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A NEW TYPE OF BIREFRINGENT FILTER

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A NEW TYPE OF BIREFRINGENT FILTER

by

J. KATZENSTEIN

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ABSTRACT

A new type of birefringent filter is proposed which uses a single birefringent plate placed between two Faraday rotation elements within the cavity of a Fabry-Perot interferometer. The interferometer is placed between crossed polarizers whose axes are parallel to the principal directions of the plate. Such an instrument, called a gyromagnetic polarizing interferometer (GYMPI), has an instrumental function which is essentially an Airy function whose finesse is approximately equal to the reflective finesse of the etalon. The argument of the Airy function is the birefringent phase shift of the retardation plate rather than the phase shift of transit between the mirrors. Thus the pass-band of the filter has that relative independence of angle of incidence that characterises all birefringent filters. The instrument is readily tuneable over a free spectral range by making the retarder a Soliel compensator or incorporating an electro-optic element. This advantage along with greater versatility and lower cost of construction should make the instrument of interest despite the low peak transmission which is the order of the reciprocal of twice the reflective finesse. The optimum of Faraday rotation is only a few degrees which means that the magnetic field requirements are not severe.

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CONTENTS

Polarizing interferometers or birefringent filters have been in use for some years, principally as astronomical instruments for photographing the sun in light of a narrow spectral width. These filters have a narrow bandpass, which is relatively insensitive to the range of angles of incidence of the light traversing it; hence all positions in the angular field to first approximation are illuminated by light with the same spectrum.

The Lyot-Ohman filter[1,2,3] was the first of these instruments. A comprehensive description of this device is given by Evans[4]. (Hereafter referred to as Evans I). Briefly, in its simplest form it consists of N birefringent plates cut with their optic axes in the plane of the faces of the plates. These plates are sandwiched between polarizers. The optic axes of the plates are parallel to each other and oriented at 45° with respect to the axes of the polarizers, which are likewise mutually parallel. Each plate is half the thickness of its predecessor. The spectral half-width of such a filter is given approximately by the retardance of its thickest plate 2^{N-1} d i.e.

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda}{[2^{N-1} d\, (\varepsilon-\omega)]} , \qquad \ldots (1)$$

where ε and ω are the extraordinary and ordinary refractive indices, d the thickness of the thinnest plate and λ the wavelength of the light. (Strictly speaking the difference in the group velocity indices ε_g, ω_g, of the ordinary and extraordinary rays should be used in equation (1) where $\varepsilon_g - \omega_g = \lambda \frac{d(\varepsilon - \omega)}{d\lambda} - (\varepsilon - \omega)$. The difference between the group and phase indices can be neglected for purpose of discussion.)

The transmission bands of the filter are separated by a free spectral range $\Delta\lambda$ given by the retardance of the thinnest plate, i.e.

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda}{d(\varepsilon-\omega)} . \qquad \ldots (2)$$

The Lyot-Ohman filter is extremely expensive to construct for a narrow passband. Furthermore, if dichoic sheet polarizers are used the absorption of the light polarized along the favoured direction reduces the transmittance of such a filter to about 10% of the theoretical value for lossless polarizers. (The use of lossless birefringent polarizers such as Rochon prisms would make the device still more costly.) The light of wavelengths falling between the passbands of the filter is not totally suppressed; there are secondary maxima which bear the same ratio to the primary maxima as the secondary maxima of a diffraction grating of 2^N rulings, a fact which we shall discuss later. While spurious light is very low, it still degrades the contrast of the filter, especially in spectral regions near the passband. Finally, a Lyot-Ohman filter is an inflexible device, designed for the isolation of a particular single spectral line ; once assembled it cannot be used for another line without rebuilding the filter. The filter can be tuned through a limited range by changing the temperature of the thermostated bath in which it is immersed, and the different transmittance maxima may be selected by changing the colored glasses required to isolate a free spectral range. Several proposals have been made by Lyot[3] and Billings[5] which would make the filter continuously tunable by varying the retardance of each element by means of elements in the form of wedges, as in a Babinet compensator or by electro-optic or photoelastic effects in additional plates added to the retarders. All of these schemes encounter problems because of the difficulty of preserving the precise geometric progression of the retardances. The most usual and satisfactory method of tuning is to incorporate rotating half wave plates or fixed quarter wave plates and rotating polarizers in the stages of the Lyot filter with the highest retardations. This serves to tune the pass-band of the filter over a limited spectral range which, for a narrow band filter, can be as large as 64 times its half-breadth. Unfortunately this method cannot be extended over the full free spectral range of the filter because the fractional wave plates are not achromatic.

