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THERE IS NO UNIQUE INTERPRETATION FOR THE SOLAR LYMAN ALPHA PROFILE

A. G. HEARN

Culham Laboratory
Abingdon Berkshire

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THERE IS NO UNIQUE INTERPRETATION FOR THE SOLAR LYMAN α PROFILE

by

A.G. HEARN

ABSTRACT

Calculations of the profiles of lines emitted by an atmosphere in which the electron temperature, the electron density and the Doppler width are allowed to vary show that the observed profile of the Lyman α line of hydrogen emitted by the sun may be produced by atmospheres which have a wide range of Doppler widths. Consequently there is no unique interpretation possible for the Lyman α profile alone. The simultaneous measurement of the Lyman β profile will give additional information, but without further observations and calculations it is not clear if a unique interpretation may be made.

U.K.A.E.A. Research Group,
Culham Laboratory,
Abingdon,
Berks.

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1. INTRODUCTION

The profile of the Lyman α line of hydrogen emitted by the sun from both quiet regions and plages was first measured in 1959 by Purcell and Tousey (1961). These profiles were interpreted by Morton and Widing (1961) who deduced Doppler temperatures for the hydrogen atoms of 90,000°K for the quiet regions and 70,000°K for the plages. Recently an interpretation has been made of the total intensities of the Lyman α and β lines which gives an upper limit for the electron temperature of the layers of the sun emitting these lines of 30,000°K (Hearn 1967). Because of the large differences between the temperatures obtained from these two interpretations, a numerical study has been made of the estimation of the Doppler widths from a Lyman α profile.

The calculations of Morton and Widing assume a particular variation of the electron temperature through the atmosphere which is specified by the expression for the Planck function. This gives a hot layer of moderate optical thickness at the edge of a cool semi-infinite atmosphere. They assume that the ratio ε of the rate of collisional de-excitation to the total rate of spontaneous radiative transitions and collisional de-excitation is constant through the atmosphere. The important part of this assumption is that the electron density is held reasonably constant in the hot outer layer. They also assume that the Doppler width is held constant through the entire atmosphere.

Morton and Widing found that for such an atmosphere the self-reversal of the profile depends only on the optical thickness of the outer hot layer, and the separation of the two peaks of the profile then depends only on the Doppler width. The profiles calculated by Morton and Widing fit the observed profiles quite well over the broad central self-reversal, but the wings of the calculated profiles fall much more sharply than in the observed profiles.

A very similar interpretation may be made from the assumption of a uniform finite plane parallel atmosphere. Profiles for such an atmosphere have been given by Hearn (1963). The semi-infinite atmosphere attached to the hot layer in the model used by Morton and Widing does not make a great deal of difference.

In these new calculations, the only assumptions built into them is that the atmosphere has plane parallel geometry and that the line is purely Doppler broadened. All the parameters specifying the atmosphere - the electron temperature, the electron density, the Doppler width and the frequency of the centre of the absorption profile - can be varied arbitrarily in the calculation. These calculations show that when the electron temperature and density are allowed to vary through the atmosphere, a given profile may be built up from any assumed Doppler width within a wide range of values.

Consequently there is no unique interpretation for a Lyman α profile taken alone. The simultaneous measurement of the Lyman β profile in many circumstances will enable a unique interpretation to be made, but it will not be clear if this is possible for the solar Lyman profiles until more observations are made simultaneously of the Lyman α and β profiles. These conclusions apply also to the profiles of the Lyman α and β lines of Helium II, but the interpretation of these lines will be simplified considerably by the measurement of the profile of the Balmer α line which, because it is optically thin, will give the mean Doppler width directly.

