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COUNTERFLOW CURRENT LEADS

J. E. C. WILLIAMS

Culham Laboratory,

Culham, Abingdon, Berkshire

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by

J. E. C. WILLIAMS

ABSTRACT

The optimisation of the length and cross-sectional area of straight current leads for minimum heat flow into a cryogenic liquid is well known. It is assumed in that analysis that no heat flows across the surface of the lead.

In the present study an optimisation is made for current leads with heat flow from the surface to a counterflowing gas stream. It is shown that a considerable reduction of heat flow into a cryogenic liquid is possible.

U.K.A.E.A. Research Group,
Culham Laboratory,
Nr. Abingdon,
BERKS.

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INTRODUCTION

1. The use of cold gas to reduce the heat flow into cryogenic liquids is an established principle and analyses of the simple case (without the intermediate generation of heat) are found in the literature(1). The importance of this principle is best illustrated by reference to the conservation of liquid Helium. This liquid has a low latent heat of evaporation, 20.9 joules per gram. Helium gas, at a pressure of one atmosphere, has a specific heat (C_p) of 6 joules per gram and is thus capable of absorbing about 1800 joules of heat per gram in warming to room temperature. If heat, which would otherwise flow down a current lead to boil off the liquid Helium, could be absorbed by the gas, the rate of loss of liquid could be reduced by a factor of about 40.

2. This present optimisation proceeds by solving the steady state heat flow equation, generalised in terms of three dimensionless quantities from which can be obtained the characteristics of the lead giving least gas flow (i.e. evaporation) for a given design current and cryogenic liquid.

THEORY

APPROXIMATIONS AND ASSUMPTIONS

3. In order to solve the equations of steady state heat flow in a current lead, some assumptions and approximations must be made.

(1) The cold gas flowing over the surface of the current lead will assume a temperature lower than that of the surface with which it is in contact. In this analysis however the temperature difference between the gas and the lead will be assumed zero at all points on the lead. This assumption gives an ideal case which may be approached but not equalled in practice.

(2) The resistivity of all electrical conductors has a complicated dependence on temperature. Here, the resistivity will be taken proportional to temperature and the thermal conductivity will be assumed constant. Since both of these quantities finally appear under a square root, the errors in the assumption are reduced.

LEAD CHARACTERISTICS AND OPTIMISATION

4. The equations of steady state heat flow along a current lead, cooled by a counter-flowing gas stream whose temperature is identical with that of the lead at all points, are:-

$$\frac{\partial^2\theta}{\partial x^2} + \frac{mC_p}{K_e a_1}\frac{\partial\theta}{\partial x} + \frac{I^2\rho}{a_1^2 K_e} = 0 \qquad (1)$$

$$K_e\, a_1 \frac{\partial\theta}{\partial x_{(x=L)}} = m\, C_L \qquad (2)$$

where :-

θ is the temperature at a point distance x from the origin

l_1 is the length of the conducting path

a_1 is the cross-sectional area of the conducting path

I is the lead current

ρ is the lead material resistivity

K_e is the effective thermal conductivity of the lead

m is the gas flow rate

C_p is the gas specific heat at constant pressure

C_L is the latent heat of evaporation

K_e is assumed constant and ρ is assumed to obey the equation:-

$$\rho = \lambda(\theta - \theta_c)$$

where θ_c is the temperature of the liquid.

5. For convenience the following substitutions will be used:-

(1)
$$\theta^1 = \theta_H - \theta_c$$

θ_H being the temperature of the hot end of the current lead;

(2)
$$A = \frac{mC_p l_1}{K_e a_1}, \qquad B = 2\sqrt{\frac{\lambda}{K_e}}\, I\, \frac{l_1}{a_1}$$

6. The general solutions to equation (1) are:-

$$\theta = \theta^1 e^{-\frac{1}{2}A\frac{x}{l_1}} \frac{\sinh\{\frac{1}{2}\sqrt{A^2 - B^2}\ .\ (1 - \frac{x}{l_1})\}}{\sinh\{\frac{1}{2}\sqrt{A^2 - B^2}\}} \qquad (3a)$$

for A > B

$$\theta = \theta^1 e^{-\frac{1}{2}A\frac{x}{l_1}}\ .\ (1 - \frac{x}{l_1}) \qquad (3b)$$

for A = B

$$\theta = \theta^1 e^{-\frac{1}{2}A\frac{x}{l_1}} \frac{\sin\{\frac{1}{2}\sqrt{B^2 - A^2}\ .\ (1 - \frac{x}{l_1})\}}{\sin\{\frac{1}{2}\sqrt{B^2 - A^2}\}} \qquad (3c)$$

for B > A

By differentiating these expressions with respect to x and inserting the resulting derivative, $\frac{\partial\theta}{\partial x}$, into equation (2) the following are obtained:-

$$\frac{C_L}{C_p\theta^1} = \frac{e^{-\frac{1}{2}A}\,\frac{1}{2}\sqrt{A^2 - B^2}}{A\,\sinh\frac{1}{2}\sqrt{A^2 - B^2}} \qquad A > B \qquad (4a)$$

$$\frac{C_L}{C_p\theta^1} = \frac{e^{-\frac{1}{2}A}}{A} \qquad A = B \qquad (4b)$$

$$\frac{C_L}{C_p\theta^1} = \frac{e^{-\frac{1}{2}A}\ \frac{1}{2}\sqrt{B^2 - A^2}}{A\ \sin\frac{1}{2}\sqrt{B^2 - A^2}} \qquad B > A \tag{4c}$$

Equations (4a,b,c), in which there is no discontinuity, inter-relate three dimensionless quantities: the heat capacity ratio $\frac{C_L}{C_p\theta^1}$, the flow parameter A and the current parameter B. This relationship is shown as a family of curves in Fig.2, in which the heat capacity ratio is plotted as a function of the current parameter for a range of values of the flow parameter. From this set of curves the solution for any one of the three parameters in terms of the other two may be found.

