Therefore current must diverge from TT' and converge again to SS'.

SUMMARY

The investigation of the dummy tab properties with a resistive analogue, shows that the asymmetries in a coil decrease when a dummy tab is incorporated opposite the feed tab and that the magnitude of the asymmetry decreases exponentially as the length of the dummy tab is increased. This result was predicted by the theory presented in the previous text. This allows us to evaluate the usefulness of a dummy tab in a magnetic field coil, within the limitations of the analogue approximation.

Furthermore it has been found that moving the boundary condition from the coil to the tab, considerably changes the asymmetry. The usefulness of changing the boundary conditions depends upon the current flow pattern in a real thetatron coil, which will be very different from that in the two dimensional resistive analogue.

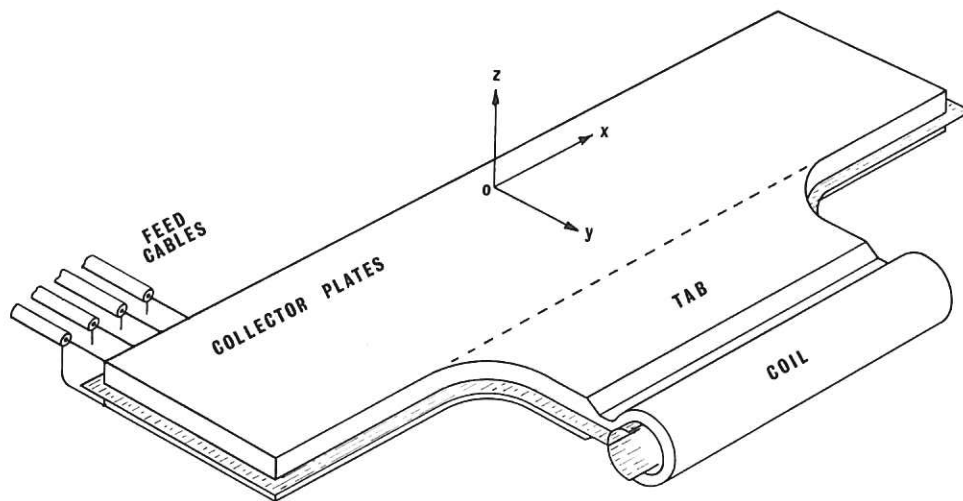


Fig.1 (CLM-M43)
Schematic of megajoule collector and coil assembly

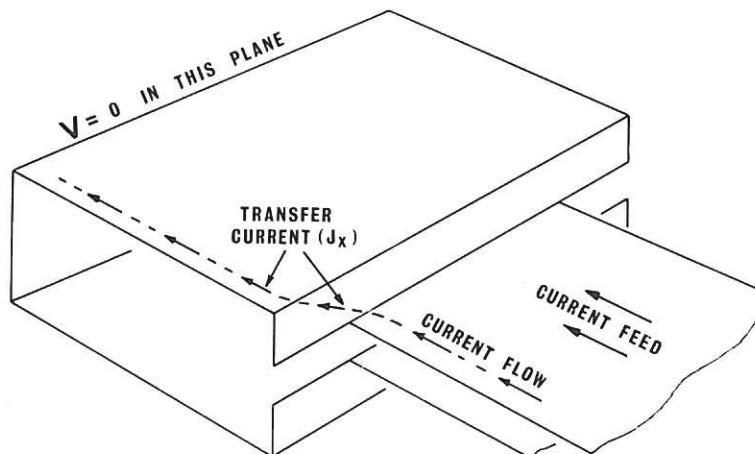


Fig.2 (CLM-M43)
Model of transfer current with rectangular section coil wider than tab

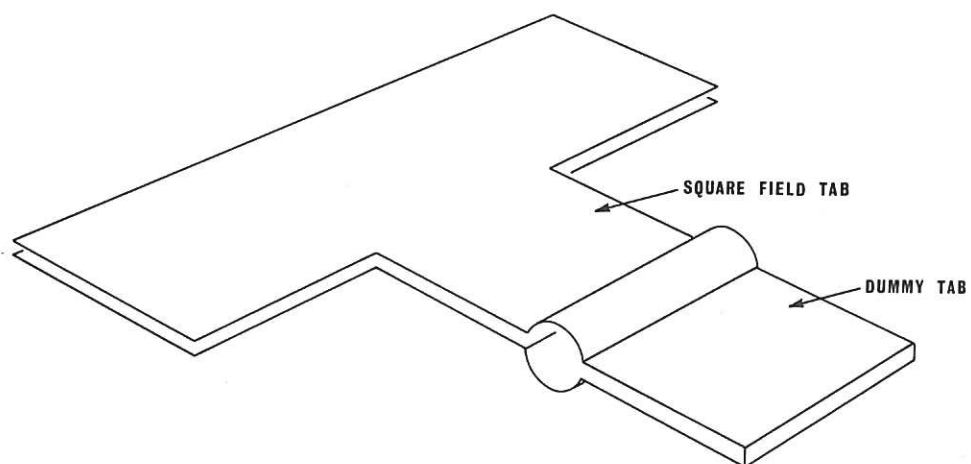


Fig. 3(a) Position of dummy tab (CLM-M43)