2. THE CALCULATIONS

Profiles were calculated for a finite plane parallel atmosphere in which it was assumed that the properties of the atmosphere depend only on the distance measured perpendicularly to the plane of the atmosphere. It was assumed that the line is purely Doppler broadened, that stimulated emission is negligible and that the source function is independent of frequency. The source function $S(\tau_{\nu_0})$ at an optical thickness τ_{ν_0} measured always at the centre frequency ν_0 of the absorption profile may be specified (Hearn 1963) by

$$S(\tau_{\nu_0}) = \frac{c}{4\pi}\frac{A(2,1)}{B(1,2)} \frac{\frac{B(1,2)}{\Delta\nu_D\sqrt{\pi}}\int_0^\infty \rho_\nu e^{-\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2} d\nu + n(c)K(1,2)}{A(2,1) + n(c)K(2,1)} \qquad \ldots (1)$$

where ρ_ν the radiation density at the point considered is integrated over the Doppler profile to give the contribution to the source function from photo-excitation by Lyman α photons. $\Delta\nu_D$ is the e^{-1} width of the Doppler profile and ν_0 is the frequency of the centre of the profile; both these parameters may vary through the atmosphere. The product of the electron density $n(c)$ and the rate coefficient $K(1,2)$ for collisional excitation from the ground level to the excited level, represents the contribution to the source function from collisional excitation.

This equation for the source function is different from the usual presentation because the term for collisional excitation is left in the equation explicitly instead of being related to the Planck function. This emphasises that it is the probability of excitation $n(c)\,K(1,2)$ sec^{-1} that is the fundamental quantity in this equation and not the electron temperature or density explicitly. The collisional rate coefficient $K(1,2)$ is a function of the electron temperature alone and is defined so that the rate of formation of excited atoms is $n(1)\,n(c)\,K(1,2)$ cm^{-3} sec^{-1} where $n(1)$ is the density of atoms in the ground level. The electron density occurs elsewhere in the equation only in the term $A(2,1) + n(c)\,K(2,1)$ which represents the processes of spontaneous transitions and collisional de-excitation depopulating the excited level. It is usual to write

$$\frac{A(2,1)}{A(2,1) + n(c)K(2,1)} = 1 - \varepsilon \qquad \dots (2)$$

For the formation of Lyman α photons ε is very small perhaps of the order of 10^{-6} and for the moderate optical thicknesses involved in these calculations the difference of this term from unity is completely negligible. Consequently the variation of ε in space is also completely negligible and the variation of electron density affects only the probability of collisional excitation.

Equation (1) has to be solved simultaneously with the calculation of the radiation density

$$\rho_\nu = \frac{1}{c} \int^{4\pi} I_\nu \, d\omega \qquad \dots (3)$$

where the intensity I_ν at the frequency ν is integrated over all solid angles, and with the solution of the equation of transfer to obtain the intensity I_ν from the source function S.

$$\frac{\cos\theta}{e^{-\left(\frac{\nu - \nu_0}{\Delta\nu_D}\right)^2}} \frac{dI_\nu}{d\tau_{\nu_0}} = - I_\nu + S \qquad \dots (4)$$

These equations combine to form a self-consistent integro-differential equation defining the variation of the source function through the atmosphere.

The parameters of the atmosphere which vary in space, namely the probability of collisional excitation $n(c)\,K(1,2)$, the Doppler width which is allowed to vary independently of any electron temperature implied in the collisional excitation rate, and the centre frequency of the profile ν_0, were specified at a number of points through the atmosphere. The number of points and their spacing have no restrictions. Between these points the parameters were assumed to vary linearly with space. With this type of representation any atmosphere can be specified.

The equations were solved by means of a Riccati transformation (Rybicki and Usher, 1966, Hummer and Rybicki 1967). This transformation changes a boundary value problem to an initial value problem and the solution involves integrating three sets of matrix differential equations. The integration of these equations presents some difficulty because the solution becomes unstable with large step lengths. An integration method described by Treanor (1966) was used because it allowed a rather longer step length than the usual Runge Kutta methods.

3. LYMAN α PROFILES

The central self-reversal of the observed Lyman α profiles may be fitted quite well by the model used by Morton and Widing and also by a simple uniform, finite, plane parallel atmosphere. But in both these cases, the wings of the calculated profile fall much more sharply than the wings of the observed profile.