7. In order to optimise a lead it is required to find an optimum value of $\frac{l_1}{a_1}$ which, for given design current I and for given heat capacity ratio $\frac{C_L}{C_p\theta^1}$ makes the flow rate m a mimimum. It can be shown that the condition for mimimum flow rate is given by:-

$$\sqrt{B^2 - A^2} = - A \tan\left\{\tfrac{1}{2}\sqrt{B^2 - A^2}\right\} \tag{5}$$

By eliminating A numerically between equations (4c) and (5), a function is obtained which relates the optimum value of the current parameter B to the heat capacity ratio. This relationship is plotted in Fig.2 together with the corresponding optimum value of the flow parameter A, (obtained by eliminating B between equations (4c) and (5) also expressed as a function of the heat capacity ratio.

8. An optimum value of $\frac{l_1}{a_1}$ is obtained from an optimum of B by substituting into B, values for the design current I, and for λ and K_e. By further substituting this value of $\frac{l_1}{a_1}$, together with C_p and K_e, into the corresponding optimum of A, the mimimum value of the flow rate m, for the given conditions, is obtained.

9. In order to compare the performances of the counterflow lead and the straight lead, a figure of merit $\frac{m}{I}\frac{C_p}{2\sqrt{\lambda\ K_e}}$ has been plotted in Fig.3 as a function of the heat capacity ratio for both types of lead. In this parameter m is the mimimum possible flow rate for a design current I and for given values of C_p, λ and K_e.

10. Slight deviation of the operating current from the design value does not greatly affect the flow rate. Typically, a decrease of the current to zero reduces the flow rate by 40% whilst an increase of the operating current to 150% of the design value increases the flow rate to 170% of the level for the design current.

TEMPERATURE PROFILE

11. It was stated above that the flow rate does not vary greatly with changing current.

The temperatures of points on the lead will vary more markedly with current however and if a counterflow lead is to be operated at currents exceeding the design value, it is desirable to know the shape of the temperature profile or, more specifically, the maximum temperature of the lead.

12. The position of the hottest point on the lead is found from equation (3c). It is given by:-

$$\left(\frac{d\theta}{dx}\right) = 0 ,$$

which leads to an expression for $\frac{x}{l_1}$ of:-

$$\tan \left\{\tfrac{1}{2}\sqrt{B^2 - A^2}\ \left(1 - \frac{x}{l_1}\right)\right\} = - \frac{\sqrt{B^2 - A^2}}{A} \qquad (6)$$

By inserting this expression into equation (3c) and by obtaining an expression for:-

$$\sin \left\{\tfrac{1}{2}\sqrt{B^2 - A^2}\right\}$$

from equation (4c), the maximum lead temperature is given by:-

$$\theta\ \max = 2\,\frac{A}{B} \cdot \frac{C_L}{C_p}\ \exp\left[\frac{A}{\sqrt{B^2 - A^2}}\ \tan^{-1}\left(- \frac{\sqrt{B^2 - A^2}}{A}\right)\right] \qquad (7)$$

Now if, for a given heat capacity ratio, A and B can be read from Fig.1, equation (7) can be solved for θmax. If, however, the required values of A and B do not lie on the curves of Fig.1, a simple approximation will enable θmax to be calculated.

13. In equation (4c), $\frac{C_L}{C_p\theta^1}$ will be positive only if:-

$$0 < \mathrm{Sin}\ \tfrac{1}{2}\sqrt{B^2 - A^2} < \pi .$$

Since the curves of Fig.1 apply only to positive values of heat capacity ratio, then:-

$$B^2 - A^2 < 4\pi^2 .$$

For large values of the current parameter B, the approximation:-

$$A \doteqdot \sqrt{B^2 - 4\pi^2} ,$$

can be used. Substituting this into equation (7), the expression for the maximum temperature becomes:-

$$\theta\max \doteqdot 2\,\frac{C_L}{C_p}\left(\sqrt{1 - \frac{4\pi^2}{B^2}}\right) \exp\left[\sqrt{\frac{B^2}{4\pi^2} - 1}\ \tan^{-1}\left(- \frac{1}{\sqrt{\frac{B^2}{4\pi^2} - 1}}\right)\right] \qquad (8)$$

If all but the most significant factors in equation (8) are neglected, then:-

$$\theta\max \propto e^{B} \qquad (9)$$

14. Equation (9) implies that a lead designed for a liquid with small heat capacity ratio, such as liquid Helium, and operated at a current greater than the design value, will have a higher peak temperature than one with the same ratio of operating current to design current but operating with a liquid of larger heat capacity ratio, such as liquid Nitrogen. In Fig.4 the maximum lead temperatures for these two cases are plotted as functions of the ratio of operating current to design current.

CONSTRUCTION

15. Counterflow leads are most easily made with the crinkled edgewound copper strip normally used as heat exchanger finning. One or more lengths of this helical strip are pushed onto a Synthetic Resin Bonded Paper tube as a single path or multiple path lead. The turns are suitably spaced apart by winding a thread or tape between them and this assembly is then encased within a tight fitting outer tube of S.R.B.P.

16. In the next section it will be seen that in order to make a lead which has the mimimum effective thermal conductivity and which behaves in close agreement with theoretical prediction, it may be necessary to construct a two-start lead in which one of the helical paths available for gas flow is blocked by a poor thermal conductor. Glass fibre, loosely wound into one of the helical channels, suffices for this purpose.

17. By slight modification, the counterflow lead can be used to reduce the quantity of a cryogenic liquid needed to cool a large body.

