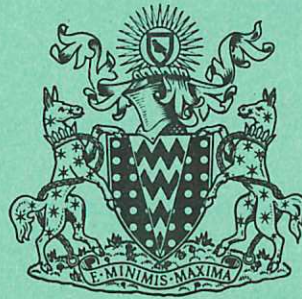
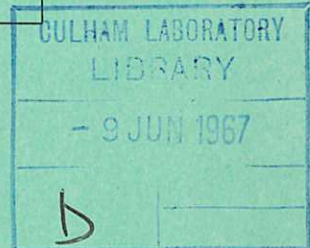


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United Kingdom Atomic Energy Authority

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# FACTORS DETERMINING BETA IN A THETA PINCH EXPERIMENT

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1967

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(Approved for publication)

FACTORS DETERMINING BETA IN A  
THETA PINCH EXPERIMENT

by

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(Submitted for publication in Nuclear Fusion)

A B S T R A C T

The containment and stability of the theta pinch depend critically upon the value of beta. However, whilst theoretical calculations predict a value of unity on the axis, experimental results show that the value on axis is substantially less than this.

The possibility that these low experimental values are due to a small field diffusion in the early stages of the current sheath formation, followed during the compression by an energy loss is examined. Using various values of  $\gamma$  to simulate the effects of energy loss, the variation of beta during the shock phase, and during the isentropic compression phase, is calculated.

The results are compared with observations on the Megajoule Theta-pinch and it is shown that the penetration of a few hundred gauss in the sheath formation stage can account for the observed values in the present experiments.

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March 1967 (ED)

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## 1. INTRODUCTION

Many of the important properties of the theta pinch depend upon the value of beta i.e. the ratio of particle pressure to the pressure of the external magnetic field. Theoretically, the particle containment time<sup>(1,2)</sup> is sensitive to beta, becoming longer as beta approaches unity; the hydromagnetic stability is also dependent on beta<sup>(3)</sup> and it has been shown<sup>(4)</sup> that the discharge can be stable for  $m = 1$  perturbations when  $\beta = 1$ . It follows that in many theta pinch experiments, using no initial applied bias field, one of the principle objectives has been to achieve a value of beta as high as possible.

Experiments have been reported<sup>(5)</sup> with an initial gas pressure of 250 millitorr in which the value of beta on the axis, measured using magnetic probes, was 0.95. There followed a trend<sup>(6)</sup> towards lower starting pressures (10-30 millitorr) in order to achieve higher temperatures, and a number of experiments<sup>(7,8,9)</sup> have been carried out in this 'low pressure regime'. Theoretical calculations<sup>(10)</sup> based on the magnetohydrodynamic equations predicted that in these conditions the value of beta on the axis would be unity, and the radial density distribution would have a sharp boundary. However, when the radial variation of electron density was first measured<sup>(7)</sup> in such an experiment, a diffuse distribution was found; further measurements suggested that the value of beta on the axis was substantially less than the theoretical value of unity. These observations have now been reported from three different experiments<sup>(7,11,12)</sup>.

As yet no satisfactory explanation for the low experimental values of beta has been proposed. A recent theory<sup>(13)</sup> predicts a diffuse current sheath of starting pressures below 5 millitorr; however, it cannot explain a current layer of thickness comparable to the plasma radius, as observed<sup>(7)</sup>, at 20-30 millitorr. It has been tentatively suggested<sup>(7)</sup> that the low values of beta might arise from field diffusion in the early stages, followed by compression and a loss of energy from the plasma (e.g. radiation). Such a process would lead to a reduction in beta.

This paper examines the last possibility in detail. Measurements of the radial density distribution are described and compared with the predictions of the Hain-Roberts code<sup>(10,14,15)</sup>; the data is then

analysed further in terms of beta. Theoretical calculations of the reduction in beta produced by shock and isentropic compression in the presence of an energy loss are presented and discussed and the results compared with the observations.

It is concluded that diffusion of the externally applied magnetic field into the plasma at early times, together with an energy loss process during the compression can account for the low values of beta observed.

## 2. APPARATUS AND DIAGNOSTIC TECHNIQUES

The Megajoule Bank was operated in the manner described elsewhere<sup>(7)</sup>. The peak field of 63 kG was reached in 6  $\mu$ sec using a coil 2 metres long and 10 cm in diameter. Deuterium gas at an initial pressure of 30 millitorr was used.

The electron density distribution was determined from absolute measurements of the continuum using a technique first described by Eberhagen and Keilhacker<sup>(16)</sup>. The plasma was viewed radially using a 10 Å bandwidth interference filter, which selected a suitable line-free region of the spectrum at 4978 Å. The light was focused onto an array of 10 fibre bundles coupled to 10 photomultipliers, so that each photomultiplier collected light emitted along one of 10 parallel chords. Assuming cylindrical symmetry, the radial density distributions were obtained numerically using an Abel inversion. The data was analysed assuming a constant electron temperature and an impurity concentration of 1% oxygen. A correction was made for background light, which was usually less than 5% of the peak intensity. The results quoted are the average values from three discharges, which were reproducible to within 10%; the overall uncertainty in the results was estimated to be from 15-20%. The assumption that the radiation in the visible region was pure continuum became valid at about 2  $\mu$ sec and the profile could not be determined at earlier times by this method. The use of this technique in the present experiment has been described in detail elsewhere<sup>(17)</sup>.

### 3. EXPERIMENTAL RESULTS AND DISCUSSION

#### RESULTS

Fig.1 shows the radial density distribution at 2  $\mu\text{sec}$  together with the theoretical profiles obtained using the Hain-Roberts code<sup>(10)</sup>; Fig.2 shows, at 6  $\mu\text{sec}$ , a comparison between experiment and theory calculated assuming 1% oxygen impurity, and full initial ionization; no impurity and full ionization; and no impurity and 10% initial ionization. Fig.3 shows the observed value of the electron density on the axis as a function of time together with the results of numerical computations carried out in the same conditions as those in Fig.2. Although the time variation of the axial density is in approximate agreement with theory, its value is 15% lower than the best theoretical estimate which lies mid-way between that for 1% impurity and 10% initial ionization. (A code including the effects of impurities and partial ionization is not available.) The effect of adding impurities is to raise the theoretical value of electron density while reducing the degree of ionization lowers it. Correcting the experimental data for an impurity content larger than 1% will decrease the final experimental values of electron density<sup>(17)</sup>.

