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THE EQUATIONS OF MAGNETOHYDRODYNAMICS USING MAGNETIC FIELD COORDINATES AND CURVATURE COEFFICIENTS

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A B S T R A C T

The equations of magnetohydrodynamics (perfect conductivity, isotropic pressure and zero Larmor radius approximation) for general equilibrium and linear stability theory are written in component form for the three orthogonal directions, $\underset{\sim}{B}$, $\underset{\sim}{\nabla}p$ and $\underset{\sim}{B} \wedge \underset{\sim}{\nabla}p$. Use is also made of the curvatures and torsions of the surfaces containing pairs of these vectors. A large reduction in the numbers of terms in the equations results and the remaining terms generally have a simple character, which is an aid to the understanding of their physical content for complex configurations. The equations lead to a general and simple form for the requirement of current parallel to the magnetic field for toroidal equilibrium, and also to a simple general definition for the local magnetic shear.

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C O N T E N T S

1. INTRODUCTION

In problems dealing with complex spatial configurations, it is normal to look for apropriate coordinate systems which will simplify the equations involved. In general magnetohydrodynamic equilibria, where no axes of symmetry occur or where no simple shape is specified for a containing vessel, one of the few remaining natural directions is that of the equilibrium magnetic field ($\underset{\sim}{B}$) and several workers[1] have found it advantageous to use $\underset{\sim}{B}$ to define one coordinate direction. In the case of isotropic pressure another natural direction is that of the equilibrium pressure gradient ($\underset{\sim}{\nabla}p$) and since p is a scalar, the surfaces of constant pressure (i.e. the magnetic surfaces) form natural coordinate surfaces[2].

In this paper both these natural directions and the third orthogonal direction, that of $\underset{\sim}{B} \wedge \underset{\sim}{\nabla}p$ are used as coordinate directions. A large reduction in the number of terms in the various equations results. Since the coordinate directions are curved the differential vector functions introduce terms involving the curvature and torsion of the magnetic surface and the surfaces containing $\underset{\sim}{B}$ and $\underset{\sim}{\nabla}p$, and $\underset{\sim}{B} \wedge \underset{\sim}{\nabla}p$ and $\underset{\sim}{\nabla}p$. Some of these are of fundamental importance to stability[3]; most of the others are related to simple gradients of equilibrium quantities.

Not least of the advantages of using these natural coordinates is that the simplicity and low number of the terms in the equations greatly facilitates the understanding of their physical content. The equations lead immediately to a general and simple form for the requirement of current parallel to the magnetic field for equilibrium. In addition a general definition for local magnetic shear becomes obvious from the equations.

2. NOTATION

The unit vector normal to the equilibrium magnetic surface in the outward direction is denoted by $\underset{\sim}{i}_n$, the unit vector parallel to the equilibrium magnetic field (B) by $\underset{\sim}{i}_\parallel$ and $\underset{\sim}{i}_\perp$ is defined as $\underset{\sim}{i}_\parallel \wedge \underset{\sim}{i}_n$. The set $\underset{\sim}{i}_n \underset{\sim}{i}_\perp, \underset{\sim}{i}_\parallel$ is right-handed and vector components in these directions will be denoted by the subscripts n, $\perp$, $\parallel$, respectively.

A component of the gradient operator $\underset{\sim}{i}_j \cdot \underset{\sim}{\nabla}$ will be written as $\frac{\partial}{\partial x_j}$. The various curvature terms which appear will be denoted by the radii of curvature R_{jk} where

$$\frac{1}{R_{jk}} \equiv \underset{\sim}{i}_j \cdot \underset{\sim}{i}_j \cdot \underset{\sim}{\nabla}\underset{\sim}{i}_k \equiv \underset{\sim}{i}_j \cdot \frac{\partial \underset{\sim}{i}_k}{\partial x_j} = - \underset{\sim}{i}_k \cdot \underset{\sim}{i}_j \cdot \underset{\sim}{\nabla}\underset{\sim}{i}_j \equiv - \underset{\sim}{i}_k \cdot \frac{\partial \underset{\sim}{i}_j}{\partial x_j} \qquad \ldots (1)$$

The torsions which occur are denoted by

$$\frac{1}{T_{jk}} = \underset{\sim}{i}_j \cdot \underset{\sim}{i}_j \cdot \underset{\sim}{\nabla}\underset{\sim}{i}_k \equiv \underset{\sim}{i}_j \cdot \frac{\partial \underset{\sim}{i}_k}{\partial x_j}$$

$$= -\frac{1}{T_{kj}} = -\underset{\sim}{i}_k \cdot \underset{\sim}{i}_j \cdot \underset{\sim}{\nabla}\underset{\sim}{i}_j \cdot \equiv -\underset{\sim}{i}_k \cdot \frac{\partial \underset{\sim}{i}_j}{\partial x_j} \qquad \ldots (2)$$

(The $1/R_{jk}$ and $1/T_{jk}$ are of course closely related to the more general Christoffel symbols.)