At the cold end of the lead the central S.R.B.P. tube is replaced by a short length of copper tube to which the lowermost turns of the lead (or of one of the leads in a multi-path unit) are soldered. The outer S.R.B.P. tube fits tightly over the entire length of the lead. The copper tube is connected thermally to the body so that heat is conducted from the body to the lead. The issuing cold gas absorbs this heat in the small heat exchanger at the lower end of the lead.

EFFECTIVE THERMAL CONDUCTIVITY

18. It is seen in section 9 that, for a given figure of merit of a counterflow lead (corresponding to a particular value of the heat capacity ratio) the mimimum ratio of flow rate m to design current I depends on K_e. For a low flow rate, K_e must be as small as possible. As a consequence, however, of the type of construction outlined in the previous section, the effective thermal conductivity of the lead is greater than the thermal conductivity of the copper alone. The increase is due in part to the presence of the S.R.B.P. tubes, which must therefore be as thin as possible, and in part to the presence of gas between turns. The presence of gas is inevitable although it may be either turbulent or stagnant in a multiple path lead. Since the transfer of heat through a turbulent gas is greater than that through a stagnant gas and since the former is also difficult to predict accurately, it is desirable to eliminate turbulent gas from at least a part of the counterflow lead. This can be accomplished in a two path lead by introducing a loose matrix of material of low thermal conductivity, such as glass fibre, into one of the two helical gas channels. A barrier of stagnant gas is thus created in the blocked channel, which greatly

reduces the heat transferred between alternate turns of the lead. The thermal conductivities of stagnant gases are known and can be incorporated in the following derivation of the effective thermal conductivity of the lead.

19. Let K denote thermal conductivity and let $\frac{l}{a}$ stand for the ratio of length to cross-sectional area. The suffices 1,2 and 3 will characterise the copper helix (or helices) the S.R.B.P. tubes and the stagnant gas, respectively. Then:-

$$K_e = K_1 + K_2 \frac{(\frac{l_1}{a_1})}{(\frac{l_2}{a_2})} + K_3 \frac{(\frac{l_1}{a_1})}{(\frac{l_3}{a_3})} \qquad (10)$$

In these ratios l_1 is always greater than l_2 and l_3 is roughly one half of l_2. Also, a_3 is the area of the annular space between outer and inner tubes and a_2 is the sum of the annular cross sectional areas of the two S.R.B.P. tubes.

20. Typical thermal conductivities for copper, S.R.B.P. and Helium gas are shown in Fig.5. It is seen that all vary appreciably with temperature whereas a constant value of K_e was assumed in the derivation of the theory. If this assumption is to hold with moderate accuracy, equation (10) should give roughly constant values for K_e when values of K_1, K_2 and K_3 corresponding to a various temperature are substituted. It can be seen that for this to transpire, the values of $(\frac{l_1}{a_1})$, $(\frac{l_2}{a_2})$ and $(\frac{l_3}{a_3})$ must be suitably chosen.

COMPARISON OF THEORY WITH PRACTICE

21. Consider the requirement for a lead operating into liquid Helium from room temperature and optimised for a current of about 8 amperes per path. It is to be a two-path lead. Graph 2 shows that for the conditions specified, the optimum current parameter is 9.4, the heat capacity ratio being 0.012. Thus:-

$$2\ I \sqrt{\frac{\lambda}{K_e}}\ (\frac{l_1}{a_1}) = 9.4 \qquad (11)$$

Details of construction necessitate an overall length of 23 cms, an outer tube of 2.54 cms 1.D. and 1.6 mm wall and an inner tube of 0.95 cms O.D. and 1.6 mm wall. Copper strip of width 0.79 cms and thickness 0.178 mm is available and λ will be taken as:-

$$0.005 \times 10^{-6}\ \Omega\ \text{cms}/^{\circ}\text{K}.$$

One of the helical gas channels will be loosely packed with glass fibre. The ratios $(\frac{l_2}{a_2})$ and $(\frac{l_3}{a_3})$ follow directly from the given dimensions:-

$$(\frac{l_2}{a_2}) = 13\ \text{cms}^{-1}$$

$$(\frac{l_3}{a_3}) = 6.7\ \text{cms}^{-1}$$

it being assumed for the latter that the blocked and free channels are the same size.

The calculation of the effective thermal conductivity of the lead is an iterative process as follows:-

22. An arbitrary value is assigned to $(\frac{l_1}{a_1})$. Equation (10) is then used, together with the curves of thermal conductivity shown in Fig.5, to determine the effective thermal conductivity at three temperatures, e.g. at 20°K, 100°K and 300°K. An average value for K_e is selected from these figures and inserted, together with λ and $(\frac{l_1}{a_1})$ into equation (11). A better value is then assigned to $(\frac{l_1}{a_1})$ and the calculation of K_e and then of I repeated: this procedure will finally give the required value for $(\frac{l_1}{a_1})$.

In the present example the value finally given for $(\frac{l_1}{a_1})$ is:-

$$(\frac{l_1}{a_1}) = 12500 \text{ cms}^{-1}$$

for the two paths together.

Hence:-

$$\begin{aligned} K_e &= 13.9 \text{ w/cm}^{\circ}\text{K at } 20^{\circ}\text{K} \\ &= 8.8 \text{ w/cm}^{\circ}\text{K at } 100^{\circ}\text{K} \\ &= 12.3 \text{ w/cm}^{\circ}\text{K at } 300^{\circ}\text{K} \end{aligned}$$

which give an average for K_e which is:-

$$K_e \text{ average} \div 10 \text{ watts/cm}^{\circ}\text{K.}$$

Equation (11) then gives:-

$$I \div 17 \text{ amperes.}$$

i.e. the optimum current per path is 8.5 amperes.

