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United Kingdom Atomic Energy Authority
RESEARCH GROUP
Preprint

# OBSERVATION OF VELOCITY-SPACE INSTABILITY

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1968

UNCLASSIFIED
(Approved for Publication)

CLM-P 159

# OBSERVATION OF VELOCITY-SPACE INSTABILITY

by

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(Submitted for publication in Plasma Physics)

## ABSTRACT

A column of plasma is generated in which electrons have a large fraction of their total energy associated with the velocity components transverse to an applied axial magnetic field. An unstable wave is found to arise spontaneously which propagates along the axis of the column. The properties of the plasma column and the dispersion characteristics of the unstable wave are studied and compared with the theory of GRUBER et al. (1965).

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January, 1968.

## 1. INTRODUCTION

Electrostatic plasma microinstabilities arising from velocity-space anisotropies have been extensively studied theoretically (BEASLEY (1967); CRAWFORD and TATARONIS (1965); DAVYDOVA et al(1967); GRUBER et al. (1965); HALL et al. (1965); SOPER and HARRIS (1965)) and have been observed in a number of experiments (ANASTASSIADES and MARSHALL (1967); ARD et al. (1966); ASHBY and PATON (1967); KUO et al. (1964); MORSE (1967); PISTUNOVICH (1963)). This communication describes an experiment in which the electrons of a plasma column are given energy in the plane transverse to the column axis and its confining magnetic field. When the electrons have sufficient transverse energy, an unstable wave is found to arise spontaneously. The theoretical model most closely approximating this situation is that of GRUBER et al. (1965) and the dispersion characteristics of the unstable wave are compared with this theory. The frequencies of interest are sufficiently high to allow the ions to be considered stationary, serving only to preserve overall charge neutrality.

## 2. EXPERIMENTAL APPARATUS

The experimental apparatus, shown in Fig.1, may be divided into three regions. The first region extends from the arc flask at the bottom of the system to the anode plate. Plasma is generated in the mercury arc source and flows along a uniform axial magnetic field of approximately 50 G as far as the insulated disc in the centre of the anode plate. The circular screen is used to suppress rotating density perturbations of the column. In this region the plasma column has a peak density of approximately $10^{10}$ cm$^{-3}$ and the electrons have a thermal energy of a few electron volts. Typically, the arc cathode current is 1·8 A, of which the circular screen collects 0·1 A, the anode plate 1·3 A, and most of the remainder flows to earth on the bottom plate at the neck of the flask. The neutral pressure of mercury vapour in this region is about $1 \times 10^{-3}$ torr.

The second region of the apparatus lies between the anode plate and the aperture plate. The plasma flows into this region through the circular slot between the 2·5 cm diameter insulated disc and the 3·8 cm diameter hole in the anode plate. Within a few centimetres of the anode plate the plasma has once more formed a solid cylindrical column of peak density approximately $10^9$ $\text{cm}^{-3}$. The plasma passes between a pair of parallel excitation plates 6·3 cm long by 5 cm wide, spaced 7·6 cm apart. A 200 MHz radio frequency signal of up to 50 volts rms amplitude is applied to one of these plates, and the other plate is connected to earth at the wall of the vacuum chamber. The plasma electrons pick up energy in the plane transverse to the magnetic field, and the plasma column then flows through the 3·8 cm diameter aperture in the aperture plate.

The third region is at the top of the system between the aperture plate and the cold trap. In this region the properties of the plasma column are measured, and the dispersion characteristics of the unstable wave are determined. The cold trap maintains a neutral pressure in the second and third regions of the system of about $3 \times 10^{-5}$torr.

