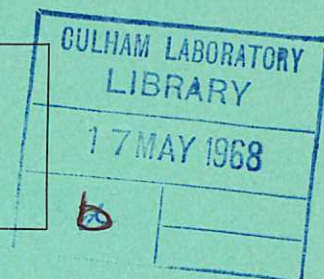


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MAGNETIC MIRROR MACHINES

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ABSTRACT

A study of the containment of particles in the loss-cone of a magnetic mirror-machine by means of RF fields whose frequency slightly exceeds the maximum value of the ion cyclotron frequency within the plasma. Computations which verify the adiabatic theory of this confinement system are reported.

The rather discouraging outcome of a number of recent thermonuclear reactor design studies based upon unaided magnetic mirror plasma confinement<sup>(1,2)</sup>, has emphasised the potential value of a device which could prevent or diminish the loss of particles along field lines in such machines, without dissipating an excessive amount of energy in the process. It has long been known<sup>(3,4)</sup> that radio-frequency fields can in principle be used for this purpose, but their usefulness has been questioned on the grounds that the RF frequency must either coincide with the (ion or electron) cyclotron frequency, in which case there is an irreversible and probably prohibitive dissipation of RF energy in the plasma, or differ substantially from it, in which case the RF fields act by exerting radiation pressure and

the required field strengths lead to a prohibitive power loss in the metal structure used to localise them. It has recently been pointed out<sup>(5)</sup> that a compromise is possible - to use RF fields whose frequency slightly exceeds the maximum value of the ion cyclotron frequency at points accessible to the plasma - and calculations based upon the conservation of certain adiabatic invariants of the particle motion have shown that the thermonuclear prospects of this approach are rather encouraging. In this note we report the outcome of a series of computations designed to test the underlying adiabatic theory.

We selected an axisymmetric parabolic magnetic mirror profile

$$\vec{B} = B_0 \left( -\frac{xz}{L_B^2}, -\frac{yz}{L_B^2}, 1 + \frac{z^2}{L_B^2} \right)$$

supplemented by a circularly polarised standing wave with a node at the centre of the well,  $\vec{E} = E_0 \frac{z}{L_E} (\cos \omega t, -\sin \omega t, 0)$  and  $B_{RF} = \frac{c}{\omega z} \vec{E}$ , where  $L_E$  and  $L_B$  are arbitrary scale lengths. The values of  $B_0$  and  $\omega$  are assumed to be such that the cyclotron frequency  $\Omega = \frac{eB}{mc}$  of an ion of charge  $e$  and mass  $m$  is lower than  $\omega$  at the mid-point  $x = y = z = 0$ . In consequence there exist resonant surfaces intersecting the  $z$  axis at  $\pm Z_{res}$  at which  $\omega = \Omega$ . According to the adiabatic theory such a field configuration should confine a plasma in the neighbourhood of the mid-point provided that the distribution function is truncated in velocity space in such a way that no particles can reach the resonant surfaces. The non-relativistic equations of motion in such fields can be reduced to a dimensionless form convenient for computing if time is measured in units of  $1/\omega$  and length in units of  $c/\omega$ :

$$\dot{v}_x = v_y \Omega_0 (1 + \alpha z^2) + v_z \Omega_0 \alpha yz + g_0 z \cos t + v_z g_0 \sin t \dots (1)$$

$$\dot{v}_y = -v_x \Omega_0 (1 + \alpha z^2) - v_z \Omega_0 \alpha x z - g_0 z \sin t + v_z g_0 \cos t \quad \dots (2)$$

$$\dot{v}_z = \Omega_0 \alpha z (v_y x - v_x y) - g_0 (v_x \sin t + v_y \cos t) \quad \dots (3)$$

where  $\Omega_0 = \frac{eB_0}{m\omega c}$ ,  $g_0 = \frac{eE_0}{m\omega^2 L_E}$  and  $\alpha = \frac{c^2}{\omega^2 L_B^2}$ . These equations are invariant under the scaling  $\vec{v} \rightarrow \beta \vec{v}$ ,  $\vec{r} \rightarrow \beta \vec{r}$ ,  $\alpha \rightarrow \alpha/\beta^2$ , and possess one exact invariant, equal to the particle energy in the (rotating) frame in which  $\vec{E}$  vanishes. This invariant was used to check the accuracy of the computations: it remained constant to within 0.1% in all the cases discussed here.