The radial variation of electron density differs markedly from the theoretical shape, in a way which cannot be explained by including the effects of partial ionization or impurities in the code. The theory predicts that the electron density decreases slowly with radius over most of the plasma and then falls rapidly, whereas the experimental profiles show a steady fall from the axis outwards.

#### ANALYSIS OF THE RADIAL DENSITY DISTRIBUTION IN TERMS OF BETA

The significance of this latter difference can best be understood by analysing the results in terms of beta, with the following assumptions:-

- (1) Electron and ion temperatures are constant across the radius; this is reasonable because of the high transverse ion thermal conductivity and short relaxation time between ions and electrons.
- (2) There is no axial loss of plasma; this has been experimentally demonstrated in the midplane for times up to 6  $\mu\text{sec}$ <sup>(7)</sup>.

Let  $n$ ,  $\beta$  and  $B_i$  be the values of electron density, beta and trapped magnetic field, all functions of the radius,  $r$ . Let  $n_0$ ,  $\beta_0$

and  $B_i$  be the corresponding values on the axis. Let  $T$  be the sum of the electron and ion temperatures and  $B$  be the intensity of the magnetic field external to the plasma.

Beta is defined by the relation

$$\beta = 1 - \frac{B_i^2}{B^2} \quad \dots (1)$$

On the axis, pressure balance gives:

$$n_0 kT = \beta_0 B^2 / 8\pi \quad (k \text{ is Boltzmann's constant})$$

At radius  $r$ ,

$$nkT = \beta B^2 / 8\pi$$

$$\therefore n/n_0 = \beta/\beta_0 \quad \dots (2)$$

Thus beta varies with radius in the same way as the electron density, and the radial distribution of  $\beta$  can be obtained from that of  $n$  if  $\beta_0$  is known. The radial distribution of  $B_i$  can also be found, using equation (1). The observed profile corresponds to a steady rise of  $B_i$  from a minimum value on the axis outwards and it follows, using Maxwell's equations, that the current sheath is diffuse.

The total flux,  $\phi_1$ , contained within any radius,  $r_1$ , is given by:-

$$\phi_1 = \int_0^{r_1} 2\pi r B_i dr$$

or

$$\phi_1 = 2\pi B \int_0^{r_1} \left(1 - \beta_0 \frac{n}{n_0}\right)^{1/2} dr \quad \dots (3)$$

In particular,  $r_1$  may be chosen to be the radius containing a fixed number of particles. The magnetic flux enclosed by a cylindrical shell containing any known mass of plasma can be determined as a function of time for various positions along the axis. Equation (3) was used to calculate the magnitude of the field diffusion (see below).

The value of  $\beta_0$  was obtained as a function of time using the measured electron density on the axis, and the experimental value of electron temperature assuming  $T_e = T_i$ .  $T_e$  was found<sup>(18)</sup> from an absorption method in the soft x-ray region and from the peak of the



continuum emission. Both these methods integrate along the axis to give an average value, but the temperature under the coil has been shown to be uniform to within  $\pm 15\%$ <sup>(18)</sup>. The maximum value of electron temperature is  $310 \pm 40$  eV; its time variation is given elsewhere<sup>(18)</sup>. An electron temperature of 300 eV and a density of  $10^{17}$  give an electron-ion energy equipartion time of 2  $\mu$ sec. The assumption that  $T_e = T_i$  is reasonable for times greater than this. The values of  $\beta_0$  obtained are shown in Table 1.

TABLE 1

$\mu$ sec	2.0	4.0	5.0
$\beta_0$	$0.8 \pm 0.2$	$0.6 \pm 0.2$	$0.6 \pm 0.2$

Before the ions and electrons reach equilibrium,  $\beta_0$  could approach unity, in which case  $T_i/T_e = 1.4$ . The time variation of the neutron yield<sup>(7)</sup> suggests that at early times  $T_i$  is greater than  $T_e$ , and so it is likely beta exceeds 0.8 before 2  $\mu$ sec.

It follows that for zero initial bias field the experimental plasma differs from that predicted by the idealised magnetohydrodynamic model in two important respects: firstly, it contains parallel trapped flux and the value of beta on the axis is 0.6 compared with the theoretical expectation of unity: and, secondly the current is distributed and does not flow in a thin sheath.

#### 4. FACTORS LEADING TO A REDUCTION IN BETA AND A DISTRIBUTED CURRENT SHEATH

The experimental profile showed the presence of trapped field from the earliest measurement at 2  $\mu$ sec (Fig.1). This could only be reproduced by the code if the preionized gas was assumed to contain trapped parallel field, or if the value of the resistivity used was much larger than that given by Spitzer<sup>(19)</sup>. It is concluded that some field diffusion took place before 2  $\mu$ sec, probably during the initial formation of the plasma. The effective resistivity necessary to account for the profile shape by anomalous diffusion in the early stages was estimated by carrying out computations assuming a

resistivity which was larger than the Spitzer value,  $\eta_S$ . For  $\eta = 10 \eta_S$  the current sheath was still sharp, but when  $\eta = 100 \eta_S$  a diffuse sheath was found.

The observed reduction in beta between two and five microseconds could have been due to anomalous field diffusion during that interval; this question was studied using equation (3) to determine the time variation of the trapped flux mixed with a fixed mass of plasma. A small increase in flux was found, but the cumulative error in the analysis was large and the result could not be established with certainty. The effect was smallest near the axis and increased as plasma at larger radii was included in the flux integral; however, it was insufficient to explain the observed reduction in beta. Effects due to classical diffusion would be three orders of magnitude too small to detect.