The mechanism by which the plasma electrons gain transverse energy from the 200 MHz exciting electric field was not thoroughly investigated, but it is believed to be due to the absorption of the extraordinary wave at the hybrid frequency $\omega_h = (\omega_{pe}^2 + \omega_{ce}^2)^{\frac{1}{2}}$

(ALLIS et al. (1963)). In this expression $\omega_{pe}$ is the electron plasma frequency and $\omega_{ce}$ is the electron cyclotron frequency. This absorption was observed by BEKEFI et al. (1962) and was found to be very broad and relatively insensitive to $\omega_{ce}$ for the case $\omega_{pe} > \omega_{ce}$ on the column axis. This last condition was always satisfied in our experiment, and the energy absorption was not found to vary significantly when the magnetic field was varied over a 20% range. Under no circumstances reported here was electron cyclotron resonance heating used: the electron cyclotron frequency was usually 133 MHz compared to the 200 MHz exciting field.

## 3. THEORY

The theory of GRUBER et al. (1965) is used to compute the expected dispersion of the unstable waves measured in this experiment. This theory considers an infinite, collisionless electron plasma in a uniform magnetic field, neutralized by stationary, positive ions. The dispersion relation for electrostatic waves is derived for several different anisotropic electron velocity distributions, and growing wave solutions are found for certain ranges of electron density, angle of propagation with respect to the magnetic field, and wave frequency. In particular, an unstable wave is expected to arise when $\omega_{pe} \gtrsim \omega_{ce}$, given a moderate anisotropy in the electron velocity distribution. The frequency of this wave is expected to lie between $\frac{1}{2}\omega_{ce}$ and $\omega_{ce}$, assuming no average drift of electrons parallel to the magnetic field. It is this wave we have measured experimentally, but with a Doppler-shifted frequency due to a small average electron drift along the magnetic field.

To compute the dispersion of the unstable wave for comparison with experiments, the following delta-function electron velocity distribution was assumed:

$$f_0(v_\perp , v_\parallel) = \frac{1}{2\pi v}\, \delta(v_\perp - v)\, \delta(v_\parallel - v_0) \; . \qquad \ldots (1)$$

Here $v_\perp$ and $v_\parallel$ refer to electron velocities perpendicular and parallel to the magnetic field respectively. All electrons are assumed to have the same transverse velocity $v$ and parallel velocity $v_0$. GRUBER et al. (1965) show that the dispersion is little affected by considering a Maxwellian distribution of perpendicular velocities. In addition, we have considered the possibility that our plasma electrons might consist of a 'hot' component (with velocity distribution given by equation (1)) and a 'cold' component with no transverse energy. For our measured <u>total</u> plasma density corresponding to $\omega_{pe}^2 \cong 3\omega_{ce}^2$, the assumption that as much as 40% of the plasma electrons are cold does not materially affect the computed dispersion. The assumption of a larger component of cold electrons would lead to an unrealistically high value of $v$ for the 'hot' component, since the measured diamagnetic signal would have to be produced by the smaller density of 'hot' electrons.

Using equation (1) the dispersion relation is

$$1 - \frac{\omega_{pe}^2}{k^2} \sum_{n=-\infty}^{\infty} \left\{ \frac{k_\parallel^2 \, J_n^2(\beta)}{(\bar{\omega} - n\omega_{ce})^2} + \frac{k_\perp^2}{2\omega_{ce}} \, \frac{J_{n-1}^2(\beta) - J_{n+1}^2(\beta)}{(\bar{\omega} - n\omega_{ce})} \right\} = 0 \qquad \ldots (2)$$

Here $k$ is the wave number, with components $k_\parallel$ and $k_\perp$ parallel and perpendicular to the magnetic field, respectively. The argument of the Bessel functions $\beta = k_\perp v/\omega_{ce}$. The frequency $\bar{\omega}$ is the Doppler-shifted frequency $\omega - k_\parallel v_0$.

Solutions of equation (2) are discussed in detail by GRUBER et al. (1965). Some general features of the dispersion for frequencies less than $\omega_{ce}$ are

(i) $\omega_{pe}/\omega_{ce}$ must be at least of order unity for instabilities to be possible.

