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CONVECTIVE AND ABSOLUTE ION CYCLOTRON INSTABILITIES IN HOMOGENEOUS PLASMAS

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CONVECTIVE AND ABSOLUTE ION CYCLOTRON INSTABILITIES IN HOMOGENEOUS PLASMAS

by

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A B S T R A C T

The properties of electrostatic ion cyclotron instabilities which can exist in mirror confinement machines are examined theoretically. Instabilities which are caused by an arbitrary combination of the loss-cone effect and the temperature-anisotropy effect are considered for an infinite plasma. The propagation characteristics (absolute or convective) of this class of instabilities are determined by numerical and analytical methods for interesting ranges of plasma parameters. Criteria on the length and lifetime of the plasma necessary for the growth of convective and absolute modes are deduced. The application of these results to present and future-generation mirror machines is discussed.

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C O N T E N T S

1. INTRODUCTION

The observation of R.F. emission at frequencies close to the ion-cyclotron frequency and its harmonics in the present generation of mirror machine experiments (BERNSTEIN et al, 1965; CORDEY et al, 1967, BELL et al, 1965 and 1966) is in keeping with theoretical predictions (SOPER and HARRIS, 1965; GUEST and DORY, 1965). Two appropriate sources of free energy are known to be available to drive such instabilities at plasma densities which have been achieved in these machines. One of these sources is the 'loss cone effect' where particles with low perpendicular energy can escape rapidly along the magnetic field lines. The other is the temperature anisotropy, a result of the method of creating the plasma by injection perpendicular to the magnetic field $\underline{B}$ so that the 'perpendicular temperature', $\alpha_\perp^2 \gg \alpha_\parallel^2$, the 'temperature' parallel to $\underline{B}$.

Theoretical studies have shown that instabilities can exist in the presence of either or both of these sources of free energy in an infinite, homogeneous plasma (that is, one which eliminates free energy sources associated with density and temperature gradients). Since all mirror confinement devices contain some combination of these two free-energy sources, as long as the wavelength of the unstable wave is significantly less than the plasma dimensions, the infinite theory should be adequate in describing the unstable behaviour of the plasma.

Instabilities associated with anisotropic temperature distributions were first examined by HARRIS (1959). Later several more extensive examinations of these modes were made by OZAWA, KAJI and KITO (1962), DNESTROVSKY, KOSTOMAROV and PISTUNOVICH (1963), HALL, HECKROTTE and KAMMASH (1965), and SOPER and HARRIS (1965). The loss-cone driven instability was first examined by ROSENBLUTH and POST (1965). A complete examination of the ion-cyclotron resonance modes ($\omega \simeq n\omega_{ci}$) for a large class of distributions including effects of the loss cone and anisotropy has been given by DORY and GUEST (1965).

Although qualitative agreement between theory and experiment exists, a more detailed examination is necessary to describe the behaviour of these modes in an experimental plasma. For example, an examination of the propagation characteristics of the unstable waves was necessary to explain threshold density observations in the DCX-2 device (BELL et al, 1966; BEASLEY, 1967).

In general a growing eigenmode can be identified theoretically by considering the sign of the imaginary part of ω for a real propagation vector k_r. However this does not imply that the amplitude of an arbitrary wave packet increases with time at every point in space or that in a finite length plasma the amplitude of the wave packet will reach an experimentally detectable amplitude. To discover whether an instability will be observable in a particular plasma, one must couple a knowledge of the propagation characteristics of the unstable wave packet (i.e. convective or absolute modes of instability) with such parameters as plasma size and plasma lifetime. If the plasma is convectively unstable, the wave packet must be able to grow to sufficient amplitude to be observed during a single traverse of the plasma*. Or if it is absolutely unstable, the plasma lifetime must be sufficiently long to allow the instability to grow to a large

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* This is based on the assumption that reflection at the ends of the machine is negligible. At least for small density gradients ($\frac{\lambda_{\parallel}}{N}$ dN/dx $\ll$ 1) this assumption is valid (AAMODT and BOOK, 1966)

enough amplitude. In addition, the unstable wave must at least fit in the plasma, regardless of the type of instability.

