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SECOND AZIMUTHAL MODE OF THE ELECTRON CYCLOTRON INSTABILITY

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by

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A B S T R A C T

Electrostatic oscillations are excited in a cylindrical shell of plasma by an electron beam which passes through the plasma with a helical trajectory. Oscillations of the first azimuthal mode ($\ell = 1$) result from a non-convective instability and have been reported previously. In this paper oscillations of the $\ell = 2$ azimuthal mode, resulting from a convective instability, are reported. The frequencies and wavelengths of the $\ell = 2$ oscillations, and the range of plasma density over which they occur, are compared with theoretical predictions.

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C O N T E N T S

1. INTRODUCTION

This paper describes an experiment to excite an electrostatic plasma microinstability which arises from an anisotropic velocity distribution of the electrons. To generate the anisotropy, an electron beam is fired into a plasma in a direction nearly perpendicular to an applied uniform magnetic field. The resulting situation is closely analogous to that described theoretically by BURT and HARRIS (1961), who considered electrostatic oscillations of a cylindrical shell of plasma excited by ions gyrating in cyclotron orbits concentric with, and of the same diameter as, the plasma shell. In the experiment described here, the gyrating ions of BURT and HARRIS are replaced by the beam electrons which have a small axial drift velocity in addition to their gyratory motion.

The excitation of the lowest order azimuthal mode ($\ell = 1$) has been reported in a previous communication (MORSE 1967), in which the properties of the instability were shown to be in agreement with the theoretical analysis of CORDEY (1965). This present paper is concerned with spontaneous oscillations of the second azimuthal mode ($\ell = 2$) which are excited by a convective instability. The behaviour of these oscillations is studied and compared with theory.

2. EXPERIMENTAL APPARATUS

The experimental apparatus is basically the same as that described previously (MORSE 1967), and is shown in Fig.1. The instability takes place in an annular region bounded on the outside and inside by conducting walls 15 cm and 5 cm in diameter, respectively. An axial magnetic field of 30 to 60 gauss is applied, and the neutral pressure of mercury vapour is maintained at approximately 3×10^{-5} Torr.

Plasma from a mercury arc source is introduced into the annular region through a slot in a plate at the bottom of the region, and it flows along the magnetic field in an annular column. The electron beam is fired into the plasma with a helical trajectory coaxial with the plasma annulus. The average axial drift velocity of the beam v_0 is approximately 0·08 of the total beam velocity, corresponding to a helix pitch angle of $4\frac{1}{2}^{\circ}$. The beam spreads axially to form a hollow cylinder of spiralling electrons of density less than 3×10^6 cm^{-3}, and the plasma density is variable in the range 3×10^7 to 3×10^9 cm^{-3}.

There are two significant changes in the apparatus from its previous form (MORSE 1967). First, the neutral pressure in the system is higher by about a factor of three, at 3×10^{-5}Torr. This results in a reduced rate of axial spreading of the electron beam from the ribbon shape shown in Fig.1. The axial spreading or 'thermal' velocity is now found to be $2{\cdot}5 \times 10^7$ cm/sec, compared with 4×10^7 cm/sec found previously.

Secondly, the interaction region, where the electron beam has spread axially to form a nearly uniform cylindrical shell of spiralling electrons, has been increased in length to approximately 15 cm by moving the electron gun up the tube by 10 cm. There is an axial decrease in plasma density of a factor of about two over the length of this region.

The diagnostic techniques for measuring the electron beam and plasma parameters, the detection of the radio-frequency oscillations associated with the instability, and the measurement of the wavelength and direction of propagation of the unstable wave, remain as described previously (MORSE 1967).

3. THEORY

The dispersion relation given by CORDEY (1965) is used to compare the results of the present experiments with the theory. Using the criterion of BERS and BRIGGS (1964) a convective instability of the second azimuthal mode is found. The azimuthal direction of propagation of the unstable wave is the same as that of the gyrating beam electrons ($\ell = +2$) and the axial direction of propagation is opposite to the axial drift velocity of the beam ($k_r v_0 < 0$, where k_r is the real part of the axial wave number k).