23. The flow rate at this current can be obtained from the figure of merit. For a heat capacity ratio of 0.012 the figure of merit of the counterflow lead is seen from Fig.3 to be 0.84. Thus:-

$$\frac{m}{I} \frac{C_p}{2\sqrt{\lambda K_e}} = 0.84$$

from which:-

$$m = 1.06 \times 10^{-3} \text{grams/sec.}$$

The flow rate at zero current is found from the curves of Fig.1. For the given heat capacity ratio and current parameter zero, the flow parameter is 4.5. Thus:-

$$m_o \frac{C_p}{K_e} (\frac{l_1}{a_1}) = 4.5$$

whence:-

$$m_o = 0.6 \times 10^{-3} \text{grams/sec.}$$

These theoretical predictions are compared in Table 1 below and in the curves of Fig.6 with observations made on a lead constructed to the specifications derived above. Also included in the table are the theoretical flow rates for a straight lead optimised for the same current.

TABLE 1

Counterflow lead	Optimum current in amperes	Gas flow rate at opt. current-gms/sec	Gas flow rate at zero current-gms/sec
Theory	18	1.1×10^{-3}	0.6×10^{-3}
Observation	18	1.5×10^{-3}	0.6×10^{-3}
Straight lead	18	34×10^{-3}	7.4×10^{-3}

DISCUSSION OF RESULTS

Some important conclusions derive from Fig.3.

24. Marked on the abscissa of this graph are values of the heat capacity ratio corresponding to three cryogenic liquids and two hot end temperatures. It is seen that the advantage of the counterflow lead over the straight lead is considerable for Helium and is significant for Hydrogen if the hot end temperature is 300°K. Furthermore, it can be seen from the figure of merit curve of Fig.3 that the flow rate for Helium with a hot end temperature of 300°K is only 14% greater than the flow rate when the hot end temperature is reduced to 78°K by cooling with liquid Nitrogen. The difference in flow rates for the straight lead under these conditions is 300%. The inference is that, since the intermediate cooling of the counterflow lead is not essential, it is ideally suited to systems which do not use Nitrogen.

25. From the "figure of merit" parameter of para.9, it can be seen that, in order to reduce to a mimimum the flow rate for a given current, the product λK_e must be a mimimum. Now λK for pure metals is a constant (the Lorentz number) and is the mimimum value that λK_e can attain. Now, in the method of construction for counterflow leads outlined in para.15 above K_e is made greater than the conductivity of the metallic helix alone by the presence of the S.R.B.P. walls. At the same time however, λ is not reduced. The mimimum possible value of λK_e could be more nearly approached by the use of thin metallic walls (e.g. stainless steel). A small part of the electrical current would flow in these walls, in consequence of which only a single path could be used.

26. Comparison of theory and practice in Fig.6 shows that the theory developed gives fairly accurate predictions of performance, within the limits of the assumption made in para.3. The best accuracy is obtained at low currents (and therefore at low gas flow rates).

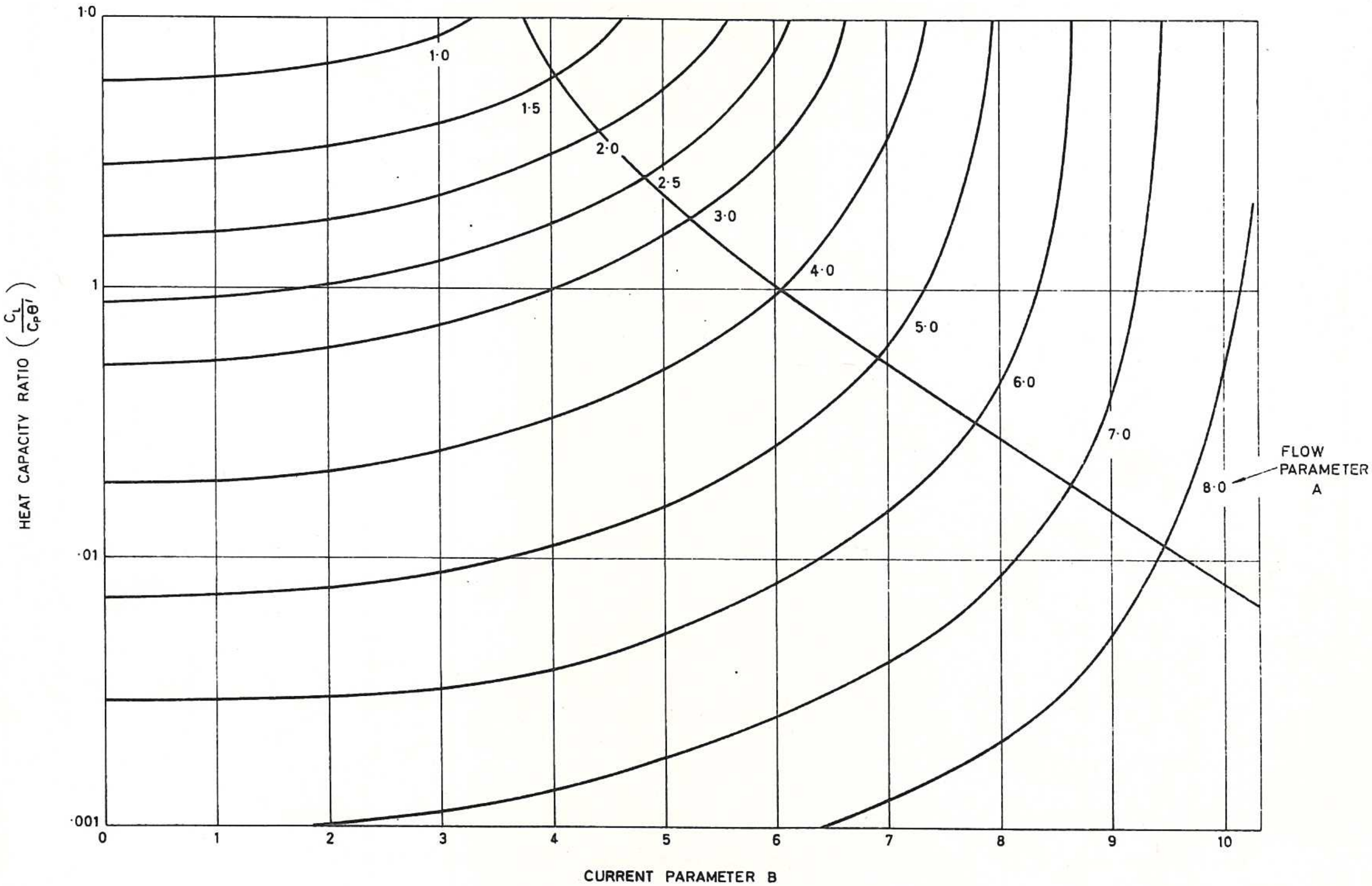
The experimentally observed optimum current is not well defined because the ratio of flow rate to current becomes roughly constant at currents above 18 amperes. This nevertheless does not vitiate the theoretical optimisation.