The three adiabatic invariants discussed in<sup>(5)</sup> here take the form

$$\epsilon_{RF} = \frac{1}{2}(v_z - v_{Ez})^2 + \mu_{RF} \Omega + \psi \quad \dots (4)$$

$$\mu_{RF} = \frac{1}{2}[(v_x - v_{Ex})^2 + (v_y - v_{Ey})^2]/\Omega \quad \dots (5)$$

$$J_{RF} = \oint (\epsilon_{RF} - \mu_{RF} \Omega - \psi)^{1/2} ds \quad \dots (6)$$

where

$$\Omega = \Omega_0 [(1 + \alpha z^2)^2 + \alpha^2 z^2 (x^2 + y^2)]^{1/2}$$

$$v_{Ex} = g_0 z \sin t + [-\Omega_0^2 \alpha^2 z^3 x g_0 (x \sin t + y \cos t) + \Omega_0 (1 + \alpha z^2) g_0 z \sin t]/(1 - \Omega^2)$$

$$v_{Ey} = g_0 z \cos t + [-\Omega_0^2 \alpha^2 z^3 y g_0 (x \sin t + y \cos t) - \Omega_0 (1 + \alpha z^2) g_0 z \cos t]/(1 - \Omega^2)$$

$$v_{Ez} = [\Omega_0^2 (2 + \alpha z^2) \alpha z^2 g_0 (x \sin t + y \cos t)]/(1 - \Omega^2)$$

$$\psi = \frac{1}{2} g_0^2 z^2 [1 + \Omega_0 (1 + \alpha z^2) - \Omega_0^2 \alpha^2 (x^2 + y^2) z^2/2]/(1 - \Omega^2)$$

and  $\oint ds$  indicates an integral along a field line of  $\vec{B}$ . It is readily shown that  $J_{RF}$  is a function of  $\chi = r^2(1 + \alpha z^2)$ , the stream function for  $\vec{B}$ , and for simplicity we have taken  $\chi$  instead of  $J_{RF}$  as the invariant.

The adiabatic theory can be used to predict the behaviour of a particle injected at the point  $x = y = z = 0$ , at  $t = 0$ , with an initial velocity  $v_z = v_{||}$ ,  $v_{\perp} = (v_x^2 + v_y^2)^{1/2}$  and  $\phi = \tan^{-1} \frac{v_y}{v_x}$ .



Let us consider first the case of a particle injected along the axis of the magnetic field  $\vec{B}_0$ , (i.e.  $v_\perp = 0$ ,  $\mu_{RF} = 0$ ). For  $g_0 = 0$ , the particle is lost from the system. For a given finite  $g_0$ , however, as Eq.(4) shows, such particles will be reflected at a point  $z \leq z_{refl}$  provided that  $\frac{1}{2} v_\parallel^2 \leq \psi(z_{refl})$  and will in consequence remain adiabatically confined. (The  $\leq$  sign here reflects the presence of the relatively small time-dependent term  $v_{Ez}$ .) As  $v_\parallel$  is increased,  $z_{refl}$  will approach  $z_{res}$ . It can never exactly reach  $z_{res}$  for any finite  $v_\parallel$ , since by definition  $\psi \rightarrow \infty$  as  $z \rightarrow z_{res}$ . However, when  $z_{refl}$  reaches some value  $z_{lim}$ , slightly inferior to  $z_{res}$ , we would expect adiabatic theory to break down, and at higher values of  $v_\parallel$  we might expect one or other of two consequences to follow:

- (i) The particle is still reflected at a point inside  $z_{res}$ , so the invariants remain defined, but their values experience a (possibly irreversible) change while the particle is in the 'non-adiabatic' region; or
- (ii) The particle passes through  $z_{res}$ , at which point the invariants are no longer even defined. If it gets well past  $z_{res}$ , it will once more enter a region where the adiabatic theory should apply, but the new values of the invariants are not simply related to their values of the other side of the resonance. Since the changes in  $\epsilon_{RF}$  and  $\mu_{RF}$  depend upon the phase with which the particles enter the resonance zone, we would expect these quantities to change in a stochastic manner at each subsequent transit through resonance and hence for the particle to be lost after a variable but finite number of transits.



To summarize, two or possibly three categories of particle are predicted: (1) adiabatically confined particles with  $z_{\text{refl}}$  significantly less than  $z_{\text{res}}$ ; (2) non-adiabatic and eventually lost particles with  $z_{\text{refl}} > z_{\text{lim}}$  or even  $> z_{\text{res}}$ ; and possibly (3) non-adiabatic but confined particles, in which the variations in the invariants are not sufficiently random to result in particle loss.

