NOT FOR PUBLICATION

CLM - M 55

TIME RESOLVED MEASUREMENTS OF THE RADIAL DENSITY DISTRIBUTION IN A THETA PINCH

by

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A B S T R A C T

The electron density distribution of the plasma is determined by measuring the intensity of the continuum radiation emitted in the visible with ten photomultipliers. These view the discharge radially via a fibre bundle array so that each photomultiplier collects the light along a chord of a circle. Assuming cylindrical symmetry, the density distribution is obtained using an Abel inversion; in addition the results give the line density and the variation of 'β' with radius. The measurement can be made at any point along the length of the discharge, and thus yields information on loss processes. This method has been used on the Megajoule Thetatron and typical results are shown to illustrate the technique.

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August 1965 (MEA)

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INTRODUCTION

In recent theta pinch experiments determination of the plasma density as a function of time has been effected by a variety of techniques:- measurement of plasma refractivity using a Mach-Zehnder interferometer[1,2]; measurement of the absolute bremsstrahlung continuum intensity in the visible[3,4]; analysis of scattered laser light[5]; and - in the pre-ionization phase - from the Stark width of suitable spectral lines[2,6,7]. Strictly speaking only the densities found from the 'side on' measurement of the continuum intensity have been radially resolved, since the radial resolution of the laser scattering results was obtained from successive discharges, and the interferometric measurements, being made 'end on', were integrated along the axis.

This paper describes an improved version of the technique described in reference (4) for making a spatially resolved determination of the electron density distribution from a measurement of the absolute continuum emission. The plasma - viewed radially through a slit in the coil - is imaged stigmatically, via a monochromator or a narrow bandwidth interference filter used to select a suitable line-free wavelength band, onto an array of 10 fibre bundles which guide the light to 10 photomultipliers. Thus each of the photomultipliers collects the light along one of 10 parallel chords. Assuming cylindrical symmetry of the plasma the radial distribution of the electron density can be determined from this measurement using an Abel inversion, and then correcting for the enhanced bremsstrahlung due to the presence of small quantities of impurities in the discharge.

As this measurement can be made at any point along the tube axis it provides information regarding the time variations in the radial density distribution at different points along the axis, and from this the line density, and the value of β (β is defined as the ratio of plasma pressure to magnetic field pressure) can be obtained. These yield information on the loss processes.

PHYSICAL PRINCIPLES OF THE METHOD

For plasma with temperatures of the order of several hundred electron volts the continuum emission in the visible can be considered as pure free-free (bremsstrahlung) radiation, since the contribution of free-bound (recombination) radiation is negligible. The intensity emitted from a plasma with ion and electron densities n_i and n_e, at an electron temperature T_e and with nuclear charge Z can be written[8]:

$$I_\lambda = \frac{32\pi^2}{3\sqrt{3}} \cdot \frac{e^6}{c^3(2\pi m)^{3/2}} \cdot \frac{n_e\, n_i Z^2}{(kT_e)^{1/2}} \cdot \frac{g}{\lambda^2}\, \exp.\left(\frac{-hc}{\lambda k T_e}\right) \text{ ergs } s^{-1} cm^{-3} sterad^{-1} cm^{-1} \quad \dots (1)$$

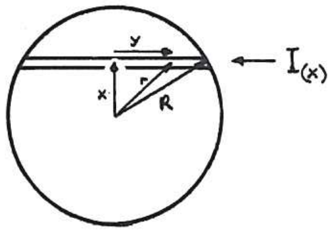
The quantity g is the Gaunt factor which represents the quantum mechanical deviation from Kramer's classical calculation and is tabulated in references (9) and (10). In the visible the exponential term is almost unity; thus for deuterium, where $Z = 1$ and $n_i = n_e = n$, equation (1) becomes:-

$$I_\lambda = A \cdot \frac{g}{\lambda^2} \cdot n^2 \; T_e^{-\frac{1}{2}} \text{ watts cm}^{-3} \text{sterad}^{-1} \text{ Å}^{-1} \qquad \ldots (2)$$

with $A = 1.5 \times 10^{-29}$ if T_e is in eV and λ in Å. For other elements the dependence on the Z^2 term becomes increasingly important. The effect of highly ionized impurities is discussed in the section on errors.

Equation (2) shows that the continuum intensity in the visible is proportional to the square of the electron density, and inversely proportional to the square root of the electron temperature. Therefore errors in the temperature have only a small effect on the accuracy of the density measurement.

The intensity recorded by each photomultiplier is integrated along a chord of circle. These intensities can be converted into a radial distribution only if the plasma has cylindrical symmetry. Assuming the plasma has cylindrical symmetry and is optically thin we have the following relations:



If $I_{(r)}$ is the intensity emitted per unit area at a given radius 'r' then $I_{(x)} = \Sigma I_{(r)}$ along the chord at (x)

$$= 2 \int_0^{\sqrt{R^2 - x^2}} I_{(r)} \, dy \qquad \ldots (3)$$

$y = (r^2 - x^2)^{\frac{1}{2}}$ from $y = 0$, $(r = x)$ to $y = (R^2 - x^2)^{\frac{1}{2}}$, $(r = R)$ and $dy = (r^2 - x^2)^{-\frac{1}{2}} \, r dr$

Hence

$$I_{(x)} = 2 \int_x^R \frac{I_{(r)} r \, dr}{(r^2 - x^2)^{\frac{1}{2}}} \qquad \ldots (4)$$

This is a form of Abel's integral equation whose formal solution is

$$I_{(r)} = -\frac{1}{\pi} \int_r^R \frac{I'_{(x)} \, dx}{(x^2 - r^2)^{\frac{1}{2}}} \qquad \ldots (5)$$

It can be solved numerically for each set of values of $I(x)$ by means of a resistive analogue or a digital computer. Both have been used but all results published were obtained using a digital computer.

