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A HIGH FREQUENCY ELECTROSTATIC INSTABILITY IN A MAGNETICALLY GUIDED PLASMA STREAM

D. E. T. F. ASHBY
A. PATON

Culham Laboratory
Abingdon Berkshire

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A HIGH FREQUENCY ELECTROSTATIC INSTABILITY IN A MAGNETICALLY GUIDED PLASMA STREAM

by

D.E.T.F. ASHBY
A. PATON

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A B S T R A C T

Hydrogen plasma from a thetatron gun has been directed into the entrance of a solenoid so that a tenuous low β plasma stream is guided along the magnetic field lines. The main parameters of the plasma and field are $n \sim 3 \times 10^{11}$ cm^{-3}, B = 2 kG and ion energy ~ 800 eV. A simple theory is presented which predicts an electrostatic instability caused by the anisotropy in the distribution of the transverse component of ion velocity; this anisotropy arises because the ion gyro radius (~ .5 cm) is of the same order as the radius of the plasma stream (~ 2 cm). The instability is similar to the "loss-cone instability" and is characterised by the following properties:

$$R(\omega) \sim \mathrm{Im}(\omega) \sim \left\{ \omega_{pi}^{-2} + (\omega_{ci}\ \omega_{ce})^{-1} \right\}^{-\frac{1}{2}}, \quad k_{\perp}/k_{\parallel} \sim \sqrt{M/m} .$$

Experimental results are described which confirm the existence of an instability and support the theoretical predictions. The frequency $\omega/2\pi$(~ 40 Mc/s) has been measured and the transverse wavelength has been estimated using double electric probes with different electrode separations. An increase in longitudinal electron energy has been demonstrated by varying the bias on an ion probe. The existence of standing waves is inferred from low frequency fluctuations on both the ion probe output and the rectified high frequency signal. It is suggested that the instability may account for the anomalous loss of diamagnetism which is often observed in similar plasma streams.

U.K.A.E.A. Research Group,
Culham Laboratory,
Nr. Abingdon,
Berks.

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CONTENTS

1. INTRODUCTION

When plasma from a plasma gun is directed into the entrance of a solenoid some plasma enters the solenoid and is guided along the magnetic field; a considerable number of experiments have been reported which make use of this effect, particularly in connection with controlled fusion research (e.g. COENSGEN et al, 1960; WAELBROECK et al, 1962; FRANCIS et al, 1964; AZOVSKII et al, 1966). In this paper we are concerned with the case when the injected plasma is tenuous and the ion energy high so that the plasma is essentially collision-free for the duration of its motion along the field. Such plasma streams do not exclude the magnetic field and β, the ratio of transverse plasma pressure to magnetic pressure, is small (typically about 10^{-2}). Experimentally it is found that the plasma radius is approximately twice the mean ion gyro radius (R_{ci}) (ASHBY and AVIS, 1966; ATKINSON and PHILLIPS, 1966). ASHBY and AVIS (1966) explained this result by showing that a "short-circuiting" effect occurs at the wall near the solenoid entrance which removes the gross space-charge electric field; as a result the ions tend to follow the same trajectories as single ions injected into a solenoid, namely gyrating so that they cut the magnetic axis (see Fig.1). Several experiments also have shown that the plasma diamagnetism, which is a measure of the transverse ion energy, decays anomalously as the plasma moves along the guide field (ATKINSON and PHILLIPS, 1966; ASHBY, 1964). The above observations have led the authors to consider whether a fast-growing electrostatic instability can occur in a collision-free low β stream when the plasma radius does not greatly exceed the ion gyro radius. This paper describes such an instability and presents supporting experimental results.

2. THEORY

Electrostatic instabilities resulting from anisotropic velocity distributions are well known (e.g. HALL et al, 1965). The guided plasma column we are considering has an ion velocity distribution similar in some respects to that originally analysed by BURT and HARRIS (1961) who predicted the existence of waves propagating almost parallel to the field lines with growth times of the order of the ion gyro period; this instability is unlikely to play a major role in the present experiment as the plasma is only in the solenoid for a time comparable with the ion gyro period ($t \sim 1$ μsec $B_z \sim 2$ kG). However the instability described by POST and ROSENBLUTH (1966), which results from a "loss-cone" type of ion velocity distribution, gives growth times short compared with the ion gyro-period and is valid when $\omega_{pi} \gg \omega_{ci}$. A similar instability can be expected in a guided plasma but in this case the anisotropy in ion velocity results from the ion diamagnetic current rather than a "loss-cone effect". The basic features of the instability are derived by making the following simplifying assumptions:

1. Consider a Cartesian frame x, y, z with a uniform magnetic field B in the z-direction

2. Assume that a homogeneous beam of ions moves in the y-direction with a constant velocity V; the ion motion near the plasma boundary in Fig.1 approximates to this case. The assumption that the ions are unaffected by the magnetic field implies that the time of interest is much less than ω_{ci}^{-1}.