To investigate the origin of the instability, it is instructive to look at the dispersion characteristics of the beam and plasma waves separately. These are shown in Fig.2 for $\ell = 2$ and no thermal motion of the beam and plasma electrons. The solid lines represent the dispersion characteristics of a waveguide filled with stationary plasma (TRIVELPIECE and GOULD 1959). The dashed lines represent the beam waves, whose coupling to the plasma waves can give rise to instabilities. The dispersion curves are plotted for the density range in which the experiment is usually operated, $\omega_{ce} < \omega_{pe} < (\omega_{ce}^2 + \omega_{pe}^2)^{1/2} < 2\omega_{ce}$. In this expression ω_{ce} is the electron cyclotron frequency and ω_{pe} is the electron plasma frequency.

The beam wave in region A of Fig.2 can be shown by the method of BERS and GRUBER (1965) to carry negative energy, and its coupling to the plasma in this region results in the instability studied here. In the frequency range $\omega_{ce} < \omega < \omega_{pe}$ the plasma supports no propagating waves, so the instability falls into the category of 'reactive instabilities' discussed by BERS and GRUBER (1965). When the dispersion relation for the coupled system of beam and plasma is solved, unstable waves with complex wave numbers are found for frequencies from ω_{ce} up to slightly below $(\omega_{ce}^2 + \omega_{pe}^2)^{1/2}$. The oscillation frequency ω and the real part of the complex wave number k_r satisfy the following approximate relation

$$\omega \approx 2\omega_{ce} + k_r v_o \quad \ldots (1)$$

with $k_r < 0$.

Next we are interested in the frequency corresponding to the maximum k_i, the imaginary part of k, which we expect to correspond to the frequency of the strongest spontaneous oscillations of the system. At low densities, $\omega_{pe} < \omega_{ce}$, the maximum k_i occurs for ω slightly greater than ω_{ce}, and the magnitude of k_i decreases rapidly with decreasing density. For the intermediate density range shown in Fig.2, the frequency corresponding to the maximum k_i is close to ω_{pe}. At high density, $(\omega_{pe}^2 + \omega_{ce}^2)^{1/2} > 2\omega_{ce}$, the frequency corresponding to maximum k_i increases only slowly with increasing ω_{pe}, and is generally between $1{\cdot}7\ \omega_{ce}$ and $1{\cdot}8\ \omega_{ce}$ for densities up to $\omega_{pe} = 3\omega_{ce}$.

At high densities, a non-convective instability appears in the frequency range $2\omega_{ce} < \omega < (\omega_{pe}^2 + \omega_{ce}^2)^{1/2}$, with $k_r > 0$ still given by equation (1). This non-convective instability is the one most closely analogous to the second azimuthal mode of the Burt-Harris instability. The convective instability studied in this paper arises only because the beam has a relatively large axial drift velocity.

In Section 4(a), the measured direction of wave propagation and wavelength are compared with equation (1). In Section 4(b), the frequency corresponding to maximum k_i as a function of ω_{pe} is compared with the measured frequencies of strongest oscillation.

Finally, in Section 4(c), the computed stability boundary in the $(\omega_{ce} - \omega_{pe})$ plane is compared with experiments. For the computations, the dispersion relation derived by CORDEY (1965) is used, including the measured axial thermal velocities of the beam and plasma electrons.

4. EXPERIMENTAL RESULTS

(a) Direction of Propagation and Wavelength

Using the previously described techniques for the measurement of the direction of propagation and wavelength of the unstable wave, it is first determined that the azimuthal mode number is 2 and the azimuthal direction of propagation is right-handed about the magnetic field ($\ell = +2$). The axial direction of propagation is found to be anti-parallel to the magnetic field ($k_r < 0$). In Fig.3 the experimental relation between ω and k_r is compared with equation (1) for three values of electron cyclotron frequency f_{ce}. The oscillation frequency ω is usually found to be slightly below $2\omega_{ce} + k_r v_o$, in agreement with theory.