EXPERIMENTAL TECHNIQUE

A. LINE-FREE WAVELENGTH BAND

In order to find a line-free wavelength band for the continuum emission measurement a spectrum of the plasma was recorded initially with a Hilger large spectrograph. An analysis of the spectrum showed that line-free regions occur at 4976^{+50}_{-20} Å, 4763 ± 10 Å and at 4526 ± 20 Å. Therefore for all experiments on the main discharge of the Megajoule Theta-pinch a wavelength band of 10 Å centred at 4976 Å was used. For the much lower energy of the pre-ionization discharge where the hydrogen had not yet been completely dissociated the region at 4526 Å is more suitable since a strong line of molecular hydrogen shows up at 4973 Å. As an additional check that the chosen bandwidth is free of impurity lines the bandwidth was altered. It proved that the measured intensity varied linearly with the bandwidth.

B. SPATIALLY RESOLVED MEASUREMENT

Fig.1 shows the optical arrangement. The plasma is imaged stigmatically on the entrance slit of a Bausch and Lomb grating monochromator (1200 lines/mm, dispersion 16 Å/mm, aperture f/4.4) by means of the lens L. At the exit slit is an array of ten glass fibre bundles each of which receives the light emitted along a chord at a different distance from the tube axis and guides it to a photomultiplier. The signals from the photomultipliers are recorded simultaneously by means of five Tektronix 551 double beam oscilloscopes. In order to guarantee a good time correlation between the 10 signals a bright-up pulse is applied to all the signals.

The radial resolution of the plasma is given by the height of each fibre bundle as projected by the optical system on the plasma. The initial experiments were done using ten light guides each 2.5 mm × 1.3 mm high and composed of several hundred glass fibres, viewing the whole 8.5 cm diameter of the discharge tube. Since the plasma usually contracts to a column 1 - 2 cm in diameter, the optical magnification was subsequently changed so that only the central region of the tube was observed giving better resolution of the plasma. Later a more sophisticated fibre bundle was used in which the four outer pipes (each 2.5 × 3 mm high) have three times the height of the inner six (each 2.5 × 1 mm high). Using a magnification of 1:1 this optical array covers the plasma by viewing the central 1.8 cm of the tube.

In the later experiments the monochromator has been replaced by a high quality interference filter with a bandwidth of 10 Å centred at 4978 Å thereby improving the light transmitting power and the ease with which the apparatus could be set up.

C. CALIBRATION

Both the relative sensitivities of the 10 channels (fibre bundle + photomultiplier + amplifier of the oscilloscope) and the absolute intensity of the continuum radiation were calibrated using a carbon arc free-running in air as a standard light source[11,12,13]. When kept burning steadily just below the so-called hissing point the arc radiates as a near black body with a crater temperature of $3800^{o}K$[13] and an emissivity ε_λ which varies only slightly with wavelength. Its intensity is therefore

$$I_\lambda = \varepsilon_\lambda \cdot \frac{2hc^2}{\lambda^5} \exp\left(\frac{-hc}{\lambda k T_e}\right) \text{ergs } s^{-1} cm^{-3} sterad^{-1} cm^{-1} \qquad \ldots (6)$$

Fig.4 shows the optical set up for the calibration. In order to get radiation from a homogeneous region of the carbon crater a magnified image of the crater is formed on the first stop S_1 which is used to cut off the outer regions. The light passing through S_1 is then imaged on the second stop S_2 via a rotating mirror (350 c/s) which chops the light in order not to saturate the photomultipliers. This second stop then becomes a pulsed light source (pulse length approx. 10 µsec) whose size can be made large enough to give an image which covers one fibre bundle. By placing the stop diametrically opposite the tube at the same distance from the mirror as the plasma and rotating the mirror through 90^{o} the same optical system (lens, mirror) can be used for both experiment and absolute calibration and the effects of absorption, reflection, and of the solid angles cancel.

Only one of the channels needs to be calibrated absolutely, the other channels being calibrated relative to this channel by homogeneously illuminating the whole array of fibre bundles and detecting a single light pulse simultaneously on all the photomultipliers.

The losses in the intensity of the carbon arc due to absorption and reflection in its optical system were measured (they turned out to be approximately 45%) and taken into account in the absolute calibration.