The above analysis applied to the case of injection along the axis; however, cases where  $v_{\perp} \neq 0$  can be easily considered, since the value of  $v_{\perp}$  determines  $\mu_{\text{RF}}$  and the equation defining the outer bound on the reflection point  $z_{\text{refl}}$  obtained from eqs(4) and (5) is

$$\frac{1}{2} v_{\parallel}^2 = \frac{1}{2} v_{\perp}^2 \left( \frac{\Omega(z_{\text{refl}}) - \Omega(0)}{\Omega(0)} \right) + \psi(z_{\text{refl}}) . \quad \dots (7)$$

Adiabatic theory does not provide internally a criterion which enables one to determine  $z_{\text{lim}}$ , which is the value of  $z_{\text{refl}}$  at which the theory breaks down. However,  $z_{\text{lim}}$  is expected to be independent of the angle of injection since a particle which is reflected at  $z_{\text{lim}}$  approaches this point with  $v \approx v_{\perp}$  and  $v_{\parallel} \approx 0$  whatever its injection angle. Consequently, if  $z_{\text{lim}}$  is determined numerically for  $v_{\perp} = 0$ , Eq.(7) then defines the theoretical outer edge of the zone in velocity space within which particles are confined adiabatically.

In comparing this adiabatic theory with the numerical results, it is necessary to take into account the fact that Eqs.(4) and (5) for the adiabatic invariants are only correct to lowest order in the expansion parameter (essentially  $v/c$ ), and in view of practical limitations on computing time, this parameter cannot be made arbitrarily small - it was typically around 0.02 in the cases studied. Consequently variations at least of the order of the expansion parameter in the computed values of  $\epsilon_{\text{RF}}$  and  $\mu_{\text{RF}}$  are expected. In default

of 'exact' expansions for the invariants, we adopted the criterion that such variations were to be regarded as compatible with conservation of the 'exact' invariants provided that they were periodic - that is, provided that after a finite number of particle transits the sequences of values of the approximate invariants repeated themselves exactly.

To test these predictions and to investigate the accuracy with which these invariants remain constant even when a particle approaches the cyclotron resonance, we integrated Eqs.(1) to (3) numerically, using a differential equation routine with automatic step length adjustment. Even with given initial conditions, Eqs.(1) to (3) define a three-parameter set of solutions depending on  $g_0$ ,  $\alpha$ , and  $\Omega_0$ . We report here the most interesting  $g_0$  dependence, taking  $\alpha = 0.357$ , and  $\Omega_0 = 0.675$ , values which, with a scaling  $\beta = 24.1$ , are relevant to an existing experiment - Phoenix II<sup>(6)</sup> at Culham.

These numerical calculations confirmed the predictions given above in considerable detail. All three categories of particle were discovered. The time dependence of  $\epsilon_{RF}$  and  $\mu_{RF}$  for two examples of adiabatically confined particles are shown in Figs.1(a) and (b), and the corresponding trajectories in phase space, projected onto the  $x-z$  plane and the  $v_{||}$  and  $v_{\perp}$  plane respectively, are shown in Figs.2(a) and (b). The variations in  $\epsilon_{RF}$  and  $\mu_{RF}$  are in both cases small, particularly when compared with the variation in  $\frac{1}{2} v^2$ , and these variations were exactly periodic, with periods of a few transit times. These figures illustrate rather perspicuously the mode of operation of near-resonance adiabatic fields. Fig.1(d) represents a typical category (2) particle, which passes through the resonance surface, at which the values of  $\epsilon_{RF}$  and  $\mu_{RF}$  become infinite.



It is seen that on both sides of the first passage through resonance, adiabatic behaviour is maintained and that a reflection results, leading to a second passage through resonance. At this and each subsequent passage through resonance, a further, apparently stochastic, change in the invariants occurs and in any real magnetic well such a particle would be lost after a rather small number of transits. For a very narrow range of injection parameters category (3) behaviour was observed. Fig.1(c) represents such a particle. It will be seen that the invariants fluctuate by a larger amount than in the case of category (1) particles; more important however is that these fluctuations are non-periodic but of apparently restricted amplitude. Particles in this class have been followed for  $10^5$  cyclotron gyration times, during which they remained confined, without any periodicity in  $\epsilon_{RF}$  and  $\mu_{RF}$  being detected. Although of theoretical interest, this category represents such a narrow band in injection velocity space that it is of little practical importance. The values of  $J_{RF}$  in the above cases are not shown, since for on-axis injection,  $\chi$  is initially zero and the observed fluctuations in its value do not give a useful measure of its invariance. In cases of off-axis injection, computations showed that its fractional variations were of the same order as those of  $\epsilon_{RF}$  and  $\mu_{RF}$ .