From 1953 to 1955 a new class of polarizing interferometers has been proposed and constructed by Solc[6]. Such a Solc filter consists of a number of identical birefringent plates, with the optic axes in the plane of the faces, which are arranged with their optic axes at small equal angles to each other, with no intervening polarizers. Only two polarizers are used at the entrance and the exit of the filter; they are crossed or parallel depending up whether the small angles between the axes of the plates alternate between equal positive and negative values or increase monotonically. Evans[7] (hereafter referred to as Evans II), who has given a full analysis of these devices with a comparison with the Lyot-Ohman filter, calls the first arrangement the 'folded filter' and the second the 'fan filter'. The optimum arrangement is for the sum of all of the angles to add up to $\pi/2$. Such a filter can be analysed by means of the Jones-matrix calculus and Evans has done this. The free spectral range of the Solc filter is equal to that of a Lyot filter whose thinnest birefringent element has the same thickness as one of the plates of the Solc filter. Its transmission bandwidth is slightly less than a Lyot filter of N elements if the number of plates $n = 2^N$. The total thickness of birefringent material in the Solc filter exceeds that of the Lyot filter of roughly equivalent band width by the thickness of one retardation plate. The absence of intermediate polarizers results in a much higher transmittance compared with the Lyot filter, but this advantage is mitigated by much higher secondary maxima, hence a lower contrast of the Solc filter compared with the Lyot filter.

Subsequent work by Harris, Ammann and Chang[8] and Solc[9] has shown that the secondary maxima of the Solc filter can be reduced by varying the angles between the optic axes of the retarders from the equal submultiples of $\pi/2$ that were used in the original design. The secondary maxima of such an 'apodized' Solc filter are found to be weaker relative to the primary maximum than those of a Lyot filter of the same bandwidth. The birefringent plates used in the Solc filter must be uniform in retardance over the aperture and from one plate to another with a tolerance that is about $1/n$ that for the plates of a Lyot filter of comparable bandwidth[10]. Hence the cost of constructing a Solc filter is generally higher than a Lyot filter of comparable bandwidth and becomes very much so for a narrow-bandwidth filter, even though the amount of birefringent material is roughly the same. The Solc filter suffers from the same limitations of inflexibility as the Lyot filter and is probably even more difficult to tune, except by temperature variation, because the number of retarders is so much greater.

In this paper, a new type of filter is proposed that overcomes most of the limitations of the Lyot or Solc filter and should be much cheaper to construct. Before proceeding to the theory of this instrument, which we have called a gyro-magnetic polarizing interferometer

(GYMPI), let us examine the Lyot and Solc filters from a point of view dffering from the conventional manner of studying these devices. Let us consider them as multiple-beam interferometers, in which the division of the incident amplitude into interfering beams is accomplished by division of polarized light passing through a birefringent medium into ordinary and extraordinary rays with differing phase velocities.

In the Lyot filter, each passage of polarized light through a birefringent plate divides the light into two partial beams differing in phase by the retardance of the plate. Each of these partial beams is subdivided into two partial beams by passage through the next plate, which has half the retardance. The net result after passage through N plates is 2^N partial beams, each differing in phase by the retardation of the thinnest plate. These partial beams when summed vectorially give an instrumental function that is equivalent to a grating of 2^N rulings, as would be expected, except that the division into partial beams by the grating occurs by wavefront division, thus requiring coherent illumination of the grating.

Now, in any multiple beam interferometer, the free spectral range in terms of the wavelength $\Delta\lambda F/\lambda$ is just the reciprocal of the order of interference k of adjacent partial beams i.e. the path difference between adjacent partial beams measured in wavelengths. The resolvance i.e. the passband width divided into the wavelength $R = \lambda/\Delta\lambda$ is just the difference, measured in wavelengths, of the optical path between the first and the last interfering beam. If the phase difference between the partial beams is uniform the resolvance is simply the free spectral range divided by the number of partical beams i.e.

$$\Delta\lambda = \frac{\Delta\lambda F}{N} ,$$

or $R = Nk$ a well-known result[11].

Now the Solc filter also can be understood qualitatively by such a partial-beam analysis. Each passage through a birefringent plate produces one partial beam in addition to those which entered, since the retardation of each plate is equal to the preceding one. The number of partial beams in the Solc filter is thus n making the Solc filter of 2^N plates equivalent in free spectral range to an N - stage Lyot filter whose thinnest retarder has the same retardance as one of the plates of the Solc filter. As the Solc filter is thicker by one retardation plate, the total difference of optical path is slightly longer hence a slightly narrower bandwidth results. The partial beams are no longer equally intense, therefore the secondary maxima are greater. In fact, the secondary maxima arise for both filters from the finite number of interfering partial beams. This is a case of the well-known Gibbs phenomenon of Fourier analysis, in which the truncation of an infinite-Fourier series expansion of a function results in a function which oscillates about the mean value of the original function with a period equal to that of the last term retained.

The preceding discussion suggests that a polarizing interferometer that divides the light into an infinite number of partial beams each of infinitesimal intensity would have an instrumental profile free of secondary maxima, as for example the Fabry-Perot or Lummer-Gehrke interferometers. Such a polarizing interferometer could be realised by placing a single retardation plate between two very flat and parallel mirrors of high but not unit reflectance, i.e. an optical cavity. The state of polarization of the light between each passage must be altered, to continue the division into interfering beams. This is most suitably accomplished by Faraday rotation, which changes sense on reversal of the direction of propagation. The simplest form of the device is indicated in Fig.1 which shows an entrance polarizer with its axis vertical, a highly but not totally reflecting mirror of interferometric quality, a Faraday rotator that provides a rotation of the plane of polarization α, a birefringent plate whose principal directions are vertical and horizontal, a secondary Faraday rotator that provides a rotation $\pm\alpha$, a second interferometric mirror rigorously parallel to the first and an output polarizer with its axis horizontal, i.e. crossed with respect to the entrance polarizer. The Faraday rotations may be in either the same or opposite senses; both possibilities give the same instrumental function. These two arrangements correspond to the fan or folded configurations of the Solc filter referred to previously, which are equivalent.