THE RESULTS

A. EVALUATION OF THE DATA

The oscilloscope signals from each photomultiplier (Fig.5a) which show the time-variations of the intensities along parallel chords are evaluated at selected points of time and plotted as histograms of intensity against distance (Fig.5b). The axis of symmetry is determined for each histogram and the data is replotted as a graph of intensity against radius (Fig.5c). On a computer these integrated intensities are first converted into a radial intensity distribution by means of an Abel inversion and then into a radial density distribution by solving equation (1). Comparison of the plots shows the variation of the plasma distribution with time. The electron temperature necessary for the calculation of the density was measured in the experiment by a soft x-ray absorption technique[6] in which measurements of the transmission through varying thicknesses of aluminium and beryllium are compared with the theoretical transmission of a bremsstrahlung continuum as computed from the appropriate absorption cross sections.

The computer plots also yield values of other important quantities, in particular, the area under each curve gives the line density; and its variation with time at different points along the coil provides information on the loss of plasma from the tube, whilst the shape of the density distribution gives the variation of 'β' with radius, which is important in theories of particle loss processes. 'β' is derived directly from the distribution curve assuming (1) that $\beta = 1$ on the axis of the discharge and (2) that pressure balance holds:

$$\beta = \frac{2n_r kT}{\frac{B_e^2}{8\pi}} = \frac{2n_r kT}{2n_r kT + \frac{B_i^2}{8\pi}}$$

hence $\beta = 1$ when $n_r = n_o \therefore B_i^2 = 0$.

Pressure balance gives

$$2n_r kT + \frac{B_i^2}{8\pi} = \frac{B_e^2}{8\pi}$$

so that

$$n_o = \frac{B_e^2}{2kT \cdot 8\pi} \quad \text{and} \quad n_o - n_r = \frac{B_i^2}{8\pi}$$

Thus

$$\beta = \frac{n_r}{n_o}$$

B. TYPICAL RESULTS

Some typical results obtained on the Megajoule bank are shown in Figs.6-8. The peak value of electron density on the axis (Figs.6,8) is in reasonable agreement with the value predicted by the Hain-Roberts Code[14,15] assuming partial ionization and 1% oxygen impurity; however in the outer regions there is a departure from the theoretical predictions. The more diffuse experimental plasma is a consequence of magnetic field diffusion in the early stages. The line density shown as a function of time in Fig.7 rises to a peak at four microseconds which is approximately equal to the initial filling pressure then after six microseconds falls due to losses from the end of the coil. This rise may be accounted for by the fact that correction has been made for 1% oxygen assuming it to be fully stripped within two microseconds (see Fig.9); if however the oxygen is only partially stripped during the first few microseconds then the effect on the bremsstrahlung is lessened and the correction should be smaller.

These measurements are discussed in detail in papers to be published[17].

C. ERRORS

The errors involved in this method fall into two groups, those due to the calibration, and those inherent in the measurement.

The carbon arc has a maximum intensity when burning steadily just below the 'hissing' point[11]. Failure to keep it burning smoothly leads to a decrease in brightness, producing an error which would tend to increase the electron density. To minimise this effect several light pulses were recorded and the maximum signal used in the evaluation of the results. Errors due to losses and absorption in the optical system of the arc were measured and corrected for as previously mentioned.

Errors in the measurement are due mainly to (1) presence of impurities in the plasma and (2) effect of background light. To minimise the effect of these and other errors several shots were analysed for each plasma condition, and the average value of the results plotted. The error bars show the extent of the scatter which is of the order of ± 15%. Errors such as those arising from the calibration, which are not affected by this process are within these limits.

The dependence of bremsstrahlung on both n_e and $n_i Z^2$ means that the presence of small quantities of oxygen, which is the most common impurity, produces a large increase in intensity. If fully stripped, as predicted by the Kolb-McWhirter programme[16] (see Fig.9) then 1% oxygen increases the electron density by 8% and the total bremsstrahlung by 77%. Thus the apparent density of the deuterium plasma is increased by some 30%. Corrections cannot be easily made since the percentage of impurity depends not only on the purity of the gas with which the tube is filled, but also on the wall material, and on the gases absorbed in it; thus the contribution of impurity continuum is very important.

At the edge of the plasma the accuracy of the results is reduced due to lack of data points on the original histogram producing an uncertainty in the shape of the tail of the intensity - distance curve, and due to the increasing effect of wall light which is present to some extent in all shots. This has an almost constant value across the tube so that its effect is small at the centre, but becomes important at the edges. A correction for this wall light can be made from examination of the original histograms from which an almost uniform light intensity can be seen on several channels, especially at later times during the discharge when the plasma has drifted towards the edge of the field of view. This background is usually small and subtraction of it from the intensity curves does not affect the values of electron density on the axis nor the variation of the peak electron density or the line density with time, its main effect being upon the line density, which is reduced to a value which is approximately that of the initial filling.

Greater accuracy could thus be obtained by covering more of the tube with the fibre array, but without increasing the number of fibres, which increases the practical difficulties of the experiment, loss of detail in the centre must result; thus a compromise has to be made.

CONCLUSION

A knowledge of the radial distribution of the electron density and its variation with time is important in the analysis of plasma behaviour. The technique described in the paper is capable of determining this distribution at different points along the length of the discharge. The measurement is an absolute one and in principle the errors are small; in practice quite large errors are likely to arise from (1) lack of axial symmetry of the plasma (2) contribution to the continuum radiation from free-free impurity radiation (3) background light. In the experiment on the Megajoule bank (1) is small, (2) is not more than 30% and is probably less, (3) varies between different shots, but can introduce large errors in line density although producing an error less than 5% in the value of electron density near the axis.

