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SINGLE PARTICLE CONFINEMENT STUDIES IN AN R.F. SUPPLEMENTED MAGNETIC MIRROR

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SINGLE PARTICLE CONFINEMENT STUDIES IN AN R.F. SUPPLEMENTED MAGNETIC MIRROR

by

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ABSTRACT

An experimental study of electron confinement in magnetic mirror geometry supplemented with r.f. quasi-potential barriers yields confinement times which appear to be a function of gas scattering only. Under minimum scattering conditions ($p \approx 5 \times 10^{-8}$ torr), 10-20 eV electrons are observed to undergo several thousand reflections before loss occurs. The magnitude of the quasi-potentials and the conditions for onset of non-adiabatic particle behaviour are found to be in good agreement with existing theory.

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1. INTRODUCTION

During the last decade numerous possibilities have been explored for the confinement of thermonuclear plasma by radio frequency fields. Careful examination of these proposals has shown that even aside from stability problems, the r.f. power requirements are so large as to preclude a net thermonuclear output(1). Indeed even the consideration of superconductors and low frequencies fails to implant any hope for success along these lines.

The picture appears rather more optimistic for hybrid combinations of static magnetic fields and r.f. fields, particularly for those open ended systems in which the bulk of the plasma pressure can be supported by the static magnetic field. We envisage long, high mirror-ratio mirror devices and cusps, for which the cross sectional area through which particles escape can be quite small relative to the total plasma area. In such machines the r.f. fields need exist therefore only over this reduced area resulting in considerable reduction in power requirements. Certainly in present day low β mirror machines the r.f. power requirements necessary to block up the loss cones as proposed by Johnston(2), are quite within reach. To block the loss of the cold plasma component resulting from charge exchange in beam injection systems is trivial in power and may offer considerable effect upon loss cone instabilities.

The theory of particle motion in an r.f. field and uniform magnetic field has evolved largely from the work of Miller(3) and for the r.f. plus non-uniform magnetic field, the work of Motz and Watson(4). If the applied r.f. frequency is above but near the cyclotron frequency in the vicinity of the mirrors, theory(1) indicates that the plasma pressure may be shared between the stationary magnetic field gradient and the r.f. field in which the major load is borne by the magnetic field. We are here concerned with such 'nearly resonant' conditions in which the applied r.f. frequency is near but slightly above the ion or electron cyclotron frequency. These are the conditions under which effective 'amplification' of the r.f. forces and corresponding power reduction are most attractive economically.

The salient features of the theory in regard to what follows is that, to order v/c, one can obtain approximate solutions of the equations of motion for particles under the influence of combined, spatially non-uniform, r.f. and stationary magnetic fields subject to certain adiabatic limits. These limits impose the following conditions. The vacuum wavelengths associated with the r.f. and cyclotron frequencies ω and ω_c must be less than or of the same order as the scale lengths of the r.f. and magnetic field non-uniformities L_ω and L_{ω_c}. Further L_ω and L_{ω_c} must be much greater than r_c, the particle gyroradius.

To order v/c the particle motion may, subject to the conditions above, be represented by the coordinate of the guiding center with superimposed small amplitude displacements of time dependences $e^{i\omega t}$ and $e^{i\omega_c t}$ at the r.f. and cyclotron frequencies. We represent the particle coordinate $\vec{r}$ by $\vec{r} = \vec{R} + \vec{\rho}_\omega + \vec{\rho}_{\omega_c}$ where $\vec{R}$ is the guiding center displacement and $\vec{\rho}_\omega$ and $\vec{\rho}_{\omega_c}$ are the terms mentioned above.

Three new adiabatic invariants emerge which describe the particle motion. They are:

$$\varepsilon_{rf} = \tfrac{1}{2} m \, (\vec{v} - \vec{v}_\omega)^2 + \psi \qquad \ldots (1)$$

$$\mu_{rf} = \tfrac{1}{2} m (\vec{v}_\perp - \vec{v}_{\perp\omega})^2/B_o \qquad \text{... (2)}$$

and in the case of mirror fields[1]

$$J_{rf} = \oint \left[\varepsilon_{rf} - \mu_{rf} B_o - \psi \right]^{\frac{1}{2}} ds \qquad \text{... (3)}$$

where $\vec{v}$ is the total velocity and $\vec{v}_\omega$ the component of velocity with frequency ω. $\tfrac{1}{2} m (\vec{v} - \vec{v}_\omega)^2$ is therefore the normal guiding center energy plus the energy associated with the cyclotron motion and ψ is analogous to an electrostatic potential. In terms of the r.f. electric field associated with a standing wave

$$\psi = \frac{e^2}{2m\omega} \left[\frac{E_+^2}{\omega - \omega_c} + \frac{E_-^2}{\omega + \omega_c} \right] \qquad \text{... (4)}$$

where E_- is the peak value of that component of field rotating in the direction of the particle cyclotron motion.