In this paper we will describe the method for determining the appropriate propagation characteristics of unstable electrostatic ion cyclotron waves. Criteria for the existence of these waves in mirror machines will be discussed. The method used for determining the propagation characteristics will be given in Section 2. In Section 3 we discuss the dispersion equation, show how analytical results can be obtained in the limit of high anisotropy ($\alpha_\perp^2 \gg \alpha_\parallel^2$), and describe numerical techniques. In Section 4 we present the numerical results for the ion distribution used by GUEST and DORY (1965). These results depict regions of stable, convectively unstable, and absolutely unstable plasma for appropriate ranges of plasma parameters. In the Section 5 we discuss the relevance of the results of Section 4 to present and future mirror machine experiments.

2. PROCEDURE FOR DETERMINING PROPAGATION CHARACTERISTICS

A rigorous method for determining whether an instability is convective or absolute was first described by BERS and BRIGGS (1963) and BRIGGS (1964).

Essentially the criterion for an absolute instability is that there is a branch point ω_B of the solution of the dispersion equation $k = k(\omega)$ occurring in the region S between the real $k_\parallel$ curve in the ω plane and the real ω axis.

To find these branch points (if any) of $k(\omega)$ for ω in region S, the whole of the region S is mapped into the k plane. A saddle point configuration in the k plane then indicates a branch point of $k(\omega)$ at ω_B.

This procedure gives precise information about the mathematical nature of the instability. However, finding the solution to $D(\omega,k)=0$ over a large region of complex k-space in order to find this saddle point is rather slow, and to do so for a large number of parameters would require a prohibitive amount of time. Hence our numerical technique uses instead one of the equivalent criteria derived by DYSTHE (1966), who shows that an absolute instability exists if the equations

$$D(\omega,\ k) = 0\ , \qquad \ldots (1)$$

$$\frac{\partial}{\partial k}\ D(\omega,\ k) = 0\ , \qquad \ldots (2)$$

have a solution for finite $(\omega,\ k)$, with $I_m(\omega) > 0$ and where the two roots belonging to the branch point are on opposite sides of the k axis for ω on the 'undeformed' Laplace contour. This latter condition is equivalent to ω_B lying in the region S.

Equations (1) and (2) were solved numerically by the iterative methods described in Section 3(c). By varying interesting plasma parameters we are able to determine the regions of convective and absolute instability in appropriate parameter space. We now turn to the appropriate expression for $D(\omega,\ k)$.

3. SOLUTIONS OF THE DISPERSION EQUATION

(a) The dispersion equation

The dispersion equation for electrostatic ion-cyclotron instabilities in a homogeneous plasma in a uniform magnetic field has been given by many authors (SOPER and HARRIS, 1965; GUEST and DORY, 1965; HARRIS, 1959); we write it as

$$D(\omega,k) = 1 - \sum_{\text{species}} \frac{\omega_p^2}{k^2} \sum_{n=-\infty}^{\infty} \int \frac{J_n^2(k_\perp v_\perp/\omega_c)}{k_\parallel v_\parallel - (\omega - n\omega_c)} \left\{ k_\parallel \frac{\partial f_o}{\partial v_\parallel} + \frac{n\omega_c}{v_\perp} \frac{\partial f_o}{\partial v_\perp} \right\} d^3v = 0. \qquad \ldots (3)$$

Here ω_p is the plasma frequency, $\underline{k}$ is the wave propagation vector with components $k_{\parallel}$ and $k_{\perp}$ parallel and perpendicular to the external magnetic field, ω_c is the gyro-frequency, ω is the wave frequency, J_n is the Bessel function of order n, and f_o is the unperturbed distribution function, normalised such that $\int f_o d^3v = 1$.

We shall consider the class of anisotropy-loss-cone distribution functions which were first used by GUEST and DORY (1965), that is

$$f_o^j = \frac{1}{\pi^{3/2}\, \alpha_{\perp}^2 \alpha_{\parallel}\; j!} \left(\frac{v_{\perp}}{\alpha_{\perp}}\right)^{2j} \exp\left(-\frac{v_{\perp}^2}{\alpha_{\perp}^2} - \frac{v_{\parallel}^2}{\alpha_{\parallel}^2}\right)$$

where $j = 1, 2, 3, \ldots$. The $\alpha_{\parallel}^2$ and $\alpha_{\perp}^2$ are the 'temperatures' in the directions parallel and perpendicular to the magnetic fields respectively. Thus there exists a temperature anisotropy, $T = \alpha_{\parallel}^2/\alpha_{\perp}^2$.