ACKNOWLEDGEMENTS

Thanks are due to H.A.B. Bodin and G.B.F. Niblett for many helpful discussions.

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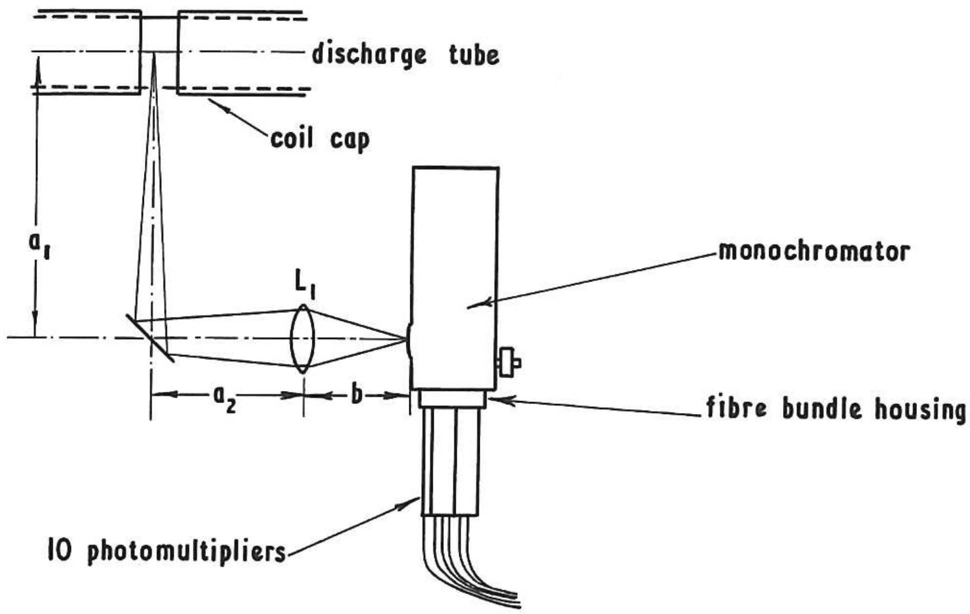