(ii) Assuming $\omega_{pe} > \omega_{ce}$ and $k_{\parallel}/k_{\perp}$ less than some critical value of order unity, two stable waves exist whose frequencies $\bar{\omega}$ lie between 0 and $\omega_{ce}$.

ii) For $\omega_{pe} > \omega_{ce}$ and $k_{\parallel}/k_{\perp}$ greater than the critical value, an unstable wave exists whose frequency $\bar{\omega}$ satisfies $\frac{1}{2}\omega_{ce} < \bar{\omega} < \omega_{ce}$. In the absence of Landau damping caused by a thermal spread of electron velocities parallel to the magnetic field, the instability persists to arbitrarily large values of $k_{\parallel}/k_{\perp}$.

## 4. MACROSCOPIC PROPERTIES OF THE PLASMA COLUMN

The plasma density in the region above the aperture plate is measured with a Langmuir probe and by recording the resonant frequency shift of the cylindrical microwave cavity formed between the cold trap and the aperture plate. The probe is only useful when no excitation is applied to the excitation plates, and hence the electrons have low energy, but it gives the density distribution within the region. With no excitation, the column is found to decrease to one-half its peak density at a radius of about 2 cm, and the density on the axis falls from about $7 \times 10^8$ cm$^{-3}$ at 7 cm above the aperture plate to about $1 \times 10^8$ cm$^{-3}$ at 2 cm below the cold trap.

When the excitation is switched on, the density of the plasma column increases as the electrons attain sufficient energy to ionize the neutral mercury vapour. The increase in density is measured by monitoring the change of the resonant frequency of the cavity which contains the plasma column. The $TE_{111}$ mode is used because its Q is higher than the $TM_{010}$, although its frequency shift for a given plasma density is smaller. The ratio of the resonant frequency of the $TE_{111}$ cavity mode (approximately 1450 MHz) to the electron cyclotron frequency was less than 0·1, and so the splitting of the mode into two separate resonances was too small to be resolved at the low densities measured here (maximum frequency shift approximately 2 MHz).

The average plasma density 10 cm above the aperture plate versus voltage applied to the excitation plates is shown in Fig.2. To compute $\omega_{pe}$, the average electron density is found by taking one-half the value of the density on the axis as measured by the Langmuir probe, and then scaling this density proportional to the resonant frequency shift as the 200 MHz excitation voltage is applied. An approximate calculation of the density directly from the resonant frequency shift data yields values of electron density within a factor of 2 of the above procedure. However, it is felt that the Langmuir probe gives the more reliable density estimate, and so the first procedure was adopted.

The average transverse energy of the electrons is measured with a diamagnetic coupling loop of 100 turns on a 3·8 cm diameter form. The 200 MHz exciting voltage is pulsed on for 500 μsec at a 1000 Hz repetition rate, and the output of the coil is fed to a high-gain selective amplifier tuned to 1000 Hz. The system is calibrated with no plasma present by applying a 1000 Hz square wave current of a few milliamperes to the main field coils. Knowing the gauss/amp rating of the main field coils, the diamagnetic signal $\Delta B$ for a given output of the selective amplifier can easily be determined.

The average transverse energy is computed from

$$\frac{\Delta B}{B_0} = \Delta \left\{ \frac{\omega_{pe}^2}{\omega_{ce}^2} \frac{eV_\perp}{mc^2} \right\} \qquad \ldots (3)$$

In this expression $B_0$ is the applied axial magnetic field, $\omega_{ce}$ is the electron cyclotron frequency associated with this field, $\omega_{pe}$ is the electron plasma frequency corresponding to the average electron density within the coupling loop, $V_\perp$ is the average transverse energy of the electrons, c is the velocity of light, and e and m are the magnitudes of the electronic charge and mass respectively. The $\Delta$ in front of the expression on the right indicates the difference between the bracketed expression evaluated with the 200 MHz excitation off and the same expression evaluated with it on. The value of $V_\perp$ with no excitation is taken as 2 eV, the value of $\omega_{pe}$ with no excitation is computed from the density measured by the Langmuir probe at the plane of the loop, and the value of $\omega_{pe}$ with the excitation on is measured by the resonant frequency shift as explained above. With the value of $\Delta B$ measured by the diamagnetic loop, $V_\perp$ with the excitation on is determined by equation (3).