The wings may be fitted with a model atmosphere of two layers, where the second deeper layer has a larger Doppler width. The temperatures obtained in this way are rather high. For the two profiles fitted by Morton and Widing, the temperature obtained from the wings of the profile from the quiet region is 550,000°K compared with 90,000°K obtained from the centre profile, and about 750,000°K from the wings of the profile from a plage compared with 70,000°K from the centre.

These temperatures are disconcertingly high, and calculations have been made to see whether the wings can be produced from a smaller Doppler width. In Fig.1, the complete profile drawn in a solid line is produced by an atmosphere of two layers; the outer layer has a Doppler width of 1.0 and the inner layer has a Doppler width of 1.5. These Doppler widths and all the other parameters except the optical depth are in arbitrary units since these calculations are concerned only with relative variations through the atmosphere. The total optical thickness of the atmosphere at the centre of the line is 20 and the distribution of the Doppler width and the probability of collisional excitation $n(c)\ K(1,2)\ sec^{-1}$ through the atmosphere are shown plotted against optical thickness in Fig.2, together with the source function. The discontinuous slope in the source function comes from the discontinuity in the Doppler width. The profile is that seen by an observer to the left of the diagram. This atmosphere produces a profile where the separation between the peaks is determined by the Doppler width of 1.0 and the wings are produced by the Doppler width of 1.5. The wings produced by a Doppler width of 1.0 are shown in Fig.1 inside the profile. They are not joined to the centre to avoid confusion.

The profile produced by the atmosphere with two Doppler widths has been reproduced by an atmosphere with a Doppler width of 1.0 throughout and this is shown in Fig.1 in dotted lines. The excitation rate and the Doppler width producing this profile are shown plotted in Fig.3 against the optical thickness, together with the source function. The broader wings are produced by having a much larger atmosphere; the optical thickness here is 200. The differences between the two profiles would be difficult to detect with the present accuracy of measuring profiles. In addition the model is a simple one and with added complexity the agreement could presumably be substantially improved.

There is no clear limit to the reduction of the Doppler width that may be obtained in explaining a given set of wings in this way. The reduction of 1.5 in Doppler width corresponds to a reduction of 2.25 in the temperature. As the Doppler width is reduced, the optical thickness of the atmosphere has to be increased. Obviously at some stage in this increase, other processes like collisional de-excitation or photo-ionization will become important, but it does not seem possible to give any general limit to this process.

In a similar way, the central self-reversal of a profile produced by a uniform atmosphere may be reproduced by an atmosphere with a smaller Doppler width when the excitation rate is allowed to vary through the atmosphere. Fig.4 shows in a solid line the profile given by a uniform atmosphere with a Doppler width of 1.6 and an optical thickness of 10. The same central self-reversal has been produced in the profile given by the dotted lines from an atmosphere with a constant Doppler width of 1.0. This reduction of 1.6 in the Doppler width corresponds to a reduction of 2.56 in the temperature. The optical thickness of this atmosphere is 100 and the excitation rate and the source function produced are shown plotted against the optical thickness in Fig.5. Once again this profile has been built up from a simple model and with added complexity the agreement could be improved. A given self-reversal is duplicated by an atmosphere with a smaller Doppler width, again at the expense of requiring an atmosphere of greater total optical thickness. The wings fall away sharply since they are the wings of a Doppler width of 1.0, but they could be extended to fit the profile of Doppler width 1.6 at the expense of increasing still further the total optical thickness of the atmosphere.

It appears that the model of Morton and Widing gives the upper limit for the Doppler temperature, for no way has been found of reproducing the central self reversal of the Lyman α line with a Doppler temperature greater than that obtained by Morton and Widing.

Calculations with the Gaussian profile centred on frequencies varying with space show that the slight asymmetries observed in the 1959 profiles may be explained by Doppler radial shifts of about 20% of the thermal Doppler width. This explanation for the asymmetry of lines was first suggested by Miyamoto (1958).