Fig. 1 (CLM-M55)
Diagram of the experimental arrangement

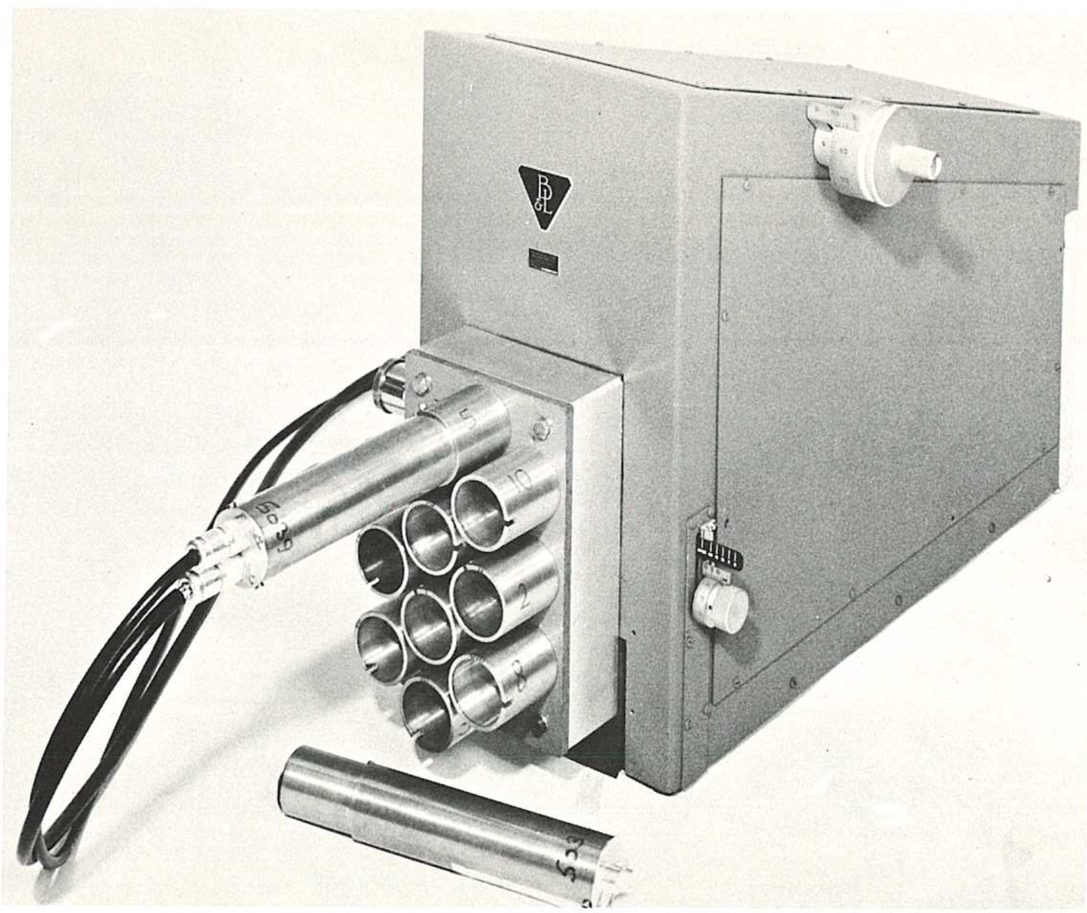

Fig. 2 (CLM-M55)
The monochromator with the ten channel attachment

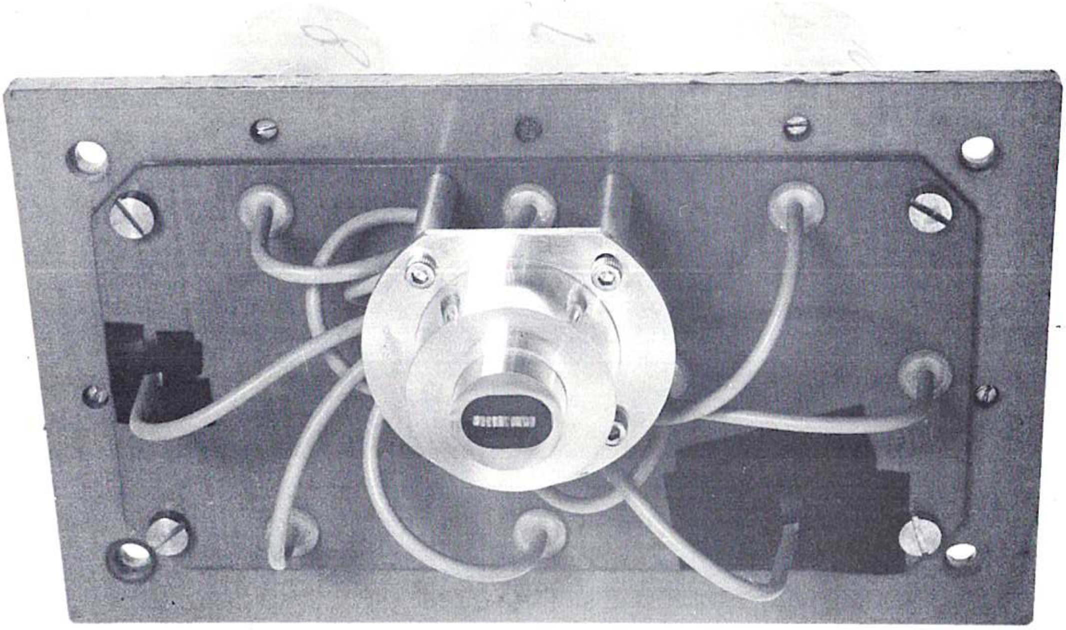

Fig. 3 Fire bundle array (CLM-M55)

