CLM - P 147

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Preprint

THE ANALYSIS OF TRANSIENTS IN PULSED LUMPED PARAMETER CIRCUITS WITH APPRECIABLE SKIN IMPEDANCE

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1967

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THE ANALYSIS OF TRANSIENTS IN PULSED LUMPED PARAMETER CIRCUITS WITH APPRECIABLE SKIN IMPEDANCE

by

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(Submitted for publication in Brit. J. of Applied Physics)

A B S T R A C T

An analytic procedure is developed for the calculation of current waveforms in pulsed lumped parameter circuits for cases where the skin impedance cannot be neglected.

Design curves are given which enable current waveforms to be determined for circuits of interest in Fusion and High Magnetic Field Research. The paper includes worked examples of the use of the curves.

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July, 1967 (ED)

CONTENTS

APPENDICES

1. INTRODUCTION

Skin effects, produced by field diffusion into conductors bounding a circuit, are not usually large enough to merit inclusion in the familiar LCR circuit equations. Consequently, methods of calculating current waveforms usually fall into two classes:

(a) those assuming uniform distribution of current throughout the conductors, and

(b) those which assume that the magnetic field is completely excluded from conductors.

There are, however, special cases where skin impedance is comparable with, or larger than, the lumped impedance of the circuit. One such case is a Multipole Magnetic Trap, used in fusion research, (DORY et al, 1966) where it is necessary to know the exact current waveform in order to calculate the '$\underline{E} \times \underline{H}$' particle drift velocity.

In this paper we develop an analytical method which should enable the circuit designer to calculate current waveforms in pulsed circuits with appreciable skin impedance. To facilitate the use of the method we present some novel design curves and worked examples.

2. ANALYSIS

The general equation of a lumped series (L·C·R) circuit may be written:-

$$\oint E d\ell + \frac{LdI}{dt} + IR + \int_0^t \frac{Idt}{c} = V \qquad \ldots (1)$$

where all quantities are in practical units and

- L is the circuit inductance, excluding skin inductance
- C is the circuit capacitance
- R is the circuit resistance, excluding skin resistance
- $\oint E d\ell$ is the voltage drop along the current path, produced by skin effects.

For a plane geometry, series-circuit composed of different conducting materials, each of resistivity η_r ohm-cm, length ℓ_r cm and current sheet width W_r cm, we may use the infinite half-plane approximation to the field diffusion equations, (see Appendix I), to obtain E, and write equation (1):-

$$\frac{I(o)}{\sqrt{t}} + \int_0^t \frac{dI(\tau)}{d\tau} \cdot \frac{d\tau}{\sqrt{t-\tau}} \left(\frac{\Lambda}{\pi}\right)^{\frac{1}{2}} + \frac{dI}{dt} + \frac{RI}{L} + \int_0^t \frac{Idt}{LC} = \frac{V}{L} \qquad \ldots (2)$$

where we define 'The Circuit Degradation Factor' as

$$\Lambda = 4\pi L^{-2}\ 10^{-9} \left\{ \sum_r \frac{\ell_r\ \eta_r}{W_r} \right\}^2 \sec^{-1} . \qquad \ldots (3)$$

Equation (2) is a Fredholm Integral Equation, (COURANT and HILBERT, 1953) which yields exact analytic solutions for some circuits (see Appendix I), but requires numerical analysis in most cases. Numerical solutions, necessarily obtained for particular values of circuit parameters, are not sufficiently general for initial design studies.

Fortunately series solutions can be obtained, to any accuracy, which allow the designer to assess the relative importance of the circuit parameters. We derive such solutions for the following cases of practical interest.

2.1 SINUSOIDAL 'SWITCH-ON' SOLUTIONS

Capacitative Energy Storage

We consider a solution of equation (2), for the case where the capacitor, initially charged to voltage V, is discharged at $t = 0$ through the circuit. The current produced is

$$I(t) = \frac{V}{L} \mathcal{L}^{-1} \left[\left(S^2 + S^{3/2}\ \Lambda^{\frac{1}{2}} + \omega^2 \right)^{-1} \right] \qquad \ldots (4)$$

where

$$\mathcal{L}^{-1} = \text{Inverse Laplace transform}$$
$$S = \text{Transform variable, and}$$
$$\omega = (LC)^{-\frac{1}{2}}$$

The contribution to $I(t)$ from the R/L term in equation (2) has not been included because it is impossible to give R a time independent value in fast circuits with appreciable skin impedance. The resistive elements appear entirely through their contributions to Λ.

A concise inversion of the Laplace Transform, in equation (4), is difficult because there is a branch point at $S = 0$ and the poles are not easy to find. We therefore, make an inversion of the asymptotic expansion of the Laplace Transform to obtain

$$I(t) = \frac{V}{\omega L} \sum_{p=0}^{\infty} (-)^p \left(\frac{\Lambda}{\omega}\right)^{p/2} \sum_{n=0}^{\infty} \frac{(-)^n \, \Gamma(n+1+p)(\omega t)^{2n+1+p/2}}{p! \, \Gamma(n+1) \, \Gamma(2n+2+p/2)} \quad \ldots (5)$$

where Γ is the Gamma Function.

The first term of the expansion, $p = 0$, is $\frac{V}{\omega L} \sin \omega t$ i.e. the trivial solution when $\Lambda = 0$ gives the current when no flux penetrates the conductors. Equation (5) may be written as:-

$$I(t) = \frac{V}{\omega L} \left\{ \sin \omega t + \sum_{p=1}^{\infty} (-)^p \frac{\Lambda}{\omega}^{p/2} S_p(\omega t) \right\} \quad \ldots (6)$$

which is a convenient form because Λ is determined solely from the physical dimensions and resistivity of the conductors, ω is the angular frequency in the absence of skin effect and S_p is a function of ωt.

Figs.1 and 2 show graphs of S_p against (ωt) which have been computed with the Culham KDF 9 computer for values of p from 1 to 8 inclusive.

Inductive Energy Storage

If the capacitor is initially uncharged and the circuit is closed at $t = 0$ when the inductance L is carrying $I(0)$ the resulting current is given by

$$I(t) = I(0) \sum_{p=0}^{\infty} (-)^p \left(\frac{\Lambda}{\omega} \right)^{p/2} \sum_{n=0}^{\infty} \frac{(-)^n \, \Gamma(n+1+p)(\omega t)^{2n+p/2}}{\Gamma(n+1)\ \Gamma(2n+1+p/2)p!} \quad \ldots (7)$$

The first term of the expansion, $p = 0$, is $I(0) \cos \omega t$ the current when no flux penetrates the conductors. Equation (7) may be written, more conveniently, as

$$I(t) = I(0) \left\{ \cos \omega t + \sum_{p=1}^{\infty} (-)^p \left(\frac{\Lambda}{\omega} \right)^{p/2} \cdot S_p(\omega t) \right\} \quad \ldots (8)$$

Figs.3 and 4 show graphs of S_p for values of p from 1 to 5 inclusive.