In Figs.3(a) and (b) we give in condensed form the results of a very large number of computations designed to map out the boundary between adiabaticity and non-adiabaticity for a fixed  $g_0$  and with varying injection parameters. From the form of Eqs.(4) and (5) for  $\epsilon_{RF}$  and  $\mu_{RF}$ , it may be seen that, within the zone of adiabaticity, the initial azimuthal phase  $\phi$  is of no importance. As the edge of this zone is approached, however, the magnitudes and periods of the

fluctuations in these invariants were found to depend on phase and correspondingly the limit of onset of non-adiabaticity is smeared into a phase dependent band which is too narrow to represent in Fig.3. The lower solid line in Fig.3(a) defines the zone of adiabatically confined particles, the shaded band above shows the region of 'non-adiabatic' confinement, and finally category (2) particles lie above this band. These curves may be compared with those in Fig.3(b) which are contours of constant  $z_{\text{refl}}$ . The line of adiabatic confinement corresponds rather closely to the contour  $z_{\text{refl}} = 0.83$ . If one uses the 'empirical' value of  $z_{\text{lim}} = 0.83$  in Eq.(7), one obtains the 'dot-dash' curve which fits the edge of the adiabatic zone as well as can be expected. The corresponding limiting value of  $(1 - \Omega)$  is 0.145.

Having confirmed that the adiabaticity boundary could be calculated from Eq.(7), given  $v_{\parallel\text{max}} \equiv [2 \psi(z_{\text{lim}})]^{1/2}$ , we investigated the dependence of  $v_{\parallel\text{max}}$  on  $g_0$  by considering parallel injection only. For  $g_0 \lesssim 0.004$  we found an approximately linear relationship  $v_{\parallel\text{max}} = 2.1 g_0$ . For larger values of  $g_0$ , a saturation phenomenon is observed, and no further improvement in the confinement is obtained if  $g_0$  is increased beyond 0.013, at which value  $v_{\parallel\text{max}} = 0.02$ .

This general result may be interpreted in the context of two classes of experiment. Firstly, if the scale lengths of electric and magnetic field,  $L_E$  and  $L_B$  respectively, are of order  $c/\omega$ , then deuterons with a parallel energy of 18 MeV could still be reflected if the required electric field strength were available. Secondly, if  $L_E \approx L_B \ll c/\omega$ , which is the case with most present-day experiments, the results can still be applied, with an appropriate scaling, provided that  $\Omega_0$  (essentially the mirror ratio), remains fixed. For example, if we take a set of parameters typical of the Phoenix II<sup>(6)</sup> experiment:



$B(0) = 12.5 \text{ kG}$ ,  $L_E = 7 \text{ cm}$ , mirror ratio  $R = 1 + \alpha = 1.357$ ,  $\Omega_0 = \frac{1}{1.1R}$ , the scaling is  $\beta = 24.1$ . The computed result -  $v_{\parallel \text{max}} = 0.02$  with  $g_0 = 0.013$  then implies that  $0.33 \text{ keV}$  protons would be confined by an electric field of  $3 \text{ kV cm}^{-1}$ . Thus experimentally realisable fields could lead to the trapping of slow ions which in turn could lead to the build-up of fast ions. Furthermore, since after scaling the unit of  $v$  is  $L_E \omega$  and that of  $E$  is  $L_E \omega^2$ , for  $L_E = 14 \text{ cm}$ , and  $B(0) = 16 \text{ kG}$ , protons of  $5 \text{ keV}$  could be confined by an electric field of  $23 \text{ kV cm}^{-1}$ .

In extrapolating the results reported here towards a thermonuclear reactor, two questions arise. Firstly, one might ask whether this method of confinement is very sensitive to collisions in the mirror region. A partial answer can be obtained by regarding a collision as equivalent to a re-injection of the particle at  $z \neq 0$  with new values of  $v_{\parallel}$  and  $v_L$ . To emphasise that a particle can easily remain confined after such an event, we have indicated in Fig.3(a) the adiabatic boundary for particles injected at  $z = 0.57$ . Secondly it may be asked how far the above discussion requires modification to take into account the influence of the plasma on the electric field profile which would exist inside it. As was shown in <sup>(5)</sup>, the self-consistent profile can be calculated if the adiabatic invariants are assumed to be conserved. Such profiles are more complicated than the linear profile adopted here, and the adiabaticity of particles in such profiles is currently under investigation.

