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THE PARTITION OF ENERGY IN A TURBULENT PLASMA

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A B S T R A C T

A theory is developed which leads to an expression for the partition of energy between mechanical and magnetic modes in a turbulent magnetohydrodynamic system in the presence of a strong external field. Modifications to this expression due to specific plasma effects are obtained and are not significant. Results are also obtained for the frequency spectra and diffusion coefficient.

Measurements of the turbulent fluctuations in the ZETA device are described, and from these, values for the partition are obtained. It is found that there is only complete agreement with the theory if an anomalous resistivity is assumed at low operating pressures.

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C O N T E N T S

1. <u>INTRODUCTION</u>

Turbulence in magnetohydrodynamic systems is not only of interest in astrophysical and geophysical applications (BATCHELOR, 1950; CHANDRASEKHAR, 1951 (A204); TATSUMI, 1960; KOVASZNAY, 1960; COWLING, 1957) but also in laboratory plasma devices (KADOMTSEV, 1965). Experiments on the containment oi nigh temperature plasmas by magnetic fields have revealed many forms of oscillation which are frequently labelled 'turbulence', and it is clear that the word is being used in a broader sense than in ordinary fluid turbulence. In addition to the usual fluid turbulence concept of a large number of interacting 'eddies', which propagate only due to their mutual interaction, a plasma exhibits a variety of wave motions which propagate independently of their mutual interaction. Magnetohydrodynamic turbulence in the presence of an applied field can exhibit these features and we shall attempt to interpret the observed fluctuations by such an approach.

The problem of homogeneous, isotropic magnetohydrodynamic turbulence has been studied by many authors (BATCHELOR, 1950; CHANDRASEKHAR, 1951 (A204); TATSUMI, 1960; LUNDQUIST, 1952; CHANDRASEKHAR, 1951 (A207); KRAICHNAN, 1958) and statistical theories of turbulence have been developed by KRAICHNAN (1957-1965), EDWARDS (1964, 1964) and HERRING (1965), which give closed equations describing the basic correlation function. We shall follow the approach of Edwards in this paper.

The assumption of isotropy is not valid for the problem we are considering as there is a natural direction of symmetry. This implies that we need to consider axisymmetric correlation tensors. The assumption of homogeneity will be justified in our case by the measured properties of the turbulent intensity fluctuations.

LEHNERT (1955) studied the effects of an applied field on MHD turbulence for the final period of decay. He concluded that turbulence elements with finite wave numbers in the field direction would be strongly damped though no non-linear terms were included. Thus the turbulence would be axisymmetric and tend to be two-dimensional. It has been suggested that turbulence would be suppressed by an applied field, and this effect has been studied by NIHOULE (1963, 1965) and MOFFATT (1961) for the isotropic case, but it does not occur if the system becomes two-dimensional as the effect of the applied field goes to zero (ROBINSON, to be published; ROBINSON, 1966). We can expect therefore that MHD turbulence will tend to the two-dimensional limit.

In MHD turbulence one of the most important problems is the partition of energy between mechanical and magnetic modes. For the case when the mean field vanishes, it has been argued by some authors (COWLING, 1957; BIERMAN and SCHLUTER, 1951) that there should be approximate partition of energy between the two modes, whereas an apparent similarity between the roles of vorticity and magnetic fields has been invoked by others (BATCHELOR, 1950; MOFFATT, 1961). There is little experimental evidence available from experiments with liquid metals as the magnetic Reynolds number is far too small (SAJBEN and FAY, 1965). In a plasma this need not be the case, but here other considerations such as the anisotropy can become important.

In the next section we describe a model for the turbulent fluctuations in the presence of a mean external field and in particular obtain an expression for the partition of energy. Possible modifications to this expression due to specifically plasma effects are considered.

In section 3 we describe measurements of electric and magnetic field fluctuations in the Zeta discharge. Results are presented for the partition of energy in the plasma and are compared with the theoretical predictions.

2. THEORY

The theory we use, termed a 'generalised random phase approximation' (EDWARDS, 1964, J. Fluid Mech.) uses a Liouville approach to consider the probability functional $P([u(\underline{r})],t)$, where $u(\underline{r})$ is the fluid velocity at $\underline{r}$. For the linear case this satisfies an equation of the Fokker-Planck form. The non-linear interaction among the wave number modes is assumed to be comprised of two parts, representing diffusion into a particular wave number from other regions of wave number space and a dynamical friction extracting energy from this mode to other modes; thus the Fokker-Planck form is retained. Equations are then derived for these two quantities, by an expansion in 'degrees of randomness'. This approach leads to a basic auto-correlation function of exponential form; this is not satisfactory for small and large times, but it is expected that the detailed behaviour of the auto-correlation function will not affect the behaviour in wave number space significantly. This defect has led to a Lagrangian approach (EDWARDS, 1964, Plasma Physics) which discusses the more realistic probability functional $P([u(r,t)])$ and the equation it satisfies.

We will study a system consisting of a uniform externally applied magnetic field and a uniform viscous conducting fluid; however, some specifically plasma effects will be considered later (e.g. Hall effect, tensor conductivity, etc.) and described by the two-fluid equations.

We do not consider Landau damping. In the application to our experimental situation the applied field is strong compared to the fluctuating magnetic and velocity fields; the system is subsonic i.e. $u \lesssim c_o$ where u is the r.m.s. velocity fluctuation and c_o the sound speed so that we can start by assuming incompressibility, and the plasma pressure is small, $\beta = 8\pi P/B^2 \ll 1$ where P is the plasma pressure and B the total magnetic field strength. Disturbances in such a situation are naturally close to two dimensional. This can be seen from equation (1) where there is no term to balance those involving B_o if $\frac{\partial}{\partial r_3} \sim \frac{\partial}{\partial r_{1,2}}$.

The incompressible MHD equations in the presence of a uniform applied field B_o (in the 3 direction) are

$$\frac{\partial u^\alpha}{\partial t} + u^\beta \frac{\partial}{\partial r^\beta} u^\alpha = - \frac{1}{\rho} \frac{\partial}{\partial r^\alpha} \left(P + \frac{b^\beta b^\beta}{8\pi} + \frac{B_o b^3}{4\pi} \right) + \frac{b^\beta}{4\pi\rho} \frac{\partial}{\partial r^\beta} b^\alpha$$

$$+ \frac{B_o}{4\pi\rho} \frac{\partial}{\partial r^3} b^\alpha + \frac{F^\alpha}{\rho} + \nu \nabla^2 u^\alpha$$

$$\frac{\partial b^\alpha}{\partial t} + u^\beta \frac{\partial}{\partial r^\beta} b^\alpha = b^\beta \frac{\partial}{\partial r^\beta} u^\alpha + B_o \frac{\partial}{\partial r^3} u^\alpha + \nu_m \nabla^2 b^\alpha \qquad \ldots \ (1)$$

where ρ is the mass density, P the pressure, ν the kinematic viscosity, ν_m the magnetic viscosity $\equiv \frac{c^2}{4\pi\sigma}$ where σ is the conductivity, and indices α and β run from 1 to 3. The magnetic field will be expressed in velocity units, $h = \frac{b}{\sqrt{4\pi\rho}}$, and as we are considering a homogeneous case we will use fourier notation. F is an external random force introduced to maintain a stationary turbulent system. In a real system it represents the input of energy due to instability, for example, in a discharge from interchange modes for which the driving force is the pressure gradient, or for flow down a tube, from the mean velocity shear (ALLEN, 1963). Because we do not know what

properties to assign to F, the only significant results are those that are essentially independent of the details of its spectrum. In addition we assume that the principal source term is purely mechanical.

The reader who is not interested in the mathematical development of this theory, but only the results, can now turn to equation (12) which gives the result for the partition of energy in terms of the basic fields, lengths and turbulent coefficients.

