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R. J. BICKERTON

A. GIBSON

Culham Laboratory,

Culham, Abingdon, Berkshire

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A NOTE ON THE SHEAR ATTAINABLE IN STELLARATOR SYSTEMS

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R.J. BICKERTON
A. GIBSON

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A B S T R A C T

An estimate is made of the maximum shear attainable in a toroidal $\ell = 3$ stellarator. The loss of plasma due to universal instabilities in such a sheared system is compared with the Bohm loss rate. The maximum shear is compared with that obtained in other toroidal systems.

U.K.A.E.A. Research Group,
Culham Laboratory,
Nr. Abingdon,
Berks.

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A comprehensive account of the shear stabilisation of a low β toroidal plasma has been given by Kadomtsev and Pogutse[1]. For a collisionless plasma showing the minimum departure from thermal equilibrium consistent with confinement, namely $T_e = T_i$, $\frac{dT}{dr} = 0$ and no net current round the torus, they find stability against universal modes if,

$$\theta > \left(\frac{m}{M}\right)^{\frac{1}{2}} \qquad \ldots (1)$$

where the shear parameter $\theta = \frac{r}{L_s}$, r is the plasma radius, L_s is the shear length defined by

$$\frac{1}{L_s} = \frac{kr}{2\pi} \cdot \frac{di_w}{dr}$$

where i_w = rotational transform/winding period, $\frac{2\pi}{k}$ = length of a winding period, and m and M are the electron and ion masses respectively. Collisional (resistive) modes will be unimportant or even stable in a plasma which is sufficiently hot[2].

We consider now the question as to how much shear can be obtained in a toroidal stellarator. We take an $\ell = 3$ helical winding, since lower values of ℓ give approximately no shear and higher values are less efficient in producing a rotational transform. In a straight $\ell = 3$ stellarator system the volume of closed magnetic surfaces is bounded by an outer surface (separatrix) on which the rotational transform per winding period, $i_w = 2\pi$. In the toroidal case, because of the lack of symmetry, the existence of magnetic surfaces cannot be rigorously proved. We have, however, made a computer study of the trajectories of field lines in a toroidal $\ell = 3$ system generated by filamentary conductors. We find that for ratios of major torus radius to minor ℓ-winding radius of < 10 the separatrix contracts so that $i_w \approx \pi/2$. As the aspect ratio is increased there is a slow asymptotic approach to the maximum $i_w = 2\pi$. This contraction of the separatrix as the toroidal curvature is increased was predicted qualitatively by Melnikov[3].

Taking the value $i_w = \pi/2$ we find a typical value for θ by taking $di_w/dr = i_w/r$, it is:

$$\theta = \frac{kr}{4} \qquad \ldots (2)$$

In a practical case the radius r_1 at which the ℓ-windings are placed will be > 2r. Configurations with $kr_1 > 1$ are unattractive because the shear (and rotational transform) is confined to a narrow zone near the separatrix. Computations show that for $kr_1 = 1.7$ only 22% of the transform occurs within 0.8 r_m (r_m = separatrix radius) compared to 55% for $kr_1 = 1$.

Further if the density distribution is such as to keep θ everywhere large enough for stability the density gradient must also be confined to a region of thickness $\delta \ll r$. Physically the dimensions of such a region must be large compared to an ion Larmor radius (a_i). The restriction

$$r \gg \delta \gg a_i \qquad \ldots (3)$$

may be acceptable in a reactor but will not be possible with the dimensions usually considered for experiments. Accordingly we shall take $kr_1 = 1$, hence

$$\theta_{max} \approx \frac{1}{8} \qquad \ldots (4)$$

Since Kadomtsev and Pogutse[1] equated the plasma radius to the scale length for the density (i.e. $(\frac{1}{n}\frac{dn}{dr})^{-1}$) the ratio $N = \theta_{max} (\frac{M}{m})^{\frac{1}{2}}$ gives the number of density scale lengths we may have in the plasma radius and retain stability. For a deuterium plasma $N \sim 8$.

Consider a model in which field lines leave the separatrix surface at an angle α before intersecting the wall of the containing vessel. If we assume that (1) is just satisfied within the separatrix then the containment time t_L obtained by taking plasma loss at sonic speed outside the separatrix is

$$t_L = \frac{r \exp{(N)}}{2 \cdot c_s \cdot \sin\alpha} \qquad \ldots (5)$$

where c_s is the sound speed in the plasma

$$\left(c_s^2 = \frac{k(T_e + T_i)}{M} \right) .$$

The empirical (Bohm) loss time found by Bishop and Hinnov[4], t_B, is

$$t_B = \pi r^2 \frac{eB}{ckT_e} \text{ (cgs units)} \qquad \ldots (6)$$

while the classical containment time due to binary collisions, t_c, is[5]

$$t_c = \text{constant.}\ \frac{r^2 \cdot B^2 \cdot T_e^{3/2}}{(T_e + T_i)} \qquad \ldots (7)$$

In the unlikely event that

$$t_L > t_c$$

we would expect a completely stable plasma with the classical containment time t_c.

If $t_c > t_L$ then we expect mild instability leading to a containment time of the order t_L, the diffusion coefficient increasing just enough to hold the plasma density gradient near the stability limit.

When B is sufficiently large the time predicted by (5) is even shorter than the Bohm time, in this case the actual confinement time will be determined by the rate at which the instability can transport plasma. Non-linear estimates[1] suggest that the rate will in fact be much smaller than the Bohm rate.

Putting typical experimental parameters into (5) and (6), namely $T_e = T_i$, $a_i/r = 0.02$, $N = 8$, $\sin L = \frac{1}{2}$ we get

$$\frac{t_L}{t_B} \simeq 10 \qquad \ldots (8)$$

If a smaller shear is accepted then a diverter can be used, in this case $\sin\alpha$ in (5) is replaced by d/L, where L is the peripheral length round the stellarator and d is the thickness of the diverted plasma. The value given in (8) may then be increased by a factor as large as 500, however in this case we have an annulus of secondary plasma moving at sonic speed along the field lines surrounding the main plasma. The stability of this whole assembly is at present unknown.

For comparison we consider the shear in other closed-line confinement systems. In the TOKOMAK the value of θ depends on the current distribution but a representative value is

$$\theta = \frac{I}{I_k} \cdot \frac{r}{R} \qquad \ldots (9)$$

where I is the plasma current, I_k the Kruskal-Shafranov limiting value for stability, and R the major radius. With typical values $I/I_k \sim 0.2$ and $r/R \sim \frac{1}{4}$, then $\theta \simeq 1/20$. For the Levitron with vacuum magnetic fields and remembering that the plasma is annular, $\theta_{max} \simeq \frac{1}{2}$, while for the stabilised, diffuse pinch $\theta_{max} \simeq 1$. The relative merits of these various approaches in other respects is well known.

We conclude that on the basis of existing theory enough shear can be obtained in a stellarator to provide containment of a suitably hot plasma for substantially longer than the Bohm time. Existing stellarators have shear parameters an order of magnitude or more lower than the value calculated here.

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