(b) Frequency of Strongest Oscillation vs. Plasma Density

We next compare the frequency of the strongest radio frequency oscillations from the instability with the frequency corresponding to the maximum computed k_i. The oscillation spectrum is observed to occupy a band of frequencies between ω_{ce} and $2\omega_{ce}$, with a well-defined peak below $2\omega_{ce}$. In Fig.4 the normalised frequency of strongest oscillation ω/ω_{ce} is plotted against the normalised plasma density ω_{pe}/ω_{ce}, for $\omega_{ce} = 2\pi \times 110$ MHz. The average plasma density in the interaction region is used to compute ω_{pe}/ω_{ce}, although we have noted that there is a significant decrease in density from the bottom to the top of the region. The frequency of strongest oscillation is seen to make discontinuous jumps at $\omega_{pe}/\omega_{ce} \simeq 1{\cdot}5$ and $2{\cdot}1$, similar to the previous experiment (MORSE, 1967). This behaviour is still unexplained, and again only the gross features of the spectrum are compared with theory. The theoretical curve of frequency for maximum k_i vs density is shown as the solid line in Fig.4.

The experimental and theoretical curves behave qualitatively in the same fashion. The measured frequency of strongest oscillation is limited to approximately $1{\cdot}5\ \omega_{ce}$, while the theory predicts a limit of about $1{\cdot}75\ \omega_{ce}$. A possible cause of this discrepancy is the limited axial length of the interaction region. According to eq.(1) the wavelength of a wave of frequency $\omega = 1{\cdot}5\ \omega_{ce}$ is approximately 5 cm, while at $\omega = 1{\cdot}75\ \omega_{ce}$ the wavelength is over 10 cm, and the interaction region is only 15 cm long. In this frequency range the computed e-folding growth length is fairly constant at 6 to 7 cm. This relatively long growth length probably accounts for the need to lengthen the interaction region before the $\ell = 2$ oscillations were observable.

(c) Stability Boundary

Finally, we investigate the stability boundary in the plasma density-magnetic field plane. In Fig.5 we show two experimental boundaries for beam densities ω_{pb}/ω_{ce} of 0·09 and 0·07, where ω_{pb} is the plasma frequency of the beam electrons. For all but the lowest values of ω_{ce} investigated, the upper density limit for unstable oscillations could not be reached experimentally. We note that there is a lower limit of ω_{ce} for which there are no $\ell = 2$ oscillations at any density.

The computed stability boundary for $\omega_{pb}/\omega_{ce} = 0{\cdot}09$ is also shown in Fig.5. The agreement with the experimental results is acceptable considering the plasma density measurement is again an average over the length of the interaction region. In the region of the $(\omega_{ce} - \omega_{pe})$ plane where theoretically predicted instability is not observed, the calculated growth length of the instability is comparable with or longer than the length of the interaction region. Detailed calculations of the growth length throughout the $(\omega_{ce} - \omega_{pe})$ plane are not available, so we are unable to compare the experimental results with contours of constant growth length.

5. DISCUSSION

In this paper we have identified and studied the properties of the second azimuthal mode of the electron cyclotron instability, and compared the results with theoretical calculations. The intensity of the $\ell = 2$ oscillations is generally 10 to 20 decibels below the $\ell = 1$ oscillations reported previously. However, the $\ell = 2$ instability is convective with a calculated minimum e-folding length of approximately 6 cm, compared to the 15 cm length of the interaction region. For the instability to grow to observable levels, it must be assumed that the wave is at least partially reflected at the ends of the interaction region.

In this experiment, the lower boundary of the interaction region is a metal plate with a circular slot to define the plasma annulus. The upper boundary, when $\omega_{pe} > \omega_{ce}$ over the entire length of the region, is the position at which the electron beam is no longer a uniform hollow cylinder of electrons but has a helical ribbon shape. At lower plasma densities, the upper boundary is approximately the position at which ω_{pe} falls below ω_{ce}.

The observed instability signal strength as a function of axial position rises rapidly in the first centimeter or two from the lower boundary, and increases slowly over the remainder of the interaction region. Above this, the signal decreases into the background noise in 5 to 10 cm. This behaviour is consistent with the assumed partial reflection of the waves at the ends of the system, and makes a direct measurement of the instability growth length impossible.