We require the probability of the system described by (1) having values of $u_{\underline{k}}$, $h_{\underline{k}}$ at time t, and for a particular system this probability will be a δ function. To obtain the average behaviour of an ensemble of systems we average with respect to the random force i.e.

$$\langle P(u_{\underline{k}}, h_{\underline{k}}, t, [F_{\underline{k}}]) \rangle = \int \cdots \int P(u_{\underline{k}}, h_{\underline{k}}, t, [F_{\underline{k}}]) \, P([F_{\underline{k}}(t)]) \pi_{\underline{k}} \delta F_{\underline{k}}$$

$$\ldots (2)$$

where

$$P(\ldots u_{\underline{k}} \ldots, \ldots h_{\underline{k}} \ldots, t) = \pi_{\underline{k}} \delta (u_{\underline{k}} - U_{\underline{k}}(t)) \, \delta(h_{\underline{k}} - h'_{\underline{k}}(t))$$

$\underline{k}$ here is the fourier index i.e.

$$U(\underline{r}) = \frac{1}{v} \sum_{\underline{k}} e^{i\underline{k}\cdot\underline{r}} \, u_{\underline{k}}$$

v being the volume of the system. A formal solution for $u_{\underline{k}}$, $h_{\underline{k}}$ can be obtained from (1), for example $u_{\underline{k}}$ is given by

$$
\begin{aligned}
u_{\underline{k}}{}^{\alpha}(t) =\ & \frac{1}{\lambda-\mu} \int_{0}^{t} [(\nu_m k^2 + \lambda) \, e^{\lambda(t-\tau)} - (\nu_m k^2 + \mu) \, e^{\mu(t-\tau)}] \, K_{\underline{k}}{}^{\alpha}(\tau) d\tau \\
& + \frac{1}{\lambda-\mu} \int_{0}^{t} (e^{\lambda(t-\tau)} - e^{\mu(t-\tau)}) \, iH_0 \, K_3 \, G_{\underline{k}}{}^{\alpha}(\tau) d\tau \\
& + [(\nu_m k^2 + \lambda) \, e^{\lambda t} - (\nu_m k^2 + \mu) \, e^{\mu t}] \, \frac{u_{\underline{k}}{}^{\alpha}(o)}{\lambda - \mu} \\
& + \frac{iH_0 \, K_3}{\lambda - \mu} \, h_{\underline{k}}{}^{\alpha}(o) \, (e^{\lambda t} - e^{\mu t}) \qquad \ldots (3)
\end{aligned}
$$

where

$$K_{\underline{k}}^{\alpha} = \sum_{\underline{j},\underline{l}}^{\Delta} M_{\underline{k}}^{\alpha}{}_{\underline{j}}^{\beta}{}_{\underline{l}}^{\gamma} (u_{\underline{j}}^{\beta} u_{\underline{l}}^{\gamma} - h_{\underline{j}}^{\beta} h_{\underline{l}}^{\gamma}) + F_{\underline{k}}^{\beta} D_{\underline{k}}^{\alpha\beta}/\rho$$

$$G_{\underline{k}}^{\alpha} = \sum_{\underline{j},\underline{l}}^{\Delta} L_{\underline{k}}^{\alpha}{}_{\underline{j}}^{\beta}{}_{\underline{l}}^{\gamma} u_{\underline{j}}^{\beta} h_{\underline{l}}^{\gamma} \quad , \quad D_{\underline{k}}^{\alpha\beta} = \delta^{\alpha\beta} - k^{\alpha}k^{\beta}/k^2$$

$$M_{\underline{k}}^{\alpha}{}_{\underline{j}}^{\beta}{}_{\underline{l}}^{\gamma} = -\frac{i}{2\nu} (k^{\gamma} D_{\underline{k}}^{\alpha\beta} + k^{\beta} D_{\underline{k}}^{\alpha\gamma})$$

$$L_{\underline{k}}^{\alpha}{}_{\underline{j}}^{\beta}{}_{\underline{l}}^{\gamma} = \frac{i}{\nu} (j^{\gamma} \delta^{\alpha\beta} - l^{\beta} \delta^{\alpha\gamma}).$$

λ and μ are the roots of $X^2 + (\nu + \nu_m)k^2 X + (H_o^2 k_3^2 + \nu\nu_m k^4) = 0$.
The summations $\overset{\Delta}{\Sigma}$ are such that $\underline{k} = \underline{j} + \underline{l}$. If (3) and the expression for $h_{\underline{k}}(t)$ are inserted into (2), using the fourier representation of the delta function i.e. $\int_{-\infty}^{\infty} e^{i\lambda x} d\lambda = \delta(x)$ we can obtain $\langle P \rangle$ and the equation it satisfies (ROBINSON, 1966). We are interested in the stationary situation and take the limit $t \to \infty$ to obtain a time independent averaged P. The result is that the random force introduces a diffusive second derivative in the Liouville equation for the averaged P, i.e. it yields the Fokker-Planck equation for a random process. The exact form of the equation depends on the auto-correlation of the random force and the simplest and least limiting assumption is that the fluctuations are instantaneous i.e. the correlation time is much shorter than that associated with the velocity correlation function; hence we use a δ function (or an exponential with time constant smaller than all other natural periods appearing). The equation then takes the form

$$\sum_{k} \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial h_{\underline{k}}^{\alpha}} \left(\nu_m k^2 h_{\underline{k}}^{\alpha} - iH_0 k_3 u_{\underline{k}}^{\alpha} - \overset{\Delta}{\underset{j,\underline{l}}{\sum}} L_{\underline{k}\,\underline{j}\,\underline{l}}^{\alpha\ \beta\ \gamma} u_{\underline{j}}^{\beta} h_{\underline{l}}^{\gamma} \right) \right.$$

$$- \frac{\partial}{\partial h_{\underline{k}}^{\alpha}} \frac{iH_0 k_3 \gamma_{\underline{k}}^{\alpha'\,\beta'} D_{\underline{k}}^{\alpha\alpha'} D_{\underline{k}}^{\beta\beta'}}{2(\nu + \nu_m)k^2} \frac{\partial}{\partial u_{\underline{k}}^{\beta}}$$

$$- \frac{\partial}{\partial u_{\underline{k}}^{\alpha}} \frac{iH_0 k_3 \gamma_{\underline{k}}^{\alpha'\,\beta'} D_{\underline{k}}^{\alpha\alpha'} D_{\underline{k}}^{\beta\beta'}}{2(\nu + \nu_m)k^2} \frac{\partial}{\partial h_{-\underline{k}}^{\beta}} + \frac{\partial}{\partial u_{\underline{k}}^{\alpha}} \left(\frac{\gamma_{\underline{k}}^{\alpha'\,\beta'} D_{\underline{k}}^{\alpha\alpha'} D_{\underline{k}}^{\beta\beta'}}{2} \frac{\partial}{\partial u_{-\underline{k}}^{\beta}} + \nu k^2 u_{\underline{k}}^{\alpha} - \right.$$

$$\left. - iH_0 k_3 h_{\underline{k}}^{\alpha} - \overset{\Delta}{\underset{j,\underline{l}}{\sum}} M_{\underline{k}\,\underline{j}\,\underline{l}}^{\alpha\ \beta\ \gamma} \left(u_{\underline{j}}^{\beta} u_{\underline{l}}^{\gamma} - h_{\underline{j}}^{\beta} h_{\underline{l}}^{\gamma} \right) \right] \langle P \rangle = 0 \qquad \ldots (4)$$

where $\gamma_{\underline{k}}^{\alpha\beta} = \langle F_k^{\ \alpha} F_{-k}^{\ \beta} \rangle$, the input correlation tensor. The factor ½ in the diffusive terms is a consequence of our δ function approximation; some other factor would appear for a different auto-correlation function. If we consider a more general input tensor, allowing for magnetic sources and their possible correlation, then additional diffusive terms will appear, of the form

$$\frac{\partial}{\partial h_{\underline{k}}} \frac{\mathcal{G}_k}{2} \frac{\partial}{\partial u_{\underline{k}}} \quad , \quad - \frac{\partial}{\partial u_{-\underline{k}}} \frac{\nu}{\nu + \nu_m} \mathcal{H}_k \frac{\partial}{\partial h_{\underline{k}}} \quad , \quad \frac{\partial}{\partial h_{\underline{k}}} \frac{\nu_m}{\nu + \nu_m} \mathcal{H}_{\underline{k}} \frac{\partial}{\partial u_{\underline{k}}}$$

where $\mathcal{G}_k = \langle G_k G_{-k} \rangle$, $\mathcal{H}_k = \langle F_k G_{-k} \rangle$ and G_k is the magnetic source term. The $\mathcal{H}_k$ represents the source term in the equation for the cross correlation $\langle u_k h_{-k} \rangle$.