The observed reduction in beta is accounted for in the present experiments as follows: a small magnetic field penetrates the plasma at early times, due to high resistivity, and the value of beta is further reduced by the radial shock wave and by the isentropic compression. This reduction becomes large in the presence of an energy loss, and the maximum fall in beta takes place when this loss is so large that the plasma compression is isothermal - that is with an effective  $\gamma$  (ratio of specific heats) of unity. This latter situation occurred in the present experiments and a constant temperature (due to thermal conduction) was observed<sup>(18)</sup>. In the appendix the variation of beta across a plane shock wave and during isentropic compression is analysed theoretically using a simple model in which there is a uniform mixture of plasma and magnetic field, and the effect of an energy loss is taken into account by using reduced values of  $\gamma$ . The results of this analysis will now be compared with the experiment.

For the shock phase only qualitative comparison is possible between the theory given in the appendix and the experimental results because of the simplifying assumptions of the model, and the uncertainty in the value of  $\gamma$ , reduced by ionization, radiation or thermal conductivity, during the implosion. The variation of  $\gamma$  during a shock wave has been discussed elsewhere<sup>(20,21)</sup>. The curves in Fig. 5

show that, for an initial beta of 0.99 and the experimental Mach number of about six beta will fall across a plane shock wave when  $\gamma$  is less than 1.3. Thus in the experiment, if the initial beta lies in the range  $0.9 < \beta < 1$ , its value will either fall or remain approximately constant across the shock wave.

During the isentropic compression, when the external field rises by a factor of seven and the value of  $\gamma$  (see above) is close to unity, it is seen from Fig.10 that a large reduction in beta will take place. The results shown in Fig.10 have been re-plotted in Fig.4 using experimental values to show the variation in beta with time during the discharge for several initial values of beta. It is seen that the experimental variation of beta can be explained in this way if the initial value is 0.99, which is equivalent to an initial field on the axis of a few hundred gauss before the plasma leaves the walls.

## 5. CONCLUSION

It was shown from an analysis of the experimentally determined radial density distribution that the plasma contained trapped parallel field although no initial bias field was applied; also, the current was distributed. The observed value of beta on the axis decreased from 0.8 at 2  $\mu$ sec to 0.6 at 5  $\mu$ sec, compared with the predictions of the theoretical model that it should remain constant at unity. It was deduced that rapid diffusion occurred in the early stages of compression. It was shown that when a plasma, containing a small trapped magnetic field is heated by shock and isentropic compression and is at the same time subject to a large energy loss (due to thermal conductivity in the present experiments) the value of beta will fall substantially during compression. The reduction in beta expected on this basis was calculated and found to agree well with the observations. It is concluded that a plasma heated by rapid compression, which is subject to a large energy loss, will result in a final value of beta much less than unity.

## 6. ACKNOWLEDGEMENTS

The authors are grateful for a number of helpful discussions with G.B.F. Niblett, J.A. Reynolds and A.A. Newton, who also made available some material on energy losses from the plasma.

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## APPENDIX

### VARIATION IN BETA FOR SHOCK WAVE AND ISENTROPIC COMPRESSION

The solutions of the equations for a plane hydromagnetic shock wave and for the isentropic compression of a uniform magnetised plasma are well known for the case when  $\gamma$ , the ratio of specific heats, equals 5/3. However, the results are sensitive to the value of  $\gamma$ , particularly as it approaches unity; the magnitude of  $\gamma$  is reduced by the presence of an energy loss process in the plasma, for example radiation or thermal conduction. Here the equations are solved for a range of values of  $\gamma$ .

The variation of beta across a plane hydromagnetic shock wave is computed for different values of  $\gamma$  and Mach number. Its variation as a function of the ratio  $B_2/B_1$  is also calculated for the case of a plasma compressed isentropically by a magnetic field which rises from  $B_1$  to  $B_2$ . The results for both cases are presented in graphical form.

#### SHOCK COMPRESSION

A plane hydromagnetic shock wave with an initial magnetic field perpendicular to the direction of flow has been analysed by Ferraro<sup>(22)</sup> and many others. Beta is defined by the relation:

$$\beta = \frac{p}{p + B_1^2/8\pi}$$

where  $p$  is the plasma pressure and  $B_1$  the magnetic field within the plasma. It is convenient to introduce  $\beta^*$ , defined as follows

$$\beta^* = \frac{8\pi p}{B_1^2} \quad \dots \text{ (A1)}$$

and related to  $\beta$  by the equation

$$\beta = \frac{\beta^*}{1 + \beta^*} \quad \dots \text{ (A2)}$$

By flux conservation

$$\frac{B_{i2}}{B_{i1}} = \frac{\rho_2}{\rho_1} \quad \dots \text{ (A3)}$$

where  $\rho$  is the density and the subscript 1 refers to the unshocked gas and the subscript 2 to the shocked gas.

From equations (A1) and (A3)

$$\frac{\beta_2^*}{\beta_1^*} = \frac{p_2}{p_1} \frac{\rho_1^2}{\rho_2^2} \quad \dots (A4)$$

from which, using equation (A2),  $\beta_2$  may be calculated if  $\beta_1$  is known.

The hydromagnetic shock wave equations, which will not be reproduced in full here, can be expressed<sup>(22)</sup> in terms of the following quantities

$$\begin{aligned} X &= \frac{\rho_2}{\rho_1} & Y &= \frac{p_2}{p_1} \\ N^2 &= \gamma M_1^2 = \gamma \frac{u_1^2}{a_1^2} & & \dots (A5) \\ Q &= \frac{1}{\beta_1^*}, & a_1 &= \sqrt{\gamma p_1 / \rho_1} \end{aligned}$$

where  $u_1$  = piston velocity and  $a_1$  the sound speed in the unshocked gas.

The Alfvén Mach number,  $M_1^*$ , is defined by the relation

$$M_1^* = \frac{u_1}{a_1^*} \quad \text{where} \quad a_1^* = \sqrt{V_{A_1}^2 + a_1^2}$$

and

$$V_{A_1} = \sqrt{\frac{B i_1^2}{4\pi\rho_1}}$$

Also, using equation (A5),

$$M_1^* = \frac{N}{(2Q + \gamma)^{1/2}} \quad \dots (A6)$$

where  $V_{A_1}$  is the Alfvén speed ahead of the shock. Note that  $M_1$  is always greater than  $M_1^*$ . It can be shown that the equations to be solved are

$$Q(2 - \gamma)X^2 + \{\gamma(Q + 1) + \frac{1}{2}(\gamma - 1)N^2\} X - \frac{1}{2}(\gamma + 1)N^2 = 0 \quad \dots (A7)$$

and

$$Y = \frac{p_2}{p_1} = 1 + N^2 \left( 1 - \frac{1}{X_0} \right) - Q(X_0^2 - 1) \quad \dots (A8)$$

where  $X_0$  is the positive root of equation (A7).