We anticipate that such an interferometer will have an instrumental function similar to an Airy function except that the phase difference between interfering beams will be determined primarily by the retardation of the birefringent plate rather than the total optical path difference between the mirrors. This can be seen once again by considering how light is divided into partial beams. Each reflection splits off a portion of the light into a partial beam differing in phase from the light emerging from the previous reflection by twice the optical path difference between the mirrors. Each of these partial beams which has made n passes through the birefringent plate is itself divided into n sub-partial beams differing in phase by the retardation of the plate. Now a necessary, but not sufficient, condition for constructive interference and hence transmission to occur is for the optical path difference between the mirrors to be close to an integer multiple of a wavelength. This condition will be satisfied by a fraction $(N_R)^{-1}$ of the incident light, where N_R is the reflective finesse of the etalon. For this fraction of light the condition for constructive interference is that the birefringent phase shift be close to an integer multiple of 2π. Thus the peak transmittance will be of the order of $(N_R)^{-1}$ and the instrumental function dependent primarily upon the birefringent phase shift rather than the transit phase shift. This point is very important as the birefringent phase shift is relatively independent of angle of incidence, while the total optical-path difference is a sensitive function of angle of incidence near normal incidence. It is just this sensitivity that is responsible for the high 'disperion' of the Fabry-Perot interferometer. Thus, if the instrumental function is determined by the retardance of the birefringent plate, the instrument can accept light over a relatively large angular field without degrading the bandpass. This tolerance of field can be increased by using one of the various special wide-field retarders (see Evans I). While the tolerance on uniformity of retardance over the aperture of the single plate is as severe as for a Solc filter,

$$\left(\delta[d(\varepsilon-\omega)] < \frac{\lambda}{2N_R}\right) ,$$

there is no need to match the retardances of several plates as in the case of the Solc or Lyot filters, hence the cost of construction should be much less. Since the filter can readily be made tuneable over the entire free spectral range by using a retarder of variable thickness such as a Soliel compensator, adding an electro-optic element, or, if the free spectral range is not too great, with a rotatable half-wave plate, the absolute value of the retardance does not have to be adjusted to correspond to a given spectral line. Finally the free spectral range and resolvance can be altered by changing the retarder for another of different thickness.

CALCULATION OF THE INSTRUMENTAL FUNCTION

The instrumental function of the device just described is calculated by the method of partial beams, as in the calculation of the Airy function. The transmitted amplitude is the superposition of the amplitude of n partial beams, the n^{th} partial beams being composed of light which has undergone $2(n-1)$ reflections in the étalon mirrors before emerging. The amplitude of the $(n+1)$ partial beam is reduced by a factor R with respect to the n^{th} where $R^{\frac{1}{2}}$ is the amplitude reflectance of the mirrors. The infinite sum of the amplitudes of the partial beams thus forms a convergent geometric series since $R < 1$.

In the present calculation,it is necessary to keep account of the states of polarization of the partial beams as well as their relative phases. By state of polarization is meant the relation between the amplitudes and phases of two orthogonal components of plane polarized light resolved along the axes of a suitable co-ordinate system. This is most readily done by use of the Jones-matrix calculus[11] in which the polarized light is represented as a two component vector with complex components. The action of the various components of polarizing optics on the light is represented by the multiplication of this vector by 2 × 2 matrix operators. Now the type of components that we will be concerned with will be ideal polarizers, Faraday rotators, and retardation plates. The polarization matrices for polarizers oriented along the x and y axes of a suitable co-ordinate system are

$$P_x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad P_y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \qquad \ldots (3)$$

The Faraday rotation matrix is the ordinary 2 × 2 orthogonal matrix for rotation of a co-ordinate system

$$S(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \qquad \ldots (4)$$

with the important proviso that the sign of α, the sense of the rotation, depends on the direction of the magnetic field with respect to the direction of propagation of the light. Thus the sign of α is unchanged by a reversal of direction of the light through the rotator. This is in counter distinction to the rotation of the plane of polarization by optically active solutions or crystals, in which the sign of α is reversed when the direction of propagation is reversed.

The birefringent plate or retarder is represented by the matrix

$$G(\Upsilon) = \begin{pmatrix} e^{i(\frac{\varphi+\Upsilon}{2})} & 0 \\ 0 & e^{i(\frac{\varphi-\Upsilon}{2})} \end{pmatrix}. \qquad \ldots (5)$$

The variables φ and Υ for normal incidence are

$$\begin{aligned} \varphi &= \frac{2\pi}{\lambda}\,[2d' + (\varepsilon+\omega)d] \\ \Upsilon &= \frac{2\pi}{\lambda}\,d(\varepsilon-\omega)\,, \end{aligned} \qquad \ldots (6)$$

where d' is the optical path between the mirrors, exclusive of the retarder of thickness d whose extraordinary and ordinary indices of refraction are ε and ω respectively; φ thus represents the mean phase difference between partial beams while γ is the phase difference introduced by the retarder between the ordinary and extraordinary rays. We have taken the x and y directions to coincide with the principal directions of the retarder. We next write down the Jones matrices for the various partial beams. We treat both cases of Faraday rotations in the same and opposite sense. The amplitudes are

<u>First Partial Beam</u>:

$$\vec{E}_1 = TP_y S(\alpha)G(\gamma)\ (-\alpha)P_x\vec{E}_o$$ equal and opposite Faraday rotations

... (7)

$$\vec{E}_1' = TP_s S(-\alpha)G(\gamma)S(-\alpha)P_x\vec{E}_o$$ equal Farady rotations in same sense.