The non-linear terms neither create nor destroy energy, but the magnetic non-linear terms transfer energy between the velocity and magnetic fields, leading to the appearance of random fields. We assume that the non-linear terms can be considered as a random force representing the non-linear input to, together with a dynamical friction representing the loss from, each mode (EDWARDS, 1964, J. Fluid Mech.). On this basis we would expect (4) to be replaced by an

equation of the form

$$\sum_{\alpha,\beta} \left\{ \frac{\partial}{\partial t} + \frac{\partial}{\partial u_k^\alpha} \left(d_k'^{\alpha\beta} \frac{\partial}{\partial u_{-k}^\beta} + w_k^{1\alpha\beta} u_k^\beta + \omega_k^{1\alpha\beta} h_k^\beta \right) + \frac{\partial}{\partial u_k^\alpha} d_k^{2\alpha\beta} \frac{\partial}{\partial h_{-k}^\beta} \right.$$

... (5)

$$\left. + \frac{\partial}{\partial h_k^\alpha} d_k^{2'\alpha\beta} \frac{\partial}{\partial u_{-k}^\beta} + \frac{\partial}{\partial h_k^\alpha} \left(d_k^{3\alpha\beta} \frac{\partial}{\partial u_{-k}^\beta} + w_k^{2\alpha\beta} h_k^\beta + \omega_k^{2\alpha\beta} u_k^\beta \right) \right\} \langle P_k \rangle = 0.$$

The diffusive coefficient d_k^1 and d_k^3 gives the input into the com-
ponents u_k, $\underline{u}_k$ and h_k, $\underline{h}_k$ respectively, arising from the combina-
tion of the external input and that arising from all other components
of u and h. The w_k^1 give the total output due to dissipative
losses and the output to all other u and h modes. The ω_k^1 are
different in that they transfer output to other u or h modes only;
these are effectively the wave frequencies which couple together a
particular u and h mode. The d_k^2 can be looked upon as represen-
ting the input arising from other modes in the equation governing the
cross correlation $\langle u_k \underline{h}_k \rangle$.

As our system is homogeneous the basic correlations possess
Hermitian symmetry

$$Q^{ij}(\underline{k}) = \langle u_{\underline{k}}^i \underline{u}_{-\underline{k}}^j \rangle = Q^{ji^*}(\underline{k}) \quad , \quad H^{ij}(\underline{k}) = \langle h_{\underline{k}}^i \underline{h}_{-\underline{k}}^j \rangle \quad \ldots (6)$$

however

$$F^{ij}(\underline{k}) = \langle u_{\underline{k}}^i \underline{h}_{-\underline{k}}^j \rangle = - F^{ji^*}(\underline{k})$$

is anti-Hermitian. (5) plainly possesses a solution of the form

$$P(u_k, h_k) = N \exp \left\{ -\frac{1}{2} \sum \left(a_k^{ij} u_k^i \underline{u}_{-k}^j + 2b_k^{ij} u_k^i \underline{h}_{-k}^j + c_k^{ij} h_k^i \underline{h}_{-k}^j \right) \right\} \ldots (7)$$

where N is a normalising factor. By forming the $Q_{\underline{k}}$'s, $H_{\underline{k}}$'s, $F_{\underline{k}}$'s
we obtain relations between the basic correlations and the $a_{\underline{k}}$, $b_{\underline{k}}$,
$c_{\underline{k}}$'s. Inserting (7) into (5) allows us to obtain the following four

equations

$$d_k^{1\alpha\beta} = w_k^{1\alpha j} Q_k^{j\beta} + \omega_k^{1\alpha i} F_{-k}^{i\beta}$$

$$d_k^{3\alpha\beta} = w_k^{2\alpha i} H_k^{i\beta} + \omega_k^{2\alpha i} F_k^{i\beta}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad \dots (8)$$

$$d_k^{2\alpha\beta} = w_k^{1\alpha i} F_k^{i\beta} + \omega_k^{1\alpha i} H_k^{i\beta}$$

$$d_k^{2'\alpha\beta} = w_k^{2\alpha i} F_{-k}^{i\beta} + \omega_k^{2\alpha i} Q_k^{i\beta} .$$

If we multiply (5) by $u_{\underline{k}}^i(t)$ on the left and $u_{-\underline{k}}^j(t')$ on the right
and average over $u_{\underline{k}}(t)$, $h_{\underline{k}}(t)$, $u_{\underline{k}}(t')$, $h_{\underline{k}}(t')$ – we obtain an equation
for the velocity auto-correlation

$$\frac{\partial}{\partial\tau} Q_{\underline{k}}^{ij}(\tau) + w_{\underline{k}}^{1i\beta} Q_{\underline{k}}^{\beta j}(\tau) + \omega_{\underline{k}}^{1i\beta} F_{-\underline{k}}^{\beta j}(\tau) = 0$$

– similarly we can obtain three other equations of the same form. The
auto-correlation functions are therefore exponentials and when the
w_k are diagonal the relevant time constants are given by the roots of

$$X^2 + (w_k^1 + w_k^2) X + (-\omega_k^1 \omega_k^2 + w_k^1 w_k^2) = 0$$

which reduce to the roots λ and μ of (3) in the linear limit.
Consider the lifetime of an 'eddy' which is small so that the local
Reynolds number is of order unity. If $H_0 k_3$ is not large and
$\nu_m \gg \nu$ then the lifetime is of order

$$\left(\nu k^2 + \frac{H_0^2 k_3^2}{\nu_m k^2} \right)^{-1} .$$

This implies that a turbulent eddy with a small scale in the field
direction (large k_3) will be damped out relatively rapidly and thus
the final state of decay will be approximately two-dimensional
(LEHNERT, 1955).