From equation (A3)

$$\beta_2^* = \beta_1^* Y/X^2$$

Values of  $X$ ,  $Y$ ,  $\beta$  were computed for  $M_1^* = 1 - 50$ , and a range of values of  $\beta_1$  and  $\gamma$ .

Figs.5-9 show  $\beta_2$  as a function of  $M^*$  for  $\beta_1 = 0.99, 0.91, 0.667, 0.333, 0.1667$  and various values of  $\gamma$ . The case of  $\gamma = 1$  is not included since the density ratio tends to infinity, and beta to zero. It is seen that beta falls across a shock wave for small Mach numbers and then rises as the shock strength increases. The reduction in beta becomes larger as  $\gamma$  is reduced.

It is always possible to find a sufficiently large Mach number to produce a rise in beta across a shock, but for low values of  $\gamma$  and high initial values of beta the required Mach number will probably be unrealistic for an experiment.

#### ISENTROPIC COMPRESSION

The variation of beta during the isentropic compression of a magnetised plasma by a magnetic field has been calculated for different values of  $\gamma$  as a function of  $B_2/B_1$ , where  $B_1$  and  $B_2$  are the values of the external magnetic field before and after the compression. The following equations are used:

$$\text{Adiabatic Compression} \quad pV^\gamma = \text{const} = c_1 \quad \dots (A9)$$

$$\text{Conservation of flux} \quad B_i V = \text{const} = c_2 \quad \dots (A10)$$

$$\text{Pressure balance} \quad B^2 = 8\pi p + B_i^2 \quad \dots (A11)$$

$$\text{Definition of } \beta \quad \beta = \frac{8\pi p}{B^2} \quad \dots (A12)$$

where  $B_i$  is the internal field,  $B$  the external field and  $V$  the volume. From which

$$B_i^2 = (1 - \beta) B^2$$

$$B_i^2 V^2 = (1 - \beta) B^2 V^2 = c_2^2$$



Substitute for  $V$

$$B^2 (1 - \beta) \left(\frac{c_1}{p}\right)^{2/\gamma} = c_2^2$$

Substitute for  $\beta$  and re-arrange

$$B^2 (1 - \beta) = \left( \frac{c_2^2}{c_1^{2/\gamma} 8\pi^{2/\gamma}} \right) \cdot (\beta \cdot B^2)^{2/\gamma}$$

Take to power of  $\gamma/2$

$$B^{\gamma-2} (1 - \beta)^{\gamma/2} = \text{const} \cdot \beta$$

Let the value of  $B$  and  $\beta$  before and after the compression be  $B_1$  and  $B_2$  and  $\beta_1$  and  $\beta_2$  respectively. Then

$$\frac{(1 - \beta_2)^{\gamma/2}}{\beta_2} = \frac{(1 - \beta_1)^{\gamma/2}}{\beta_1} \left( \frac{B_2}{B_1} \right)^{2-\gamma} \quad \dots \text{(A13)}$$

Equation (A13) is solved to give  $\beta_2/\beta_1$  for a range of values of  $B_2/B_1$  and  $\gamma$ . Fig.10 shows beta after compression as a function of  $B_2/B_1$  for  $\gamma = 1.0, 1.333$  and  $1.667$ . The maximum initial value of beta shown on this figure is 0.99 and the final value can be read off directly for this case; for other initial values the result can be obtained by scaling the abscissa. Fig.11 shows the values of beta after compression as a function of  $\gamma$  for  $B_2/B_1 = 5$  and several initial values of beta, which are shown on the graph at  $\gamma = 2$ , when there is no change on compression. It is seen that the fractional reduction in beta is greatest for intermediate initial values, since in the limits of  $\beta_1 = 1$  and 0 it does not change.

#### CONCLUSION TO APPENDIX

The results show that the variation in beta is sensitive to the value of  $\gamma$ . The value of beta falls across a hydromagnetic shock wave at low Mach numbers and rises as the shock strength is increased. The fall is most pronounced for low values of  $\gamma$  and high initial values of beta; in this case beta will fall across a shock wave unless the Mach number is very high ( $> 10$ ). Beta always decreases during an isentropic compression when  $\gamma < 2$  and the reduction is greatest when  $\gamma$  is small; for example when  $\gamma = 1$  and the compressing magnetic

field is raised by a factor of 5 beta will fall from 0.95 to 0.56. It is concluded that in the presence of an energy loss from the plasma sufficient to reduce the value of  $\gamma$ , the final value of beta in a fast magnetic compression experiment can be substantially less than unity.

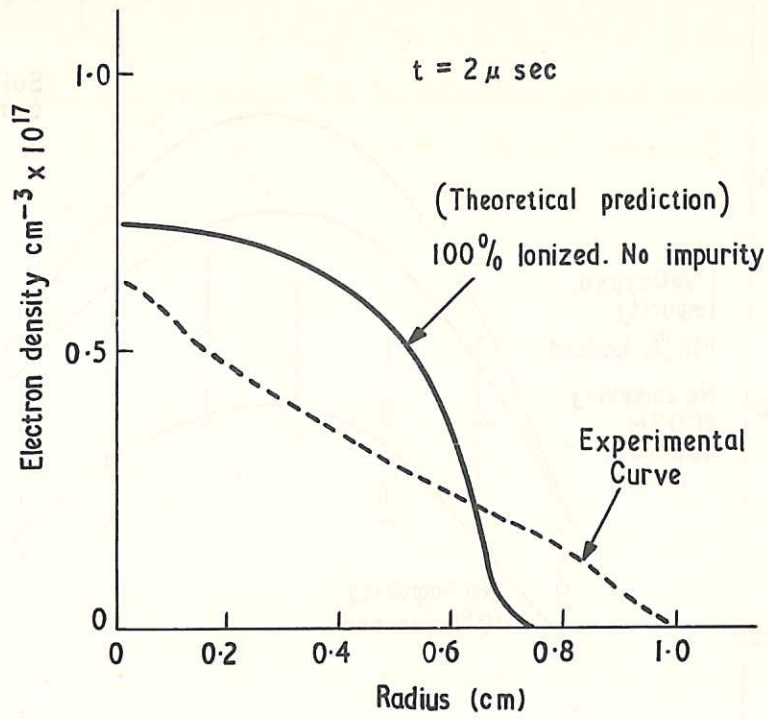


Fig. 1 (CLM-P133)  
Radial density distribution at  $2 \mu \text{ sec}$  (Theory and experiment)