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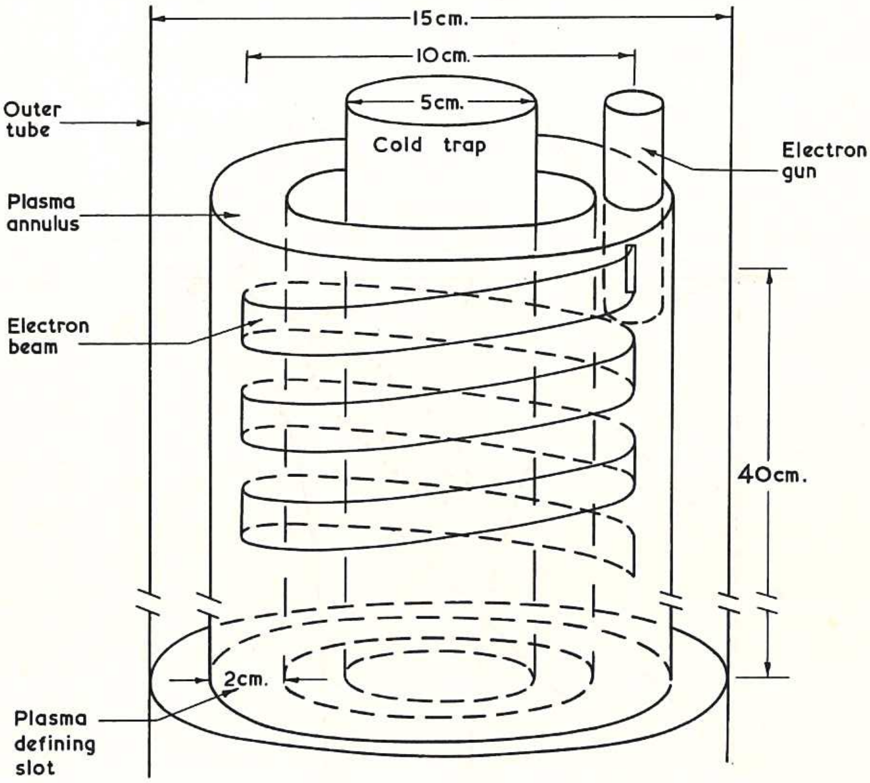

Fig. 1 (CLM-P 143)
Experimental configuration. Axial magnetic field directed downward

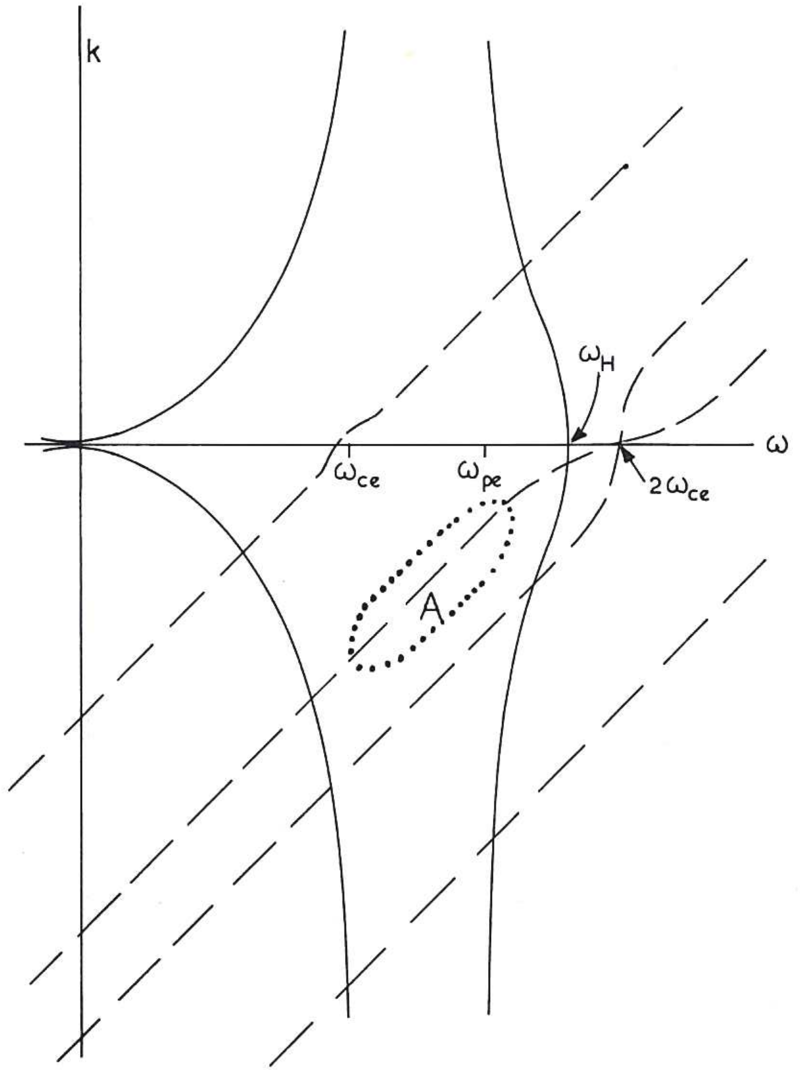

Fig.2 (CLM-P 143)