2.2 EXPONENTIAL DECAY SOLUTIONS

A solution of equation (2), for a 'clamped' circuit, when an inductance L_o, carrying current $I(0)$, is short-circuited at $t = 0$ by a clamp-switch of resistance R_1 and inductance L_1, is

$$I(t) = \frac{I(0)\ L_o}{(L_o + L_1)}\ \mathcal{L}^{-1} \left[\left(s + s^{\frac{1}{2}} \Lambda^{\frac{1}{2}} + a \right)^{-1} \right] \quad \ldots (9)$$

where $a = \dfrac{R_1}{L_o + L_1}$ has been assumed time independent, a reasonable assumption for the long time scale of most practical clamp-circuits.

Using the asymptotic expansion of the Laplace Transform we find

$$I(t) = \frac{I_o L_o}{L} \sum_{p=0}^{\infty} (-)^p \left(\frac{\Lambda}{a} \right)^{p/2} \sum_{n=0}^{\infty} \frac{(-)^n \, \Gamma(n+1+p)(at)^{n+p/2}}{\Gamma(n+1)\ \Gamma(n+1+p/2)p!} \quad \ldots (10)$$

where $L = L_o + L_1$.

The first term, $p = 0$, is $(I_o L_o/L)\ e^{-at}$ the solution when the circuit conductors have zero resistivity. When $R = 0$ it can be shown that the series summation is $\frac{L_o}{L} \cdot I_o \exp(\Lambda t)\,\mathrm{Erf}_c \sqrt{\Lambda t}$, a known analytic solution (see Appendix I).

Equation (10) may be written, more conveniently, as

$$I(t) = \frac{I_o L_o}{L} \left\{ e^{-at} + \sum_{p=1}^{\infty} (-)^p \left(\frac{\Lambda}{a}\right)^{p/2} S_p \right\} \qquad \ldots (11)$$

Fig.5 shows graphs of S_p against (at) for values of p from 1 to 5 inclusive.

2.3 CONVERGENCE OF THE SERIES-SOLUTIONS

Equations (6), (8) and (11) all involve the summation of series which should be tested for convergence using D'Alembert's Ratio Test.

Provided

$$\left(\frac{\Lambda}{\omega}\right)^{\frac{1}{2}} \left| \frac{S_{p+1}}{S_p} \right| < 1$$

for sufficiently large p, the series converge. The number of terms necessary to sum, for a given accuracy, is dependent upon the actual parameters of the circuit and time.

3. TYPICAL DESIGN CALCULATIONS

The examples, given below, demonstrate the usefulness of the design curves.

Example 1

A 100 μF capacitor, charged to 40 kV, is discharged at $t = 0$ through an aluminium parallel plate transmission line (20 cm long, 10 cm wide, insulation spacing 2 mm) into a 9·9 nH single turn stainless steel coil (10 cm diameter, 10 cm effective length wrapped around a 10 cm long high-conductivity copper core of 9·4 cm diameter). What is the current waveform over the first 7 μsec?

The inductance of the transmission line is 5·0 nH, giving a total inductance of 14·9 nH and angular frequency

$$\omega = (14\cdot9 \times 10^{-9}\ 100^{-4})^{-\frac{1}{2}} = 0\cdot82 \times 10^{6}\ \mathrm{sec}^{-1}$$

The circuit degradation factor, ignoring skin effect in the high conductivity copper, is

$$\Lambda = \frac{4\pi \times 10^{-9}}{(14\cdot9 \times 10^{-9})^2} \left\{ \frac{20}{10} \sqrt{5\cdot75 \times 10^{-6}} + \sqrt{68 \times 10^{-6}} \right\}^2 = 5.06 \times 10^{5}\ \mathrm{sec}^{-1}$$

$$\therefore\ \frac{\Lambda}{\omega} = 6\cdot2 \times 10^{-2}$$

Inserting these values into equation (6) gives

$$I(t) = \frac{V}{\omega L} \left\{ \sin t + \sum_{1}^{\infty} (-)^{p}\ (6\cdot2 \times 10^{-2})^{p/2}\ S_p \right\} \quad \ldots (12)$$

This series converges for the appropriate S_p's given in Figs.1 and 2. The current waveform with skin effects included is shown in Fig.6.

Example 2

A stainless steel coil (10 cm long, 5 cm radius, inductance 93·5 nH) is carrying a current of 1MA when it is short-circuited at t = 0, by a clamp switch (L = 6·5 nH, R = 0·2 mΩ). What is the resulting current over the first 200 seconds? The current, neglecting skin effects, is

$$I(t) = 0\cdot94\ \exp\left[- t \frac{R_1}{L} \right]\ \mathrm{MA},$$

where

$$L = L_1 + L_0 \quad \text{and} \quad R_1/L = 2\cdot10^{3}\ \mathrm{sec}^{-1}\ .$$

If we assume that the switch, and associated feeder, is made of low resistivity materials we can ignore skin effect in these parts, and

$$\Lambda = \frac{4\pi}{(100\times 10^{-9})^2} \times 10^{-9} \left\{ \frac{10\pi}{10} \sqrt{68\times 10^{-6}} \right\}^2 = 0{\cdot}84\times 10^{+3}\ \text{sec}^{-1}$$

i.e.

$$\left(\frac{\Lambda}{a}\right) = 0{\cdot}42$$

Inserting these values into equation (11) gives

$$I(t) = 0{\cdot}94 \left\{ \exp\left[-2{\cdot}10^3\, t \right] + \sum_{1}^{\infty} (-)^p\, (0{\cdot}42)^{p/2}\, S_p \right\} \quad \ldots (13)$$

This series converges for the appropriate S_p's given in Fig.5. The current waveform with skin effects included is shown in Fig.7.

Example 3

An inductance consisting of a straight rod, radius 8 cm, placed centrally inside a cylindrical return conductor of radius 16 cm is 1 metre long and made of stainless steel; $\eta = 68\times 10^{-6}$ ohm-cm. The coaxial inductor is carrying 1MA when it is connected across a circuit consisting of a 12 nH, active-clamp circuit comprising a 1F capacitor bank charged to 430 volts and a clamp-switch with an almost constant voltage drop of 100 volts. What is the current waveform of the clamped current?