Because $\underset{\sim}{i}_n$ is always perpendicular to the surface containing $\underset{\sim}{i}_\perp$, $\underset{\sim}{i}_\parallel$, the two geodesic torsions of the magnetic surface are related[4]

$$T_{n\perp} = T_{n\parallel} \qquad \ldots (3)$$

but there is no corresponding relation with $T_{\perp\parallel}$.

In those cases where coordinates are required for integration the three coordinates which can be used are

$$\psi, \ \alpha = \int_0^{x_\perp} \frac{B}{|\nabla\psi|} dx_\perp \quad \text{and} \quad \varphi = \int_0^{x_\parallel} B dx_\parallel$$

where ψ labels the magnetic surface and is the flux going the short way round within the magnetic surface[5]. For a given magnetic surface, $x_\parallel$ is the length measured along a magnetic line of force from a particular $\perp$ line which defines the zero of $x_\parallel$. Similarly $x_\perp$ is the distance along a $\perp$ line from the zero magnetic line of force. The Jacobian is

$$J = 1/\left(|\nabla\psi| \frac{\partial\alpha}{\partial x_\perp} \frac{\partial\varphi}{\partial x_\parallel} \right) = \frac{1}{B^2} \qquad \ldots (4)$$

The coordinates ψ, α are line labelling coordinates corresponding to the α, β coordinates of Taylor (see reference 1).

Rationalised electromagnetic units are used throughout.

3. THE DIFFERENTIAL VECTOR FUNCTIONS

If $\underset{\sim}{F}$ is any vector

$$\nabla\underset{\sim}{F} = (F_n \underset{\sim}{i}_n + F_\perp \underset{\sim}{i}_\perp + F_\parallel \underset{\sim}{i}_\parallel)$$

$$= \Sigma_j \, (\underset{\sim}{i}_j \underset{\sim}{\nabla} F_j + F_j \underset{\sim}{\nabla}\underset{\sim}{i}_j)$$

$$= \Sigma_j \left(\underset{\sim}{i}_j \underset{\sim}{\nabla} F_j + F_j \left[\underset{\sim}{i}_n \frac{\partial \underset{\sim}{i}_j}{\partial x_n} + \underset{\sim}{i}_\perp \frac{\partial \underset{\sim}{i}_j}{\partial x_\perp} + \underset{\sim}{i}_\parallel \frac{\partial \underset{\sim}{i}_j}{\partial x_\parallel} \right] \right)$$

and since for example

$$\frac{\partial \underset{\sim}{i}_n}{\partial x_n} = \underset{\sim}{i}_\perp \left(\underset{\sim}{i}_\perp \cdot \frac{\partial \underset{\sim}{i}_n}{\partial x_n}\right) + \underset{\sim}{i}_\parallel \left(\underset{\sim}{i}_\parallel \frac{\partial \underset{\sim}{i}_n}{\partial x_n}\right)$$

$$= - \frac{\underset{\sim}{i}_\perp}{R_{n\perp}} - \frac{\underset{\sim}{i}_\parallel}{R_{n\parallel}}$$

and

$$\frac{\partial \underset{\sim}{i}_\perp}{\partial x_n} = \underset{\sim}{i}_\parallel \left(\underset{\sim}{i}_\parallel \cdot \frac{\partial \underset{\sim}{i}_\perp}{\partial x_n}\right) + \underset{\sim}{i}_n \left(\underset{\sim}{i}_n \cdot \frac{\partial \underset{\sim}{i}_\perp}{\partial x_n}\right)$$

$$= \frac{\underset{\sim}{i}_\parallel}{T_{\parallel\perp}} + \frac{\underset{\sim}{i}_n}{R_{n\perp}}$$