If we substitute f_o^j in equation (3), we may then write the dispersion equation in the form (see GUEST and DORY, (1965))

$$D = 1 + 2 \sum_{\text{species}} \frac{\omega_p^2}{k^2\alpha_{\parallel}^2} \left\{ 1 + \sum_{n=-\infty}^{\infty} \left[C_n^j(\lambda) \left(\frac{\omega - n\omega_c}{k_{\parallel}\,\alpha_{\parallel}}\right) Z\left(\frac{\omega - n\omega_c}{k_{\parallel}\,\alpha_{\parallel}}\right) \right.\right. \qquad \ldots (4)$$

$$\left.\left. + \, T\, D_n^j(\lambda) \left(\frac{n\omega_c}{k_{\parallel}\,\alpha_{\parallel}}\right) Z\left(\frac{\omega - n\omega_c}{k_{\parallel}\,\alpha_{\parallel}}\right) \right] \right\} = 0$$

where

$$C_n^j(\lambda) \equiv 2\pi \int_0^{\infty} v_{\perp}\, dv_{\perp}\, J_n^2\, (k_{\perp}\, v_{\perp}/\omega_c)\; g_o^j(v_{\perp}) \quad ,$$

$$D_n^j(\lambda) \equiv -\pi\alpha_{\perp}^2 \int_0^{\infty} v_{\perp}\, dv_{\perp}\, J_n^2\, (k_{\perp}\, v_{\perp}/\omega_c)\; \frac{1}{v_{\perp}}\, \frac{dg_o^j(v_{\perp})}{dv_{\perp}} \quad ,$$

$$g_o^j(v_{\perp}) \equiv \frac{1}{\pi\alpha_{\perp}^2 j!} \left(\frac{v_{\perp}}{\alpha_{\perp}}\right)^{2j} \exp\,(-\, v_{\perp}^2/\alpha_{\perp}^2)$$

$$\lambda = \frac{k_{\perp}^2\, \alpha_{\perp}^2}{2\omega_c^2} \quad , \qquad T = \frac{\alpha_{\parallel}^2}{\alpha_{\perp}^2}$$

and

$$Z(\zeta) \equiv \pi^{-\frac{1}{2}} \int_{-\infty}^{\infty} \frac{e^{-x^2}}{x-\zeta}\, dx$$

is the plasma dispersion function (FRIED and CONTE, 1961). The infinite series in equation (4) is convergent for all values of $|k_{\parallel r}| > |k_{\parallel i}|$. Outside this domain one must either put a cut-off in the ion velocity distribution or else find a convergent expansion. However the non-convergence of the series is not bothersome unless one attempts to calculate (ω_B, k_B) at very high densities.

The remarks on convective and absolute instability apply to the direction parallel to the magnetic field. In the direction perpendicular to the magnetic field we assume the plasma boundary determines normal modes which are discrete and real. This is a valid assumption as long as the radial wavelength is not significantly less than the plasma radius.

(b) Analytic solution for limiting case of high anisotropy ($T \ll 1$)

In the limit of very small T, the 'C_n' term dominates in equation (4); in this case, it is possible to simplify equation (4) so that we may investigate analytically whether the instability is convective or absolute. We further assume

(1) $\left|\frac{\omega - n\omega_{ci}}{\omega_{ci}}\right| \ll 1$ (we consider instabilities in which the frequency is close to the n^{th} harmonic of ω_{ci})

(2) $\left|\frac{\omega - n\omega_{ci}}{k_{\parallel}a_{\parallel}}\right| \gg 1$ (ions have small velocity spread parallel to the field lines)

(3) $\lambda_e \left(= \frac{k_\perp^2 \, a_{\perp e}^2}{\omega_{ce}^2}\right) \ll 1$ (perpendicular wavelength much greater than an electron gyro radius)

(4) $\alpha_{\parallel e} = 0$ (electrons assumed to have zero velocity spread along field lines).