Separate dispersion characteristics of electrostatic waves in plasma and electron beam. Solid lines: plasma waves. Dashed lines: beam waves. Hybrid frequency $\omega_H = (\omega_{ce}^2 + \omega_{pe}^2)^{1/2}$. Asymptotic slope of beam wave curve is v_0^{-1}. Coupling in region A gives rise to instability reported here.

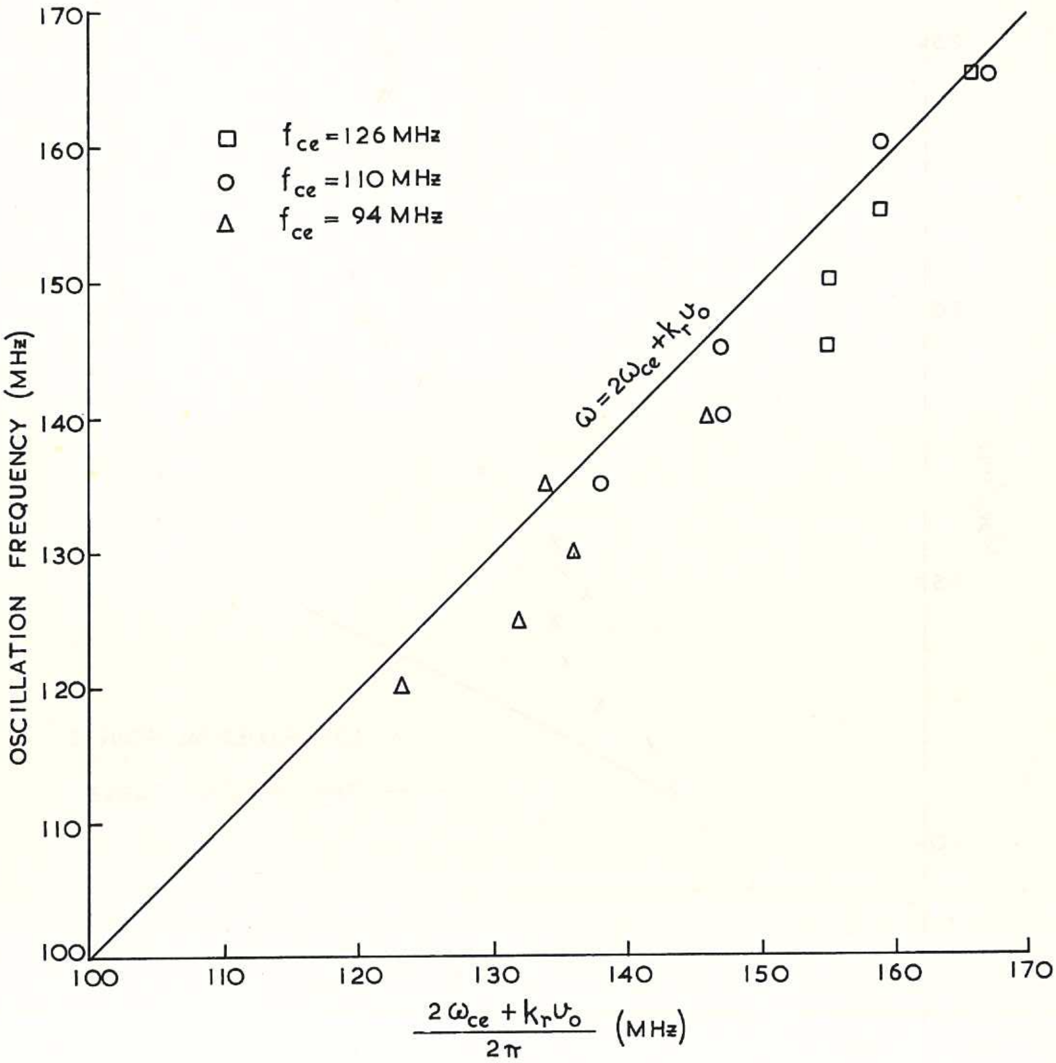

Fig. 3 (CLM-P 143)

Oscillation frequency vs axial wave number k_r.
Wave number plotted as Doppler shift $k_r v_o$ below $2\omega_{ce}$.