The $w_{\underline{k}}$ can be written in the form

$$w_{\underline{k}}^{1\alpha\beta} = \nu_k^{\ 2}\delta^{\alpha\beta} + R_k^{1\alpha\beta} \quad , \quad w_k^{2\alpha\beta} = \nu_m k^2\,\delta^{\alpha\beta} + R_{\underline{k}}^{2\alpha\beta} \quad , \quad \omega_{\underline{k}}^{\alpha\beta} = -\,iH_o k_3 \delta^{\alpha\beta} + R_{\underline{k}}^{\alpha\beta}$$

$$\cdots (9)$$

The $R_{\underline{k}}$'s represent specifically turbulent contributions to the eddy frequencies w and $R_{\underline{k}}^1/k^2$ is effectively a turbulent viscosity. We can justify the assumption that $\omega_k^1 = \omega_k^2 = \omega_k$ in terms of the equal transfer of energy to and from the velocity and magnetic fields by the applied field terms. The $d_{\underline{k}}$ are given by

$$d_k^{1\alpha\beta} = I_k^{\alpha\beta} + S_k^{1\alpha\beta} \quad , \quad d_{\underline{k}}^{3\alpha\beta} = S_{\underline{k}}^{3\alpha\beta} \quad , \quad d_{\underline{k}}^{2\alpha\beta} = S_{\underline{k}}^{2\alpha\beta}$$

where the $S_{\underline{k}}$ represent effective inputs due to the action of the non-linear terms, and I_k a physical input into the velocity field. Using (8) we can now obtain an expression for the partition of energy between the two fields. As our system is axisymmetric the tensor form of $Q_k^{\alpha\beta}$ and $d_k^{\alpha\beta}$ is (ROBINSON, 1966)

$$Q^{ij}(\underline{k}) = A(k,k_3)\left(\frac{k^i k^j}{k^2} - \delta^{ij}\right) + c(k,k_3)\left[\delta^{ij}\,\frac{k_3^2}{k^2} + \lambda^i\lambda^j - \frac{k_3}{k^2}(\lambda^i k^j + \lambda^j k^i)\right]$$

where λ is a unit vector in the direction of symmetry. $W_k^{\alpha\beta}$ is diagonal but with different elements. Using these properties we obtain

$$\frac{Q_k^{ii}}{H_k^{ii}} = \frac{w_k^{2i\alpha}d_k^{1\alpha i} - \omega_k^{i\alpha}d_k^{2'\alpha i}}{w_k^{1i\alpha}d_k^{3\alpha i} - \omega_k^{i\alpha}d_k^{2\alpha i}} \quad . \qquad \cdots (10)$$

It is difficult to obtain the closed equations for R_k and S_k in such a system (ROBINSON, 1966); however, we can make an estimate of all the above quantities in the energy containing region of the turbulent spectrums. This is the region where $K \sim 1/L$ where L is the correlation length. In this region where we only have a direct input

to the velocity field, $I_{\underline{k}} \gg S_{\underline{k}}$, and we obtain from (10)

$$\frac{Q_k^{ii}}{H_k^{ii}} = 1 + \frac{w_k^2(w_k^1 + w_k^2)}{-\omega_k \omega_k} \geqslant 1 \qquad \ldots (11)$$

where the w_k are the transverse components. We see that in this approximation the energy residing in the velocity field is always greater than or equal to that in the magnetic field. In addition if the applied field is made very strong and the system cannot tend to the two-dimensional limit at the same rate as the field is increased (perhaps due to the finite length of the apparatus) then

$$Q_k^{ii}/H_k^{ii} \rightarrow 1 \ .$$

This represents a system of weakly interacting Alfvén waves. If the system is close to two-dimensional then there is virtually no transfer of energy to the magnetic field by the non-linear term in the velocity equation arising from the magnetic force term, nor is there much transfer due to the non-linear term in the induction equation, hence we expect w_k^2 and ω_k to have approximately their linear values $\nu_m k^2$ and $- iH_o k_3$, so the 'turbulent conductivity' component is small. The nett transfer of energy by these two non-linear terms can be shown to vanish in the two-dimensional limit. As $w_k^1 = \nu k^2 + R_k$ and for the above region of wave number space, the turbulent frequency R_k is approximately u/L; or we can consider an 'eddy viscosity' $R_k/k^2 \sim uL$, which we call $\tilde{\nu}$. Thus (11) becomes

$$\frac{\overline{U_k^2}}{\overline{h_k^2}} \left(k \sim \frac{1}{L} \right) = \frac{H_o^2 k_3^2 + \nu_m k^2 (\tilde{\nu} + \nu_m) k^2}{H_o^2 k_3^2} \ . \qquad \ldots (12)$$

It is possible to obtain a result similar to this by a linear calculation (ROBINSON, 1966), but the above calculation shows how the

expression is modified by non-linear effects. The major contribution to the energy in the two fields arises from this small wave number region so that measured ratios will not be sensitive to the value at higher wave numbers (unless we frequency discriminate) where a possible inertial range might exist and basic theories can predict a value for the partition (KRAICHNAN, 1965).

If we consider the two-dimensional limit $\omega_k^2 \ll w_k^1 w_k^2$ then the frequency spectrum of the velocity field is determined by w_k^1 as

$$Q_k^{\alpha\beta}(w) = \frac{I_k^{\alpha\beta}(w)}{w^2 + w_k^{12}} \quad (I_k^{\alpha\beta}(w) = I_k^{\alpha\beta} \text{ for the delta function input}).$$

We can therefore consistently approximate w_k^1 by the characteristic frequency of the velocity fluctuations, this frequency is observed to be of the same order as that given by a turbulent viscosity, $\sim u/L$. Similarly for the magnetic field spectrum we have

$$H_k^{\alpha\beta}(w) = \frac{I_k^{\alpha\beta}(w) \, \omega_k^2}{(w^2 + w_k^{12})(w^2 + w_k^{22})}$$

so that if $w_k^2 < w_k^1$ the characteristic frequency is w_k^2 and could be used to obtain w_k^2. Thus at high frequencies $Q_k \sim w^{-2}$, $H_k \sim w^{-4}$. In the wave limit $\overline{U^2} \to \overline{h^2}$, the above considerations are not valid.

A diffusion coefficient associated with the turbulence can be calculated from

$$D_{\underline{k}} = \tfrac{1}{2} \, Q(k, w)\Big|_{w \to 0}$$

which in the limit of $\overline{u^2}/\overline{h^2} \gg 1$ gives $\sim u_{\underline{k}}^2/w_k^1$ and so $D \sim u_\perp L$. In the opposite limit the diffusion coefficient is considerably reduced as the plasma is frozen to the lines of force (effectively $\nu_m \to 0$). Thus the simple calculation of diffusion as a random walk (GIBSON and MASON, 1962) is valid only if $\overline{u^2}/\overline{h^2} \gg 1$; otherwise the observed displacements do not represent steps of a random walk.

We can obtain some estimates of the effect of departures from the basic equations (1) on the partition value (12) by solving the problem in the linear limit and using the above ideas to extend the expressions to a turbulent limit.

The linear Hall effect arises from a coupling of the fluctuating current density with the mean applied field. The equations to be solved are

$$\frac{\partial h_{\underline{k}}^{\alpha}}{\partial t} + \nu_m k^2 h_{\underline{k}}^{\alpha} = iH_o k_3 u_{\underline{k}}^{\alpha} + \sqrt{\frac{mc^2}{4\pi ne^2}}\; H_o k_3 \varepsilon^{\alpha\beta\gamma} k^{\beta} h_{\underline{k}}^{\gamma}$$

$$\dots (13)$$

$$\frac{\partial u_{\underline{k}}^{\alpha}}{\partial t} + \nu k^2 u_{\underline{k}}^{\alpha} = iH_o k_3 h_{\underline{k}}^{\alpha} + F_{\underline{k}}^{\lambda} D_{\underline{k}}^{\lambda\alpha}/\rho$$

where m is the ion mass, $\varepsilon^{\alpha\beta\gamma}$ the Levi-Civita symbol and $\sqrt{\frac{mc^2}{4\pi ne^2}} = R_s$ is the ion collisionless skin depth. To solve these equations for the stationary correlation functions of the fields produced by the random force involves a considerable amount of algebra and only the results will be quoted. In the wave limit when $H_o^2 k_3^2 \gg \nu\nu_m k^4$ we obtain

$$\frac{\overline{u_{\underline{k}}^2}}{\overline{h_{\underline{k}}^2}} = 1 + \frac{\nu_m}{\nu + \nu_m} k^2 R_s^2 \qquad \dots (14)$$

and in the opposite limit the previous result holds. Consideration of the non-linear terms again suggests that $\nu k^2 \to w^1 k$, $\nu_m k^2 \to w^2 k$.