The behaviour of the average transverse energy with excitation voltage is shown in Fig.2. The average transverse energy appears to level off between 40 eV and 50 eV. This levelling off may be due to an over-estimating of the density at higher transverse energy. We have assumed the density to vary with the exciting voltage as indicated by the resonant frequency shift when the coupling loop is withdrawn from the cavity, but with the loop in place amount of plasma passing through the loop may be reduced as the cyclotron radius of the electrons (0·5 cm at 50 eV transverse energy) becomes comparable with the loop radius (1·9 cm).

The distribution of electron velocities parallel to the magnetic field is measured with an electrostatic energy analyser mounted directly under the cold trap. The average drift velocity of the plasma electrons is found to correspond to 10 eV energy, and the high-energy tail of the distribution is nearly Maxwellian with a temperature of approximately 3 eV. These figures remain essentially constant as the 200MHz exciting voltage is varied from 0 to 50 V rms.

The electrostatic oscillations resulting from the instability are detected with a VHF communications receiver of 5 μV sensitivity, connected to any of the number of antennae located between the aperture plate and the cold trap. The direction of propagation and wavelength of the unstable waves are determined by summing the signals from pairs of these antennae, as reported in an earlier communication (MORSE (1967)).

## 5. MEASUREMENT OF THE UNSTABLE WAVE

The variation of signal strength from the instability with exciting voltage is shown in Fig.2, along with the curves of plasma density and average transverse energy. The signal strength in this case is measured at 105 MHz, near the centre of the band of unstable frequencies. The instability signal has risen well above the noise level by the time the average transverse energy has reached 10 to 12 eV. Remembering that the parallel velocity distribution has a thermal spread of 3 eV, the instability is seen to become well-developed when the average transverse energy is approximately 4 times the parallel thermal energy.

The field pattern of the unstable wave is found to be azimuthally symmetric in phase (m = 0), and the axial direction of wave propagation is found to be up the tube, toward the cold trap. To compare the experimental dispersion with the theory, it is next necessary to estimate the transverse wave number $k_\perp$. The intensity of the signal from the instability is found to be azimuthally symmetric, strongest on the system axis, and mainly confined to within 4 cm of the axis. Therefore $k_\perp$ is taken to satisfy $J_0(k_\perp r) = 0$ at $r = 4$ cm, leading to $k_\perp = 0{\cdot}6\ \mathrm{cm}^{-1}$.

The measured and theoretical variations of axial wave number $k_\parallel/k_\perp$ with frequency are shown in Fig.3. The theoretical curves shown as solid lines are calculated from equation (2) assuming $\omega_{pe}^2/\omega_{ce}^2 = 3$ and that the axial velocity $v_0 = 1{\cdot}9 \times 10^8$ cm sec$^{-1}$, corresponding to the measured average axial drift velocity of the plasma electrons. Only the real part of the complex frequency $\omega = \bar{\omega} + k_\parallel v_0$ versus $k_\parallel/k_\perp$ is shown, in the range of $k_\parallel/k_\perp$ corresponding to unstable solutions of equation (2). As previously noted, two stable waves propagate in this frequency range for $k_\parallel/k_\perp$ less than the critical value for instability. The curve for $\beta = 0{\cdot}3$ corresponds to the measured values of $k_\perp$, $V_\perp$, and $\omega_{ce}$, while the curve for $\beta = 0{\cdot}7$ demonstrates the sensitivity of the dispersion to this parameter.

The experimental points shown in Fig.3 represent the measured dispersion of the unstable wave with $\omega_{ce} = 2\,\pi \times 133$ MHz, and

50 V rms excitation voltage. The values of plasma density and average transverse energy are determined from Fig.2, and correspond to the theoretical curve for $\beta = 0{\cdot}3$. The error brackets represent the day-to-day repeatability of the experiment. Changing the magnetic field by 10% in either direction yielded approximately the same experimental curve.