4. DISCUSSION

These calculations show that the Lyman α profile by itself contains surprisingly little unique information about the variation of the Doppler width and other properties through the atmosphere. It would appear that the profile alone contains little more information than the total intensity of the line. This must come from the fact that all Lyman α photons which are generated eventually leave the atmosphere, so that when a Lyman α photon is observed it is difficult to tell from which part of the atmosphere it comes originally.

For a Lyman β photon it is different, for the emission of a Balmer α photon is about as probable as the emission of a Lyman β photon. So that the Lyman β photons that are observed are weighted in favour of those leaving at their first attempt. This means that the source function for Lyman β is much more strongly controlled by the local property of the collisional excitation rate than it is for Lyman α. It follows then that the profile of the Lyman β line may show much more strongly the variations of the local properties of the atmosphere, although it is not clear how much the coupling between levels 2 and 3 will destroy this. There is a second difference in the formation of the Lyman α and β profiles; the scale in optical thickness in Lyman β is 6.24 times less than the scale in Lyman α. Even if the source functions of the two lines were completely coupled together, the measurement of the Lyman β profile would give very useful additional information for the interpretation of the Lyman α profile.

It requires further detailed calculations and more observations to find out how much information may be extracted uniquely from the profiles of Lyman α and β measured and interpreted simultaneously.

It has been shown that a given Lyman α profile can be reproduced for any assumed Doppler width within a wide range. If the Doppler width is known then a certain amount of unique information can be extracted even from a Lyman α profile. The total optical thickness of the layers emitting the line can be estimated and a crude picture of the variation of the rate of collisional excitation through the atmosphere may be obtained.

Calculations on the interpretation of the total intensities of the Lyman α and β lines (Hearn 1967) show that the layers emitting these lines are optically thin in Balmer α. Unfortunately the emission of Balmer α photons related to the emission of Lyman β photons of hydrogen is completely masked by the strong emission in the visible region of the photosphere so that there is no obvious way of determining independently the Doppler width of these lines.

However for Helium II, the profile of the Balmer α line at 1640 Å can be observed directly and this could profitably be combined with the measurements of the Lyman α and β lines at 304 Å and 256 Å.

5. ACKNOWLEDGEMENTS

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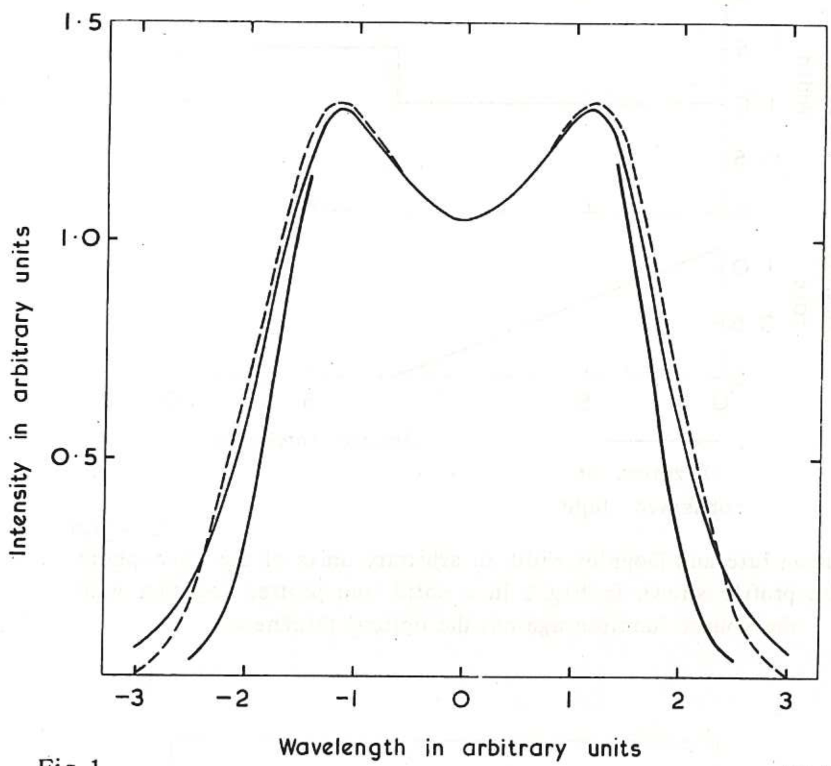