Compressibility can also introduce modifications to the expressions, but in the limit when k_3 is small the principal effect is to produce a parallel component of the magnetic field fluctuations, which we do not consider as our experimental results only give the partition for transverse fields (the full value is unlikely to differ significantly from the transverse). These calculations are most

useful in predicting the size of the density fluctuations (ROBINSON and RUSBRIDGE, 1966). We can also consider pressure tensor effects as $w_i \tau_i$ can be large (w_i is the ion cyclotron frequency and τ_i the ion-ion collision time). Again the algebra is complicated but the final result is that there is no radical alteration to (12) except that if k_3 is small, ν should be taken to have its perpendicular value (MARSHALL, 1957)

$$\nu = \frac{\mu}{\rho \left(1 + \frac{16}{9} w_i^2 \tau_i^2 \right)} . \qquad \ldots (15)$$

We can also consider the effect of inhomogeneity and in particular the effect of a magnetic field possessing shear e.g. $\underline{B} = (o, By, Bo)$ and the effective shear length is given by $L_s = Bo/\partial By/\partial x$. In this case we find that if k_3 is small, $1/k_3$ is replaced in the expression for the partition by a length related to the shear length.

3. EXPERIMENT

Measurements of turbulence were carried out in the Zeta discharge (BUTT et al., 1958; BURTON et al., 1962; GIBSON and MASON, 1962) at a gas current of 150 kA, applied magnetic field of 370 gauss and filling pressure between 0·5 and 5 mtorr D_2. Radial magnetic fields were measured using small search coils of 50 turns with high frequency compensated Miller integrators, with the integrating resistor mounted close to the coil. The overall frequency response was about 1 MHz and the sensitivity about 1 mvolt/gauss. The coils were placed in a 7/8" diameter quartz tube silvered on the inside, that served as a vacuum envelope. Electric fields were measured by observing the voltage developed between two platinum pins, 4 mm long by 2 mm diameter

separation 1 cm. The frequency response in this case extends up to 500 kHz.

The variance of both these fields was measured by a long period integrator (LEES and RUSBRIDGE, 1965). It measures $\langle |b| \rangle$ rather than the variance, but provided the probability distribution of b is gaussian

$$\langle |b| \rangle = 0 \cdot 796 \sqrt{\langle b^2 \rangle} \ .$$

In measuring b a filter passing frequencies above 10 kHz was used, as it is known that this is sufficient to exclude the large scale motions of the entire discharge (RUSBRIDGE et al., 1962) leaving only the more localised turbulence. The signals were gated, the gate being open for 400 μs centered on peak current for 20 successive discharges, which is sufficient to reduce statistical errors to about ±5%, the exact error depending on the auto-correlation function.

The amplitude of the radial magnetic field fluctuations, b_r is found to be independent of radius over most of the discharge (Fig.1). The amplitudes of b_r and b_θ are equal within experimental error. The density fluctuations behave similarly to the b_r fluctuations (ROBINSON and RUSBRIDGE, to be published), which leads us to believe that there is a central core region where the turbulence is homogeneous and an edge region where loss and interaction with the walls are dominant processes. If the electric field $\underline{E}$ is derived solely from a potential φ, then if $\langle \varphi \varphi \rangle_r = \chi$ where our correlation function notation is defined by

$$\langle E_\theta \ (\underline{x}) \ E_\theta (\underline{x} + r) \rangle = \langle E_\theta \ E_\theta \rangle_r ,$$

then

$$\langle E_\theta \ E_\theta \rangle_\theta = - \frac{\partial^2 \chi}{\partial r^2} \ , \quad \langle E_\theta \ E_\theta \rangle_r = - \frac{1}{r} \frac{\partial \chi}{\partial r} \qquad \dots (16)$$

These relations are compared with experimental data in Fig.2 and give reasonable agreement after allowing for the imperfect correlation at small separations, which is a constant feature of our electric field measurements at low pressures. This may be due to small scale turbulence effects or some surface phenomena on the probes. Measurements of the two magnetic field correlation functions $\langle b_r b_r \rangle_r = f(r)$ and $\langle b_\theta b_\theta \rangle_r = g(r)$ are shown in Figs.3 and 4. For isotropic turbulence the condition div B = 0 requires that

$$g = f + \frac{r}{2} \frac{\partial f}{\partial r} \qquad \ldots (17)$$

while in the two-dimensional limit (the correlation length in the field direction is much greater than that transverse to it)

$$g = f + r \frac{\partial f}{\partial r} . \qquad \ldots (18)$$

The two relations are compared with experiment in Fig.4 using a form for f derived from Fig.3 and we see that (18) is a good fit to the experimental points. Note that Figs.2 and 3 both show transverse correlation lengths of about 5 cm. Attempts to measure the correlation length along the magnetic field are shown in Fig.5 and give estimates of 50-100 cm for this length. As the paths of lines of force are not known with certainty, and in any case fluctuate during the period of measurement we can only obtain a lower limit for this quantity. Using order of magnitude estimates for the terms in equation (1) and the known value of β, it can be shown that the parallel length cannot be less than 50 cm.

By considering a generalised Ohm's law we can show that the fluctuating electric field perpendicular to the mean magnetic field is related to perpendicular fluid velocity by $\overline{u_\perp^2} Bo^2 = \overline{E_\perp^2}$ to within 25% at the lowest filling pressures we have used, where the departures are

worst. We have confirmation of values for this velocity from spectro-
scopic measurements of Doppler broadening (JONES and WILSON, 1962),
and microwave scattering (WORT and HEALD, 1965). Using the known
experimental variation of this $u_\perp$ with filling pressure and the
calculated variations of ν and ν_m using known electron and esti-
mated ion temperature (RUSBRIDGE, to be published), we deduce the
variation of the ordinary and magnetic Reynolds numbers (R, R_m) of
the turbulence with filling pressure, shown in Fig.6. The Reynolds
numbers associated with the turbulence are large, of the same order
as those attained in laboratory fluid turbulence experiments
(BATCHELOR, 1960; GIBSON and SCHWARTZ, 1963; GRANT et al., 1962).

As we have already noted the correlation length along a field
line may be limited by the shear length $L_s^{-1} = \dfrac{\mu'}{\mu(1 + \mu^2\, r^2)}$ where
$\mu = \dfrac{B_\theta}{rB_z}$; except very near the axis of the discharge the shear length
is not greater than 150 cm. The most unstable hydromagnetic pertur-
bations are those whose wave numbers coincide with the inverse pitch
$(2\pi\mu^{-1})$ of the mean field configuration at the magnetic centre of the
plasma (WHITEMAN, 1965; ROBINSON, to be published). From experimen-
tal results this length is about 220 cm, and it is plausible that this
length determines the actual parallel correlation length $(\Lambda_\parallel)$, because
this perturbation could be one of the dominant ones driving the more
localised turbulence.

Calculated values of $\overline{u_\perp^2}/\overline{h_\perp^2}$ using (12) and the above estimates
of ν, ν_m are shown in Fig.7 for a particular $\Lambda_\parallel$. The lower curve 1
shows the partition as a function of pressure using the usual values
for the transport coefficients. Curve 2 shows the result of using
w_k^1 in place of the usual kinematic viscosity and 3 the effect of
also using w_k^2. This is less than w_k^1 from the measured frequency

spectra and has been taken to be a constant fraction of it. The effect of decreasing k_3 (increasing $\Lambda_\parallel$) is shown on curve 4. Curve 1 should be comparable with experiment at high pressures, while 2 and 3 are likely to be valid at low pressures. The effect of the Hall term on the partition as expressed in (14) is demonstrated in Fig.8, where we have used w_k^1 and w_k^2 in place of the usual dissipative coefficients.