3 Assume that $T_e = 0$.

4. Assume that an electrostatic wave exists given by the potential

$$\Phi = \Phi_o \exp i\,(\omega t - k_z z - k_y y).$$

Using the above assumptions we derive the following expressions for the perturbed ion and electron velocities:

$$(v_y)_{ion} = (e/M)\, k_y\, (\omega - k_y V)^{-1}\, \Phi \qquad \ldots (1)$$

$$(v_z)_{ion} = (e/M)\, k_z\, (\omega - k_y V)^{-1}\, \Phi \qquad \ldots (2)$$

$$(v_x)_{electron} = i(e/m)\, k_y\, \omega_{ce}^{-1}\, \Phi \qquad \ldots (3)$$

$$(v_y)_{electron} = (e/m)\, k_y\, (\omega/\omega_{ce}^2)\, \Phi \qquad \ldots (4)$$

$$(v_z)_{electron} = -(e/m)\, (k_z/\omega)\, \Phi \qquad \ldots (5)$$

From equations (1) to (5), the continuity equations and Poisson's equation we obtain the dispersion relation:

$$1 + \left(\frac{k_y}{k}\right)^2 \left(\frac{\omega_{pe}^2}{\omega_{ce}}\right) = \frac{\omega_{pi}^2}{(\omega - k_y V)^2} + \frac{\omega_{pe}^2}{\omega^2}\left(\frac{k_z}{k}\right)^2 \qquad \ldots (6)$$

Equation (6) is similar to the dispersion relation given by POST and ROSENBLUTH (1966); it can be obtained from the complete dispersion relation given by (BURT and HARRIS, 1961; HARRIS, 1963) by making the appropriate approximations.

By assuming $k_y/k \sim 1$ and defining

$$\omega_o = \left[\omega_{pi}^{-2} + (\omega_{ce}\ \omega_{ci})^{-1} \right]^{-\frac{1}{2}}$$

$$k_z/k = \cos\theta$$

$$\Omega = \omega/\omega_o$$

$$K = k_y\ V/\omega_o$$

equation (6) may be written

$$1 = (\Omega - K)^{-2} + \Omega^{-2}\ (M/m)\cos^2\theta \qquad \dots (7)$$

The case $(M/m)\cos^2\theta = 1$ is easily solved; other cases have been solved numerically and the results are presented in Fig. 2. It will be seen that high frequency, high growth rate electrostatic instabilities are found which are characterised by

$$Im(\omega) \sim R(\omega) \sim k_y\ V \sim \omega_o$$

and

$$k_z/k_y \sim (m/M)^{\frac{1}{2}}.$$

The validity of the calculation requires $\omega_{ce} \gg |\omega| \gg \omega_{ci}$ which is satisfied if $\omega_o \gg \omega_{ci}$. This condition in turn is satisfied if $\omega_{pi} \gg \omega_{ci}$ which also ensures that the perpendicular wavelength λ_y is much less than the ion gyroradius V/ω_{ci}.

The assumption $T_e = 0$ is valid provided $\omega/k_z c_e \gg 1$ and $k_y c_e/\omega_{ce} \ll 1$, where c_e is the electron thermal speed. These requirements allow moderately high electron temperatures before the model becomes invalid; for example with $(M/m)\cos^2\theta = 1$ they require $T_e \ll 1/6\ MV^2$. Smaller values of $\cos^2\theta$ allow still higher values of T_e.

It is clear that these unstable waves grow at the expense of the ion transverse energy and, as some of the wave energy is due to electron motion along the field lines, loss of diamagnetism will result. Non-linear effects and thermalization of the electrons will eventually limit the oscillations but, as there is no containment of electron energy along the field lines, a continual loss of ion energy seems likely as long as a large anisotropy exists in the ion transverse velocity. These aspects of the instability are not considered in the following experimental work which was aimed at verifying the existence of the above instability with regard to the frequency and wavelength characteristics.

3. APPARATUS

The apparatus shown in Fig.3 used the gun developed by Cruddace (CRUDDACE and HILL, 1966). The solenoid was supported inside a large vacuum tank so that most of the field lines closed on themselves without intercepting a wall unless the movable conducting cylinder shown, was placed at the mouth of the solenoid. The apparatus was designed to extend the work of Ashby and Avis on plasma injection (ASHBY and AVIS, 1966) by comparing the case when field lines are "short-circuited" with the case when a large radial electric field can exist in the plasma stream (ASHBY and BURCHAM, 1966). The experiments to be described in this paper were all carried out with the cylinder at the entrance of the solenoid thus ensuring that "short-circuiting" of the flux lines did occur. A burst of hydrogen plasma, with a directed energy centred about 800 eV (4×10^7 cm sec^{-1}), was produced by the thetatron gun. The aperture limiter selected the central portion of plasma. Near the solenoid the plasma density was about 3×10^{11} cm^{-3} and the ions were moving essentially parallel to the solenoid axis.

Fig.4a shows the probe used for measuring the longitudinal and transverse plasma velocities and plasma density. The collecting electrodes were biased negatively to collect ions. The ion current, $I_{\parallel}$ or $I_{\perp}$, is given by (neV) $\times$ area, where V is the mean velocity component of the ions entering the probe (ASHBY and AVIS, 1966; ASHBY, 1963). Because $V_{\parallel}$, the longitudinal component of V, is known from time-of-flight, n and $V_{\perp}$ can be calculated from the two ion currents.

Fluctuating electric fields were measured with the type of probe illustrated in Fig.4b. The one shown was used to detect E_θ and E_z. Considerable care was necessary in electrically balancing these probes; in addition it was essential to make the electrodes physically small compared with the wavelengths of the instability (~ 2mm). The need for small probes coupled with the low plasma density and the magnitudes of the signals observed (~ 4 volts) made it difficult to use as high a probe impedance as desired. The probe used represents a compromise between the conflicting requirements. The probe output was fed, via a balanced-to-unbalanced transformer (3-300 Mc/s bandwidth) and a simple T-network high pass filter, to either a high frequency oscilloscope (Tektronix type 585) or to a square law crystal detector and amplifier with an integrating time constant of 0.1 μsec.