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#### REFERENCES

1. SIVUKHIN, D.V., Plasma Physics 8, 607 (1966)
2. FOWLER, T.K. and RANKIN, M., Plasma Physics 8, 121 (1966).
3. VEDENOV, A.A., et al., Proc. Intern. Conf. Peaceful Uses Atomic Energy, Geneva, 32, 239 (1958).
4. CONSOLI, T., Phys. Letters 7, 254 (1963).
5. WATSON, C.J.H., Proc. Intern. Colloquium Interaction RF Fields with Plasma, Saclay (1968).
6. BERNSTEIN, W. et al., Proc. I.A.E.A. Conference on Plasma Physics and Controlled Nuclear Fusion Research, Culham 1965, 2, 23 (1966).
7. CONSOLI, T., *ibid*, 2, 483 (1966).



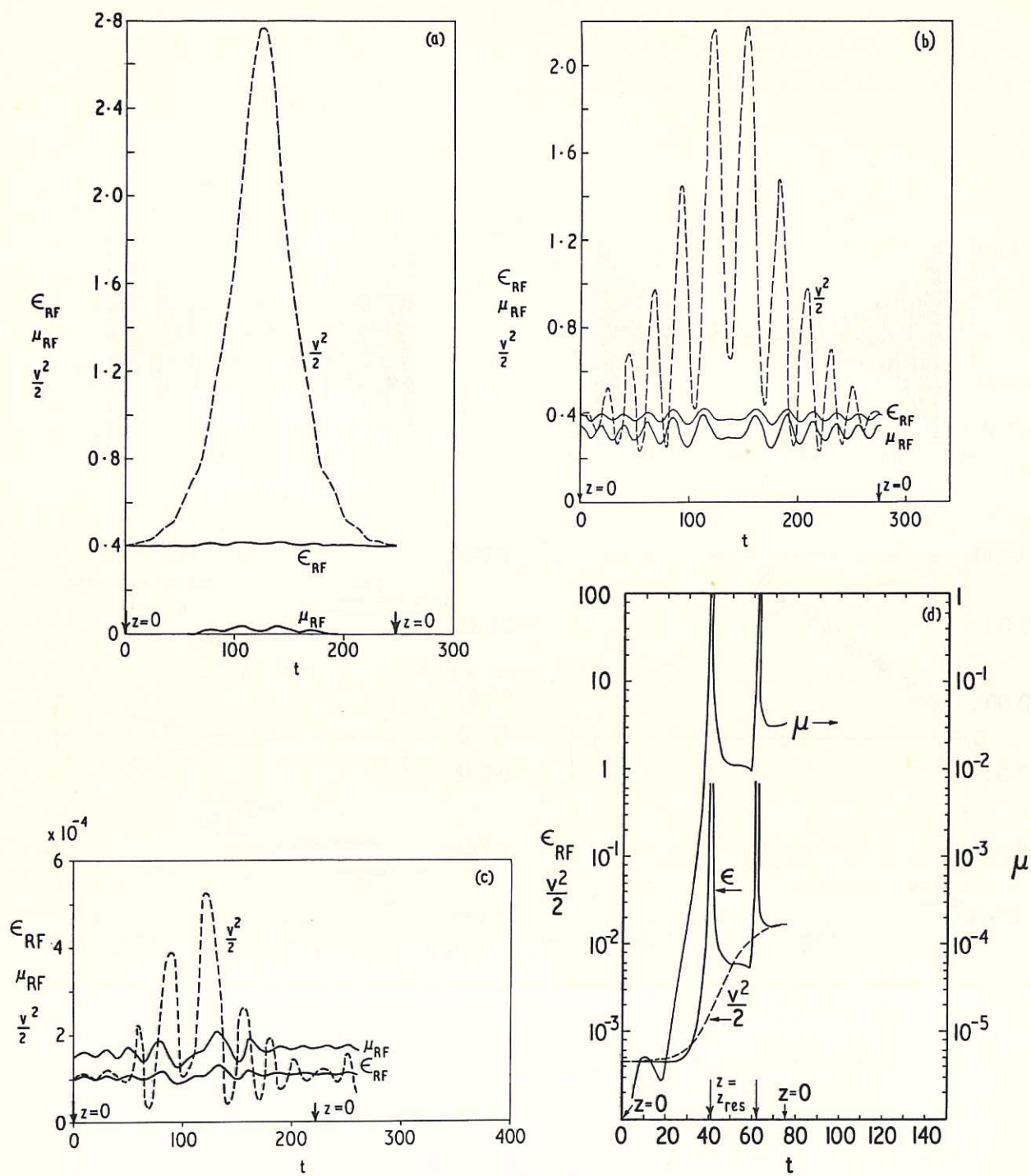


Fig. 1

(CLM-P165)