The electric field fluctuations are known to scale as I^2/ρ (RUSBRIDGE et al., 1962) where I is the gas current and ρ the mass density, so the magnetic field fluctuation should vary as $I/(\rho Y)^{\frac{1}{2}}$, where $Y = \overline{u^2}/\overline{h^2}$ is the energy partition. Thus if $\sqrt{Y}$ does not vary strongly we expect approximately $h \sim I/\rho^{\frac{1}{2}}$. A summary of a number of results for the absolute radial magnetic field fluctuations demonstrates that this relationship is approximately valid, as shown in Fig.9.

The partition Y was determined in detail as a function of filling pressure for a fixed current of 150 kA in a D_2 discharge by measuring $(\overline{E_\theta^2})^{\frac{1}{2}}$ and $(\overline{b_r^2})^{\frac{1}{2}}$. The results are shown in Figs.10 and 11. Note that the ρ^{-1} and $\rho^{-\frac{1}{2}}$ variations do not hold at high pressures but nevertheless there is a strong correlation between the absolute values of the fluctuating quantities. From other methods of measurements of the turbulent velocity it is unlikely that $u_\parallel > u_\perp$ so that our measured partition is probably close to the total partition of energy. We note that the values range from 6·5 at the lowest pressures to 1·0 at the minimum, rising to 1·9 at the high pressure end. (It is interesting to note that these values are comparable with those obtained by Russian astronomers (KAPLAN, 1964) in the solar photosphere, where values of 5 to 10 were measured.)

Values for the partition have also been measured in Tiber (REYNOLDS et al., 1966), which is a linear pinch device. Here values in the region of unity have been observed. The operating pressure used in these measurements was higher than the pressures at which we have made measurements. Consequently as k_3 is restricted by the finite length of the machine we would expect values close to unity from our theory. The fluctuations in Tiber were considered to be of a magnetohydrodynamic type.

The variation of this partition with initial pressure can be explained by our model for pressure $\gtrsim 3 \cdot 5$ mtorr, for measurable values of k_3. At low pressures we only obtain agreement if in addition to using w_k^1 we use w_k^2 which is a sizeable fraction of w_k^1, and even then the k_3's are rather small compared with our suggested lower limits. Reference to Fig.8 suggests that the Hall effect cannot give the correct size, but does give a more correct pressure variation for the partition.

An anomalously high ν_m could account for our results at these lower pressures; this might arise from some non-hydromagnetic effect (e.g. electron runaway). If plasma motion across the magnetic field followed the Bohm law microscopically, i.e. within each individual eddy, rather than as a macroscopic average over eddies, the effect would be to increase the effective value of ν_m sufficiently to explain the results.

In a homogeneous system, high values of the partition might arise either from a high ν_m or from a small value of k_3. However, as shown above, in the presence of shear the maximum valve of the partition is obtained for $k_3 \approx 1/L_s$. no further increase occurs if k_3 is

reduced below this value. High values of the partition can therefore only arise if ν_m is large; and with our parameters ν_m must be appreciably larger than the value corresponding to the Spitzer resistivity.

The minimum observed in the fluctuations, Fig.10, may reasonably be regarded as the minimum predicted by the theoretical model. This arises because at the higher pressures the Reynolds number is relatively small and if viscosity is the principal damping mechanism then the intensities will be proportional to its inverse. The perpendicular component has a maximum when $w_i \tau_i \approx 1$ which for our parameters occurs at about 6 mtorr of D_2. Hence this may account for our observed minimum. No such minimum has been observed in discharges in neon where we believe $w_i \tau_i \ll 1$ over the whole pressure range of observation.

The radial variation of $(\overline{E_\theta^2})^{\frac{1}{2}}$ has been measured and falls continuously from the centre. If we combine this variation with that of the total field strength then we can estimate the velocity fluctuations as a function of radius; with the result that these are homogeneous out to some 35 cm (c.f. Fig.1). The dynamics of the edge region may determine the integral scale

$$\Lambda = \int_0^\infty f(r)dr . \qquad (\sim 5 \text{ cm})$$

For example, KADOMTSEV (1965) has suggested that by analogy with a turbulent jet one might expect $\Lambda \sim R/10$ where R is the radius of the tube; this gives about the right value and moreover Λ is very nearly independent of the discharge parameters (ROBINSON et al., to be published).

The cross correlation $F_{\underline{k}}^{ij}$ is an odd function of k_3 so that the spatial correlation of u and h vanishes for zero separation. The tensor properties of $F_{(r)}^{ij}$ are the same as those for $Q_{(r)}^{ij}$ except that the defining scalars are now odd functions of r. The derivative of the correlation in the direction of the applied field, taken in the limit of zero separation, yields the energy transfer term associated with the applied field. As $F_{(r)}^{ij}$ is odd in k_3 we can then show that $\langle u^1(\underline{x}) \, h^1(\underline{x} + r_1) \rangle = 0$. This has been confirmed experimentally by measuring $\langle E_\theta(\underline{x}) \, b_r(\underline{x} + r) \rangle$, where for separations in the range 3–7 cm we have obtained a value for the correlation of $- 0 \cdot 007 \pm 0 \cdot 031$, i.e. zero within the limits of error.

4. CONCLUSIONS

A theory has been developed which leads to an expression for the partition of energy between mechanical and magnetic modes in a turbulent magnetohydrodynamic system in the presence of a strong external field. The assumptions used in obtaining this expression have been justified by experimental observations. Alterations to this expression due to specific plasma effects have been obtained and are not very significant. The result can be interpreted in terms of a number of competing turbulent frequencies which characterise the two modes. In addition results are obtained for the frequency spectra and the diffusion coefficient.

A number of basic properties of the turbulent fluctuations in the Zeta device have been described, in particular it has been shown that the electric field correlation functions can be derived from a scalar potential and the magnetic field correlation functions correspond to a two-dimensional limit. The electric field measurements are used to

determine a turbulent velocity from which we have obtained estimates of the Reynolds numbers, which can be sufficiently large to be comparable with those obtained in laboratory fluid flow experiments.

Observations on the fluctuations give values for the partition in the region of unity for high operating pressures which can be explained by the theory. However at low pressures, where values for the partition of up to six are obtained, there is no agreement with the theory unless the plasma motion across the magnetic field follows a possible Bohm law microscopically and leads to an anomalous resistivity. Under these conditions values for the partition comparable with experiment are obtained.