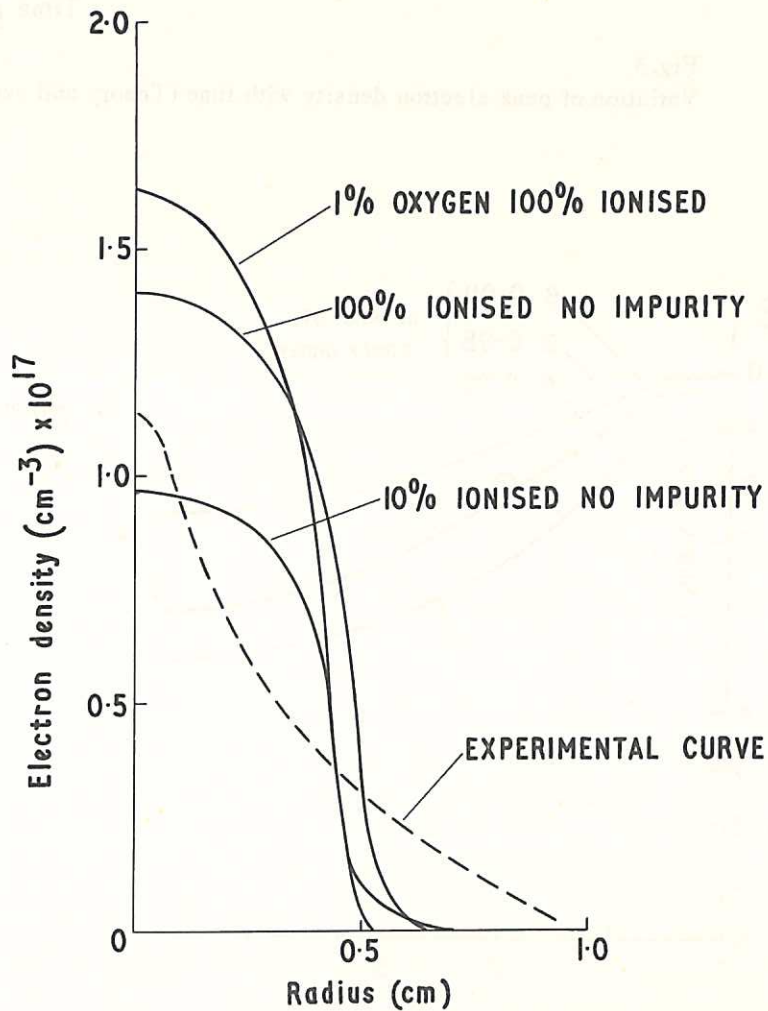


Fig. 2 (CLM-P133)  
Radial density distribution at  $6 \mu \text{ sec}$  (Experiment and more detailed theory)