4. EXPERIMENTAL RESULTS AND DISCUSSION

The measurements to be described were made at a position 35 cm from the solenoid entrance; thus the plasma moved through a region of uniform field for a distance of some 25 cm before measurements were made. Fig.5 shows the variation of plasma density with radius at a flux density of 2 kG. The plasma density was measured with an ion probe.

The anisotropy in the ion velocity distribution is indicated by Fig.6. The transverse $V_\perp$ refers to the azimuthal velocity deduced from the two ion currents collected by a probe of the type shown in Fig.6. The points 'A' were obtained when the side hole in the probe was set so that it would receive a positive ion orbiting freely about the magnetic axis; the points 'B' were obtained with the probe rotated through 180^o. The azimuthal plasma velocity is 10^7 cm sec^{-1} near 3 cm and decreases to 5×10^6 cm sec^{-1} near 1 cm radius. It exceeds $\omega_{ci} r/2$ by a factor of two or three and indicates that the ion trajectories do not cut the magnetic axis; this fact is consistent with presence of an instability which alters the ion transverse energy and causes radial diffusion.

Fluctuating electric fields were detected by the E-probe shown in Fig.4b. The following oscillograms illustrate the main features of these fluctuations. In each figure the E-probe signal is shown together with the output from an ion probe at the same axial position but at a different radius. The distance between the probes was 4.3 cm.

Fig.7 compares $\overline{\Phi_\theta^2}$ and $\overline{\Phi_z^2}$ at a radius of 3 cm, where $\Phi = \int E d\ell$ from one electrode to the other. To obtain these oscillograms the output from the E-probe was passed, via a 30 Mc/s high pass filter, to a square law detector. The distance separating the probe electrodes was 0.2 cm. The oscillograms show that the probe output commenced when plasma entered the vacuum tank and encountered the fringing field surrounding the solenoid. In the case of $\overline{\Phi_\theta^2}$ a large increase of signal is shown at the time when the plasma passed the probe; the corresponding $\overline{\Phi_z^2}$ signal is considerably less.

Calibration of the whole probe receiver system showed that the amplitude of the Φ_θ signal is approximately 4 volts R.M.S. The receiver calibration was verified by using a high speed oscilloscope to view Φ_θ direct. As a check on the validity of the measurement a probe was used with its electrodes short-circuited; it gave effectively no output. The E-probe also gave effectively no output when the flux density of the guide field was zero; at the same time the ion probe signal was smooth and very reproducible (see lower oscillogram Fig.7). The latter observation indicates that the irreproducibility, evident from the size of error bar in Figs.5 and 6 and the ion probe signals in Fig.7, is not due to erratic gun behaviour but results from the interaction of the plasma and the magnetic field.

The probe output was fed direct to a high speed oscilloscope and compared with the output obtained when first a 10 Mc/s and then a 30 Mc/s high-pass filter was inserted into the circuit; in both cases the output was essentially the same. The magnitude of $\overline{\Phi_\theta^2}$ was reduced by 80% when a 50 Mc/s high pass filter was used in the receiver circuit.

Fig.8 shows that $\overline{\Phi_\theta^2}$ is smallest near the axis of the plasma column and also compares the $\overline{\Phi_\theta^2}$ signal from probes with different electrode separations; these latter signals have essentially the same amplitude and indicate that $\lambda_\theta/2 \lesssim 0.2$ cm.

The above results agree with the predictions of theory on the following points:

(1) The observed high frequencies are around 30-50 Mc/s, which greatly exceeds the ion gyrofrequency of 3 Mc/s but is comparable with $\omega_0/2\pi$. (100 Mc/s near 2 cm radius).

(2) The ratio of E_θ/E_z should be $\sim (M/m)^{1/2}$. The measured value of $\overline{\Phi_\theta^2}$ is much greater than $\overline{\Phi_z^2}$ at frequencies above 30 Mc/s but their ratio is only about ten. However the probe impedance is rather low and consequently $\overline{\Phi_\theta^2}$ could be much bigger than is indicated. Alternatively the measured $\overline{\Phi_z^2}$ could result from a large amplitude low frequency signal which, although attenuated, is not eliminated by the high-pass filter.

(3) The measured $\overline{\Phi_\theta^2}$ at frequencies above 30 Mc/s, is greatest near the edge of the plasma column where the anisotropy is greatest.

(4) The azimuthal wave length should be approximately equal to the azimuthal plasma velocity divided by the frequency of oscillation, namely 0.25 cm. This value is consistent with the results obtained by varying the electrode separation.

An important feature of the theory is that the longitudinal electron energy increases as a result of the instability; the following results confirm that an increase does occur. The electron temperature was measured, in the absence of a guide field, by varying the bias voltage on an ion probe. A special probe was used; it was sleeved inside a glass tube for most of its length and electrically isolated from the vacuum tank so that it could assume the plasma potential. A flat plate was used to collect current instead of the cylinder shown in Fig.4. Fig.9 shows a log linear plot of probe current against bias voltage. The electron temperature indicated is 0.8 eV and is consistent with the expansion of the plasma as it moves from the gun to the solenoid.

The presence of a magnetic field completely altered the above results. Fig.10 shows typical signals at four bias voltages. At nine volts bias the current fluctuates violently and even becomes negative momentarily. As the voltage is increased the signal increases but the amplitude of the fluctuation decreases. This result argues that the fluctuations are due to variations of electron energy within the plasma encountering the probe rather than variations in density; electron energies greater than 9 eV are indicated. If the electrons have acquired this energy at the expense of the transverse ion energy (~ 50 eV) then the associated loss of diamagnetism will exceed 20%.