the array making up $\nabla \underset{\sim}{F}$ is

$$\nabla \underset{\sim}{F} = \left| \begin{array}{lll} \frac{\partial F_n}{\partial x_n} + \frac{F_\perp}{R_{n\perp}} + \frac{F_\parallel}{R_{n\parallel}}, & \frac{\partial F_\perp}{\partial x_n} + \frac{F_\parallel}{T_{\perp\parallel}} - \frac{F_n}{R_{n\perp}}, & \frac{\partial F_\parallel}{\partial x_n} + \frac{F_\perp}{T_{\parallel\perp}} - \frac{F_n}{R_{n\parallel}} \\ \frac{\partial F_n}{\partial x_\perp} + \frac{F_\parallel}{T_{n\parallel}} - \frac{F_\perp}{R_{\perp n}}, & \frac{\partial F_\perp}{\partial x_\perp} + \frac{F_\parallel}{R_{\perp\parallel}} + \frac{F_n}{R_{\perp n}}, & \frac{\partial F_\parallel}{\partial x_\perp} + \frac{F_n}{T_{\parallel n}} - \frac{F_\perp}{R_{\perp\parallel}} \\ \frac{\partial F_n}{\partial x_\parallel} + \frac{F_\perp}{T_{n\perp}} - \frac{F_\parallel}{R_{\parallel n}}, & \frac{\partial F_\perp}{\partial x_\parallel} + \frac{F_n}{T_{\perp n}} - \frac{F_\parallel}{R_{\parallel\perp}}, & \frac{\partial F_\parallel}{\partial x_\parallel} + \frac{F_n}{R_{\parallel n}} + \frac{F_\perp}{R_{\parallel\perp}} \end{array} \right| \quad \ldots (5)$$

If the particular component of $\nabla \underset{\sim}{F}$ containing $\frac{\partial F_j}{\partial x_k}$ is denoted by A_{jk} then

$$\nabla \cdot \underset{\sim}{F} = A_{nn} + A_{\perp\perp} + A_{\parallel\parallel} \quad \ldots (6)$$

and

$$\nabla \wedge \underset{\sim}{F} = \underset{\sim}{i}_n (A_{\parallel\perp} - A_{\perp\parallel}) + \underset{\sim}{i}_\perp (A_{n_\parallel} - A_{\parallel n}) + \underset{\sim}{i}_\parallel (A_{\perp n} - A_{n_\perp}) \quad \ldots (7)$$

4. EQUILIBRIUM EQUATIONS

The components of the equilibrium quantities are clearly

$$\underset{\sim}{B} \equiv (0, 0, B)$$

$$\underset{\sim}{j} = (0, j_\perp, j_\parallel)$$

$$\nabla p = \left(\frac{dp}{dx_n}, 0, 0\right)$$

and hence the equilibrium equations have the following components

$\underset{\sim}{j} = \nabla \wedge \underset{\sim}{B}$

$$0 = j_n = \frac{\partial B}{\partial x_\perp} + \frac{B}{R_{\parallel\perp}} \quad \ldots (8)$$

$$j_\perp = -\frac{\partial B}{\partial x_n} - \frac{R}{R_{\parallel n}} \quad \ldots (9)$$

$$j_\parallel = \frac{B}{T_{\perp\parallel}} - \frac{B}{T_{n\parallel}} \quad \ldots (10)$$

$\nabla \cdot \underset{\sim}{B} = 0$

$$\frac{\partial B}{\partial x_{\parallel}} + \frac{B}{R_{n_{\parallel}}} + \frac{B}{R_{\perp\parallel}} = 0 \qquad \ldots (11)$$

$\nabla \cdot \underset{\sim}{j} = 0$

$$\frac{\partial j_{\parallel}}{\partial x_{\parallel}} + \frac{j_{\parallel}}{R_{n_{\parallel}}} + \frac{j_{\parallel}}{1/R_{\perp\parallel}} + \frac{\partial j_{\perp}}{\partial x_{\perp}} + \frac{j_{\perp}}{R_{n_{\perp}}} + \frac{j_{\perp}}{R_{\parallel\perp}} = 0 \qquad \ldots (12)$$

$\underset{\sim}{j} \wedge \underset{\sim}{B} = \nabla p$

$$j_{\perp} B = \frac{\partial p}{\partial x_n} \qquad \ldots (13)$$

$\nabla \wedge (\underset{\sim}{j} \wedge \underset{\sim}{B}) = 0$

$\perp$ component

$$\frac{\partial (j_{\perp} B)}{\partial x_{\parallel}} + \frac{j_{\perp} B}{R_{n_{\parallel}}} = 0 \qquad \ldots (14)$$