The coaxial inductance

$$L_o = 200 \log 2 = 178\ \text{nH} \quad \text{and} \quad L = L_o + L_1 = 190\ \text{nH}.$$

If we assume that the switch and associated active clamp bank make a negligible contribution to the Circuit Degradation Factor, then

$$\Lambda = \frac{4\pi\times 10^{-9}}{(190\times 10^{-9})^2} \times \left\{ \left(\frac{100}{2{\cdot}8\pi} + \frac{100}{2{\cdot}16\pi} \right) \sqrt{68\times 10^{-6}} \right\}^2 = 0{\cdot}21\times 10^3\ \text{sec}^{-1}$$

$$\omega = (LC)^{-\frac{1}{2}} = (190\times 10^{-9})^{-\frac{1}{2}} = 2{\cdot}3\times 10^3\ \text{sec}^{-1}$$

i.e.

$$\left(\frac{\Lambda}{\omega} \right) = \cdot 091$$

Using the infinite half-plane approximation (see Appendix I), the current in the clamped circuit is given by

$$I(t) = \mathcal{L}^{-1} \left[\left(\frac{SL_o}{L} \times 10^6 + \frac{(430-100)}{L} \right) \left(S^2 + S^{3/2} \ \Lambda^{1/2} + \omega^2 \right)^{-1} \right] \quad \ldots (14)$$

The asymptotic expansions (6) and (8) then yield

$$I(t) = 0{\cdot}94 \times 10^6 \left\{ \cos(\omega t) + \sum_{p=1}^{\infty} (-)^p \, (\cdot 091)^{p/2} \; {}_L S_p \, (\omega t) \right\}$$

$$+ \; 0{\cdot}752 \times 10^6 \left\{ \sin(\omega t) + \sum_{p=1}^{\infty} (-)^p \, (\cdot 091)^{p/2} \; {}_c S_p \, (\omega t) \right\} \quad \ldots (15)$$

where ${}_L S_p$ and ${}_c S_p$ are found from Figs.3 and 4, and Figs.1 and 2 respectively; subscripts L and c are used to separate contributions made to I from inductive and capacitative energy storage. The clamped current and its components are shown in Fig.8.

4. CHECKS ON THE ACCURACY OF THE DESIGN CURVES

All the design curves were plotted from the series expansions, $S_p(\omega t)$, with the aid of the Culham KDF 9 Computer.

$S_p(\omega t)$ defined by equations (5) and (6), (7) and (8), (10) and (11) may be expressed analytically, for p = 2, as

$$S_2 = \frac{\Gamma(\frac{1}{2}) \omega t^{3/2}}{8\sqrt{2}} \left\{ 3J_{\frac{1}{2}}(\omega t) + \omega t \, J_{-\frac{1}{2}}(\omega t) \right\}$$

$$S_2 = \frac{\Gamma(\frac{1}{2})}{8\sqrt{2}} \left\{ (3\omega t^{\frac{1}{2}} - \omega t^{5/2}) \, J_{\frac{1}{2}}(\omega t) + 5\omega t^{3/2} \, J_{-\frac{1}{2}}(\omega t) \right\}$$

$$S_2 = (2\omega t - \omega t^2) \, e^{-\omega t}$$

These tabulated expressions, together with similar expressions for $p = 4$, evaluated from tables of Bessel Functions, agreed with the computed values to within 1 part in 1000.

5. CONCLUSIONS

The circuit equation for a series LCR circuit, possessing appreciable skin-impedance, is a Fredholm Integral Equation. By introducing a new electrical constant, the 'Circuit Degradation Factor', and using asymptotic expansions we have demonstrated that:-

(a) Current waveforms may be easily obtained for practical circuits

(b) The method advocated is a simple algebraic procedure which uses constants obtained from general design curves

(c) The method enables the designer to assess the relative significance of the circuit parameters. Note the critical dependence of the series-solutions upon (Λ/ω).

The factor Λ/ω, although usually associated with a dissipation of energy in the circuit (hence our designation 'Circuit Degradation Factor') can sometimes have useful properties. For example, parallel arrangements of 'skin elements' (see Appendix II), or even specially tailored elements incorporated into transmission lines could be used deliberately to control the waveform in a desired manner.

6. REFERENCES

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COURANT, R. and HILBERT, D., 1953, Methods of mathematical physics (New York: Interscience), 2, 299-303.

DORY, R.A., et al, 1966, Phys. Fluids, 9, 997-1009.

SYMBOLS

R	circuit resistance in ohms
L	circuit inductance in henries
C	circuit capacitance in farads
ω	angular frequency $= (LC)^{-\frac{1}{2}}\ sec^{-1}$
ℓ	lengths of plane-conductor in cm
r	defines the rth conductor
w	width of current - sheet in cm
η	resistivity in ohm-cm
Λ	'circuit degradation factor' in sec^{-1}, defined by equation (3)
$\mathcal{L}$	Laplace transform
S	Laplace transform variable
δ	delta function
Γ	gamma function
S_p	summation of series
$H(x,t)$	magnetic field in oersted at depth x cm time t secs
$E(o,t)$	electric field, in c.g.s.(em), at surface, time t
$I(t)$	total current in amperes at time t
τ	integration variable
p,n,r	integers used in series
ρ	radius of curvature of conductor

APPENDIX I

Derivation of the circuit equation

The diffusion equation for a good conductor (displacement current neglected and permeability assumed to be unity) is

$$\nabla^2 \underline{H} = \frac{4\pi}{\eta} \frac{\partial \underline{H}}{\partial t} 10^{-9} \qquad \ldots \text{(A1)}$$

where H is in c.g.s. (em) units, and the scalar resistivity (in ohm cm^{-1}) is a constant independent of position. The treatment for magnetic materials ($\mu \neq 1$) is more complicated because in general μ depends on H, and thus varies in space and time. The treatment given here may be used, (with μ inserted in the appropriate places) provided that the metal has a narrow hysteresis B-H curve and operates over the linear region; but in high current applications this condition is rarely satisfied.

In the infinite half plane approximation equation (A1) becomes

$$\frac{\partial^2 H}{\partial x^2} = \frac{4\pi}{\eta} \frac{\partial H}{\partial t} 10^{-9} \qquad \ldots \text{(A2)}$$

where $x = 0$ is the metal-space interface and x measures the depth into the metal.

If a current $I(t)$, which produces a magnetic field $H(0,t)$ at the interface $x = 0$, is established in the conductor a solution of equation (A2), (see CARSLAW and JAEGER, (1943) paras.12.3 and 12.2, equation (7)) is

$$H(x,t) = \int_0^t \left\{ \frac{\partial}{\partial\tau} \Big[H(0,\tau) \Big] + \delta(\tau)\, H(x,0) \right\} \mathrm{Erf}_c \left\{ x \sqrt{\frac{\pi}{\eta\, 10^9\, (t-\tau)}} \right\} d\tau \qquad \ldots \text{(A3)}$$

Since E in c.g.s. (em) units is given by $\underline{E} = \frac{\eta\, 10^9}{4\pi}$ curl $\underline{H}$ we can derive an integral for E(0,t) the electric field at the metal-space interface

$$E(0,t) = \frac{\eta\, 10^9}{4\pi} \int_0^t \frac{d}{d\tau} [H(0,t)] \frac{2\; d\tau}{\sqrt{\eta 10^9}\;(t-\tau)} + \frac{\sqrt{\eta\; 10^9}}{2\pi} \frac{H(0,0)}{\sqrt{t}} \quad \ldots (A4)$$

Taken over the integration path, shown in Fig.9, Ampere's circuital rule yields

$$H(0,t) = \frac{4\pi I}{10W} \text{ oersted} \quad \ldots (A5)$$

where I is the current, in amperes, carried by a current sheet of width W.