The $\overline{\Phi_\theta^2}$ signals in Figs.7 and 8 show signs of similar modulation, although not as pronounced as the fluctuations in ion current. The frequency in both cases is between 5 to 10 Mc/s and can be explained by postulating the existence of standing waves in the plasma column. If pairs of waves varying as $\exp i(\omega t - k_y y \pm k_z z)$ exist then standing waves of the form $2\cos k_z z \exp i(\omega t - k_y y)$ will result. Because the column is moving with a velocity $V_\parallel$, all quantities will vary with time as $2\cos(k_z V_\parallel t) \exp(i\omega t)$ in a stationary frame of reference. The velocity of the plasma column is 4×10^7 cm/sec, so a dominant wavelength along the field lines of 8 cm, for example, would cause the $\overline{\Phi_\theta^2}$ signal to display a modulation of 10 Mc/s; this value for the wavelength along the magnetic field lines is consistent with the theory. The absence of high frequency fluctuations (~ 40 Mc/s) on the ion probe signal is not unexpected. The ion probe will rectify the high frequency electron current and leave the demodulated signal, of angular frequency $2k_z V_\parallel$, superimposed on the ion current.

5. CONCLUSION

A high frequency electrostatic instability characterised by the frequency ω_o, where $\omega_o^{-2} = \omega_{pi}^{-2} + (\omega_{ce}\,\omega_{ci})^{-1}$, and by $k_\perp \gg k_\parallel$ has been observed in a low density plasma stream. These oscillations are attributed to the anisotropy in the azimuthal ion velocity distribution and may account for the anomalous loss of diamagnetism which has been observed in similar plasma streams. It is probable that the instability will exist at the edge of any magnetically confined plasma to a depth of one ion gyro radius.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

ASHBY, D.E.T.F. (1964) Proc. 6th Int. Conf. Ionization Phenomena in Gases, Paris, 1963. 4, 465.

ASHBY, D.E.T.F. and AVIS, B.E. (1966). J. Nucl. Energy Pt C, 8, 1.

ASHBY, D.E.T.F. and BURCHAM, J.N. (1966). Unpublished information.

ATKINSON, D. and PHILLIPS, J.A. (1966). Private communication.

AZOVSKII, Yu.S., GUZHOVSKII, I.T., MAZALOV, Yu.P. and PISTRYAK, V.M. (1966). Sov. Phys.-Tech. Phys., 10, 1092.

BURT, P. and HARRIS, E.G. (1961). Phys. Fluids, 4, 1412.

COENSGEN, F.H., SHERMAN, A.E., NEXSEN, W.E. and CUMMINS, W.F. (1960). Phys. Fluids, 3, 764.

CRUDDACE, R.G. and HILL, M. (1966). Mechanism of plasma acceleration in a conical thetapinch gun. Culham Laboratory, 1966. CLM-M52.

FRANCIS, G., MASON, D.W. and HILL, J.W. (1964). Nature, 203, 623.

HALL, L.S., HECKROTTE, W. and KAMMASH, T. (1965). Phys. Rev., 139, A1117.

HARRIS, E.G. (1963). The effect of finite ion and electron temperatures on the ion cyclotron resonance instability. London, H.M.S.O., 1963. CLM-R32.

POST, R.F. and ROSENBLUTH, M.N. (1966). Phys. Fluids, 9, 730.

WAELBROECK, F., LELOUP, C., POFFÉ, J.P., EVRARD, P., DER AGOBIAN, R. and VÉRON, D. (1962). Proc. I.A.E.A. Conf. on Plasma Physics and Controlled Nuclear Fusion Research, Salzburg, 1961. Nucl. Fusion 1962 Suppl., 2, 675.

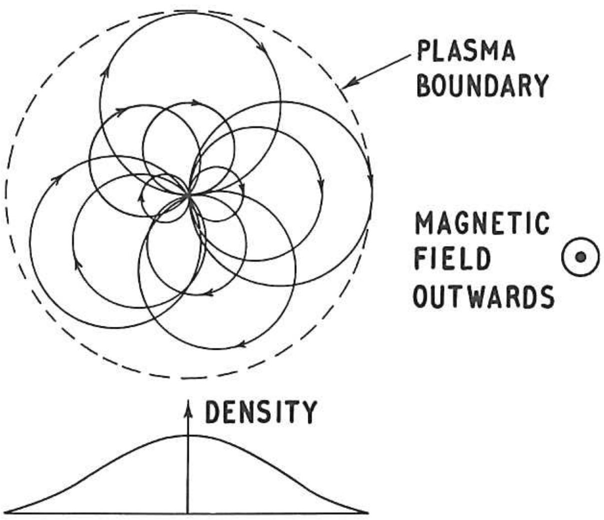

Fig. 1 (CLM-P 126)
Typical ion trajectories viewed from the rest frame of the plasma