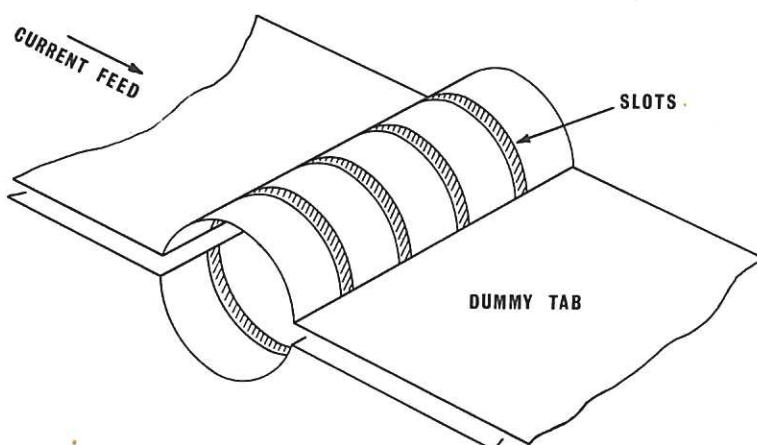


Fig. 3(b) Slotting of coil. (Suggested modifications of coil-collector assembly.) (CLM-M43)

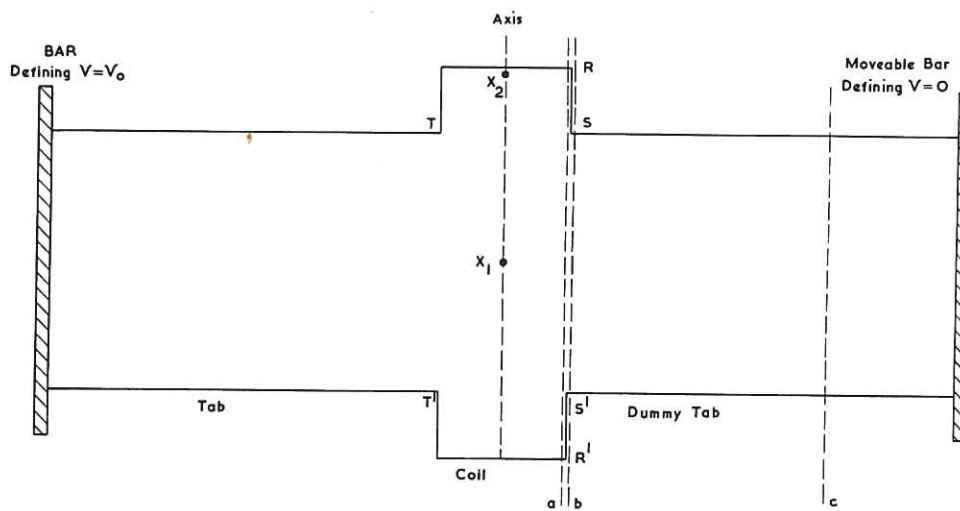


Fig. 4 (CLM-M 43)
Configuration of tab-coil-dummy tab analogue

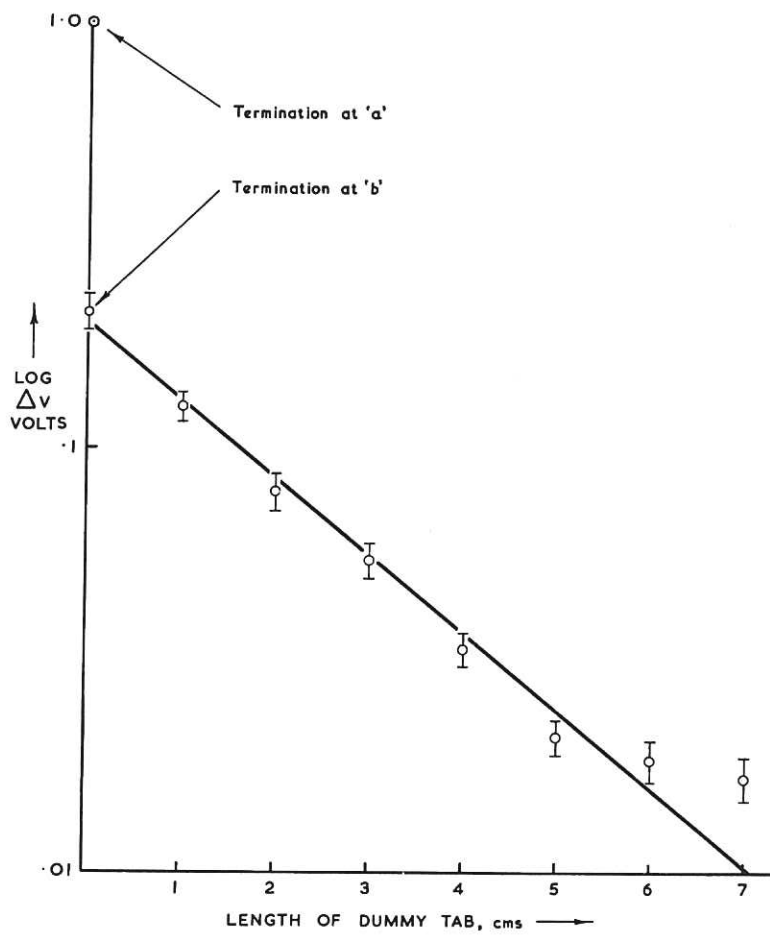
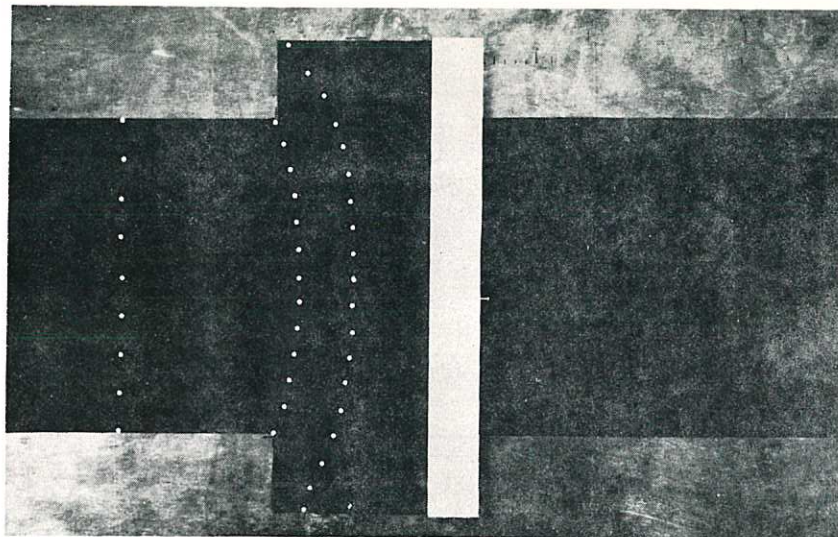