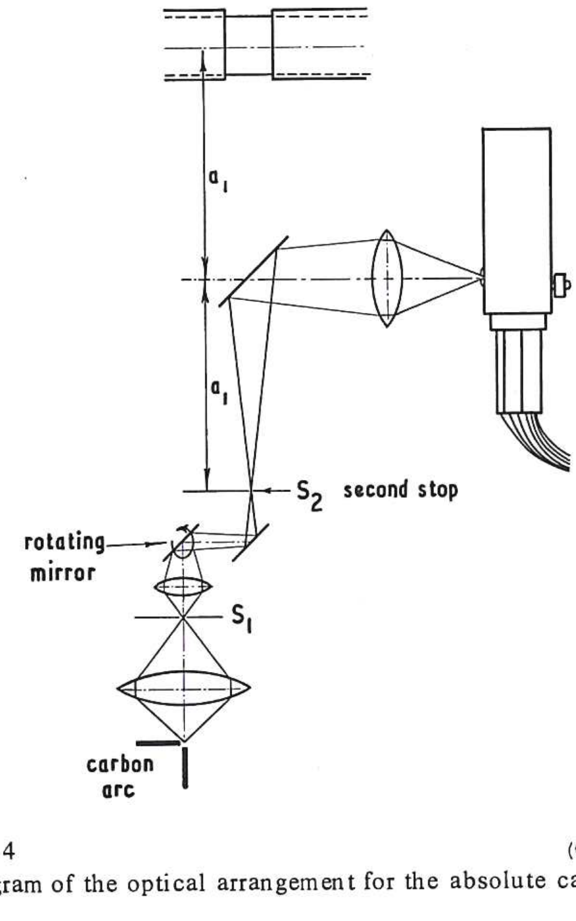

Fig. 4 (CLM-M55)
Diagram of the optical arrangement for the absolute calibration

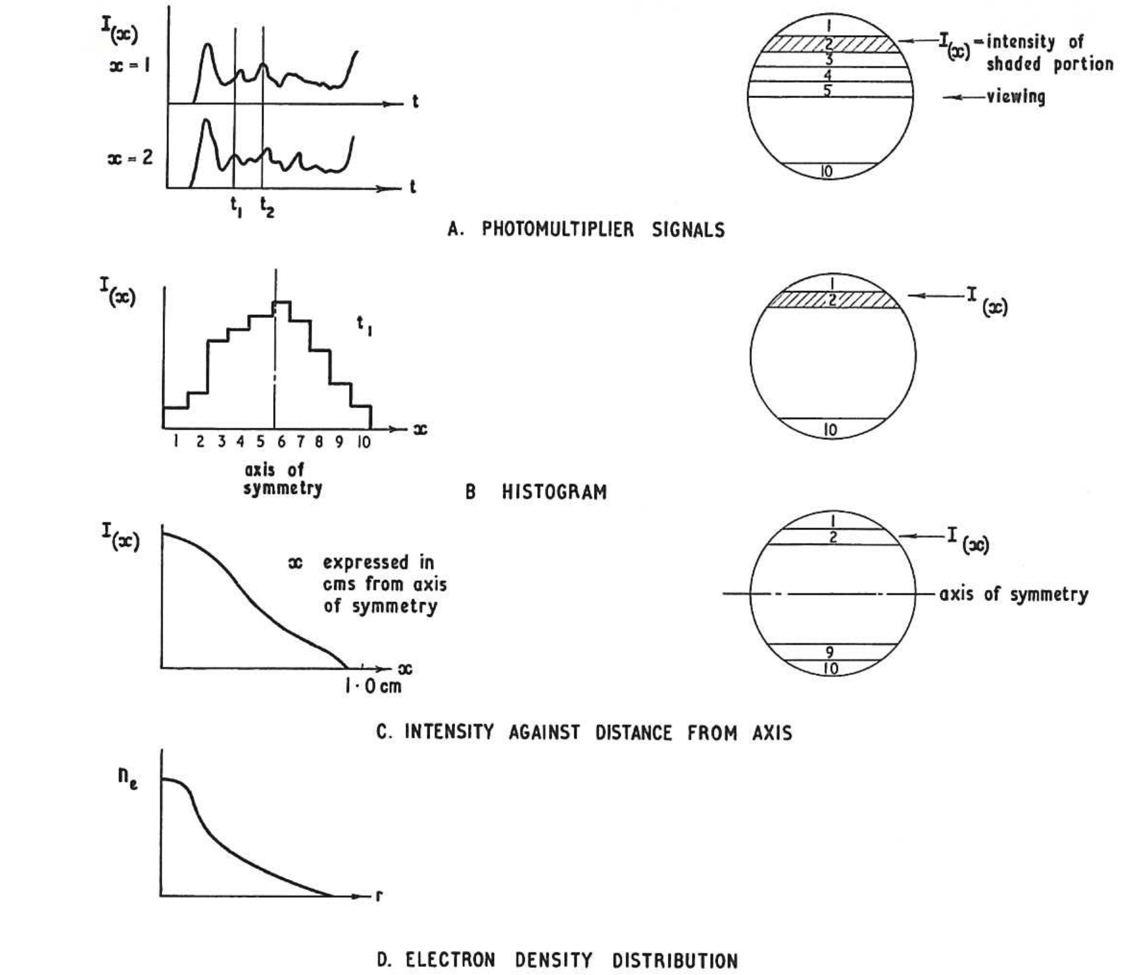

Fig. 5 Steps in the evaluation of the data (CLM-M55)