5. ACKNOWLEDGEMENTS

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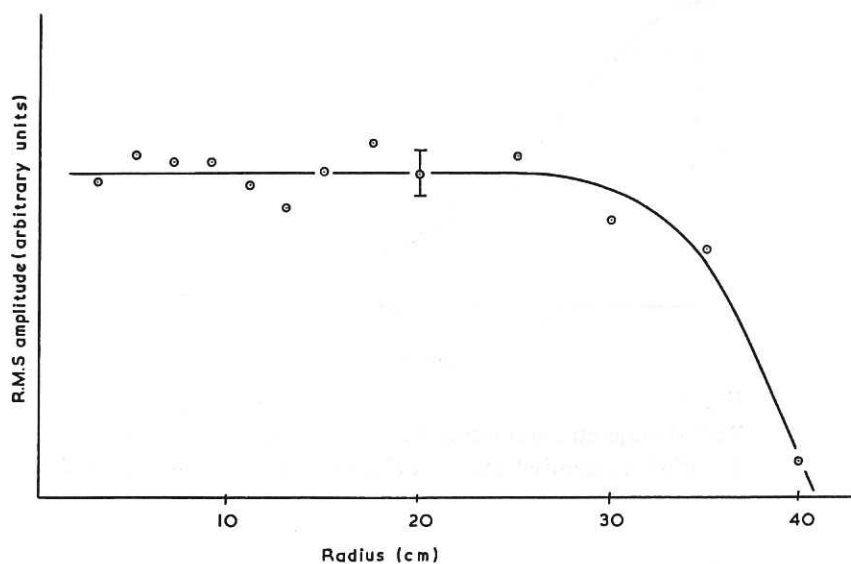
Fig. 1   (CLM - P 167)
Rms radial magnetic field fluctuations as a function of radius
0.5 m torr D_2, I = 150 kA, applied axial field 290 gauss
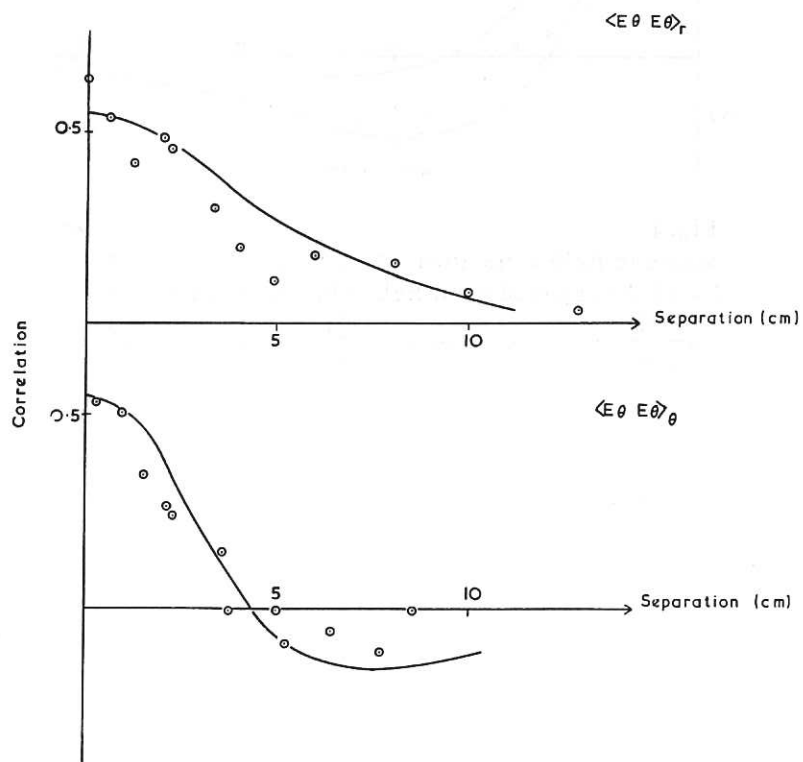
Fig. 2   (CLM - P 167)
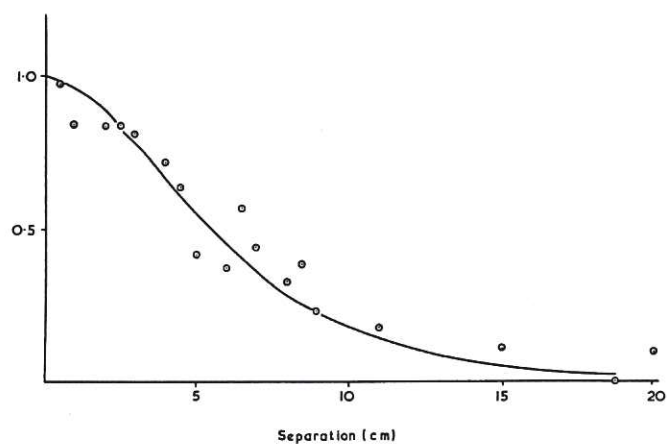Electric field correlation, ½ m torr D_2, I = 150 kA,
applied axial field 370 gauss

Fig. 3   (CLM-P 167)
Radial magnetic field correlation $<b_r\, b_r>_r$, ½ m torr D_2,
$I = 150\,kA$, applied axial field 370 gauss, —— sech$(x/4.2)$
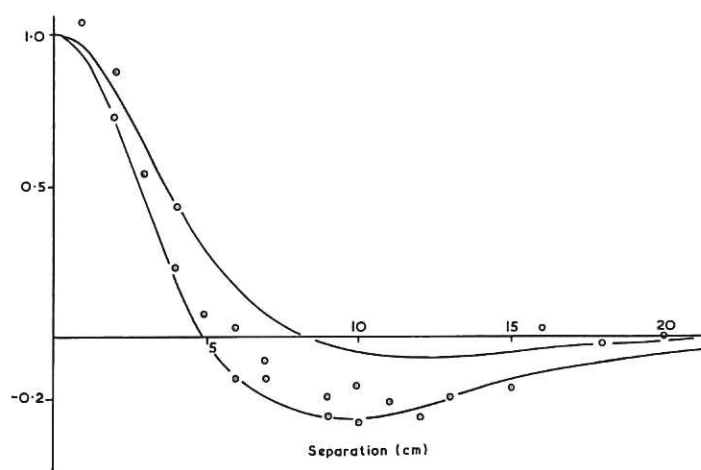
Fig. 4   (CLM-P 167)
Magnetic field correlation $<b_\theta\, b_\theta>_r$, 0.5 m torr D_2,
$I = 150\,kA$, applied axial field 370 gauss upper curve
$f + \dfrac{r}{2}\dfrac{\partial f}{\partial r}$ isotopic, lower $f + r\dfrac{\partial f}{\partial r}$ two dimensional
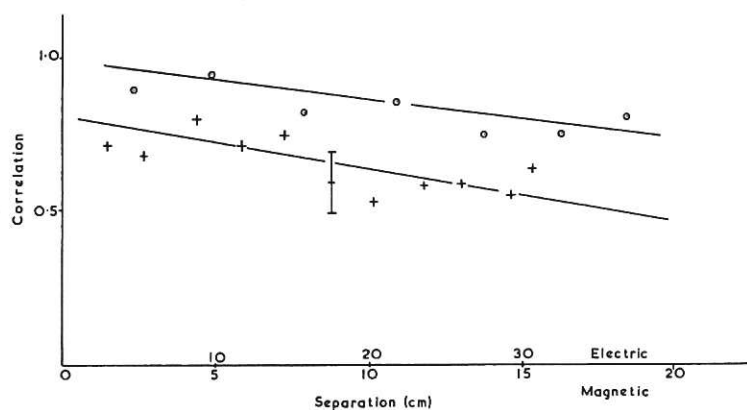
Fig. 5   (CLM-P 167)
Magnetic and electric field correlations in the axial direction
⊖ magnetic   + electric
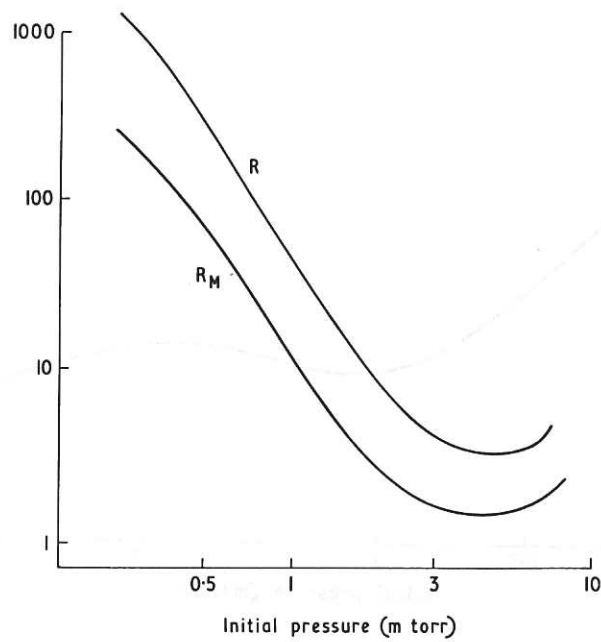
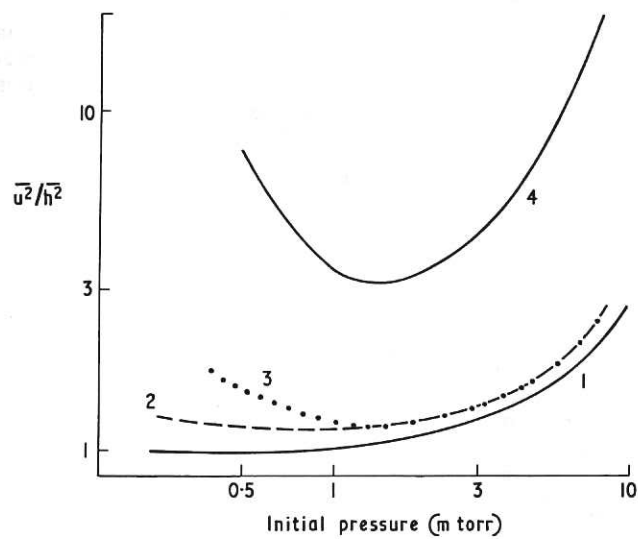Fig.6 Reynolds numbers as a function of initial pressure (CLM-P 167)