Time evolution of  $\epsilon_{RF}$ ,  $\mu_{RF}$  and energy  $v^2/2$  for particles injected at  $x=y=z=0$ ,  $t=0$ , for half of a transit period. The injected velocities  $(v, \theta, \phi)$  of the particles are given below: (a) adiabatically confined particle  $(0.009, 0.0, 0.0)$ ; (b) adiabatically confined particle  $(0.009, 0.57, 0.0)$ ; (c) 'non adiabatically confined' particle  $(0.01414, 0.7954, \pi)$  (d) 'non adiabatic lost' particle  $(0.03, 0.0, 0.0)$

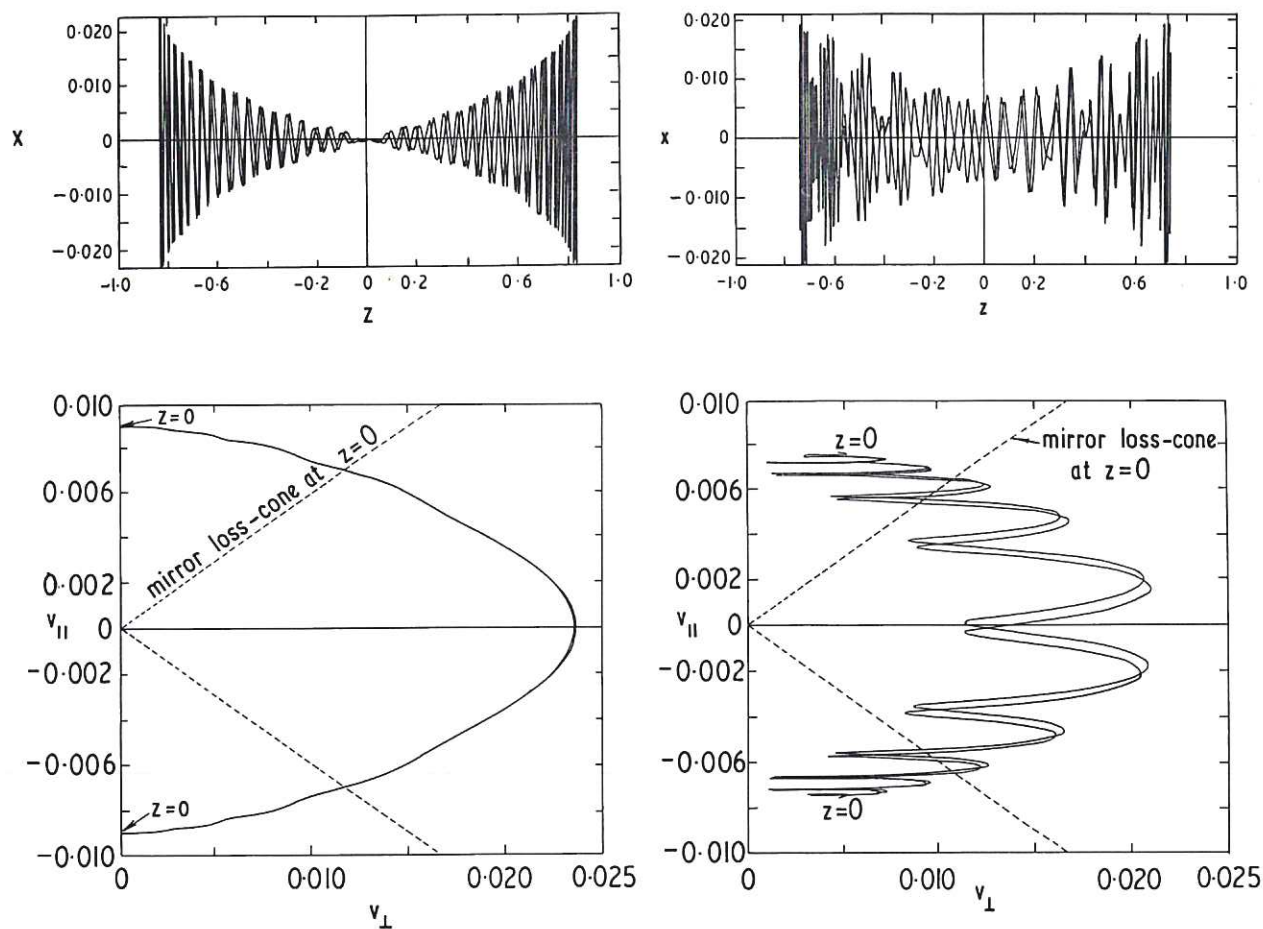


Fig. 2 (CLM-P 165)  
Particle trajectories in  $x$  -  $z$  and  $v_{||}$  -  $v_{\perp}$  planes for one complete transit period  
for the adiabatically confined particles (a) (0.009, 0.0, 0.0) (b) (0.009, 0.57, 0.0)