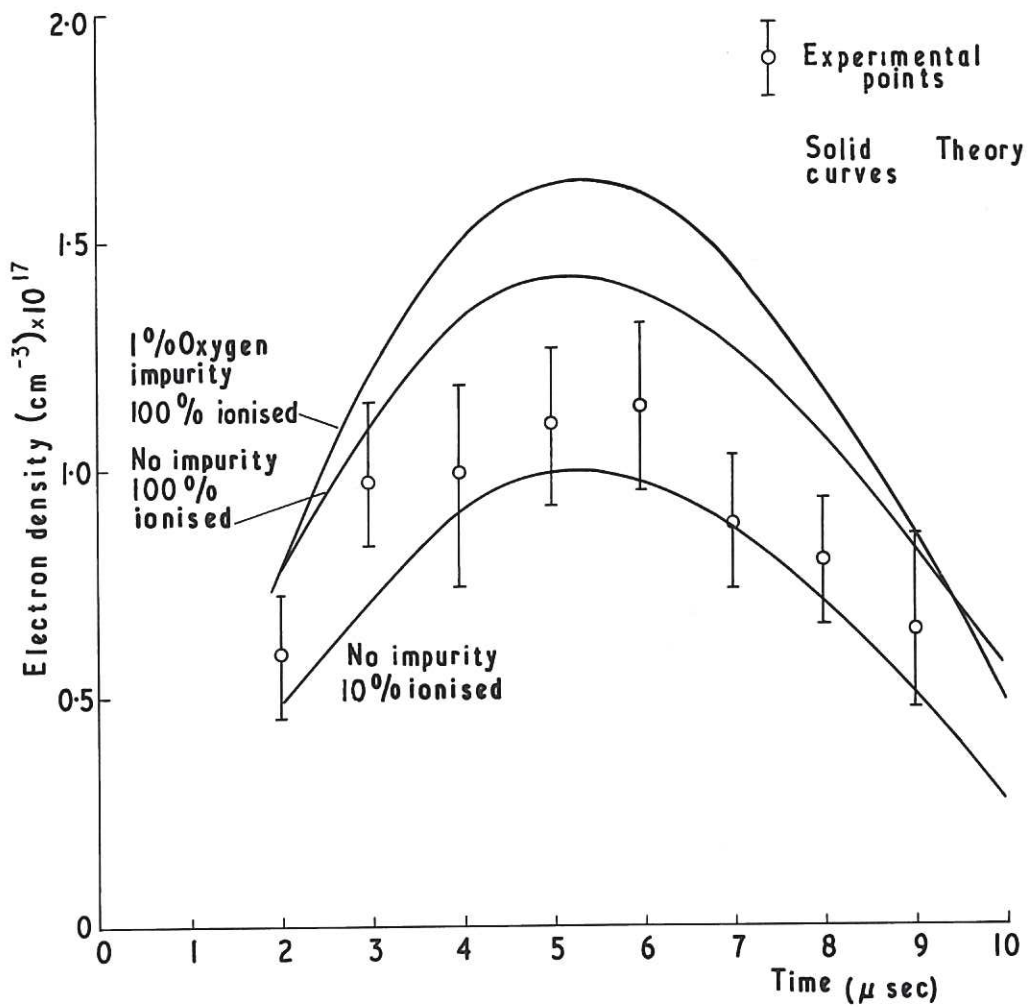


Fig. 3 (CLM-P 133)  
 Variation of peak electron density with time (Theory and experiment)

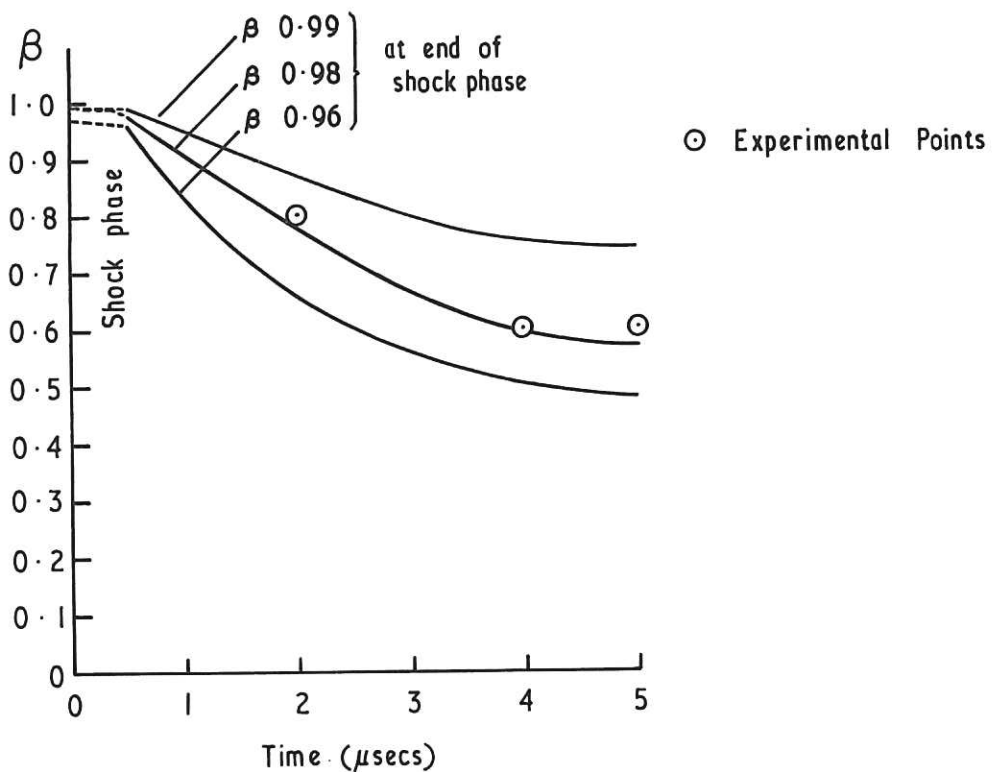


Fig. 4 (CLM-P 133)  
 Variation in  $\beta$  on axis in Megajoule experiment

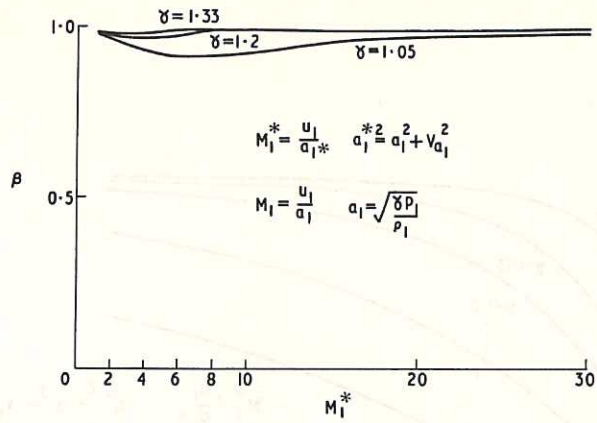


Fig. 5 (CLM-P 133)  
 $\beta$  as function of  $M_1^*$  for various  $\gamma$  ( $\beta_1 = 0.99$ )

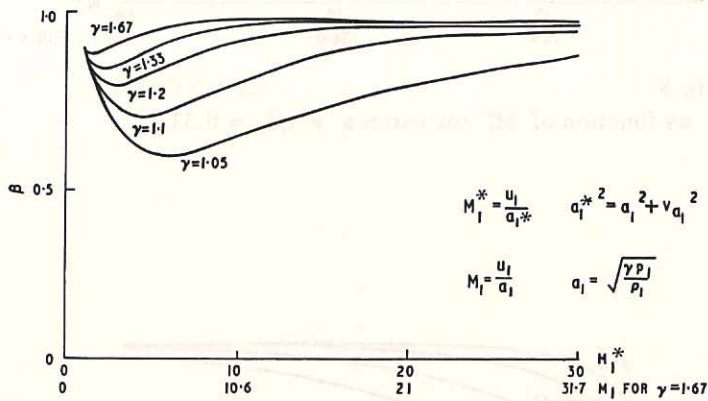


Fig. 6 (CLM-P 133)  
 $\beta$  as function of  $M_1^*$  for various  $\gamma$  ( $\beta_1 = 0.91$ )