Fig. 1

(CLM-P 128)

The profile of a line whose wings are formed by a Doppler width of 1.5 is shown in a solid line. The wings of this line are reproduced by the profile in a dotted line which is formed by an atmosphere of Doppler width of 1.0. The wings formed by a Doppler width of 1.0 corresponding to the profile in a solid line are shown separately

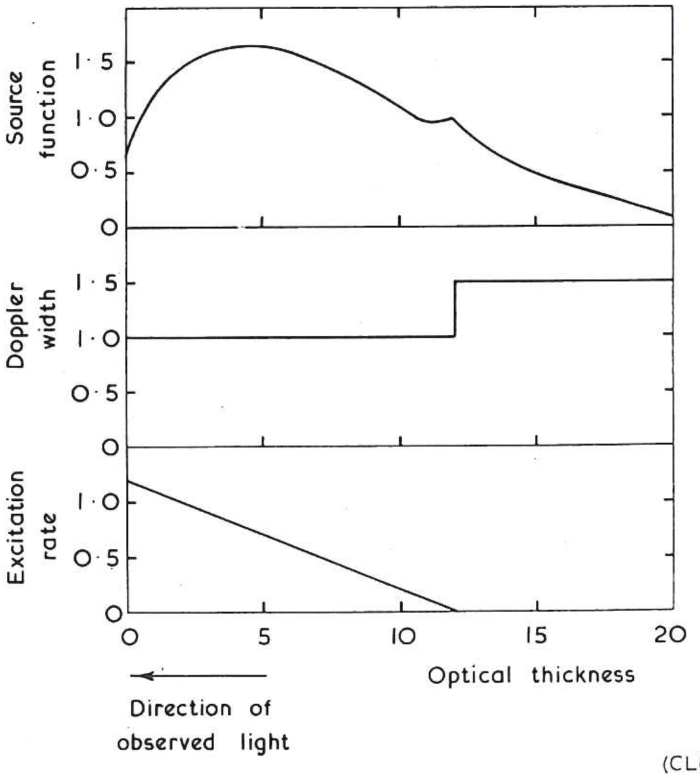

Fig. 2 (CLM-P 128)

The excitation rate and Doppler width in arbitrary units of the atmosphere emitting the profile shown in Fig. 1 in a solid line plotted together with the source function against the optical thickness

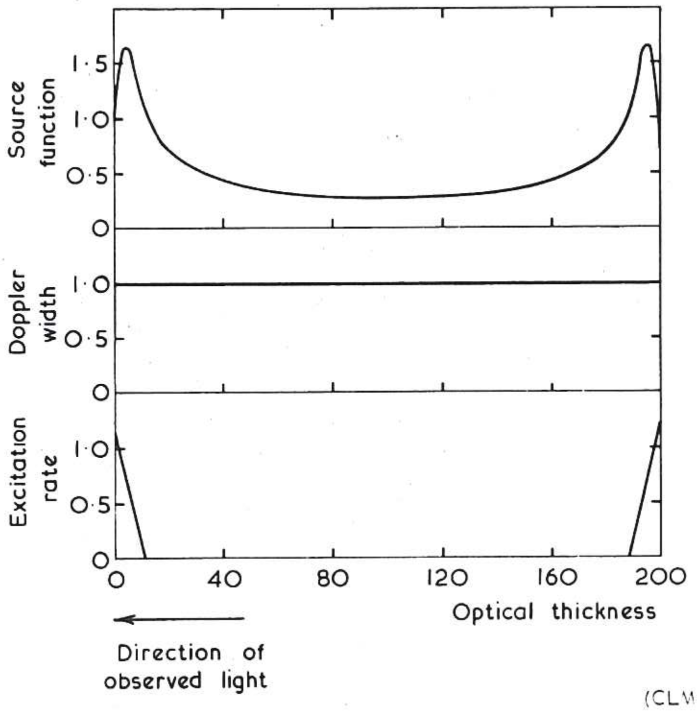

Fig. 3 (CLM-P 128)