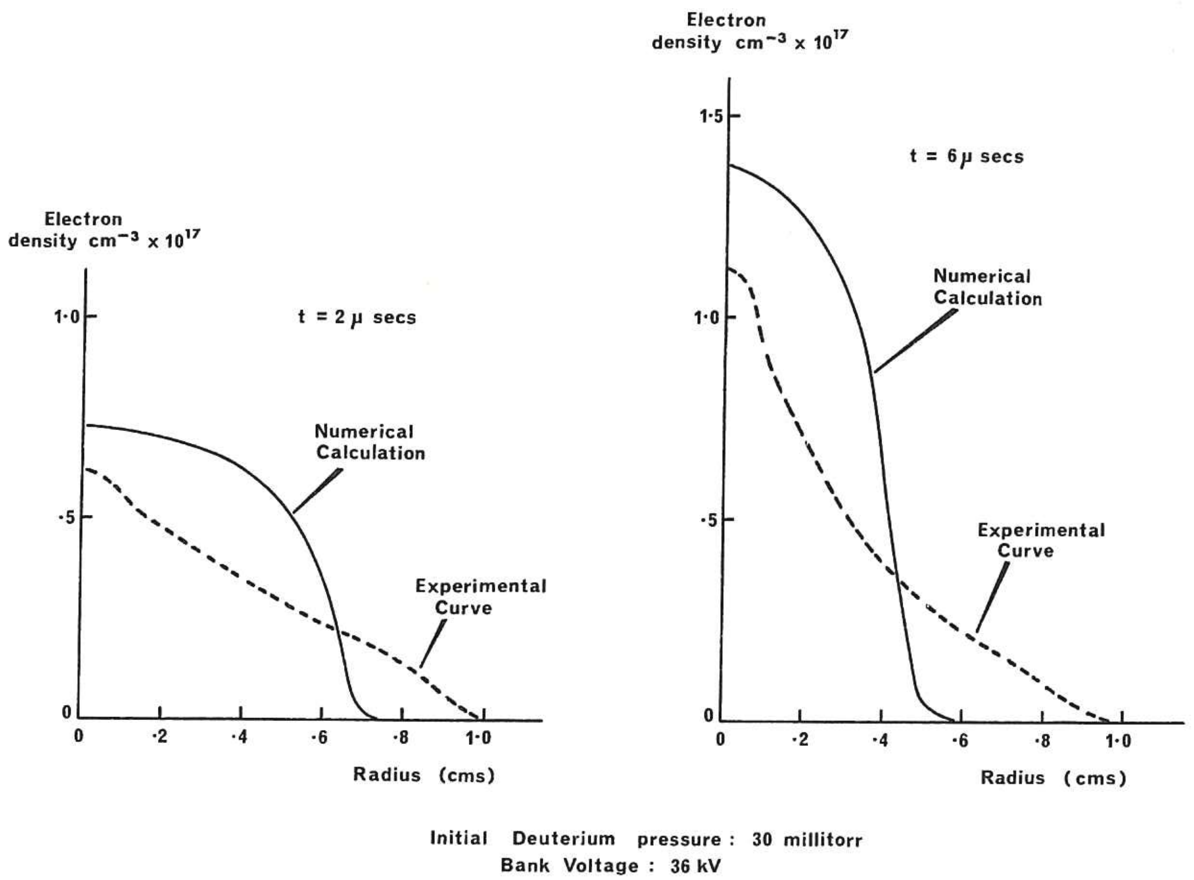

Fig. 6 (CLM-M55)
Theoretical and experimental radial density distribution at 2 μsec and 6 μsec