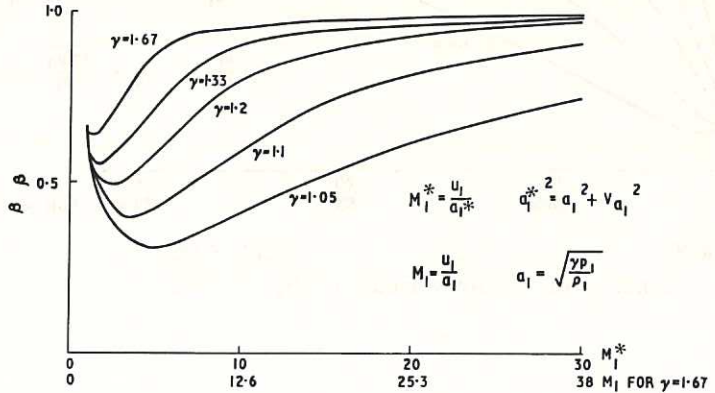


Fig. 7 (CLM-P 133)  
 $\beta$  as function of  $M_1^*$  for various  $\gamma$  ( $\beta_1 = 0.667$ )

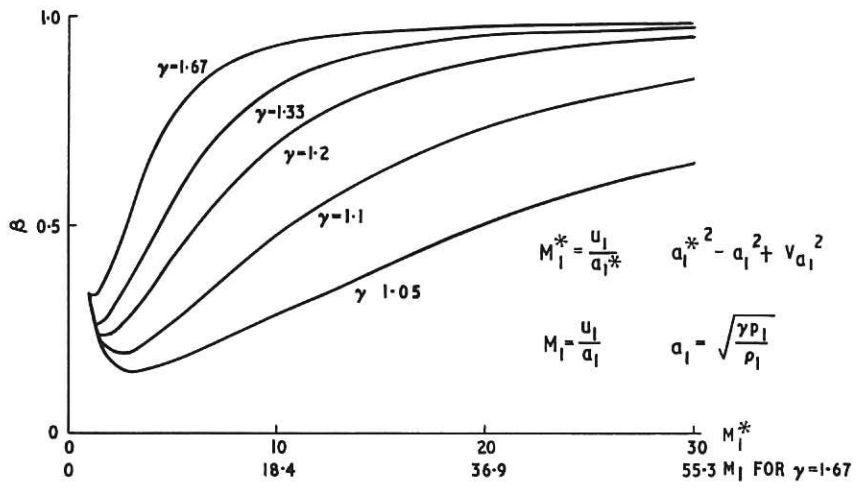


Fig. 8 (CLM-P 133)  
 $\beta$  as function of  $M_1^*$  for various  $\gamma$  ( $\beta_1 = 0.333$ )

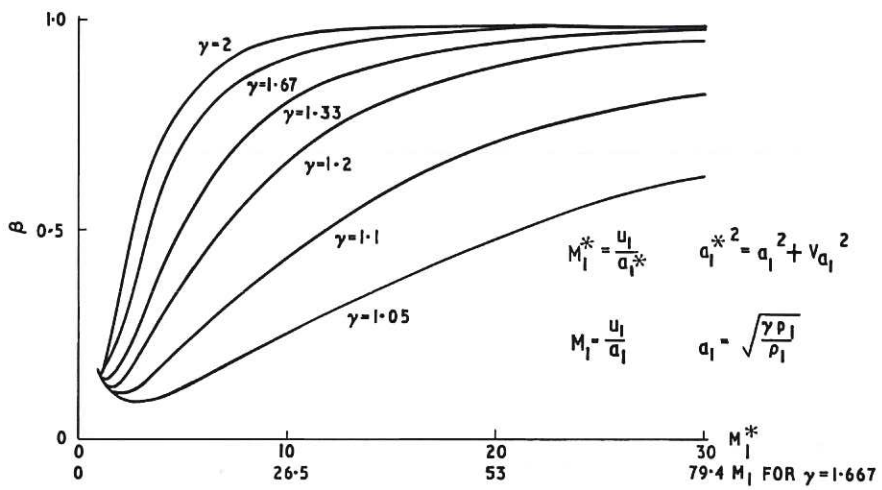


Fig. 9 (CLM-P 133)  
 $\beta$  as function of  $M_1^*$  for various  $\gamma$  ( $\beta_1 = 0.1667$ )

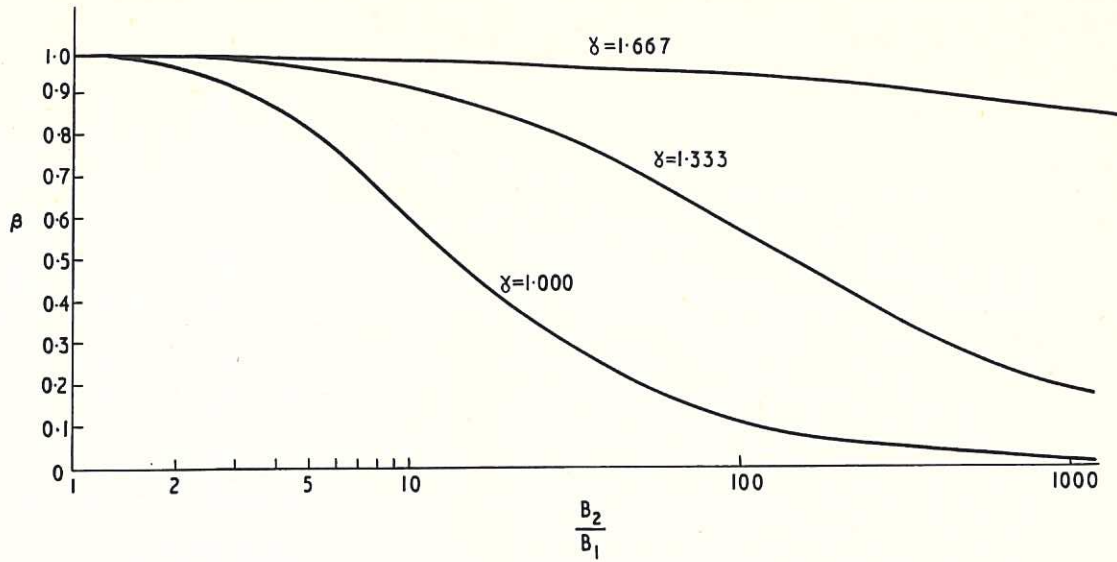


Fig. 10 (CLM-P133)  
 Variation in  $\beta$  with magnetic field compression ratio for various  $\gamma$ .