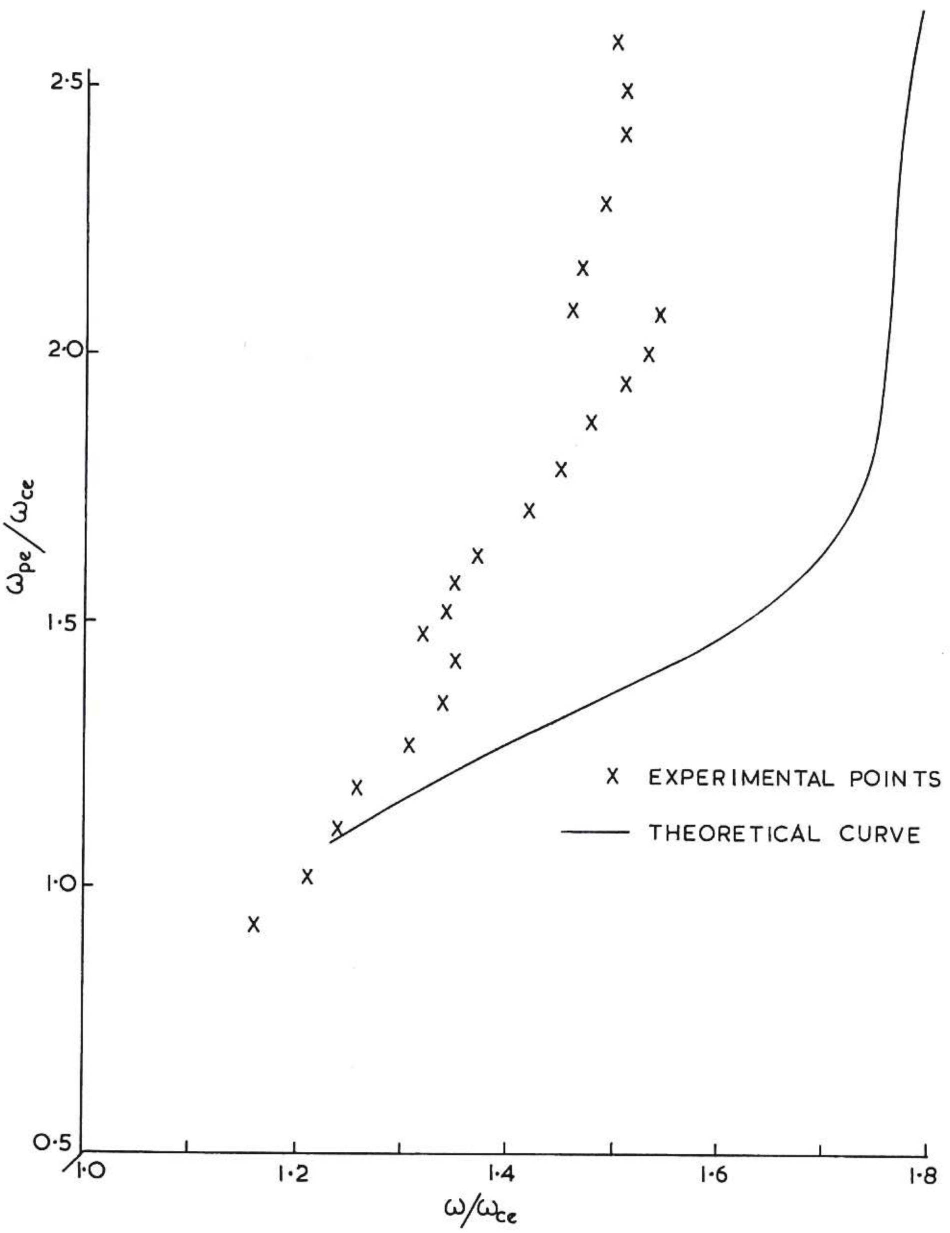

Fig. 4 (CLM - P 143)

Frequency of strongest oscillation and calculated frequency corresponding to maximum k_i vs plasma density.