We shall hereafter denote the case of Faraday rotations in the same sense with a prime.

<u>Second Partial Beam</u>:

$$\vec{E}_2 = TRP_y S(\alpha)G(\gamma)S(-2\alpha)G(\gamma)S(2\alpha)G(\gamma)S(-\alpha)P_x\vec{E}_o$$

... (8)

$$\vec{E}_2' = TRP_y S(-\alpha)G(\gamma)S(-2\alpha)G(\gamma)S(-2\alpha)G(\gamma)S(-\alpha)P_x\vec{E}_o$$

<u>n^{th} Partial Beam</u>:

$$\vec{E}_n = TR^{n-1}P_y S(\alpha)G(\gamma)[S(-2\alpha)G(\gamma)S(2\alpha)G(\gamma)]^{n-1}S(-\alpha)P_x\vec{E}_o$$

... (9)

$$\vec{E}_n' = TR^{n-1}P_y S(-\alpha)G(\gamma)[S(-2\alpha)G(\gamma)S(-2\alpha)G(\gamma)]^{n-1}S(-\alpha)P_x\vec{E}_o\ ,$$

$T^{1/2}$ is the amplitude transmittance for the etalon mirror.

The total amplitude is just the sum to infinity of the expression (9).

It may be written as:

$$\vec{E} = \sum_{n=0}^{\infty} R^n T P_y S(\alpha) G(\Upsilon) A^n S(-\alpha) P_x \vec{E}_o ,$$

$$\vec{E}' = \sum_{n=0}^{\infty} R^n T P_y S(-\alpha) G(\Upsilon) A'^n S(-\alpha) P_x \vec{E}_o . \quad \ldots (10)$$

The matrices A and A′ are defined as

$$A = [S(-2\alpha) G S(2\alpha) G], A' = [S(-2\alpha) G]^2 . \quad \ldots (11)$$

These matrices are unimodular therefore since $R < 1$ the infinite series defined by Eq.(10) may be summed to give:

$$\vec{E} = T P_y S(\alpha) G(\Upsilon) [1-RA]^{-1} S(-\alpha) P_x \vec{E}_o ,$$

$$\vec{E}' = T P_y S(-\alpha) G(\Upsilon) [1-RA']^{-1} S(-\alpha) P_x \vec{E}_o . \quad \ldots (12)$$

We now proceed to calculate the matrices A,A′, explicitly.

$$A = e^{i\varphi} \begin{pmatrix} e^{i\frac{\Upsilon}{2}} [\cos\frac{\Upsilon}{2} + i\sin\frac{\Upsilon}{2}\cos 4\alpha] & -ie^{-i\frac{\Upsilon}{2}} \sin\frac{\Upsilon}{2} \sin 4\alpha \\ -ie^{i\frac{\Upsilon}{2}} \sin\frac{\Upsilon}{2} \sin 4\alpha & e^{-i\frac{\Upsilon}{2}} [\cos\frac{\Upsilon}{2} - i\sin\frac{\Upsilon}{2}\cos 4\alpha] \end{pmatrix} \ldots (13a)$$

$$A' = e^{i\varphi} \begin{pmatrix} e^{i\frac{\Upsilon}{2}} [\cos\frac{\Upsilon}{2}\cos 4\alpha + i\sin\frac{\Upsilon}{2}] & e^{-i\frac{\Upsilon}{2}} \cos\frac{\Upsilon}{2} \sin 4\alpha \\ -e^{i\frac{\Upsilon}{2}} \cos\frac{\Upsilon}{2} \sin 4\alpha & e^{-i\frac{\Upsilon}{2}} [\cos\frac{\Upsilon}{2}\cos 4\alpha - i\sin\frac{\Upsilon}{2}] \end{pmatrix} .$$

... (13b)

The calculation is completed by inserting the expressions (13) into Eqs.(12) and carrying out the indicated operations. The square of the absolute value is computed to obtain the transmitted flux, I for a given incident flux I_o.

The instrumental function $f(\varphi,\Upsilon)$ defined by $I = f(\varphi,\Upsilon) I_o$ for both cases is

$$f(\varphi,\Upsilon) = \frac{\tau_E}{2}(1-R)^2 \frac{(1+R^2-2R\cos\varphi)}{[1+R^2-2R\cos(\varphi+\chi)][1+R^2-2R\cos(\varphi-\chi)]} \sin^2\frac{\Upsilon}{2} \sin^2 2\alpha$$

$$\cos\chi = \cos^2\frac{\Upsilon}{2} - \sin^2\frac{\Upsilon}{2}\cos 4\alpha , \quad \ldots (14a)$$

$$f'(\varphi,\Upsilon) = \frac{\tau_E}{2}(1-R)^2 \frac{(1+R^2+2R\cos\varphi)}{[1+R^2-2R\cos(\varphi+\chi')][1+R^2-2R\cos(\varphi-\chi')]}\cos^2\frac{\Upsilon}{2}\sin^2 2\alpha$$

$$\cos\chi' = \cos^2\frac{\Upsilon}{2}\cos 4\alpha - \sin^2\frac{\Upsilon}{2} \; . \qquad \ldots (14b)$$

We have introduced the transmittance τ_E of the etalon according to the usual definition:

$$\tau_E = \left[1 - \frac{Q}{(1-R)}\right]^2 , \qquad \ldots (15)$$

which derives from the energy conservation expression, $R + T + Q = 1$, where Q is the absorption coefficient of the etalon mirror. The factor ½ comes from the fact that the first polarizer transmits only half of the incident flux.