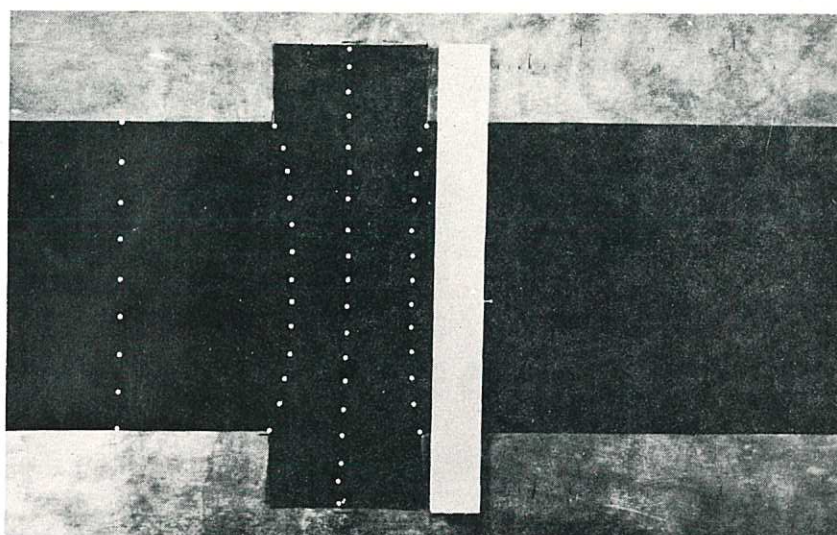


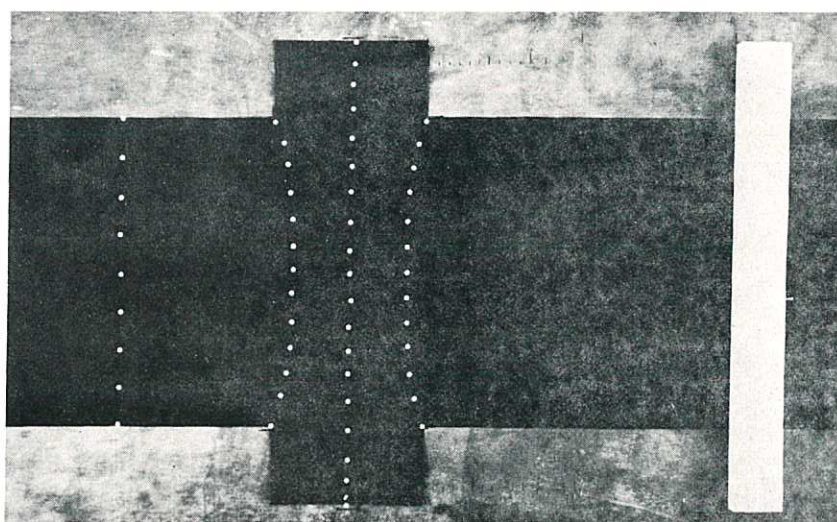
Fig. 5 (CLM-M 43)
Asymmetry voltage / length of dummy tab



(a) No dummy tab



(b) Short but finite dummy tab



(c) Dummy tab length equal to coil width

Fig. 6 (CLM-M43)
Equipotentials in analogue of tab-coil-dummy tab system