27. Theory predicts that, at currents above the opimum, the peak temperature of the lead should rise rapidly to the point at which the lead would be destroyed: thus, by the use of the appropiate curve of Fig.4 it can be seen that in theory, a current of 27 amperes in the present lead should give a peak temperature of 1,000^{o}K, sufficient to melt the copper. This theoretical limiting current is indicated in Fig.6. The experimental observations show however that currents in excess of the theoretical limit could be passed without apparent damage to the lead.

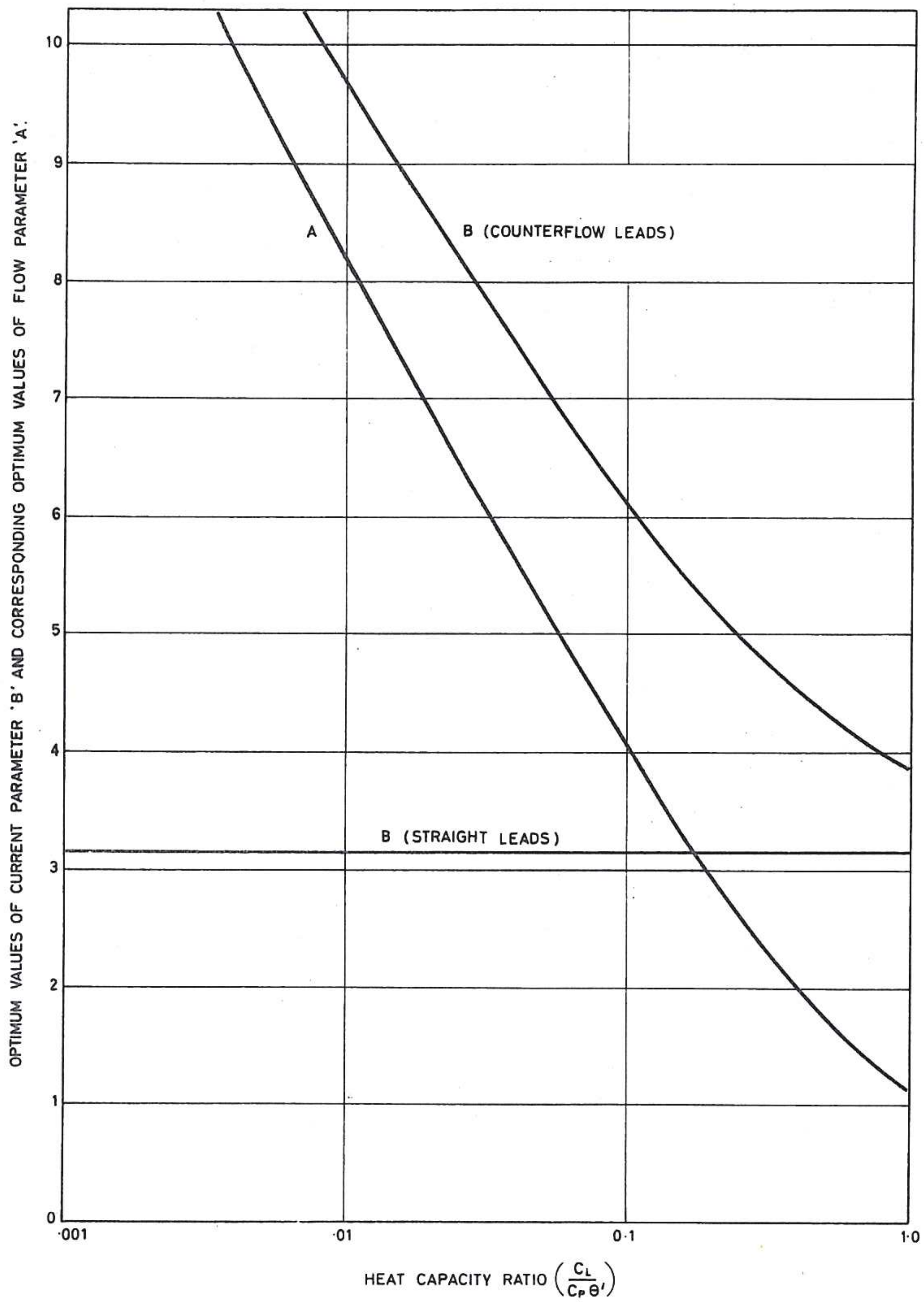
28. A possible explanation for this effect and for the departure of the experimental curve of Fig.6 from the theoretical is as follows:- Increasing lead current causes an increase in lead temperature and hence a rise in the values of both λ and K_e. Also K_e may be increased by turbulence created in the blocked channel by imperfect sealing between the S.R.B.P. and the copper. Thus the ratio $\frac{\lambda}{K_e}$ may remain roughly constant whilst λK_e and K_e increase. Thus the optimum current would be as predicted, the flow rate at zero current would be as predicted, but the flow rates at other points would be greater than predicted.