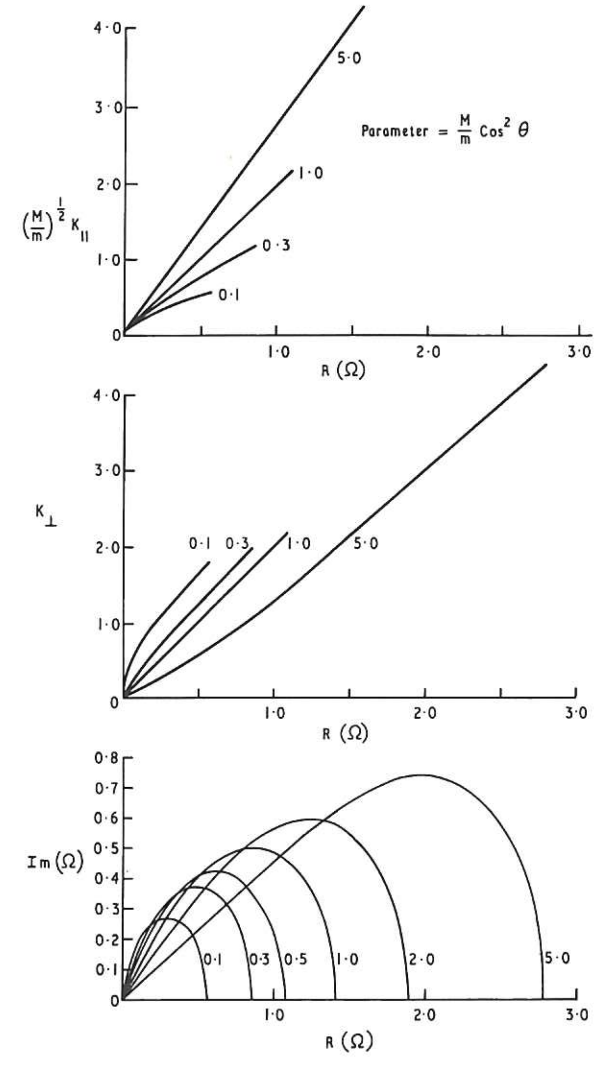

Fig. 2 (CLM-P 126)
Curves derived from the dispersion equation
$$1 = (\Omega - K)^{-2} + \Omega^{-2}(M/m)\cos^2\theta$$

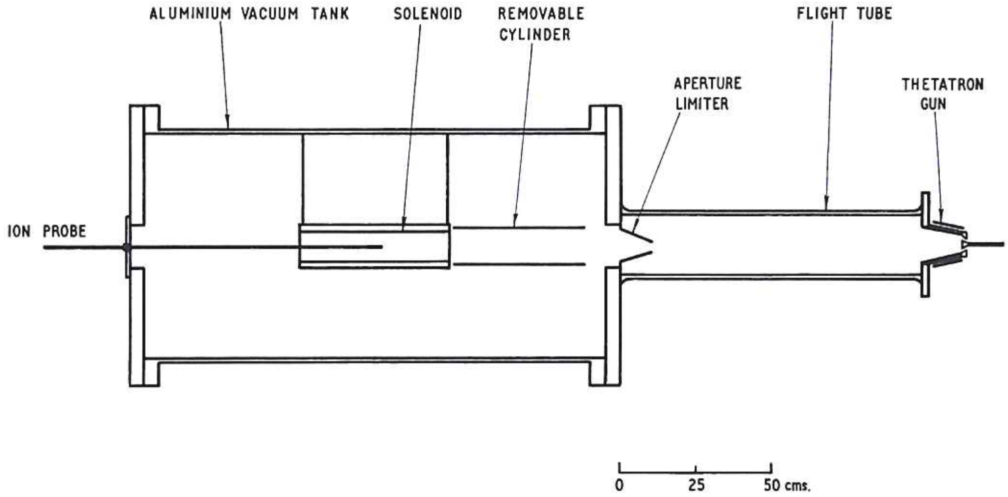

Fig. 3 Schematic diagram of the apparatus (CLM-P 126)

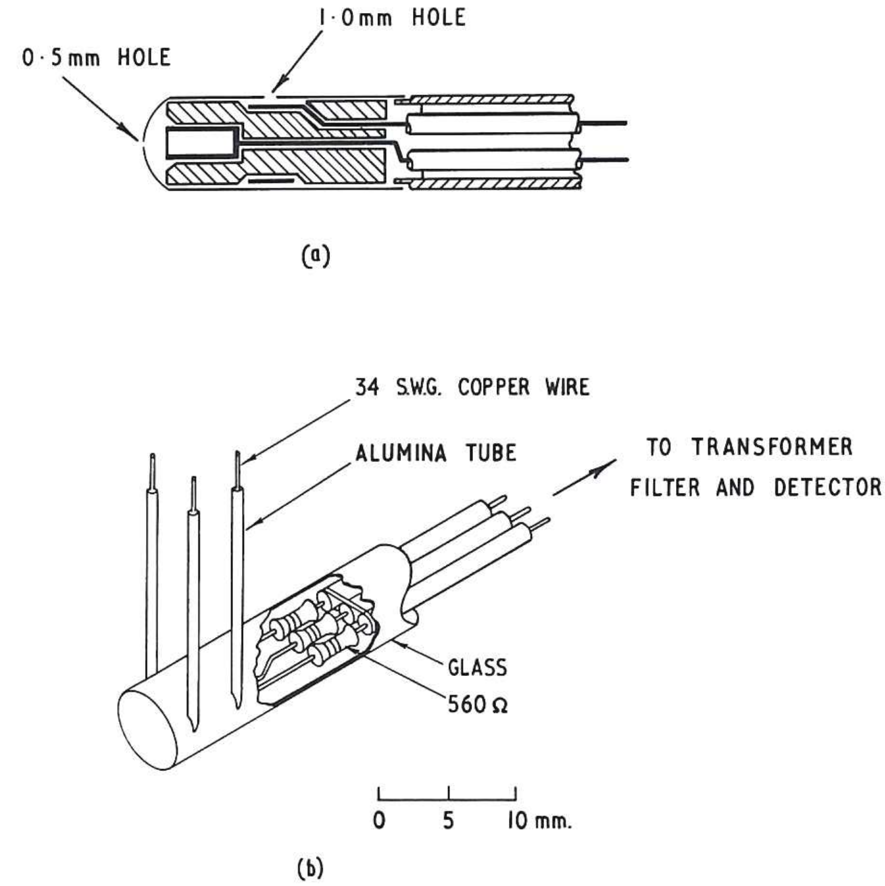

Fig. 4 (CLM-P 126)

(a) Ion probe used for measuring $V_{\parallel}$, $V_{\perp}$ and n. The collecting electrodes are biased negatively.