The spontaneously-excited waves are seen to arise for smaller values of $k_{\parallel}/k_{\perp}$, and at lower frequencies, than theoretically predicted. The theory, however, is for a uniform plasma and does not take into account the radial or axial density gradients. The spontaneously—excited waves do not arise for values of $k_{\parallel}/k_{\perp}$ greater than about $1{\cdot}6$, presumably because the shorter wavelength waves are subject to thermal damping. The theory does not predict this damping because of the assumption of a delta-function distribution of axial velocities. The theory does predict further instabilities in the neighbourhood of the Doppler-shifted higher cyclotron harmonics, but we did not observe these waves.

## 6. CONCLUSIONS

In this experiment we have shown that spontaneously-excited waves arise in a plasma column whose electrons have an anisotropic velocity distribution. The dispersion of these waves agrees qualitatively with the unstable waves predicted by GRUBER et al. (1965). For the plasma column of this experiment, a ratio of average transverse energy to parallel thermal energy of the electrons of 3 to 4 is sufficient to excite the unstable waves.

## 7. ACKNOWLEDGEMENT

The author would like to express his appreciation to Dr P. C. Thonemann for encouragement and advice throughout the course of these experiments, and to Dr J.G. Cordey for many helpful theoretical discussions. Dr. S. Gruber kindly supplied some unpublished computations. The assistance of W.H.W. Fletcher and N.R.G. Ainsworth in the design, maintenance, and operation of the experiment is gratefully acknowledged.

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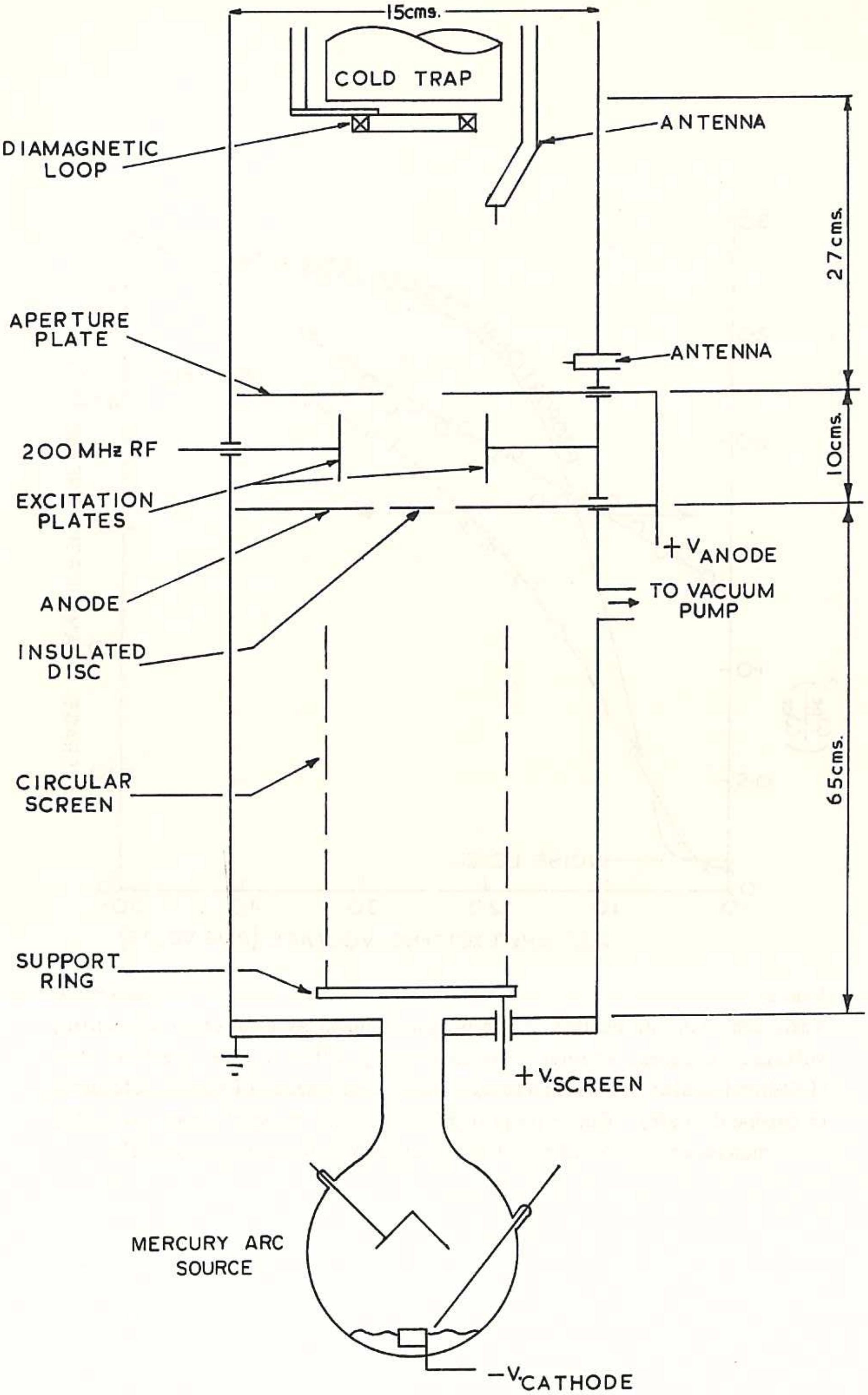