Using the assumptions (1) - (4) and the asymptotic form of the plasma dispersion function, equation (4) reduces to:

$$D(\omega, k_\parallel) = \frac{k^2}{k_\parallel^2} - \frac{\omega_{pe}^2}{n^2\omega_{ci}^2} - \frac{\omega_{pe}^2 \, C_n^J(\lambda_i)}{(\omega - n\omega_{ci})^2} \left\{ 1 + \frac{3}{2} \frac{k_\parallel^2 \, \alpha_{\parallel i}^2}{(\omega - n\omega_{ci})^2} \right\} = 0 \qquad \dots (5)$$

for frequencies ω close to the n^{th} harmonic of ω_{ci}.

To find the branch points of $k_\parallel(\omega)$ for the above dispersion equation we solve equations (1) and (2). Using the D of equation (5) we obtain

$$\omega_B = n\omega_{ci} \pm \frac{i\omega_{pi} \{C_n^j(\lambda_i)\}^{1/2}}{\left(\frac{\omega_{pe}^2}{n^2\omega_{ci}^2} - 1\right)^{1/2}} \left\{ 1 \pm i(6)^{1/2} \frac{k_\perp \, \alpha_\parallel}{\omega_{pi} \, C_n^J(\lambda_i)} \right\} \qquad \dots (6)$$

and

$$k_{\parallel B} = \left[\pm i \frac{\{C_n^j(\lambda_i)\}^{1/2} k_\perp \, \omega_{pi}}{\alpha_{\parallel i}\left(\frac{\omega_{pe}^2}{n^2\omega_{ci}^2} - 1\right)} \left\{ \left(\frac{2}{3}\right)^{1/2} \pm 2i \frac{k_\perp \, \alpha_{\parallel i}}{\omega_{pi} \, \{C_n^j(\lambda_i)\}^{1/2}} \right\} \right]^{1/2} \qquad \dots (7)$$

Thus we see that there exists a branch point of $D(\omega, k_\parallel) = 0$ in the Im $\omega > 0$ region (unstable region) of the ω plane. Moreover after a careful examination of the real $k_\parallel$ curve, one can show that provided ω_{pe} is large enough the branch point with the two upper signs lies between the real $k_\parallel$ curve and the real ω axis (i.e. in the region S). Hence the ion-cyclotron instability caused by strong anisotropy is absolute. These values of $(\omega_B, k_{\parallel B})$ (equations (6) and (7)) were found to agree quite well with the numerical results discussed in Section 4.

(c) Numerical solutions

As a precaution against numerical errors, two completely different numerical techniques were employed to obtain the solutions $(\omega_B, k_{\parallel B})$ to equations (1) and (2) for the various parameters. These methods will now be briefly described.

The first method made use of the fact that the roots of equations (1) and (2) are also the minima of

$$F(\omega, k_{\parallel}) = \left[|D|^2 + \left|\frac{\partial D}{\partial k_{\parallel}}\right|^2 \right]^{\frac{1}{2}}$$

The minima of the function F were found by using an existing programme written by M.J. POWELL (1964). This programme minimises a function of several variables by changing the value of one parameter at a time and does not require the derivatives of F. When a minimum value of F of order 10^{-6} was found, the corresponding $(\omega_B, k_{\parallel B})$ was said to be a root of equations (1) and (2).

The other method made use of two functions,

$$F_1 = |D|^2 \quad \text{and} \quad F_2 = \left|\frac{\partial D}{\partial k_{\parallel}}\right|^2$$

This program first used F_1 for a given set of parameters to find a solution of the dispersion equation, $\omega(k_{\parallel})$; then, F_2 was used to obtain the saddle point. After each iteration on F_2, F_1 was again used to ascertain the new ω for the new $k_{\parallel}$. The solutions $(\omega_B, k_{\parallel B})$ were deemed to be 'good' solutions when their fractional change after an iteration was less than a prescribed amount (usually 10^{-6}). The functions were approximated by an appropriately dimensioned paraboloid in order to obtain iteration equations. Analytic derivatives of F_1 and F_2 were used in the programs. This iteration technique was written as a subroutine usable for any arbitrary dispersion relation and was first tested on the three-stream dispersion relation, the results of which are well known in certain limits (BERS and BRIGGS, 1963; BRIGGS, 1964).