The instrumental function $f(\varphi,\Upsilon)$ depends upon both of the variables φ and Υ, which have entirely different scales of variation with wavelength as indicated by the expression (6). Since

$$d' + d\,\frac{(\varepsilon+\omega)}{2} \gg d(\varepsilon-\omega),$$

the variable Υ is essentially constant over an interval of wavelength for which φ changes by 2π. Thus the dependence of the expression (14) on φ may be eliminated by averaging this variable over one cycle, which corresponds to the wavelength interval of a free spectral range of the etalon. This procedure and the assumptions justifying it are the same as those employed by Shoenberg[12] in his treatment of the white-light fringes formed by a pair of nearly identical etalons in series. In fact, the physical situation here is very similar to that treated by him, except the ordinary and extraordinary rays in a single etalon play the role of the two sets of interfering beams in a pair of nearly identical etalons. We thus want to calculate

$$F(\Upsilon) = \frac{1}{2\pi}\int_0^{2\pi} f(\varphi,\Upsilon)\,d\varphi \; . \qquad \ldots (16)$$

This calculation involves the evaluation of the following integrals, which is most readily accomplished by converting them to contour integrals about the unit circle in the complex plane and using the method of residues. They are

$$\frac{1}{2\pi}\int_0^{2\pi} \frac{d\varphi}{[1+R^2-2R\cos(\varphi+\chi)][1+R^2-2R\cos(\varphi-\chi)]} = \frac{1+R^2}{1-R^2}\,\frac{1}{1+R^4-2R^2\cos 2\chi} \quad \ldots (17)$$

$$\frac{1}{2\pi}\int_0^{2\pi} \frac{\cos\varphi\, d\varphi}{[1+R^2-2R\cos(\varphi+\chi)][1+R^2-2R\cos(\varphi-\chi)]} = \frac{2R\cos\chi}{1-R^2}\,\frac{1}{1+R^4-2R^2\cos 2\chi}\,.$$

Using these results, we obtain for the instrumental function the expressions

$$F(\Upsilon) = \frac{\tau E}{2}\,\frac{(1+R^2)^2}{(1+R)^3(1-R)}\,\frac{1-\frac{4R^2}{(1+R^2)^2}\cos\chi}{1+\frac{4R^2}{(1-R^2)^2}\sin^2\chi}\left(\frac{1-\cos\Upsilon}{2}\right)\sin^2 2\alpha. \quad \ldots (18a)$$

$$F'(\Upsilon) = \frac{\tau E}{2}\,\frac{(1+R^2)^2}{(1+R)^3(1-R)}\,\frac{1+\frac{4R^2}{(1+R^2)^2}\cos\chi'}{1+\frac{4R^2}{(1-R^2)^2}\sin^2\chi'}\left(\frac{1+\cos\Upsilon}{2}\right)\sin^2 2\alpha. \quad \ldots (18b)$$

Note that the second expression goes over to the first one by the substitution of $\Upsilon + \pi$ for Υ. This is similar to the situation of a Solc filter of the 'folded' or 'fan' type, which have identical instrumental functions except for the above change of variable. As will be discussed later the functions (17) assume their maximum values for those values of Υ for which the factor $\sin^2\chi$ becomes small while the numerators are non-zero. These are odd multiples of π for (17a) and even multiples of π for (17b). Thus the wavelengths of maximum transmittance are those for which the retarder is a half-wave plate for Faraday rotations in the opposite sense and those for which the retarder is a whole-wave plate for Faraday rotations in the same sense. We shall continue the discussion of the instrumental function for the former case, only; the other case follows upon substitution of $\Upsilon + \pi$ for Υ.

For all cases of interest, R will be a number close to unity. We can simplify expression (17a) by discarding terms of the second or higher order in (1-R). Introducing the reflective finesse of the etalon N_R in its usual definition

$$N_R = \frac{\pi R^{\frac{1}{2}}}{1-R}, \qquad \ldots (19)$$

we may simply (17a) to the form

$$F(\Upsilon) = \frac{\tau_E}{2} \frac{1}{4}\left(\frac{N_R}{\pi}\right) \frac{(1-\cos\chi)(1-\cos\Upsilon)}{1+\left(\frac{N_R}{\pi}\right)^2 \sin^2\chi} \sin^2 2\alpha \,. \qquad \ldots (20)$$