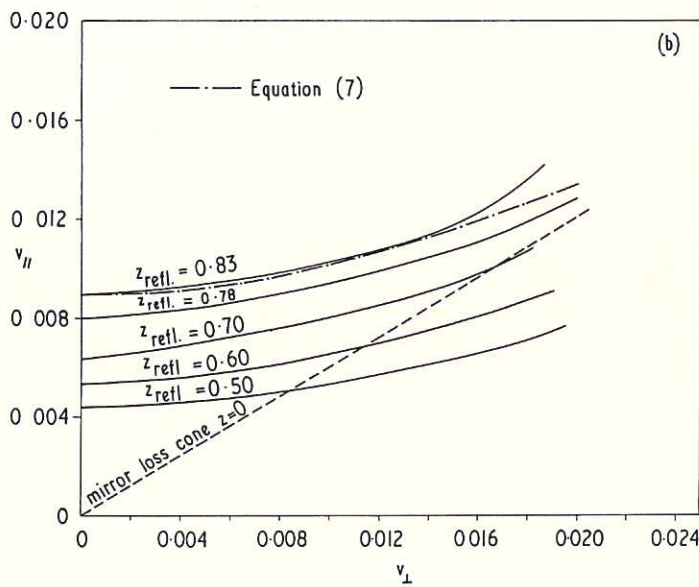
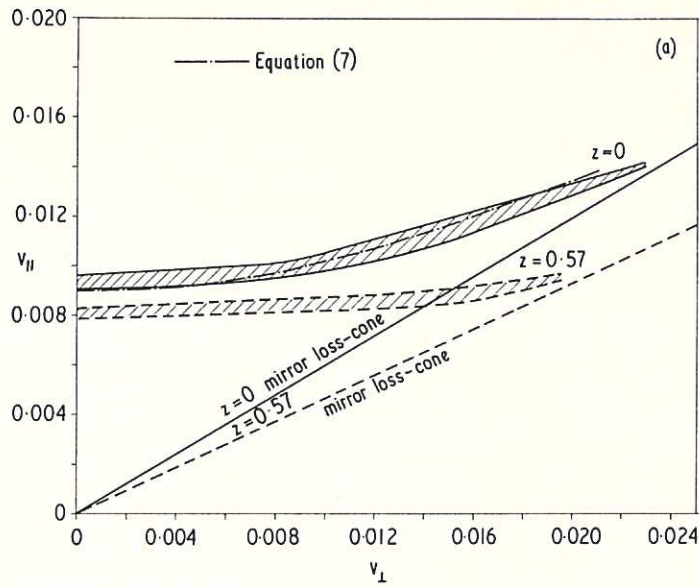


Fig. 3 (CLM-P 165)  
 (a) Boundaries of adiabaticity for injection at  $z = 0$  and  $z = 0.57$   
 (b) Contours of  $z_{\text{refl}}$  for different injections at  $z = 0$  in  $v_{||} - v_{\perp}$  plane