4. NUMERICAL RESULTS

Stable regions and regions of convective and absolute instability were determined from the dispersion relation equation (4) for $j = 1$ distribution using the numerical technique described in Section 3(c). Results are shown in Fig.1 as families of pairs of curves in $\frac{\omega_{pe}}{\omega_{ci}}$ - T - space. The dashed curves denote the neutral stability curves, and the solid curves the convective-absolute (C-A) boundary. Each of the family of curves represents a different value of λ_i; Fig.1 gives a portrayal of unstable regions for three of the four parameters. In Table 1 we give the value of $k_{\parallel}\rho_i$ ($\rho_i = \alpha_{\perp i}/\omega_{ci}$) on the C-A boundaries of Fig.1. Dependence of the results on the electron temperature, α_e^2 can be described in terms of these results and will be given below. Also the very important dependence of the results on harmonic number will be discussed in the light of the results for the fundamental frequency. Stability criteria based on these results and incorporating all parameters will be given below.

The most important result is that provided certain criteria on plasma size are met, the absolute instability can exist at sufficiently high density in any mirror-confined plasma, regardless of the value of T. Hence, even though the energy source is completely different in the regions of low or high T, and even though the mathematical term characterising the instability is different, the qualitative behaviour of the unstable wave is the same: it will be absolutely unstable at sufficiently high densities with a frequency and growth rate $\omega = \omega_r + i\omega_i$; where $0.95\,\omega_{ci} \leqslant \omega_r \leqslant \omega_{ci}$, $\omega_i \sim (.02)\,\omega_{ci}$.

As can be seen from Table 1 and Fig. 1, the characteristics of the instability are somewhat different in the two regimes. Threshold densities are considerably lower for the anisotropy-dominated modes than for the loss-cone modes. On the other hand, the wavelengths are generally much longer for the loss-cone modes. In both regimes, $k_{\parallel r}$ decreases as density is increased, or in other words, the characteristic wavelength becomes longer.

The convectively unstable region extends to a density an order of magnitude lower than the absolutely unstable region. In this region, maximum spatial growth rates occur at the C-A boundary. On the basis of these growth rates and assuming that to be observed the wave packet must grow ten e-foldings in one traversal of the plasma (see footnote page 2) we may write a stability criterion against these convective modes. The plasma will be stable if

$$L_p \leqslant 10^3 \sqrt{T} \rho_i \qquad \ldots (8)$$

where L_p is the plasma length and ρ_i the ion gyroradius. Thus the convective modes will be observable only in very long plasmas.

The absolute instabilities have a sufficiently fast growth rate to assure their being observed in almost any mirror injection experiment, since in order to be observed, the plasma life-time,

$$T_L \geqslant 10^3/\omega_{ci} \ . \qquad \ldots (9)$$

However, these absolute modes have rather long axial wavelengths. Since these waves must fit in the plasma, the plasma length must also satisfy a criterion for the absolute modes

$$L_p \geqslant 57 \sqrt{T} \rho_i \ (1 - \log_{10} T) \ . \qquad \ldots (10)$$

The absolute instability cannot occur if this criterion is not satisfied. Although infinite plasma convective modes per se would not exist on the basis of growth length considerations, it does not necessarily mean that the system will be stable. In this case a correct treatment of such a problem must include boundary effects.

Very important scalings in all quantities become evident if one considers instabilities at higher frequencies $\omega \simeq n\omega_{ci}$, $n = 1, 2, 3, \ldots$. In order to make a proper comparison of instabilities at the different harmonics, one must not fix λ_i, but rather consider the instability at the most unstable λ_i, λ_u. The value of $k_\parallel$ associated with this λ_u is approximately equal to the maximum $k_\parallel$ for all unstable λ_i for a given n, and of course the density is by definition the minimum density for a given n. We find that λ_u scales with n^2. For these λ_u both $k_\parallel$ and ω_i scale approximately with n, and the threshold density scales slightly faster than n^2. The scaling of $k_\parallel$ with n means that the criterion (10) must be divided by n.