REFERENCES

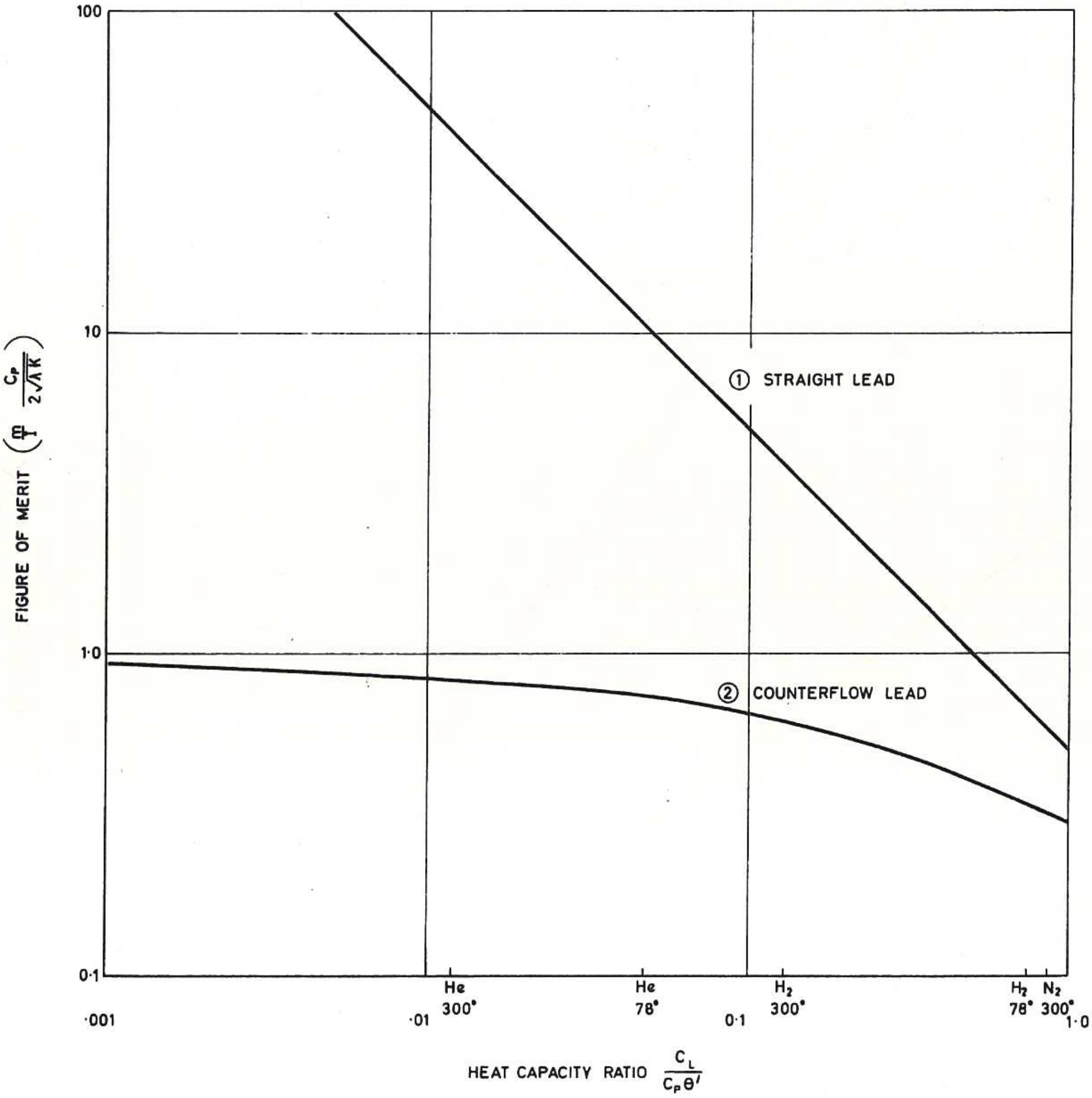
SCOTT, R.B., Cryogenic Engineering, p.239.