$\parallel$ component

$$\frac{\partial (j_{\perp} B)}{\partial x_{\perp}} + \frac{j_{\perp}}{R_{n_{\perp}}} = 0 \qquad \ldots (15)$$

Other Relationships

From (11) and (14)

$$\frac{\partial j_{\perp}}{\partial x_{\parallel}} = \frac{j_{\perp}}{R_{\perp\parallel}} \qquad \ldots (16)$$

and from (8), (12) and (15)

$$\nabla \cdot (j_{\perp} \underset{\sim}{i}_{\perp}) = \frac{2 j_{\perp}}{R_{\parallel\perp}} \qquad \ldots (17)$$

$1/R_{\parallel\perp}$ is the geodesic curvature of the magnetic surface parallel to $\underline{B}$, which is the same thing as the curvature of $\underline{B}$ in the plane of the magnetic surface. Thus the general form of the well known condition that a $j_{\parallel}$ is required for equilibrium in a torus[6] is the following theorem:-

Theorem

If in a magnetohydrodynamic equilibrium the magnetic field has any curvature in the plane of the magnetic surface, a non-zero $j_{\parallel}$ is required for that equilibrium.

The Curvatures and Torsions

From the above equations, five of the six curvatures $1/R_{jk}$ have simple relationships with equilibrium quantities. Thus

$$\frac{1}{R_{n_\perp}} = -\frac{1}{|\nabla p|}\frac{\partial|\nabla p|}{\partial x_\perp} = -\frac{1}{|\nabla\psi|}\frac{\partial|\nabla\psi|}{\partial x_\perp}$$

$$\frac{1}{R_{n_\parallel}} = -\frac{1}{|\nabla p|}\frac{\partial|\nabla p|}{\partial x_\parallel} = -\frac{1}{|\nabla\psi|}\frac{\partial|\nabla\psi|}{\partial x_\parallel}$$

$$\frac{1}{R_{\perp\parallel}} = \frac{1}{j_\perp}\frac{\partial j_\perp}{\partial x_\parallel} = -\frac{1}{\underset{\sim}{i}_\perp\cdot\nabla\alpha}\frac{\partial(\underset{\sim}{i}\cdot\nabla\alpha)}{\partial x_\parallel}$$ (geodesic curvature of the magnetic surface in the $\perp$ direction.)

$$\frac{1}{R_{\perp n}}$$ (normal curvature of magnetic surface in the $\perp$ direction.)

$$\frac{1}{R_{\parallel n}} = -\frac{1}{B}\frac{\partial B}{\partial x_n} - \left(\frac{\beta}{2}\right)\frac{1}{p}\frac{\partial p}{\partial x_n}$$ (normal curvature of magnetic surface in $\parallel$ direction.)

$$\frac{1}{R_{\parallel\perp}} = -\frac{1}{B}\frac{\partial B}{\partial x_\perp}$$ (geodesic curvature of magnetic surface in $\parallel$ direction.)

... (18)

where the relations involving $\nabla\alpha$, $\nabla\psi$ follow because $|\nabla\psi|$ is proportional to $|\nabla p|$ for gradients in the $\perp$ and $\parallel$ directions.

Since there are only two independent torsions, the following simpler notation is introduced

$$T_M \equiv T_{n_\parallel} = T_{n_\perp} = -T_{\parallel n} = -T_{\perp n}$$
$$T_n \equiv T_{\perp\parallel} = -T_{\parallel\perp} \qquad \ldots (19)$$

T_M is a measure of the geodesic torsion of the magnetic surface in the $\parallel$ and $\perp$ directions. T_n measures the rotation of $\underset{\sim}{B}$ in the plane of the magnetic surface as one proceeds along $\underset{\sim}{i}_n$. For reasons given in Section 6 below, the sum of these two torsions is defined as the <u>local magnetic shear</u> S.

$$S \equiv \frac{1}{T_M} + \frac{1}{T_n} \qquad \ldots (20)$$

whence from (10), (19) and (20)

$$\frac{1}{T_n} = \tfrac{1}{2}\left(S + \frac{j_\parallel}{B}\right) \qquad \ldots (21)$$

$$\frac{1}{T_M} = \tfrac{1}{2}\left(S - \frac{j_\parallel}{B}\right) \qquad \ldots (22)$$