The voltage drop along the direction of current flow is obtained from (A5) and (A4); for a current carrying element length ℓ, constant width W, we obtain

$$V = 10^{-8} \oint \underline{E} \cdot \underline{d\ell} = 2 \cdot 10^{-9/2} \frac{\ell}{W} \sqrt{\eta} \left\{ \int_0^t \frac{d}{d\tau} [I(\tau)\;] \frac{d\tau}{\sqrt{t-\tau}} + \frac{I(0)}{\sqrt{t}} \right\} \text{volts} \quad \ldots (A6)$$

If the summation of voltage drops due to different circuit elements is allowed for in the familiar L, C, R circuit equation we obtain equation (2) of Section 2.

Application to non-planar conductors and conductors of finite thickness

When the infinite half-plane approximation is used to obtain solutions for curved conductors a reasonable criterion is that the penetration depth, at the time of interest, should be smaller than the conductor's radius of curvature. Caution should, of course, be exercised in applying this criterion but the validity of the procedure may be numerically tested, with the aid of the formula

$$\text{Error} = 0 \left\{ 50 \cdot \frac{x\ 10^{-9}}{(x-\rho)^{3/2}} \sqrt{\frac{\pi t}{\eta\rho}}\ \% \right\}$$

where ρ is the conductor radius of curvature, t is time in seconds and x is the penetration depth in cm.

It has also been tacitly assumed that the skin depth, for times of interest, is always less than the thickness of the metal (d). Therefore calculations are only valid for

$$t < \frac{\pi}{4} \frac{d^2}{\eta} 10^{-9} .$$

APPENDIX II

MISCELLANEOUS EXACT SOLUTIONS

The Leplace Transform of the circuit equation, including skin effects, will sometimes yield exact solutions for the current. A few solutions for simple circuits are given below.

1. Resistance-free clamp-circuit

Circuit equation $\overline{I(s)}\ (S + S^{\frac{1}{2}}\ \Lambda^{\frac{1}{2}}) = I_o(L_o/L)$ ... (A1a)

Solution $I(t) = \frac{I_o L_o}{L}\ \exp\ (\Lambda t)\ \mathrm{Erf}_c\ \sqrt{\Lambda t}$ ... (A1b)

2. Clamp-circuit, switch with constant voltage-drop, V

Circuit Equation $\overline{I(s)}\ (S + S^{\frac{1}{2}}\ \Lambda^{\frac{1}{2}}) = I_o(L_o/L) - \frac{V}{S}\ (L_o/L)$ ... (A2a)

Solution $I(t) = \left(\frac{I_o L_o}{L} - \frac{V}{\Lambda L} \right) \exp\ (\Lambda t)\ \mathrm{Erf}_c \sqrt{\Lambda t} - \frac{V}{L\Lambda} \left(2 \sqrt{\frac{t\Lambda}{\pi}} - 1 \right)$

... (A2b)

3. Constant voltage applied to inductor

Circuit equation $\bar{I}(s)\ (S + S^{\frac{1}{2}}\ \Lambda^{\frac{1}{2}}) = \frac{V}{LS}$ ... (A3a)

Solution $I(t) = \frac{V}{L\Lambda} \left\{ 2 \sqrt{\frac{t\Lambda}{\pi}} - 1 + \exp\ (\Lambda t)\ \mathrm{Erf}_c\ \sqrt{\Lambda t} \right\}$

... (A3b)

4. Constant current fed to two parallel inductors

Inductor L_1 has degradation factor zero,
inductor L_2 has degradation factor Λ .

CIRCUIT EQUATIONS

$SL_1\overline{I_1} = (S + S^{\frac{1}{2}}\ \Lambda^{\frac{1}{2}})\ L_2\ \bar{I}_2$, and

$I = (\bar{I}_1 + \bar{I}_2)S$ ... (A4a)

SOLUTION

$$I_2(t) = \frac{IL_1}{L_1 + L_2} \exp\left[\left(\frac{L_2}{L_1 + L_2}\right)^2 \Lambda t\right] \mathrm{Erf}_C\left[\frac{L_2}{L_1 + L_2}\sqrt{\Lambda t}\right]$$

$$I_1(t) = I - I_2(t) \qquad \ldots \text{(A4b)}$$

The above list is by no means exhaustive; useful heuristic exercise may be had finding other circuits which yield analytic solutions.