Now the value of $\cos\chi$ ranges from $+1$ to $-\cos 4\alpha$ as Υ varies from zero to 2π. These values correspond to minimum values of $\sin\chi$. Now $F(\gamma)$ vanishes for $\cos\chi = +1$; therefore the maxima of the function occur when $\cos\chi = -\cos 4\alpha$. Now we shall see that the most satisfactory operating regime of the instrument is for α very small; therefore for values of γ well removed from an odd multiple of π, we may substitute Υ for χ in the expression (20). Since the magnitude of $F(\Upsilon)$ is proportional to $\sin^2 2\alpha$, we must retain terms of the second order of $\sin 2\alpha$ in the expression of $\sin\chi$ in the neighbourhood of the maxima. The factor $(1-\cos\chi)$, however, is slowly varying in this region so we may substitute $(1-\cos\Upsilon)$ for it. The approximate expression for $\sin^2\chi$ in the vicinity of $\Upsilon = m\pi$ (m odd) is

$$\sin^2\chi \approx \sin^2\Upsilon + 4\sin^2 2\alpha \,. \qquad \ldots (21)$$

The final result for the instrumental function can then be written as

$$F(\Upsilon) = \frac{\tau_E}{2} \frac{1}{16}\left(\frac{\pi}{N_R}\right) \frac{y^2}{1+y^2} \frac{(1-\cos\Upsilon)^2}{1+\left(\frac{1}{1+y^2}\right)\left(\frac{N_R}{\pi}\right)^2 \sin^2\Upsilon}$$

$$y = \frac{2N_R}{\pi} \sin 2\alpha \,. \qquad \ldots (22)$$

The instrumental function is essentially an Airy function whose maximum value is $\frac{1}{4}\frac{\pi}{N_R}\frac{y^2}{1+y^2}$ and whose finesse is $N = \frac{N_R}{[1+y^2]^{½}}$. The optimum value for the parameter y is that which maximizes the transmittance-resolvance product, which is proportional to $\frac{y^2}{[1+y^2]^{3/2}}$. This value is $y = \sqrt{2}$; with the corresponding value of α is equal to

$$\sin 2\alpha \approx 2\alpha = \frac{\pi}{2\left(\frac{N_R}{\sqrt{2}}\right)} \quad . \qquad \ldots (23)$$

The angle of Faraday rotation between successive passes is 2α. The optimum value of this angle is thus seen to be the same as the optimum angle of rotation between the individual retarders of a Solc filter of $N_R/\sqrt{2}$ plates. The value of this angle for values of the reflective finesse between 30 and 40 is between 2^{o} and 3^{o}. The required amount of Faraday rotation is thus extremely small. Larger angles of rotation would give negligible increase of peak transmittance and reduce the finesse of the instrumental function. Inserting the optimum value for α, we obtain for the final instrumental function

$$F(\Upsilon) = \frac{\tau_E}{2}\frac{1}{6}\left(\frac{\pi}{N_R}\right)\frac{\sin^2\frac{\Upsilon}{2}}{1+\frac{1}{3}\left(\frac{N_R}{\pi}\right)^2\sin^2\Upsilon} \quad . \qquad \ldots (24)$$

The ideal peak transmittance is thus $\frac{1}{6}\left(\frac{\pi}{N_R}\right) \approx \frac{1}{2N_R}$.

A reflection configuration of the (GYMPI) is also possible. This would differ from the arrangement of Fig.1 in that the second mirror would be of unit reflectance and the first polarizer would be replaced by a polarizing beam splitter such as a Rochon or Wollaston prism, which would also serve as the exit polarizer. Such a configuration is found to have the same instrumental function as the previously described transmitting version, but with twice the finesse and twice the peak transmission. The optimum value of α and, hence the required magnetic field, is half that of the transmitting version. On the other hand the contrast requirement of the polarizing beam splitter is very severe, as it must discriminate against unwanted light more intense by a factor N_R than the wanted light, while the polarizers of the transmitting instrument need discriminate only between wanted and unwanted light of the same intensity. The reflection version is also more awkward to incorporate into a telescopic system, since the optical path is no longer straight. Nonetheless, the reflecting GYMPI may be of interest, if a polarizing beam splitter of sufficient contrast can be achieved.

THE FIELD-WIDENING PROPERTY OF THE GYMPI

The ideal transmittance of the GYMPI is $\frac{1}{2N_R}$ and the ideal finesse $\frac{1}{\sqrt{3}}$ that of the Fabry-Perot etalon that is a part of it. On this basis alone, the instrument would have little utility either as a monochromatic filter or a monochromator. The chief virtue of the instrument is the field-widening property i.e. the relative independence of the quantity Υ on the angle of incidence θ. If we call Υ_o the value of Υ for normal incidence and ψ the azimuthal angle of the plane of incidence with respect to the optic axis of the retarder the value of Υ to second order in θ is

$$\Upsilon = \Upsilon_o \left[1 + \frac{\theta^2}{2\omega} \left(\frac{\cos^2\psi}{\varepsilon} - \frac{\sin^2\psi}{\omega} \right) \right] \qquad \ldots (25)$$

(see Evans I, equation (III.19)).