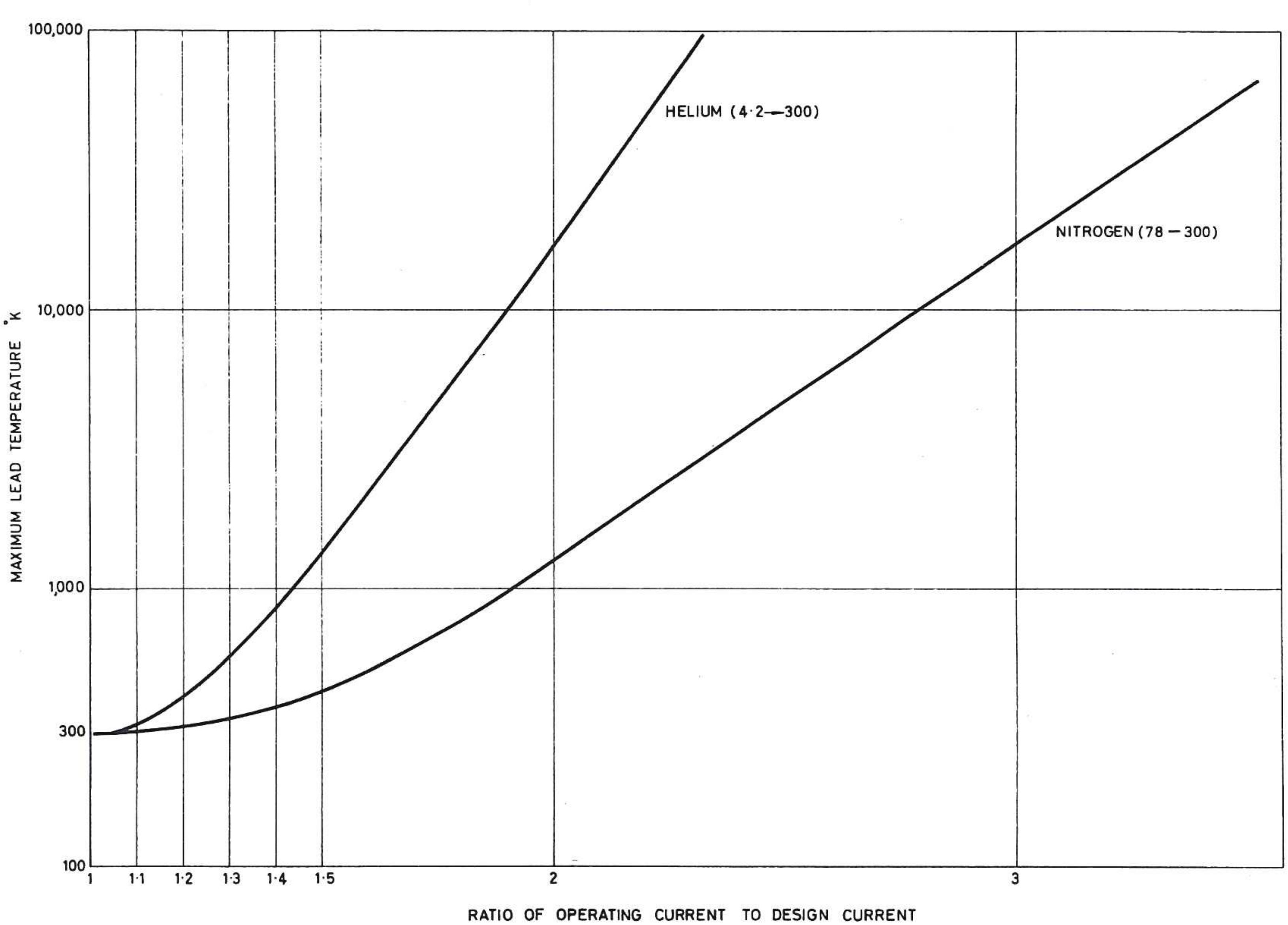
CLM-P 15   Fig. 1   Curves describing the general operating conditions of the counter flow current lead in terms of the flow parameter, the current parameter and the heat capacity ratio.



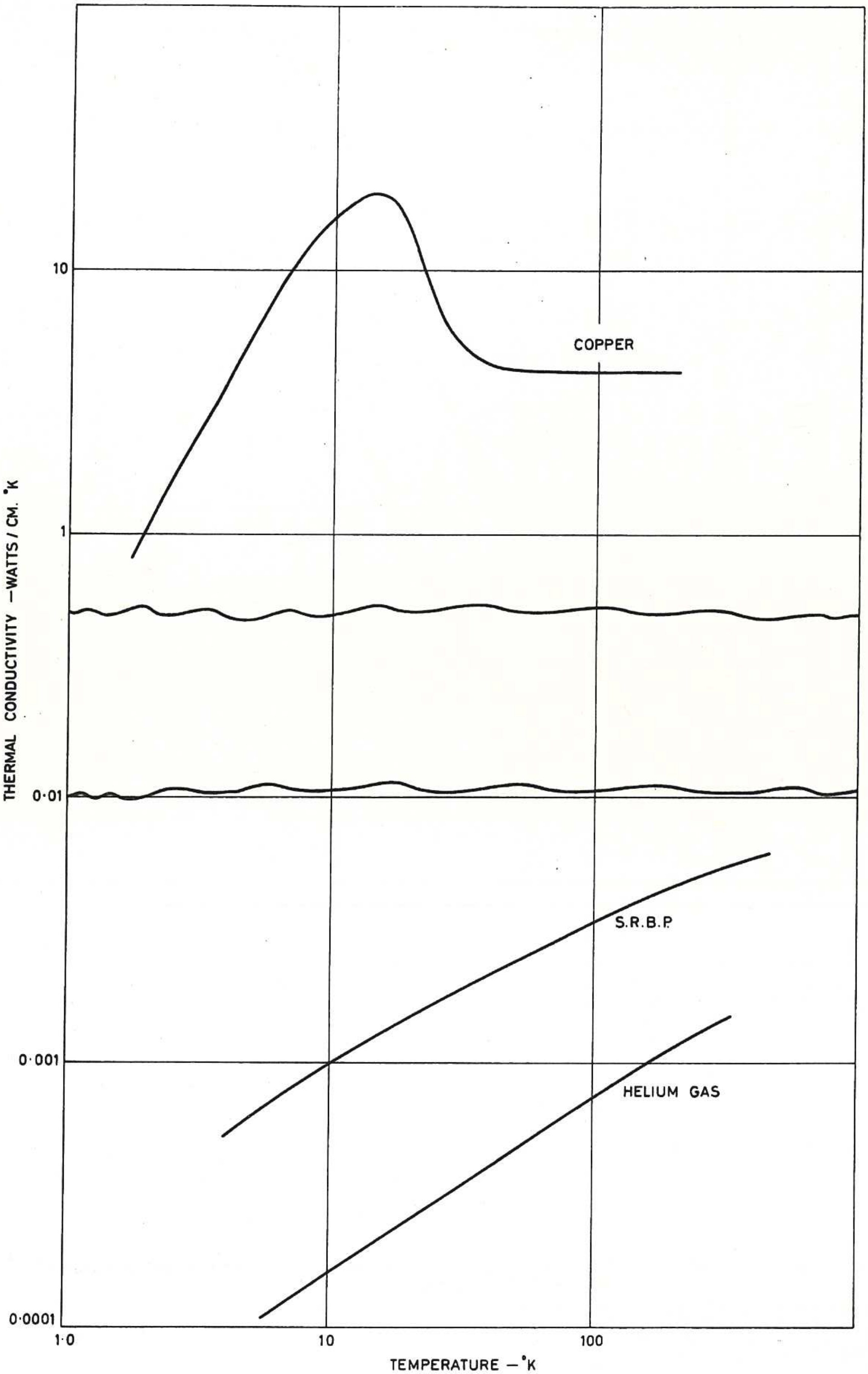
CLM-P 15 Fig. 2 The optimum values of the flow and current parameters for the counter flow lead as functions of the heat capacity ratio. Also shown is the optimum current parameter for the straight lead.