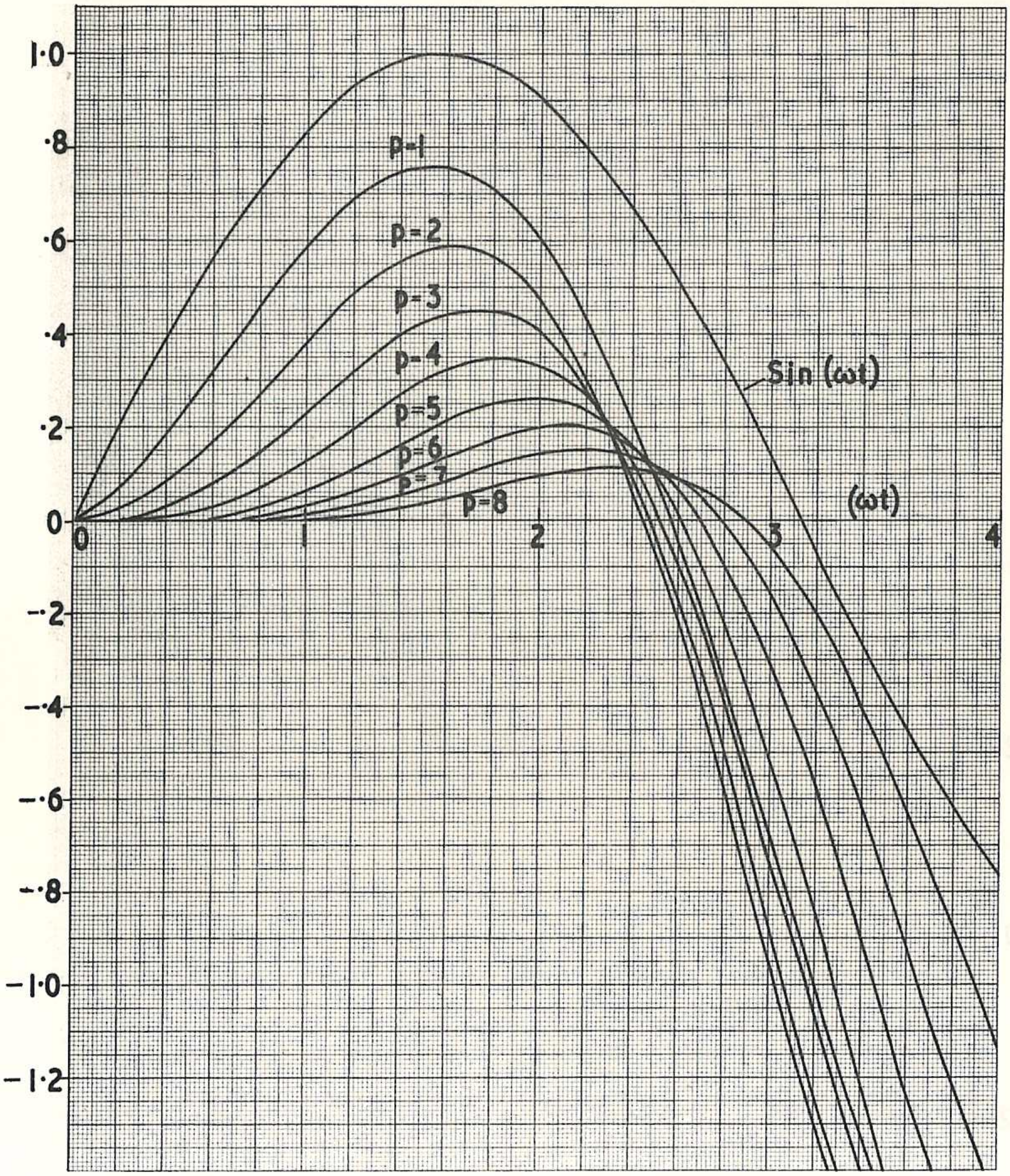

Fig. 1 (CLM-P 147)
$S_p(\omega t)$ for sinusoidal "switch-on" solutions: capacitive energy storage

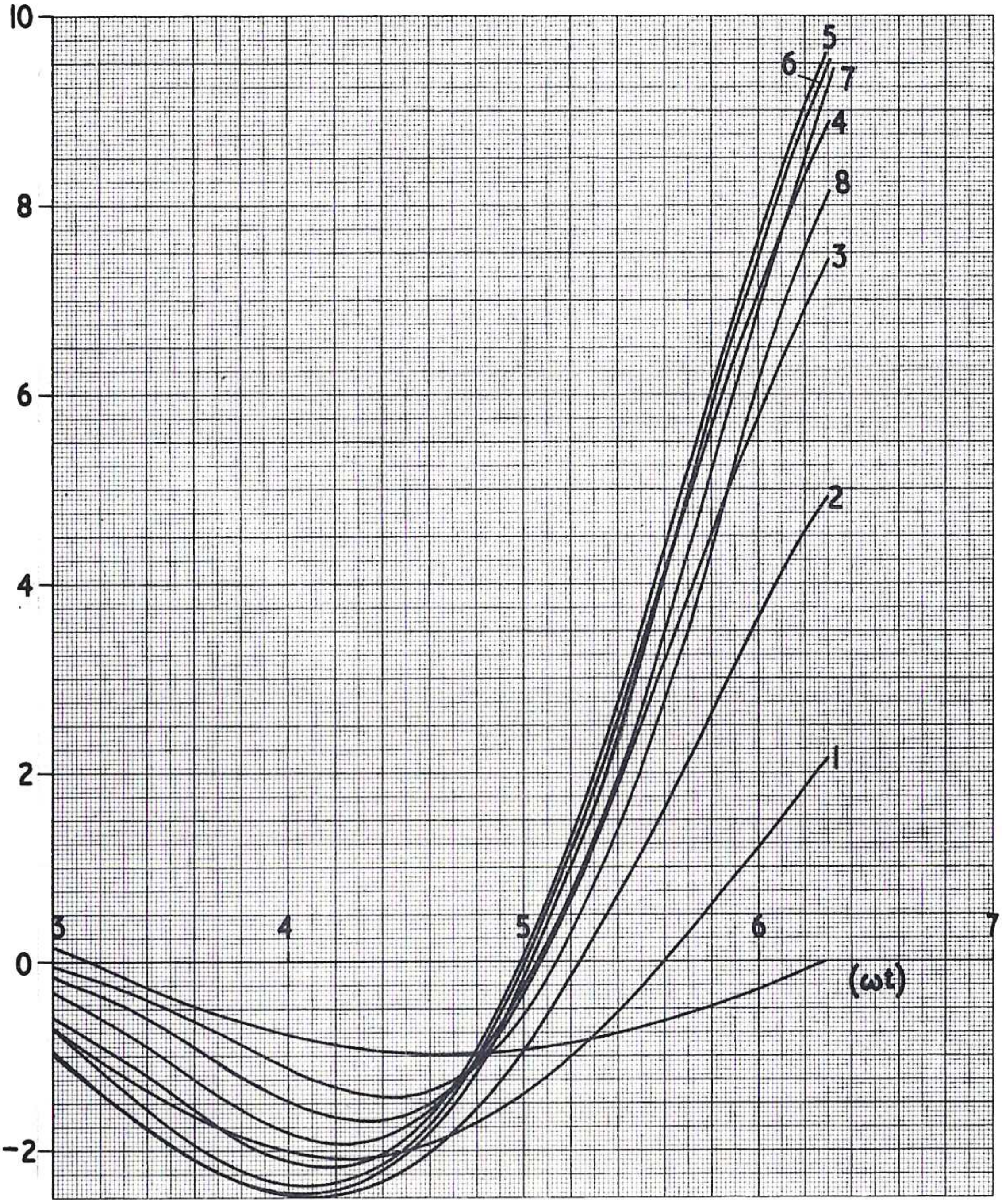

Fig. 2 (CLM-P 147)