Fig. 1 Experimental apparatus (CLM-P 159)

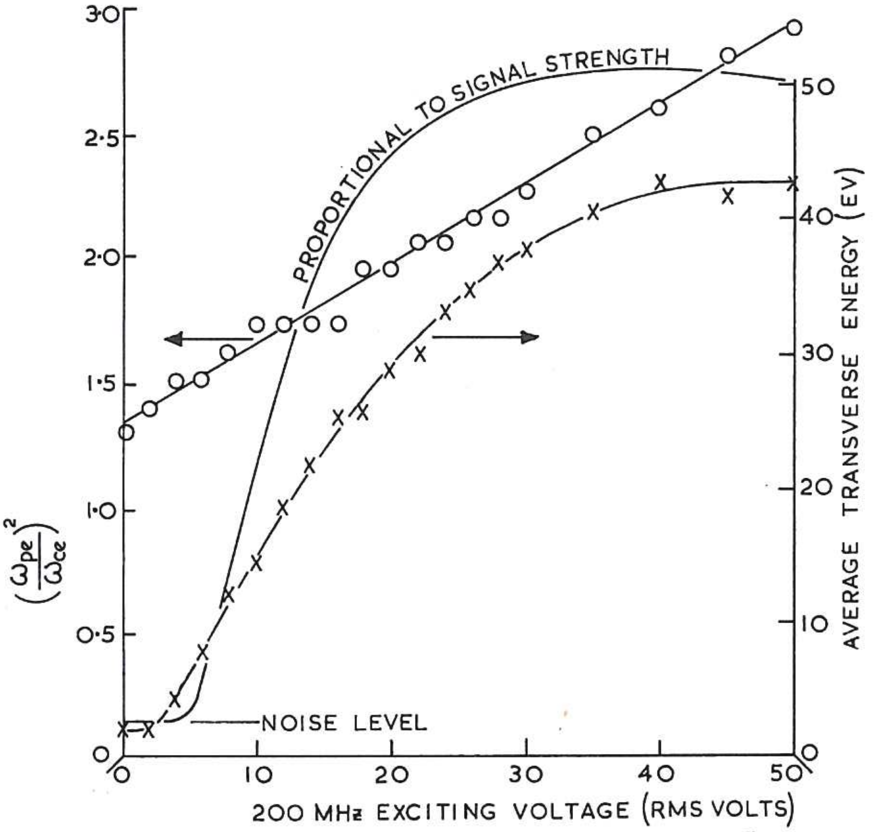

Fig. 2

(CLM-P 159)