(b) Electric probe used to detect fluctuating electric fields

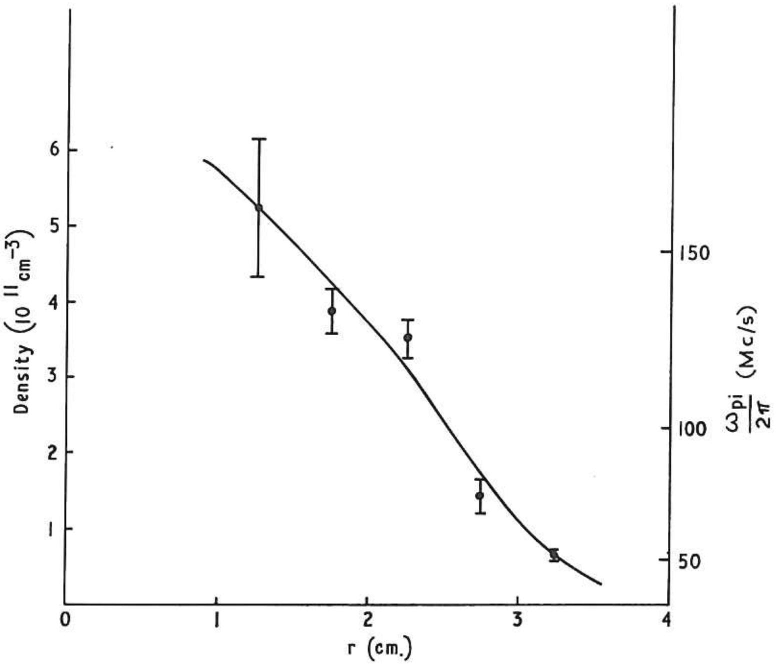

Fig. 5 (CLM-P 126)
The variation of plasma density with radius when $B_z = 2$ kG

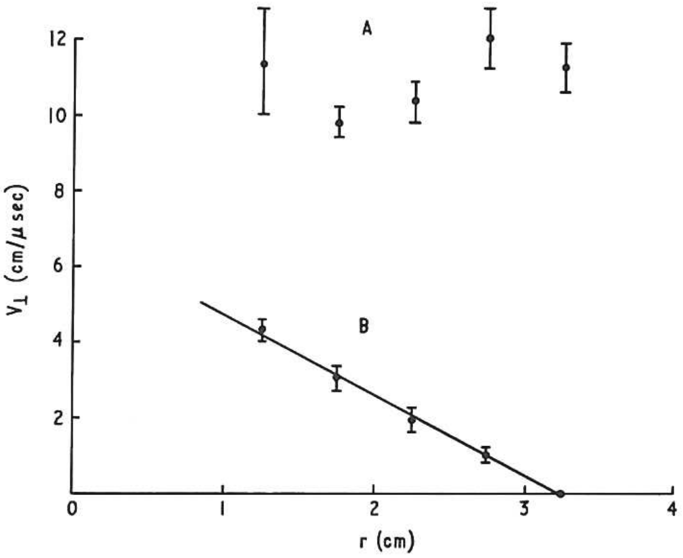

Fig. 6 (CLM-P 126)
The variation of azimuthal velocity $V_\perp$ with radius when $B_z = 2$ kG. Points 'A' refer to $V_\perp$ in the direction of a freely orbiting positive ion ; points 'B' refer to the opposite direction