Fig.7 Partition of energy as a function of initial pressure (CLM-P 167)

1 —— $k_3 = 0.0067$, ν , ν_m 2 – – – $k_3 = 0.0067$, w_k^1 , ν_m

3 •••• $k_3 = 0.0067$, w_k^1 , w_k^2 4 —— $k_3 = 0.002$, w_k^1 , w_k^2
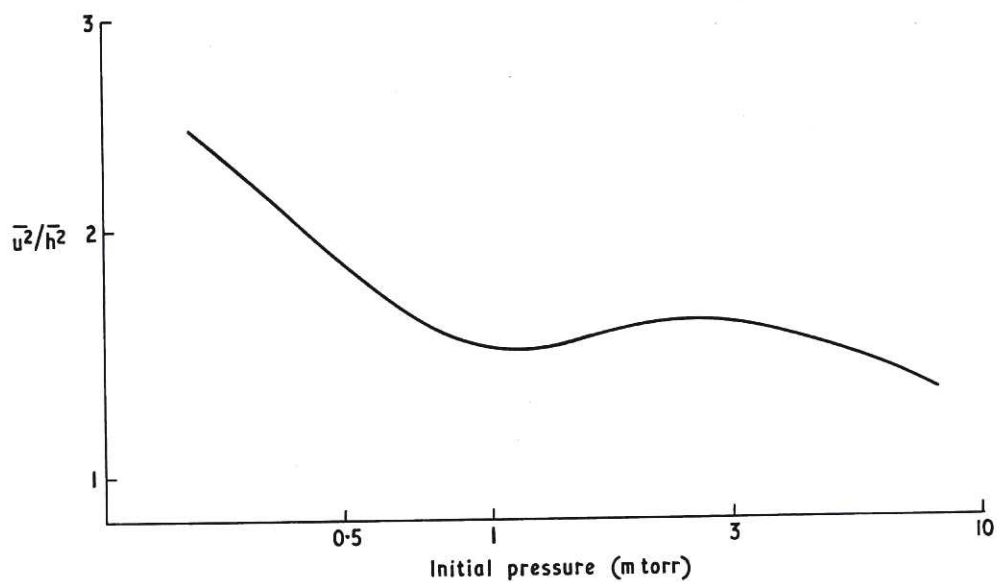
Fig. 8   (CLM - P 167)

Partition of energy for the Hall effect. Lower w_k^1, w_k^2

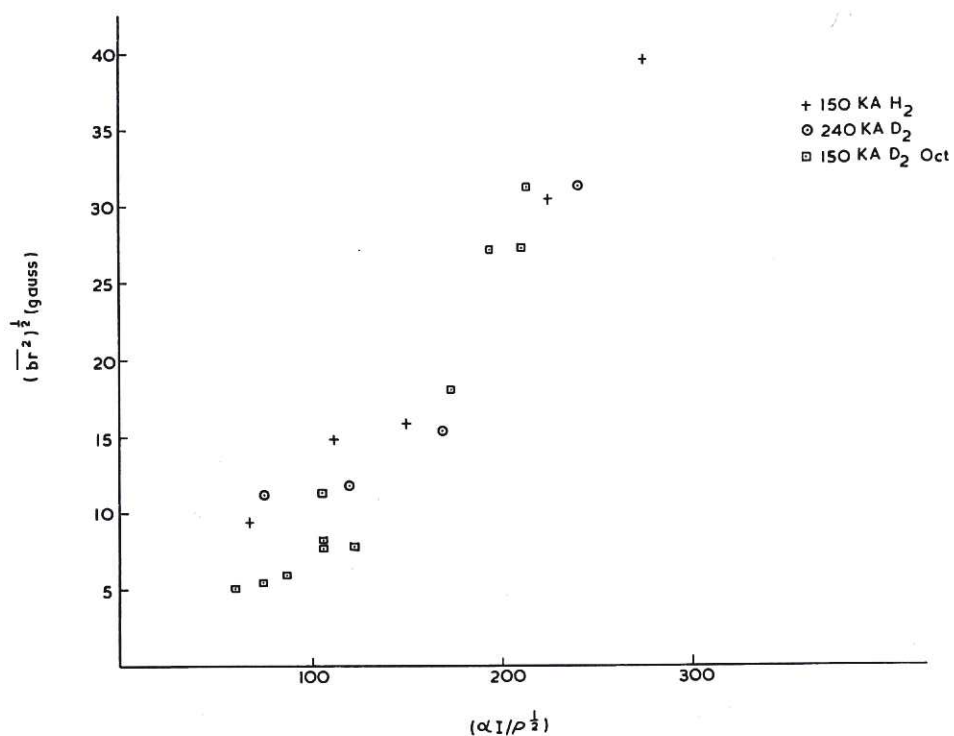Fig. 9   (CLM - P 167)

Variation of rms magnetic field fluctuations with $I/\rho^{1/2}$
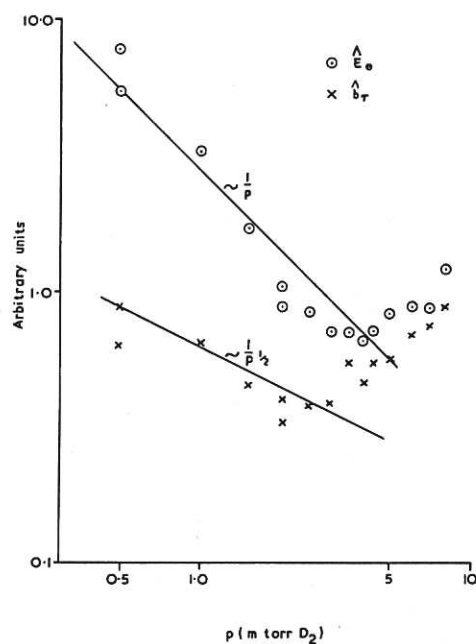
Fig. 10   (CLM - P 167)
Variation of rms electric field and radial magnetic field with
initial pressure $I = 150 \, kA$, axial field 370 gauss

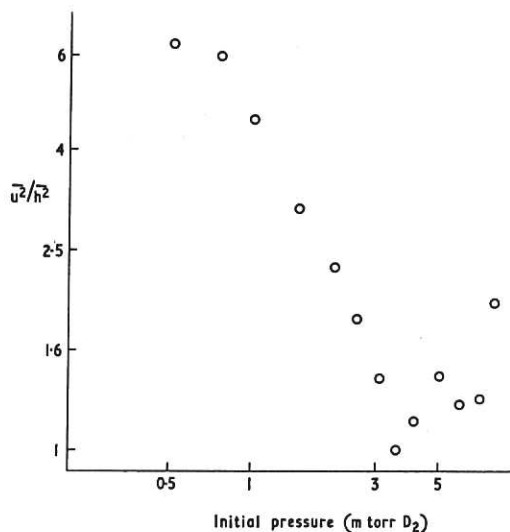Fig. 11   (CLM - P 167)
Partition of energy as a function of initial pressure,
$I = 150 \, kA$, applied axial field 370 gauss