Fortunately the electron temperature provides a stabilizing influence (Landau damping) which we shall now examine. In all calculations reported so far, the ratio of electron temperature (assumed to be isotropic) to parallel ion temperature

$$\Theta = \frac{m_e \, \alpha_e^2}{m_i \, \alpha_{\parallel i}^2}$$

has been kept constant ($\Theta = 0.05$, typical of present experiments).

Hence one must keep in mind that Θ is not a measure of absolute electron temperature but that it does depend on the anisotropy in that constant α_e implies constant ΘT. However, the effect of electron temperature does not depend solely on α_e , but rather on the quantity

$$\zeta_e = \frac{\omega_B}{k_{\parallel B}\,\alpha_e} \simeq \frac{n\omega_{ci}}{k_{\parallel B}\,\alpha_e}$$

since it is this quantity which is the argument of the Fried-Conte function through which the electron temperature makes its contribution. But that contribution will be significant only when $\zeta_e \lesssim 2.5$; for large ζ_e, the effect on $k_{\parallel B}$, ω_B, or the position of the C-A curve is nil. As ζ_e decreases below about 2.5, the electron contribution becomes increasingly significant in that the product $k_{\parallel B}\,\alpha_e$ approaches a constant, or $k_{\parallel B} \propto \Theta^{-\frac{1}{2}}$. (Thus at $\Theta = 0.05$ for a $T = 1$ plasma the electron contribution is the same as at $\Theta = 0.39$ for a $T = 0.01$ plasma.) An empirical approximation for the electron contribution is

$$\frac{k_{\parallel B}\,\alpha_e}{\omega_{ci}} \simeq \frac{\Theta^{\frac{1}{2}}}{.4(1-\log_{10}T) + \Theta^{\frac{1}{2}}}$$

This wavelength dependence turns out to be the sole stabilizing contribution of the electron temperature. The change in the position of the C-A curve over a wide range of Θ is essentially nil.

We may use the scaling of results with harmonic number and electron temperature to rewrite our criterion on plasma length necessary to allow absolute modes:

$$L_p > \frac{57\sqrt{T}\rho_i}{n}\left[\, 2.5\sqrt{\Theta} + (1-\log_{10}T)\right] \qquad \ldots (11)$$

The threshold density for a given harmonic may then be related by

$$\omega_{pe} \gtrsim n\omega_{ci}\left(\frac{m_i}{m_e}\right)^{\frac{1}{2}}\left[\frac{T}{T^{3/4} + 0.025}\right] \qquad \ldots (12)$$

Thus the two conditions (11) and (12) are able to describe the infinite-plasma results for anisotropy-loss-cone instabilities.

5. DISCUSSION

We may compare the results of Section 4 with experiments. Such comparisons are compatible but somewhat inconclusive at this stage. Perhaps the easiest comparison can be made with the DCX-2 machine (BELL et al, 1965). The plasma is accurately described by a $T = 0.001$ distribution and the length of plasma is such that absolute modes should exist but convective modes would not be expected. The observed threshold density (LAZAR et al, 1966) for onset of the instability does coincide with the position of the C-A curve and not the neutral stability curve.

The case of Phoenix II ($T = 0.02$) is not as easily interpreted as that of DCX-2 since the wavelength of the instability is such as to cast doubt on the applicability of the infinite theory. Nevertheless, the observed threshold density for onset of the fundamental cyclotron frequency appears to be close to that predicted. It is also observed that as the density is further increased, the plasma becomes stable to this mode. It also becomes stable when the electron temperature is increased (through action of the instability). Both observations are in agreement with the theory; however, the wavelength is not sufficiently sensitive to plasma density to permit meaningful comparison, and electron temperature cannot be measured sufficiently accurately at present.

For a mirror machine to be in the region of thermonuclear interest the ion density must be of order 10^{14} particles/cm^3. This means that with presently available magnetic fields (10^5 gauss) $\omega_{pi} \sim 14\omega_{ci}$, so for the most stable plasma, $T = 1$, $\Theta = 1$, using condition (12) at least 14 harmonics of ω_{ci} could be absolutely unstable.