$S_p(\omega t)$ for sinusoidal "switch-on" solutions: capacitive energy storage

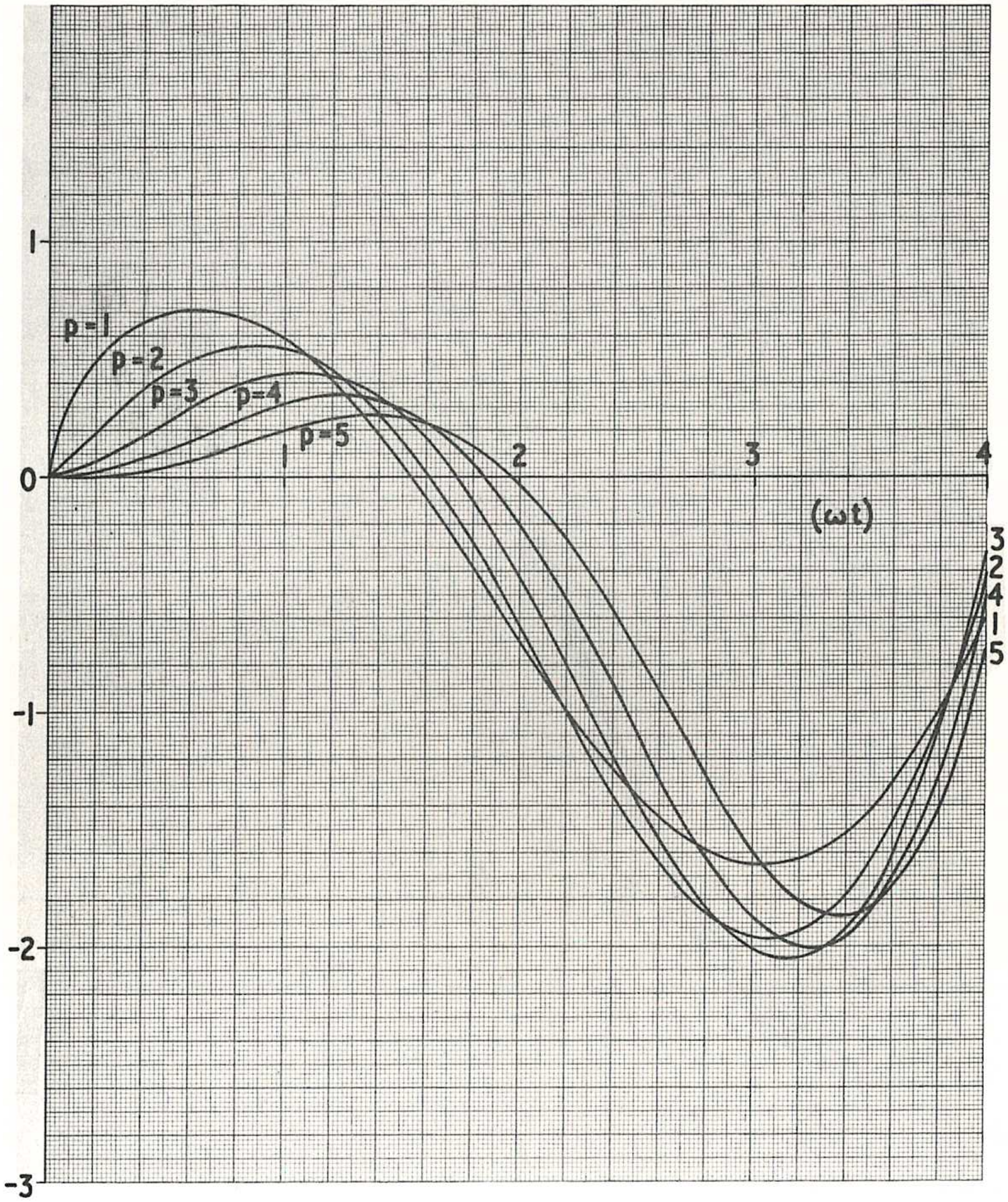

Fig. 3 (CLM-P 147)

$S_p(\omega t)$ for sinusoidal "switch-on" solutions: inductive energy storage

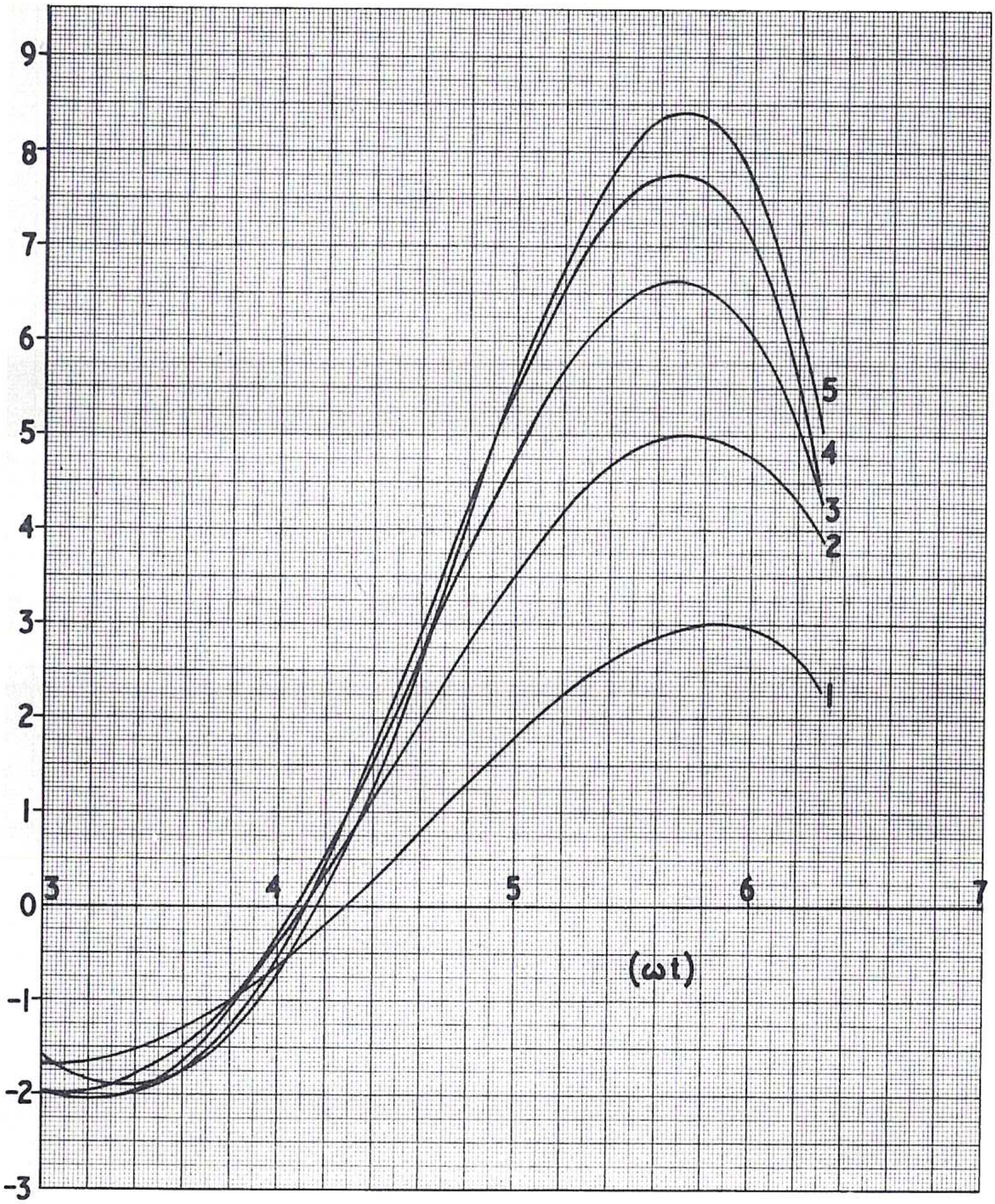

Fig. 4 (CLM-P 147)