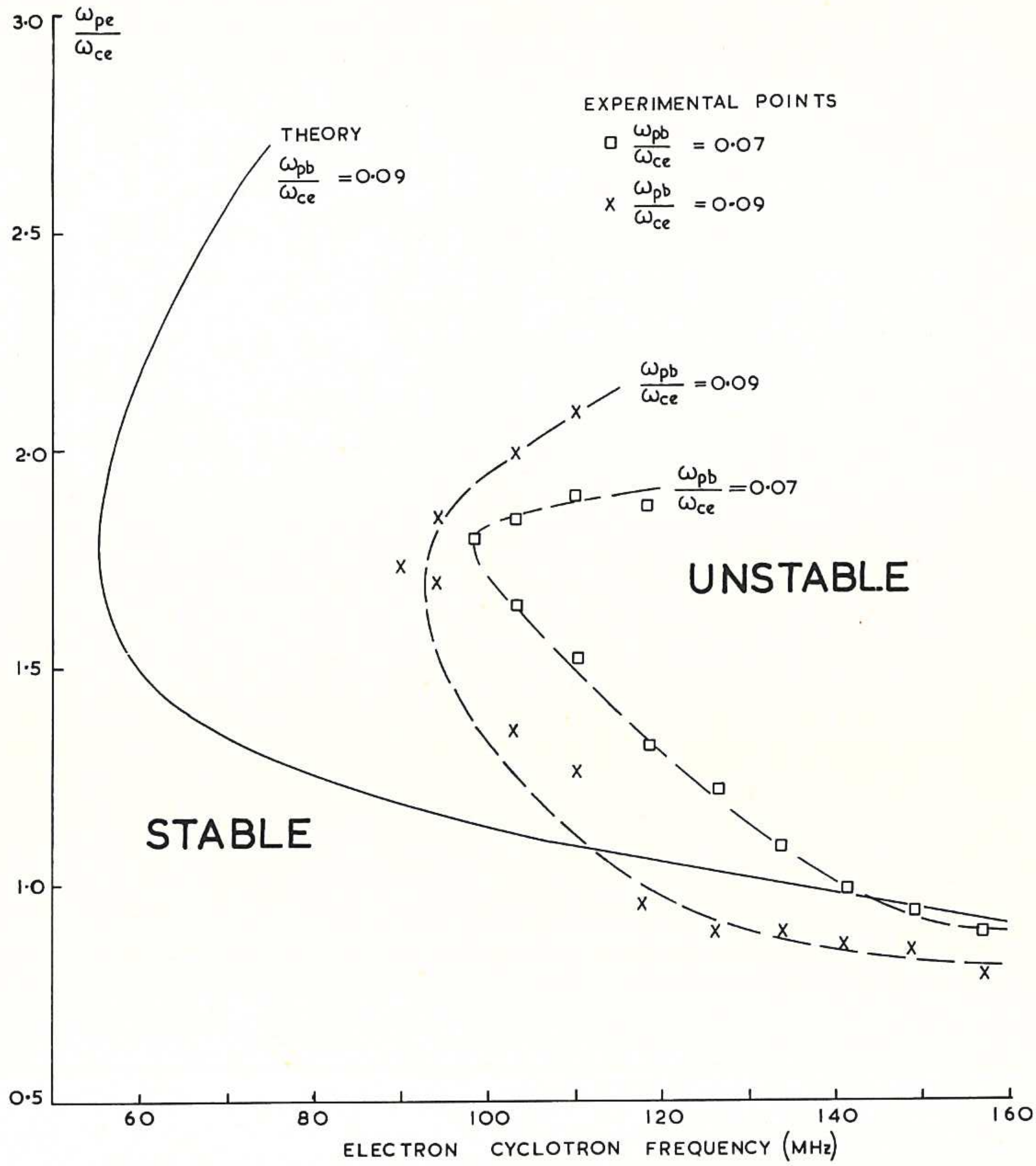

Fig. 5 (CLM-P 143)
Stability boundaries in the magnetic field - plasma density plane.
Dashed lines: Experimental curves. Solid line: Theoretical curve.