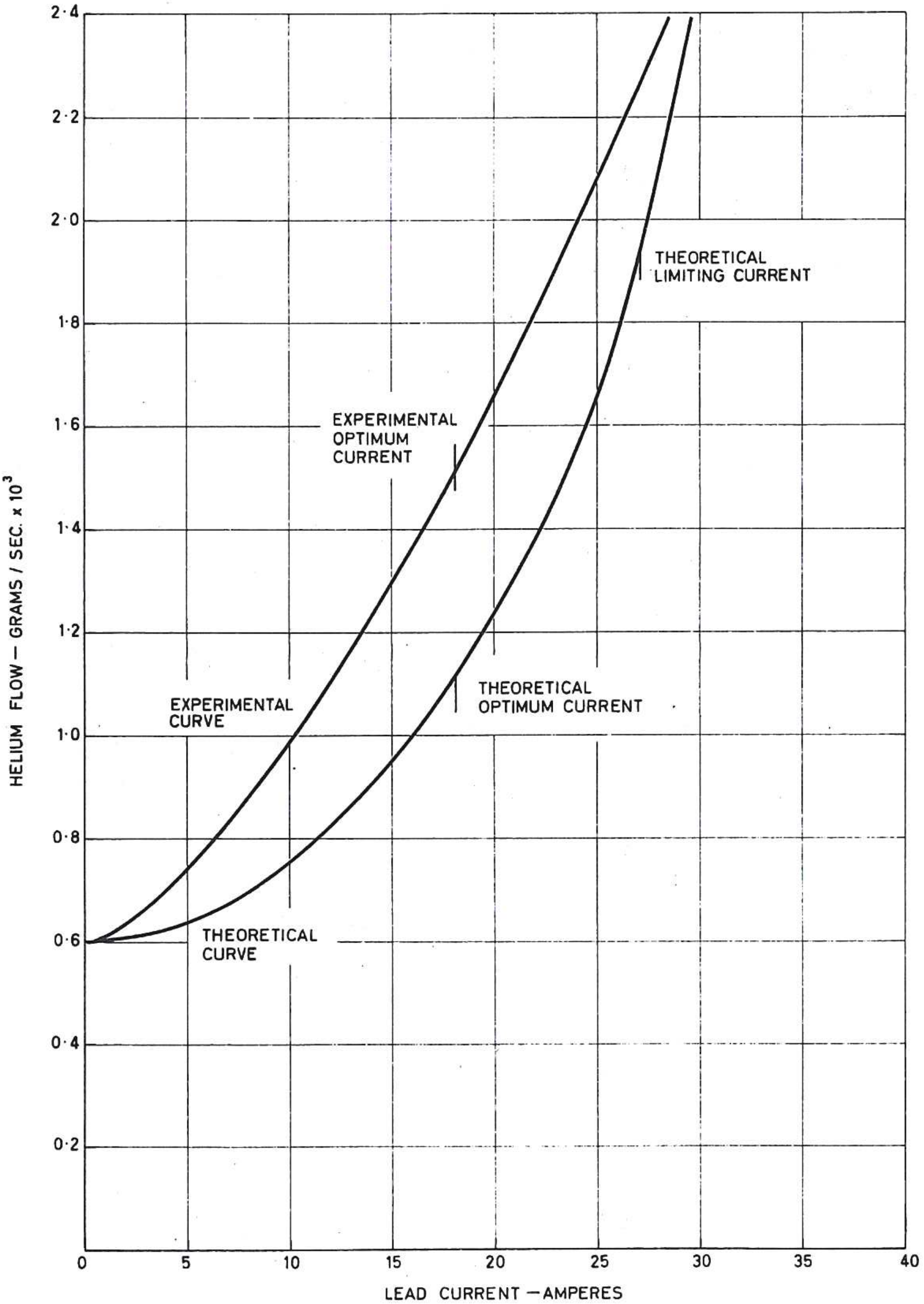
CLM-P 15 Fig. 3 The figures of merit of the counter flow lead and the straight lead as functions of the heat capacity ratio.



CLM-P 15 Fig. 4 The maximum temperature of the counter flow lead as a function of the ratio of operating current to design current for the conditions, helium-to-room-temperature and nitrogen-to-room-temperature.



CLM-P 15 Fig. 5 The thermal conductivities of copper and S.R.B.P.



CLM-P 15 Fig. 6 The gas flow rates as functions of current for a counterflow lead in theory and in practice.