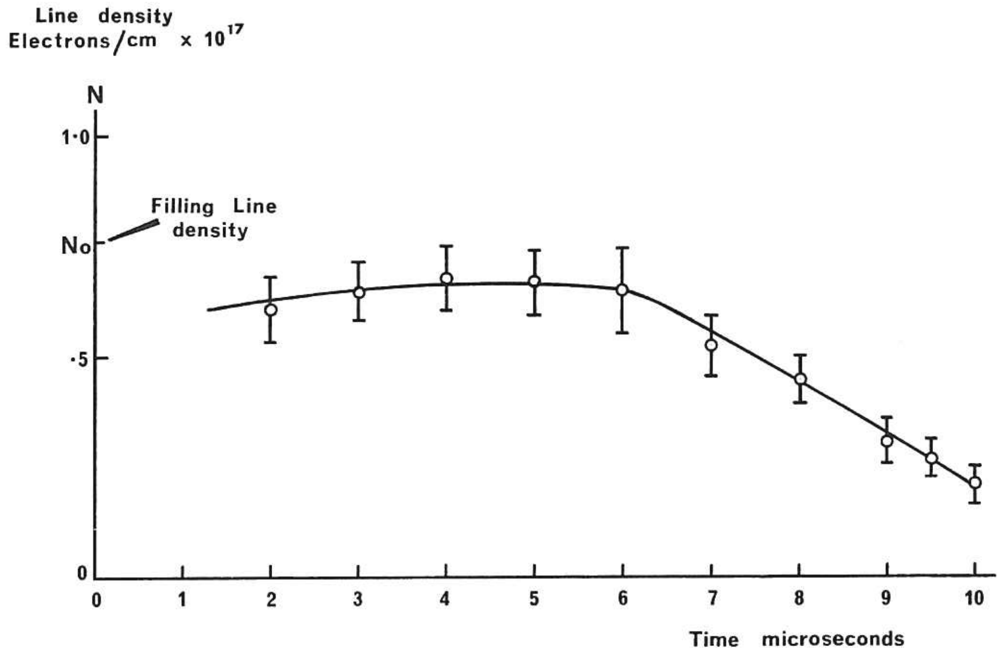

Fig. 7 (CLM-M55)
Line density in the mid-plane of the coil as a function of time

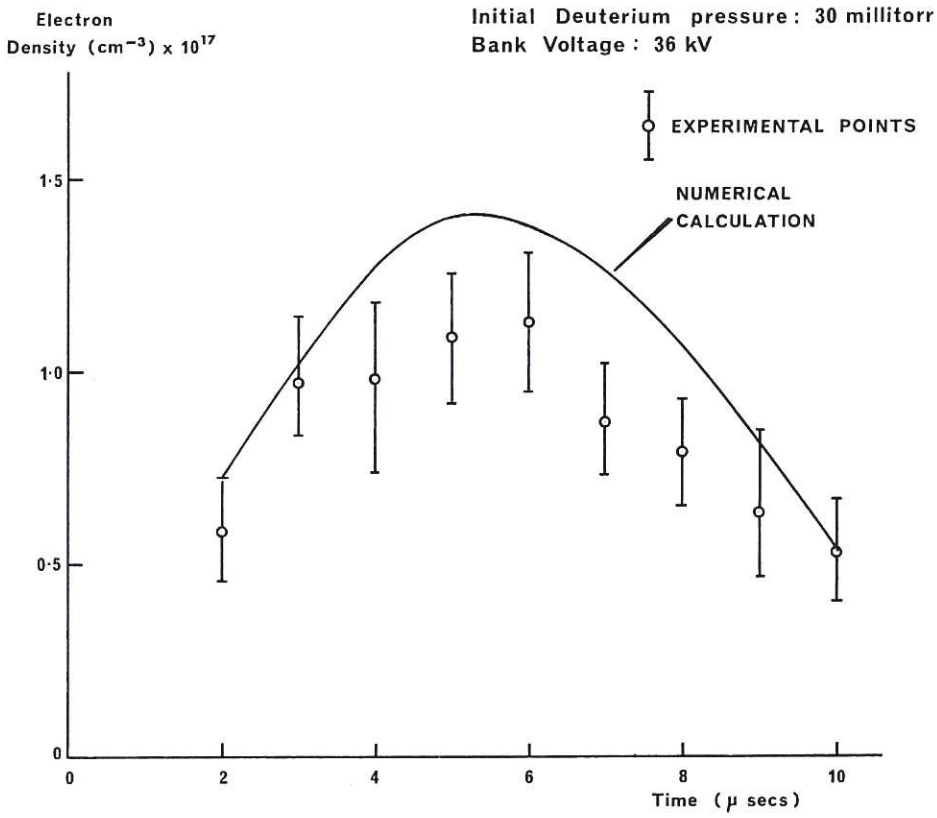

Fig. 8 (CLM-M55)
Variation of peak electron density with time, experiment and theory

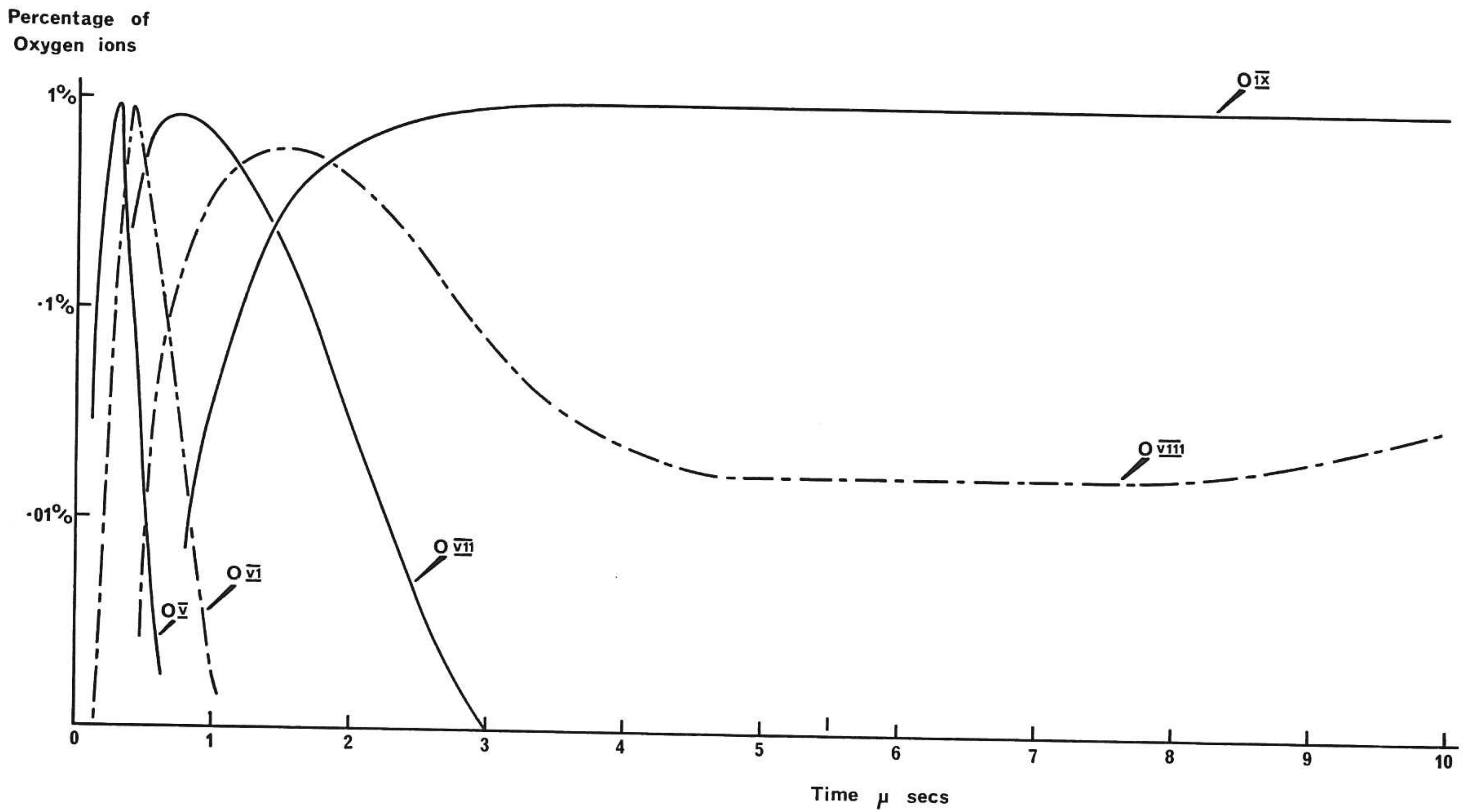

Fig. 9

(CLM-M55)

Percentage population of oxygen ions in a D_2 + 1% O_2 plasma