5. LINEARISED STABILITY EQUATIONS

Assuming an infinitesimal plasma displacement $\underset{\sim}{\xi}$, the components of the MHD equations when linearised are

$$\rho\frac{\partial^2\xi_n}{\partial t^2} = \delta j_\perp B + j_\perp\delta B_\parallel - j_\parallel\delta B_\perp - \frac{\partial\delta p}{\partial x_n} \equiv F_n \qquad \ldots (23)$$

$$\rho \frac{\partial^2 \xi_\perp}{\partial t^2} = j_\parallel \delta B_n - \delta j_n B - \frac{\partial \delta p}{\partial x_\perp} = F_\perp \qquad \dots (24)$$

$$\rho \frac{\partial^2 \xi_\parallel}{\partial t^2} = - j_\perp \delta B_n - \frac{\partial \delta p}{\partial x_\parallel} = F_\parallel \qquad \dots (25)$$

$$\delta p = - \xi_n \frac{\partial p}{\partial x_n} - \gamma p \underset{\sim}{\nabla} \cdot \underset{\sim}{\xi} \qquad \dots (26)$$

$$\delta \underset{\sim}{B} = \underset{\sim}{\nabla} \wedge (\underset{\sim}{\xi} \wedge \underset{\sim}{B}) \qquad \dots (27)$$

Writing out the components of $\delta \underset{\sim}{B}$ with the aid of (5) and the various equilibrium relations in section 4,

$$\delta B_n = - \frac{\xi_n B}{R_{n\parallel}} + B \frac{\partial \xi_n}{\partial x_\parallel}$$

$$= \frac{B}{|\nabla \psi|} \frac{\partial (\xi_n |\nabla \psi|)}{\partial x_\parallel} \qquad \dots (28)$$

$$\delta B_\perp = - \frac{\xi_n B}{T_n} - \frac{\xi_\perp B}{R_{\perp\parallel}} + B \frac{\partial \xi_\perp}{\partial x_\parallel} - B \frac{\xi_n}{T_M}$$

$$= - \xi_n \left(j_\parallel + \frac{2B}{T_M}\right) + \frac{\xi_\perp B}{R_{\perp\parallel}} + B \frac{\partial \xi_\perp}{\partial x_\parallel}$$

$$= - \xi_n S B + \frac{B}{\left(\frac{\partial \alpha}{\partial x_\perp}\right)} \frac{\partial \left(\xi_\perp \frac{\partial \alpha}{\partial x_\perp}\right)}{\partial x_\parallel} \qquad \dots (29)$$

$$\delta B_\parallel = - \left(\sum_i \xi_i \frac{\partial B}{\partial x_i}\right) + B \frac{\partial \xi_\parallel}{\partial x_\parallel} + \frac{B \xi_n}{R_{\parallel n}} + \frac{B \xi_\perp}{R_{\parallel\perp}} - B \underset{\sim}{\nabla} \cdot \underset{\sim}{\xi}$$

$$= \xi_n \left(j_\perp + \frac{2B}{R_{\parallel n}}\right) + \frac{2 \xi_\perp B}{R_{\parallel\perp}} - B \underset{\sim}{\nabla} \cdot (\underset{\sim}{i}_n \xi_n + \underset{\sim}{i}_\perp \xi_\perp) \qquad \dots (30)$$

With the aid of (26), (28), (29) and (30) and the relations of section 4, the following forms for the components $\underset{\sim}{F}$ can be obtained

$$F_n = \frac{\partial}{\partial x_n} (\gamma p \underset{\sim}{\nabla} \cdot \underset{\sim}{\xi}) + B |\underset{\sim}{\nabla} \wedge \underset{\sim}{Y}|_\perp - \underset{\sim}{j} \wedge \underset{\sim}{i}_n \cdot \underset{\sim}{Y} - 2 \xi_n B \left(\frac{j_\parallel}{T_M} + \frac{j_\perp}{R_{\parallel n}} \right) \qquad \dots (31)$$

$$F_\perp = \frac{\partial}{\partial x_n} (\gamma p \underset{\sim}{\nabla} \cdot \underset{\sim}{\xi}) - B |\underset{\sim}{\nabla} \wedge \underset{\sim}{Y}|_n \qquad \dots (32)$$

$$F_\parallel = \frac{\partial}{\partial x_\parallel} (\gamma p \underset{\sim}{\nabla} \cdot \underset{\sim}{\xi}) \qquad \dots (33)$$

where

$$\underset{\sim}{Y} = \delta \underset{\sim}{B} + \underset{\sim}{j} \wedge \underset{\sim}{i}_n \xi_n$$