The usable angular field is thus ω times larger than that of a Fabry-Perot etalon for rays incident in the plane normal to the optic axis and $(\varepsilon\omega)^{\frac{1}{2}}$ times larger for rays incident in the plane containing the optic axis. The usable field can be greatly increased by use of one of Lyot's wide-field retarders (see Evans I). The simplest of these is Lyot's type-I element in which the retardation plate is split into two plates of half the required thickness with their optic axes at 90° and a half-wave plate between them with its axis at 45° to those of the crystal plates. The phase shift of such a retarder is the sum of two expressions (25) with $\cos\psi$ and $\sin\psi$ interchanged. The expression is

$$\Upsilon = \Upsilon_o \left[1 - \frac{\theta^2}{2} \left\{ \frac{\varepsilon - \omega}{\varepsilon\omega^2} \right\} \right] . \qquad \ldots (26)$$

The expression in curly brackets is 0·025 for calcite and 0·002 for quartz. Even larger angular fields are possible with the type-II and type-III Lyot wide field elements.

This field-widening property is shared by all polarizing filters. Because of the capacity to accept a much larger solid angle without loss of resolvance the overall throughput of the GYMPI will exceed that of a Fabry-Perot etalon of the same resolvance by as much as an order of magnitude despite the reduction of the peak transmittance by $\left(\frac{1}{2N_R} \right)$.

COMPARISON OF THE GYMPI WITH OTHER TYPES OF FILTERS

A reasonable figure of merit of a spectral device is its throughput resolvance product. By throughput we mean the fraction of radiant energy of the appropriate wavelength emitted by the source which falls on the detector, which may be an image area on a photographic emulsion or a photo-electric device. The throughput is conveniently considered as a product of two factors: the transmittance which is the ratio of the radiance of the source to the radiance of the image and the solid angle or étendue which is the fraction of isotropically radiated flux from the source which is collected by the instrument. Now this étendue can be taken as the same for all birefringent filters, because they share the field-widening property. The appropriate figure of merit for intercomparing birefringent filters is the transmittance-resolvance product. If $k = d\,\frac{(\varepsilon - \omega)}{\lambda}$ is the order of interference of the thinnest retarder of a Lyot filter of N stages or one of the n identical plates of a Solc filter or half the order of interference of the retarder of the GYMPI we may write these products as

$k(2\tau_p)^N \frac{\tau_p}{2}$	Lyot Filter
$k(2\tau_p^{\frac{1}{2}})^N \frac{\tau_p}{2}$	Split-element Lyot Filter
$k\ n\ \tau_p^2$	Solc Filter
$\frac{k\ \tau_E\ \tau_p^2}{\sqrt{3}}$	GYMPI .

The quantity τ_p is the transmittance of the polarizers for radiation polarized in the favoured direction. Now if $\tau_p < 50\%$ as is the case for dichroic sheet polarizers in the ultra-violet, the transmittance resolvance product of a simple Lyot filter may well be less than that of the GYMPI especially for a very narrow-band filter having a large number of stages. The situation is different for a split-element Lyot filter which uses only half as many polarizers as the simple filter[13], though such a filter is more complex and hence more expensive to build. The transmittance of the Solc filter is clearly greater than that of the GYMPI in all regions of the spectrum. On a practical basis, however, it is not feasible to use compensated wide-field elements in a narrow-band Solc filter because of the large number of plates required. Thus if the comparison is made between a simple Solc filter of n plates and a GYMPI with a type I wide-field retarder the factor $\frac{\sqrt{3}n}{\tau_E}$ of increased transmittance is off-set by a factor $(\frac{\varepsilon - \omega}{\varepsilon})$ in étendue so the overall throughput of the two instruments will be comparable.

If we compare the GYMPI with a Fabry-Perot etalon of the same resolvance, the Fabry-Perot will have a transmittance $2\sqrt{3}\, N_R$ larger than the GYMPI but will have an éntendue less by a factor $\left(\frac{\varepsilon - \omega}{2\varepsilon\omega^2}\right)$ for a type-I wide-field element. Thus, with a quartz retarder, the overall luminosity of the GYMPI will exceed that of the Fabry-Perot by an order of magnitude. This advantage can be extended still further by use of a type-III wide-field element, which for a restricted wavelength interval can have a retardance that is independent of angle of incidence up to fourth-order terms in θ.

It can thus be concluded that the low transmittance of the GYMPI does not rule it out as a practical instrument compared with the other devices in current use.

The cost of the GYMPI should be appreciably less than either a Lyot or a Solc filter. Fabry-Perot etalons are relatively cheap compared with birefringent plates of suitable quality. The use of a magnetic field may limit the use of the GYMPI for certain applications but, as we shall discuss in the final section, recent developments in the high Verdet-constant low-loss materials make the required magnetic field of quite moderate magnitude so that it could be provided by permanent magnets.

Note that if the retarder is as thick as the thickest element of a Lyot filter the pass-band can be reduced to $1/N_R$ of the pass-band of the corresponding Lyot filter. Thus the GYMPI can have an extremely narrow pass-band, one or two orders of magnitude smaller than is currently achieved in birefringent filters. Such a filter would, however, require a relatively narrow-band pre-monochromator to isolate the desired order. This could be another GYMPI, a Solc filter or perhaps a suitable interference filter, if the requirements on usable angular field are not too severe.