The excitation rate and Doppler width in arbitrary units of the atmosphere emitting the profile shown in Fig. 1 in a dotted line plotted together with the source function against the optical thickness

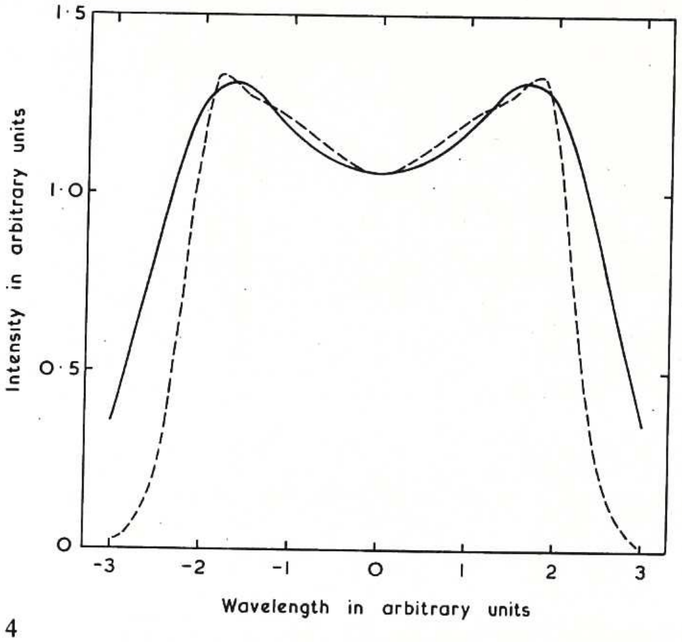

Fig. 4

(CLM-P 128)

The profile of a line formed in an atmosphere with a Doppler width of 1.6 is shown in a solid line. The central self-reversal of this line is reproduced by the profile in a dotted line which is formed by an atmosphere of Doppler width 1.0

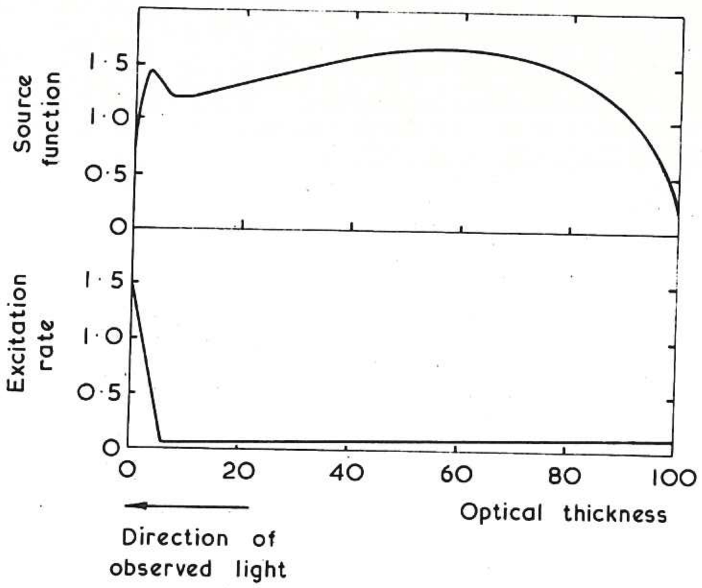

Fig. 5

(CLM-P 128)

The excitation rate of the atmosphere with a Doppler width of 1.0 which emits the profile shown in a dotted line in Fig. 4 plotted together with the source function against optical thickness