For the 14th harmonic to be absolutely unstable we see from inequality (11) that

$$L_p > 15\ \rho_i\ .$$

Hence in all but the very short thermonuclear plasmas the absolute instability should exist.

In summary, we find that the instability associated with the temperature anisotropy-loss-cone effect is convective at the instability threshold but absolute at a plasma density an order of magnitude higher than the threshold density. In the convective region, the growth length is very long; hence in the absence of strong reflections from the ends of the plasma, one should not expect to see an instability until the density was raised to the threshold of the absolute instability.

The absolute instability occurs with a very long axial wavelength. This sets a criterion on the plasma length necessary for those modes to fit in the plasma. This criterion is dependent on anisotropy, electron temperature, and mode number. While this criterion is valid for defining an absolutely unstable region, it is not valid as a stability criterion, which cannot be obtained from the infinite plasma theory. To examine stability, one must consider the finite-plasma problem. However, on the basis of the low convective growth rates and long wavelengths from the infinite theory one might expect this criterion to be closely related to a stability criterion.

TABLE 1

Values of $k_{\parallel} \rho_i$ at C-A boundary

$$\left(\rho_i = \frac{a_{\perp i}}{\omega_{ci}} \right)$$

$k_{\parallel} \rho_i$	$\lambda = 1$	$\lambda = 3$	$\lambda = 5$	$\lambda = 11$
T = 0.0001	2.9 - 0.77i	1.7 - 0.171i	1.3 - 0.5i	1.0 - 0.37i
T = 0.001	0.632 - 0.231i	0.443 - 0.171i	0.379 - 0.145i	0.316 - 0.12i
T = 0.01	0.17 - 0.066i	0.15 - 0.058i	0.14 - 0.052i	0.12 - 0.045i
T = 0.1	Convective	0.066 - 0.025i	0.066 - 0.024i	0.06 - 0.023i
T = 1.0	Stable	0.035 - 0.0109i	0.036 - 0.011i	0.034 - 0.0108i

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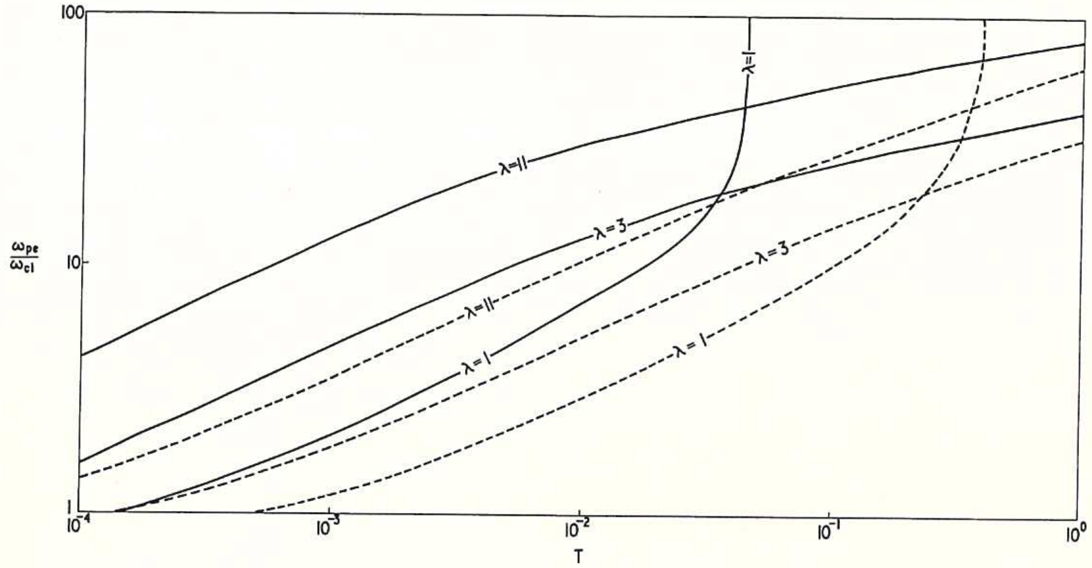

Fig. 1.

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Stable, convective, and absolutely unstable regions of anisotropy-loss-cone instabilities.

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