It should be noted that the peak transmission of two GYMPIs in series will be of order N_R^{-1} instead of N_R^{-2}, provided that the mean optical-path differences in the two etalons are equal within a few wavelengths. This is the condition for the formation of white light fringes in a pair of étalons. The difference of mean optical paths between the two étalons should be small compared with the difference of optical path between the extraordinary and ordinary ray of the retarders of the individual GYMPIs. The unwanted orders can thus be eliminated by using two GYMPIs in series, whose retarders have slightly different thicknesses d_1 and d_2, but both of which are integer or half-integer numbers of wavelengths thick at the desired wavelength. Thus the maxima will coincide at the wanted wavelength λ_o but will not coincide except for wavelengths spaced at an interval $\Delta\lambda = \lambda_o^2(\varepsilon - \omega)^{-1}(d_2 - d_1)^{-1}$. Thus if d_2 and d_1 are sufficiently close, the transmitted maxima of the tandem GYMPI will be sufficiently far separated so that all but the desired wavelength can be suppressed with a simple absorption filter. While the peak transmittance of the tandem GYMPI can be made of order N_R^{-1} as explained above, the transmissions of the two etalons in series will be proportional to $\tau_{E1}\,\tau_{E2}$. With the low-loss multilayer dielectric coatings now available, this should not result in an unacceptably low transmittance.

PRACTICAL CONSIDERATIONS

The practical realization of the GYMPI should be within the capabilities of modern optical techniques. Faraday rotation of 2° to 3° is quite moderate and should be achievable with fields of a few thousand ampere turns for paramagnetic Faraday elements (Verdet constant 0·1 to 0·2) or a few tens of thousand ampere-turns for diamagnetic elements (Verdet constant 0·01 to 0·02). A simplification of construction may be possible in which the etalon plates are themselves used as the Faraday-rotation elements, the reflecting coatings being on the outside of the cavity. While rather large fields would be required for the usual étalon plates of fused quartz, etalon plates of certain synthetic crystals such as the rare earth aluminum garnets[14] or yttrium aluminum garnet (YAG) could be used. These substances are paramagnetic and hence have Verdet constants one to two orders of magnitude greater than quartz. Since these materials are grown as single crystals and are capable of being worked to high precision, acceptable étalon plates probably could be made of them. They have high transmittance throughout the entire spectrum, except for relatively narrow absorption bands. Whether it would be advantageous to combine the etalon plate and Faraday rotator in the same optical element rather than using separate plates for each function would have to be determined by experiment.

The retarder could be of calcite, quartz or any other birefringent material of suitable homogeneity. A particularly useful crystal in the visible spectrum might be lithium niobate or potassium lithium Niobate. Such crystals[15] have differences in the ordinary and extraordinary indices of refraction which are comparable to calcite. They show a strong transverse electro-optic effect so that the retardance could be changed over a wavelength by an applied electric field, thus tuning the filter over a free spectral range.

The GYMPI can be used in the ultraviolet down to the quartz limit. Recent developments[16] in the use of the Fabry-Perot étalon in this region have shown that a usable finesse is achievable with an acceptable transmittance. Since the Verdet constant of all substances increases strongly toward the ultraviolet end of the spectrum the magnetic field requirements become less severe there. The use of the GYMPI in the infrared is also feasible up to the 3μ region with a quartz or calcite retarder. This limit could be extended with a retarder of rutile. The Verdet constants of all substances, however, decrease strongly toward the infrared which increases the magnetic field requirements.

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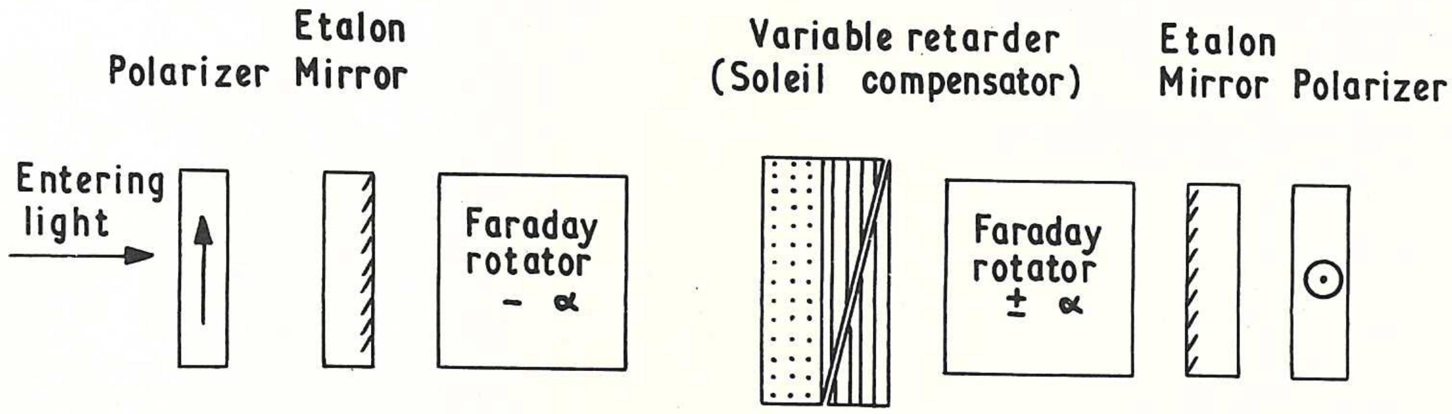

Fig.1 Optical train of the GYMPI (CLM-P 162)