Variation of the properties of the plasma column as a function of exciting voltage. Circles: average plasma density, 10 cm above aperture plate (left-hand scale). Crosses: average transverse energy of plasma electrons (right-hand scale). Curve proportional to signal strength from instability measured at 105 MHz. Electron cyclotron frequency: 133 MHz

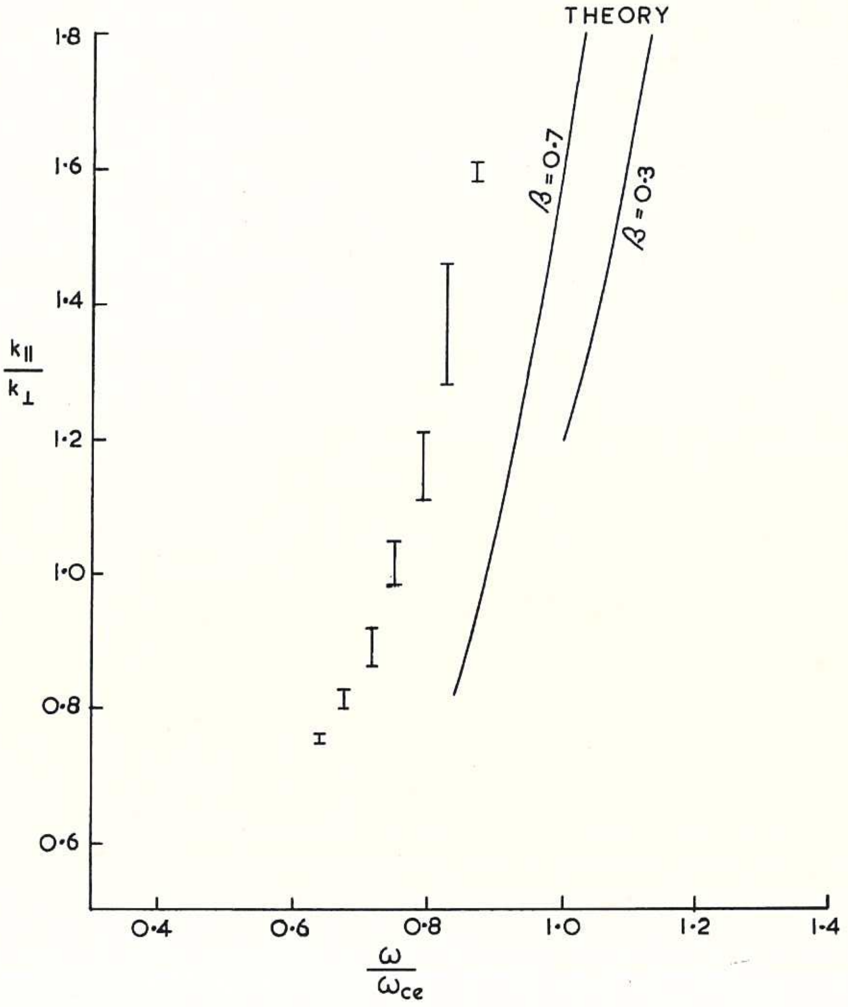

Fig. 3 (CLM-P159)

Dispersion characteristics of the unstable wave. Experimental conditions $\omega_{ce} = 2\pi \times 133$ MHz, exciting voltage = 50 V rms, $\omega_{pe}^2/\omega_{ce}^2 = 3$, average transverse energy $V_{\perp} = 43$ eV, $k_{\perp} = 0.6$ cm$^{-1}$. Theoretical curve for $\beta = 0.3$ corresponds to experimental parameters, curve for $\beta = 0.7$ shows sensitivity of theory to the value of $\beta$