$S_p(\omega t)$ for sinusoidal "switch-on" solutions: inductive energy storage

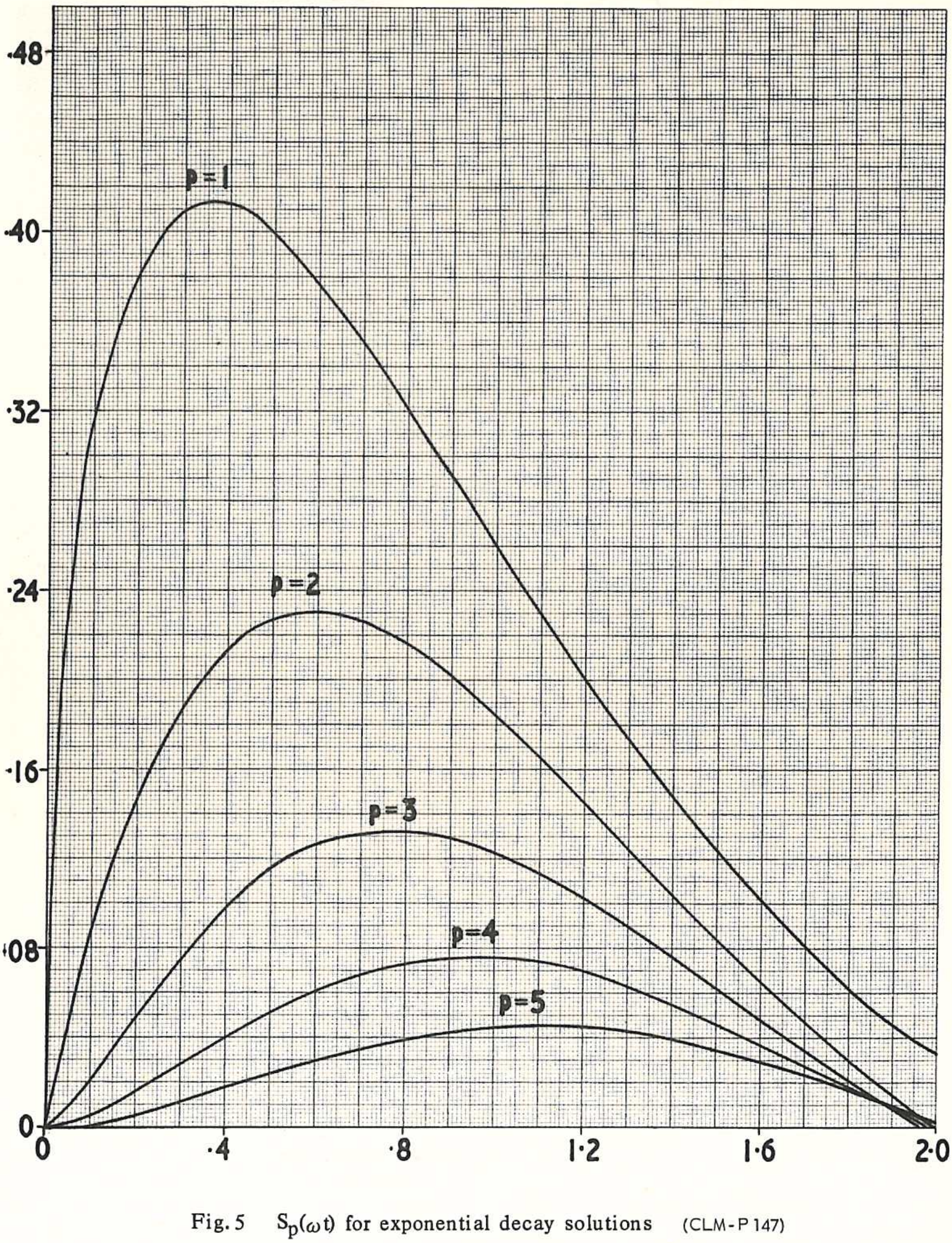

Fig. 5 $S_p(\omega t)$ for exponential decay solutions (CLM-P 147)

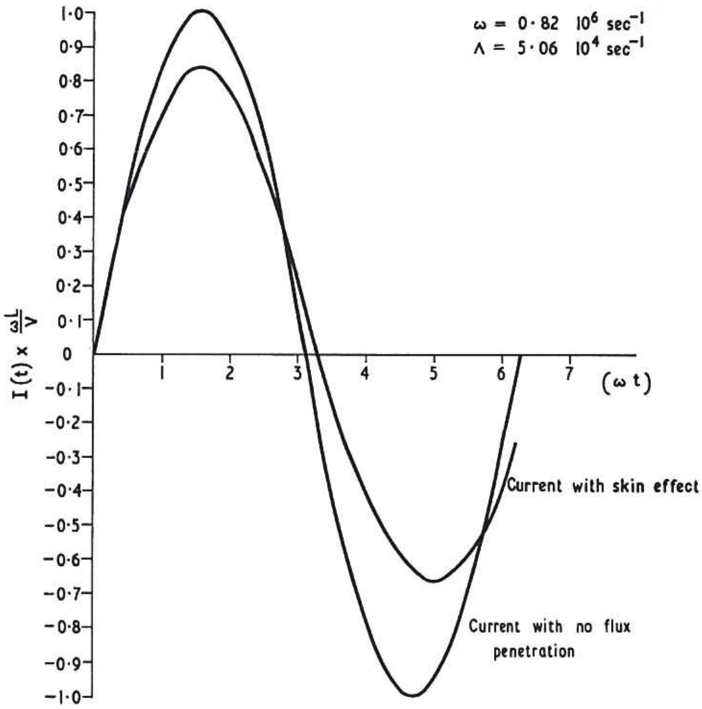

Fig.6 Waveforms: example 1 (CLM-P 147)

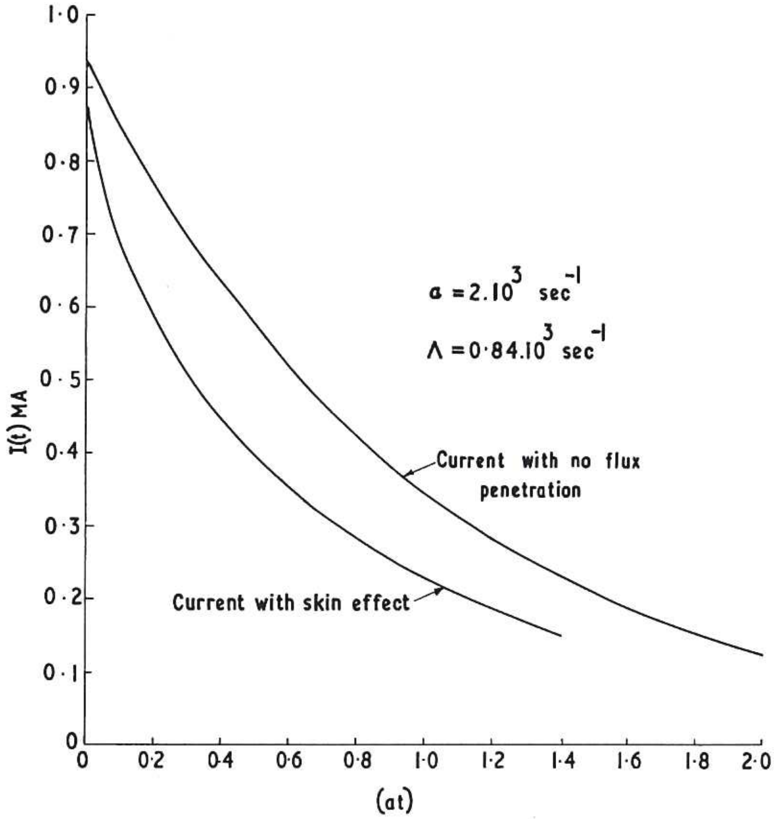

Fig.7 Waveforms: example 2 (CLM-P 147)