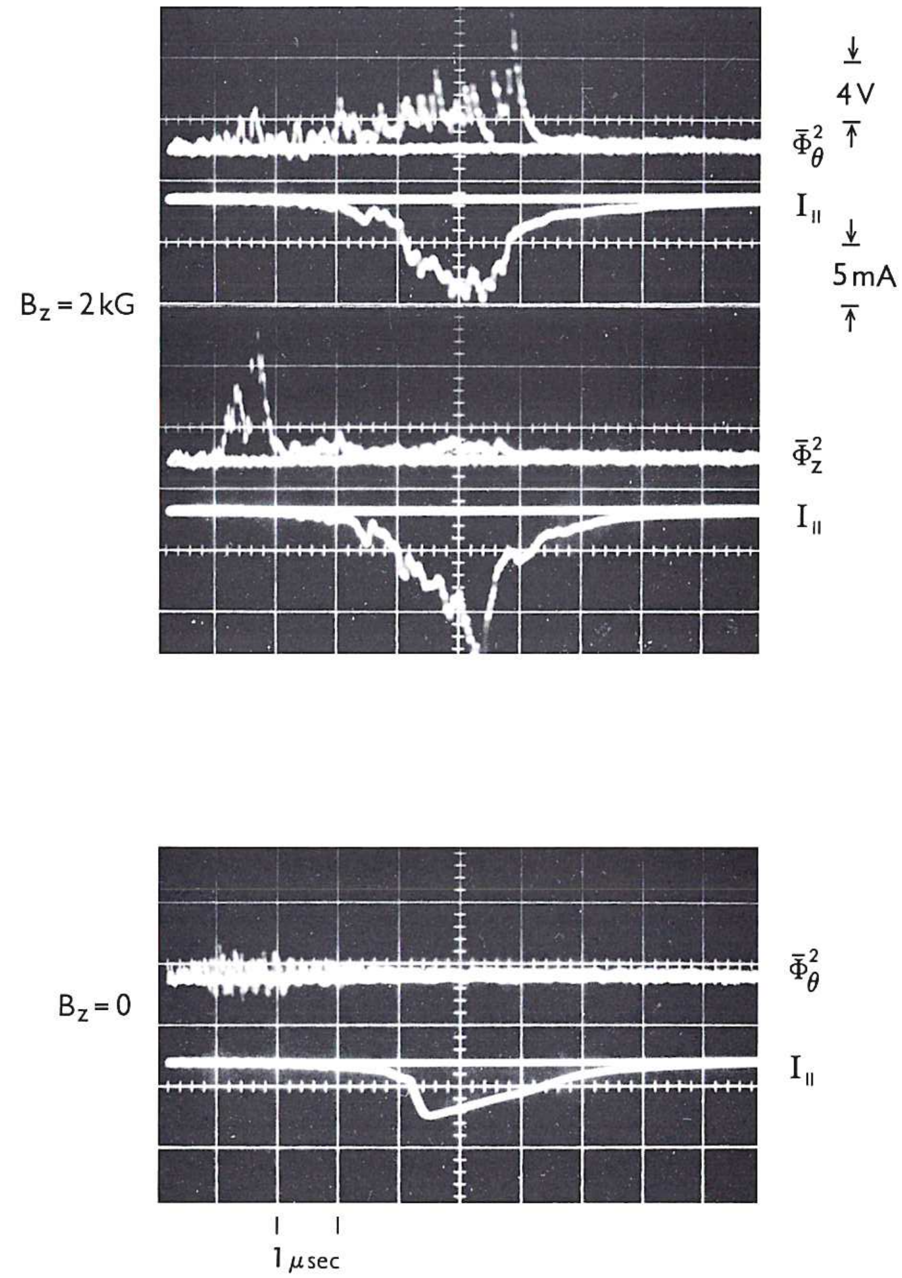

Fig. 7 (CLM-P 126)

Oscillograms of $\overline{\Phi_\theta^2}$ and $\overline{\Phi_z^2}$ for r = 3 cm with B_z = 2 kG and $\overline{\Phi_\theta^2}$ with B_z = 0. Bandwidth is 30-300 Mc/s. Signals from an ion probe at 1.3 cm radius are also shown