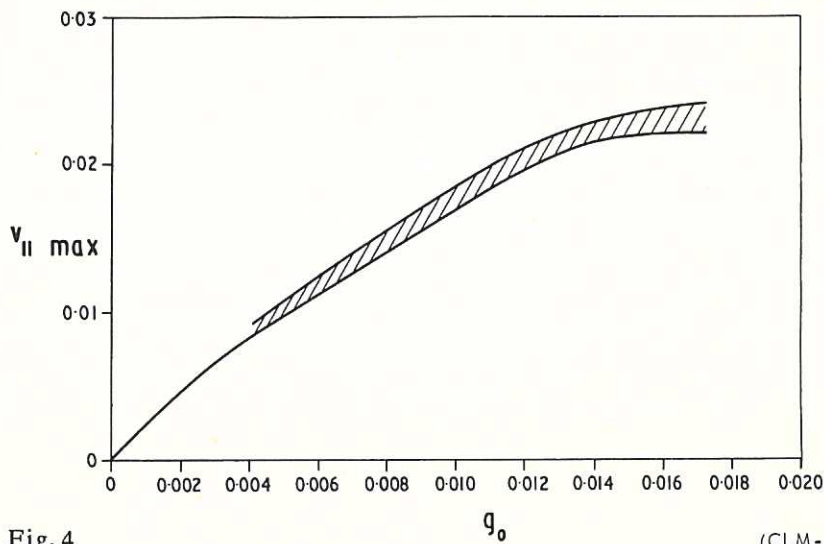
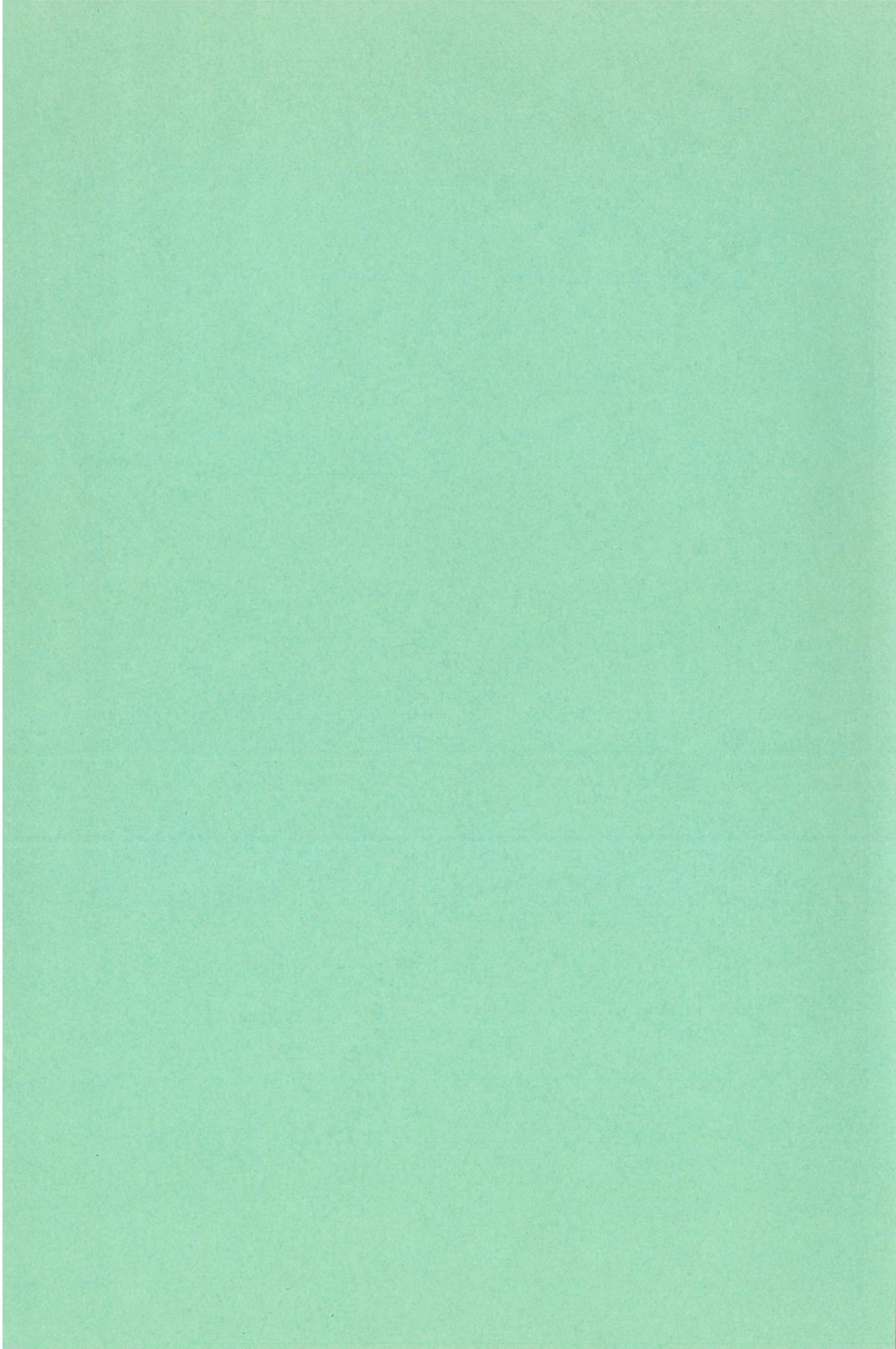


Fig. 4 (CLM-P 165)  
 Variation of maximum confined  $v_{||}$  at  $z = 0$  with applied RF electric field









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