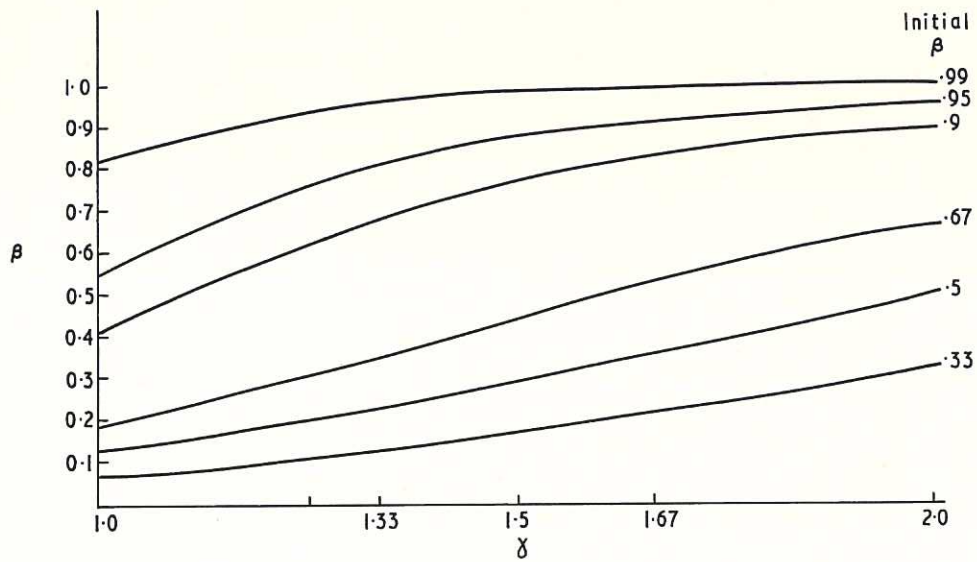
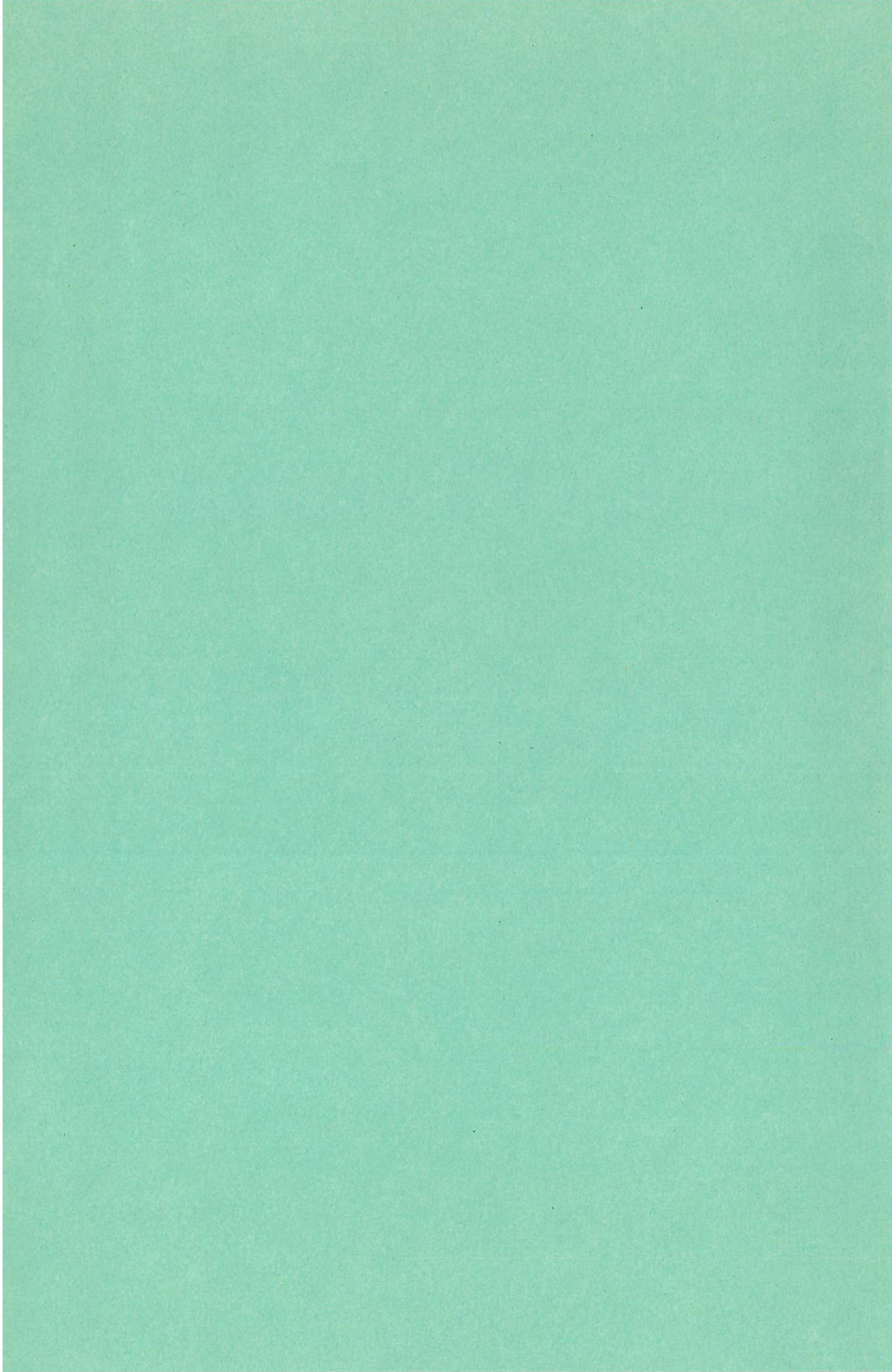


Fig. 11 (CLM-P133)  
 Variation in  $\beta$  with  $\gamma$  for various initial values of  $\beta$ . ( $B_2/B_1 = 5$ )







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