This form of $\underset{\sim}{F}$, which has been given previously by Bineau[7], readily yields one of the standard forms of the energy principle[8]

$$\delta W = -\tfrac{1}{2}\int d\tau\, \underset{\sim}{\xi}\cdot \underset{\sim}{F}$$

$$= \tfrac{1}{2}\int d\tau \left[\underset{\sim}{Y}^2 + \gamma p\,(\nabla\cdot\xi)^2 + 2\xi_n^2\, B\left(\frac{j_{\parallel}}{T_M} + \frac{j_{\perp}}{R_{\parallel n}}\right)\right] \qquad \ldots (34)$$

This is one of the most useful forms of δW for toroidal stability[5]. The last term in the square brackets has been used to obtain sufficient conditions for stability against $j_{\parallel}$ - driven and $j_{\perp}$ - driven modes[3].

6. MAGNETIC SHEAR

From (29) it is seen that S is a measure of $\delta B_{\perp}/B$ produced per unit displacement ξ_n in the negative n-direction. For

$$\delta B_{\perp} = \delta B_{\perp}\,(\xi_n, \xi_{\perp})$$

the quantity S is defined by

$$S = \frac{1}{B}\,\frac{\partial\,\delta B_{\perp}}{\partial \xi_n}\ .$$

Apart from a multiplying factor (see below) this is one of the roles played by magnetic shear as defined for plane or cylindrical magnetic field configurations. For this reason it is proposed that S be called the local magnetic shear.

For cylindrically symmetric equilibria

$$S = \frac{j_{\parallel}}{B} - \frac{2B_\theta B_z}{rB^2} = \frac{B_\theta B_z}{B^2}\left(\frac{1}{\mu}\,\frac{d\mu}{dr}\right) \qquad \ldots (35)$$

where $\mu = B_\theta/rB_z$. Hence S contains the extra factor $B_\theta B_z/B^2$ compared with the normal definition of shear for a cylindrical plasma[9], namely $\frac{1}{\mu}\frac{d\mu}{dr}$. The form contained in S is considered preferable, not only because of its role in determining the $\delta B_{\perp}$ produced by ξ_n, but also because it measures more accurately the stabilising effect of magnetic shear for Suydam type perturbations, i.e. where $\underset{\sim}{\xi}$ is localised in the vicinity of the singular magnetic surface. For such a perturbation, the stabilising term is

$$\frac{\tfrac{1}{4}\xi_n^2}{(\frac{m}{r})^2 + k^2}\left(\frac{dk_{\parallel}B}{dr}\right)^2_{k_{\parallel}=0} = \tfrac{1}{4}\,\frac{B_\theta^2 B_z^2}{B^2}\left(\frac{1}{\mu}\,\frac{d\mu}{dr}\right)^2 = \tfrac{1}{4}\,S^2B^2$$

and Suydam's necessary stability condition[9] becomes

$$\tfrac{1}{4}\,S^2B^2 \geqslant -\frac{2B_\theta^2}{B^2}\,\frac{dp}{dr} = -\frac{2}{R_{\parallel n}}\,\frac{dp}{dr} \qquad \ldots (36)$$

where the right hand side is now proportional to the ∇p destabilising term, as seen for the expression for δW in equation (34).

For plane geometry (no field curvature) S reduces to

$$S = \frac{1}{T_n} = \frac{j_{\parallel}}{B} = \frac{1}{B}\left(\frac{d\underset{\sim}{k}\cdot\underset{\sim}{B}}{dy}\right)_{\underset{\sim}{k}\cdot\underset{\sim}{B}=0} = F' \qquad \ldots (37)$$

where the coordinates and nomenclature of the last two terms are those of Furth et al[10]. (The normalising field magnitude B used by Furth et al. has been taken as the local field magnitude).

In general equilibria, as opposed to cylindrically symmetric and plane systems, the presence of a non-zero S is not necessarily stabilising, since $\delta B_{\perp}$ can be zero if $\xi_{\perp}$ has a suitable variation along $\underset{\sim}{B}$ (see equation 29). This cancelling is possible, however, only if S is periodic along $\underset{\sim}{B}$, since the $\xi_{\perp}$ term is of necessity periodic. The important quantity is the average value of S; this is proportional to the gradient of the rotational transform.

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