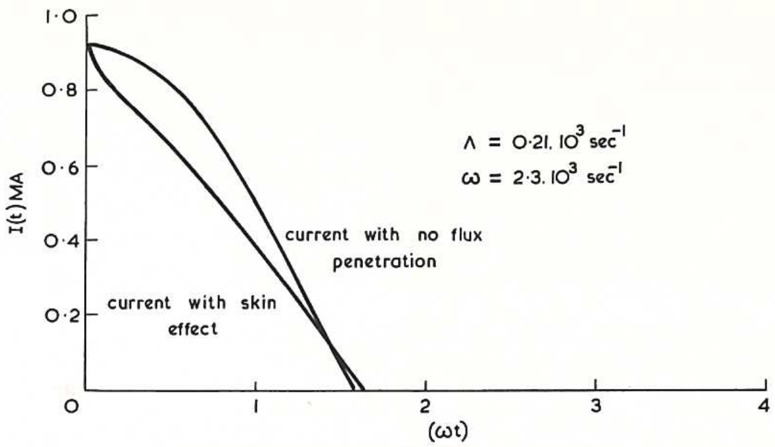

Fig. 8a (CLM-P 147)
Contribution to "clamped-current" from energy stored in inductor

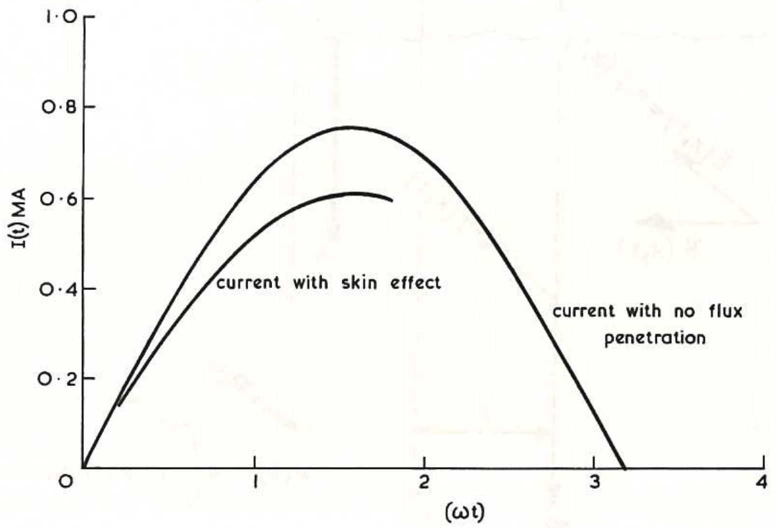

Fig. 8b (CLM-P 147)
Contribution to "clamped-current" from energy stored in capacitor

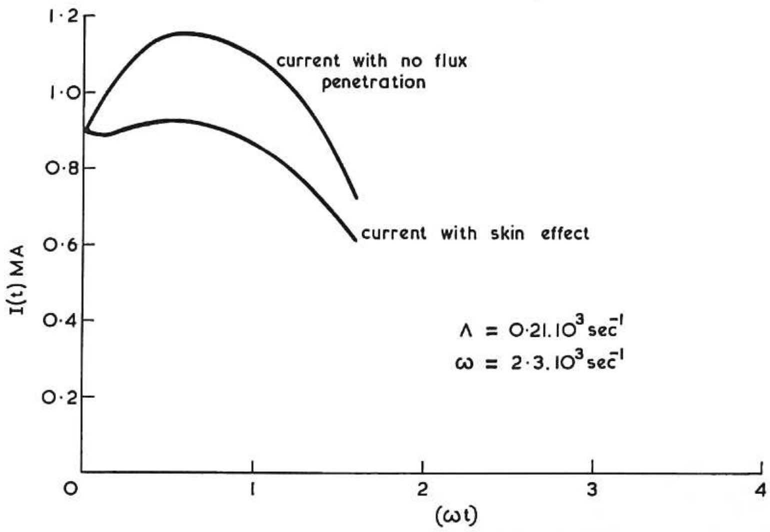

Fig. 8c Total "clamped-current" (CLM-P 147)

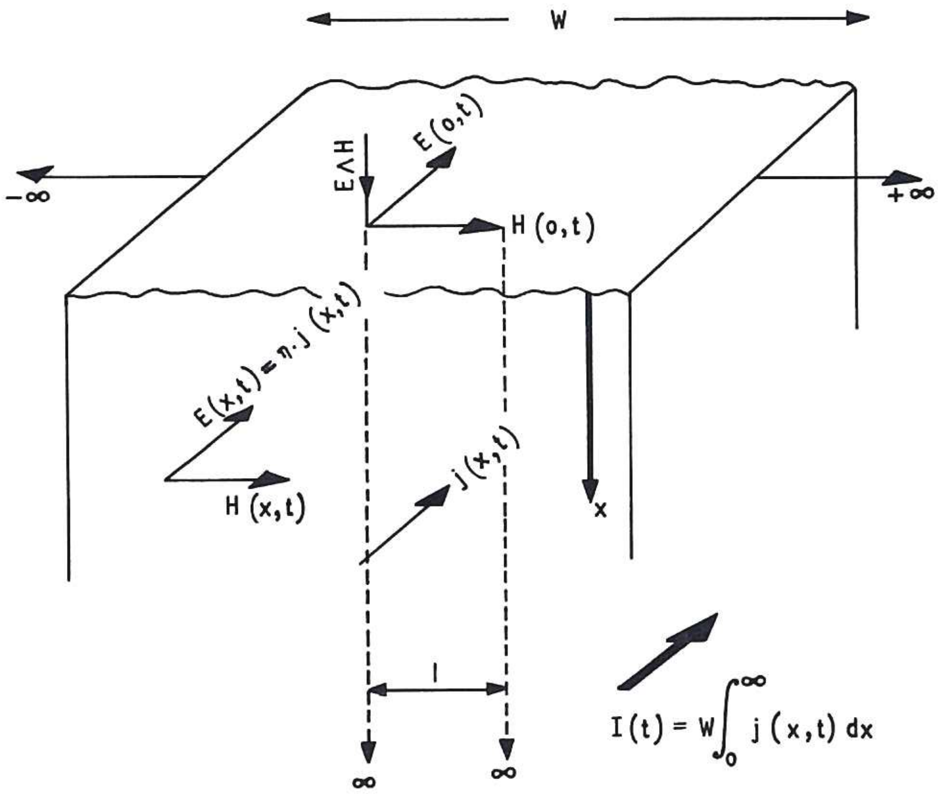

Fig. 9 Field configuration (CLM-P 147)