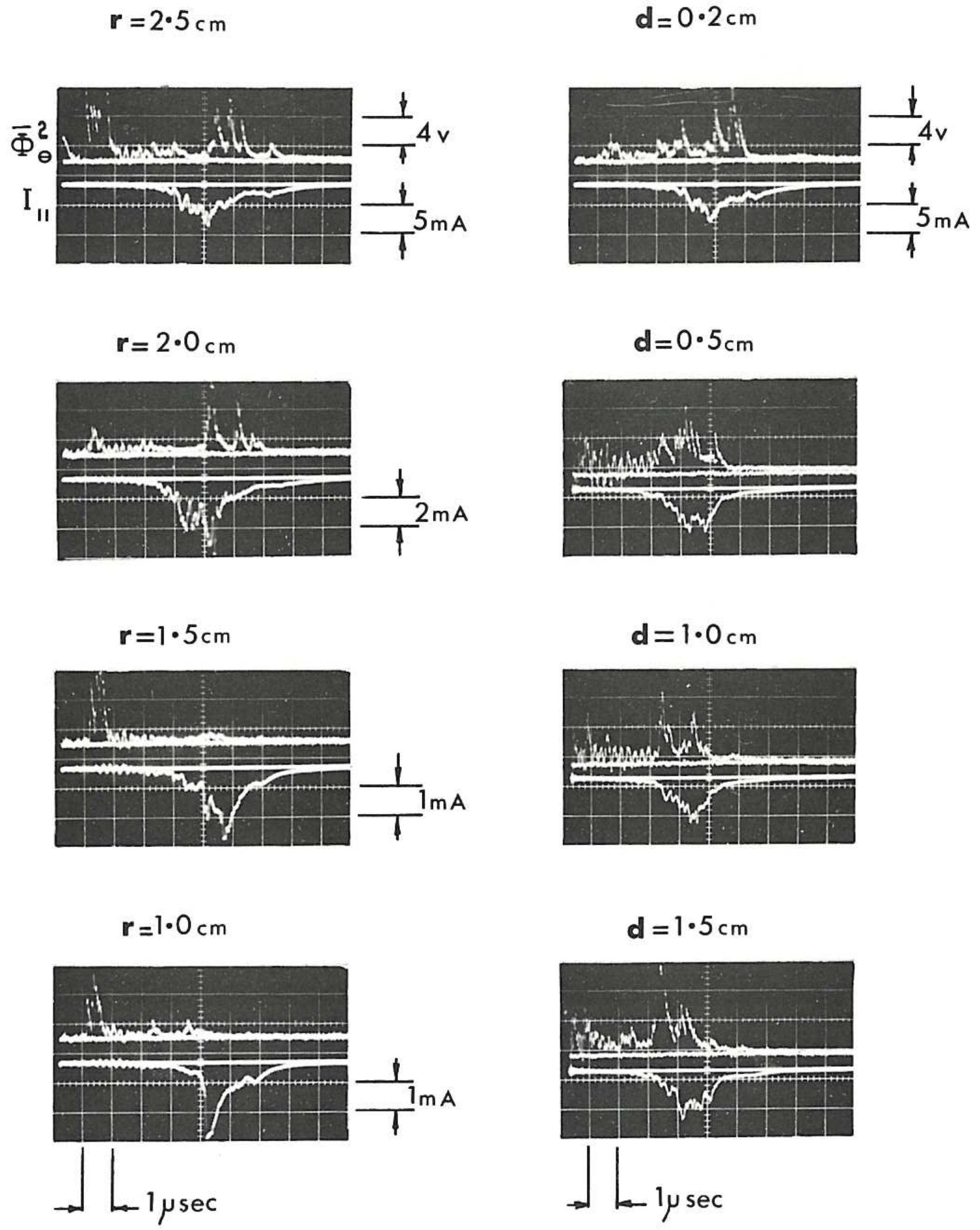

Fig. 8 (CLM-P 126)

Oscillograms of $\overline{\Phi_\theta^2}$ from an E-probe at different radii and $\overline{\Phi_\theta^2}$ from probes with different electrode spacings 'd'. Bandwidth is 30 - 300 Mc/s

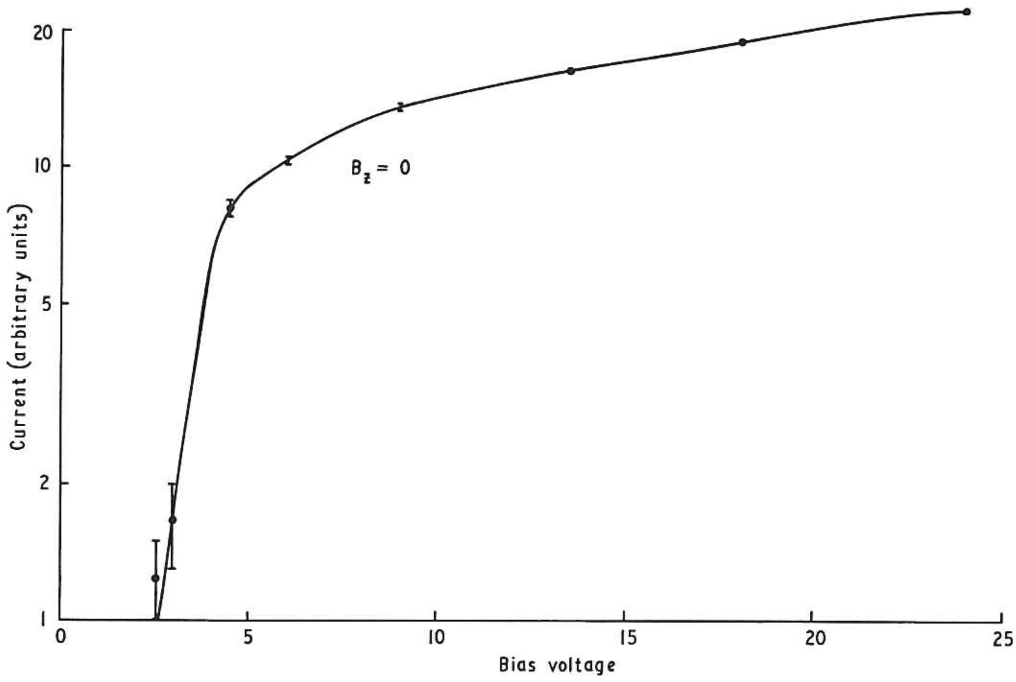

Fig. 9 (CLM-P 126)
The variation of ion probe current with bias voltage when $B_Z = 0$

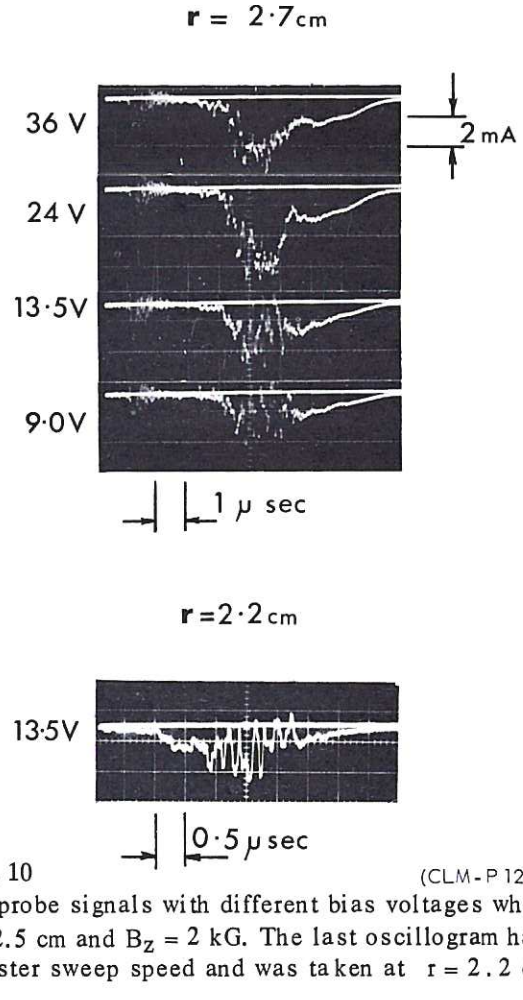

Fig. 10 (CLM-P 126)
Ion probe signals with different bias voltages when $r = 2.5$ cm and $B_Z = 2$ kG. The last oscillogram has a faster sweep speed and was taken at $r = 2.2$ cm