Eqs.(1) and (2) indicate that the total energy and ordinary magnetic moment $\mu = \tfrac{1}{2} m v_\perp^2/B_o$ are not conserved but in fact grow irregularly in time as the particle moves into regions of increasing r.f. field strength. Of more importance is the implication that μ and the total energy are local functions of this field and hence return to their original values when the particle returns to its original position. This is quite distinct from the case of cyclotron resonance where irreversible effects occur.

Extensive numerical calculations[5] have confirmed the invariance of ε_{rf} and μ_{rf} to within the expansion parameter v/c. The experimental results to be reported carry this a step further in establishing the invarance implied by eq.(1) and the well depth and adiabaticity conditions for actual particle orbits. They provide the magnetic field counterpart to the work of Bravo-Zhivotovsky et al[6] who have confirmed the r.f. quasipotential for $B_o = 0$.

The present studies have employed an electron beam in magnetic mirror geometry where the r.f. potential ψ is derived from a microwave cavity. They consists of two parts: (a) single reflections where the theoretically predicted r.f. potential can be related to beam cutoff and where non-adiabatic effects in particle energy are expected when $\omega - \omega_c/\omega$ become of order v/c; and (b) multiple reflections where fine scale departures from adiabaticity would be more evident.

2. SINGLE REFLECTION

The experimental arrangement is shown in Fig.1 and consists of a simple magnetic mirror of ratio $1{\cdot}5/1$ with the electron gun located on the axis in the center of one mirror and the microwave cavity in the center of the opposite mirror. The beam is injected along the field line with $v_\perp/v_\parallel \ll 1$. The cavity is of copper and made from 10 cm wave guide with the dimensions shown in Fig.1. It is operated in the TE_{10} mode from a 2450 MHz, 100 watt CW magnetron. Two small tubes beyond cutoff for the microwave radiation allow passage of the electron beam through the cavity to a multigrid retarding potential analyser. The cavity quality factor Q was measured by standing wave phase shift and found to be 6000. Of the scale lengths L_ω and $L_{\omega c}$, L_ω is the smaller and is ~ 5 cm although more than two orders of magnitude greater than $\frac{v_\perp}{\omega_c} = r_c$.

The value of E^2 within the cavity is simply related to the net power input P and the loaded Q of the cavity. With the power available about 2 kV/cm could be obtained. Microwave power was measured from the temperature rise of the cavity cooling water but the product PQ is not known to better than ± 15%.

This part of the experiment consists therefore, of a measurement of the electron beam transmission through the cavity as the magnetic field is slowly raised so that ω_c approaches ω. Over the range of injection energies 10–100 eV, cutoff of the beam to better than a factor 10^3 could be achieved under conditions in which non-adiabatic effects were not observable in regard to particle energy. Higher energy electrons can be reflected with the E^2 available but only under conditions such that $\frac{\omega - \omega_c}{\omega} \lesssim v/c$ for which the electron energy no longer remain adiabatic. Within the energy range 10–100 eV, the detectable onset of energy deviation occurs when $\frac{\omega - \omega_c}{\omega}$ approaches to within a factor 2 to 3 of v/c and first manifests itself in the appearance of a small tail to a still reasonably monoenergetic beam. As $\omega - \omega_c$ decreases still further the energy dispersion becomes quite large extending to more than 10 times the injection value. Fig.2 shows recorded tracings of the energy distribution of the transmitted beam for various magnetic field coil currents corresponding to the values of $\frac{\omega - \omega_c}{\omega}$ indicated in parenthesis. Energy deviation is apparent at $\frac{\omega - \omega_c}{\omega} \approx 3\ v/c$.

If one sets the bias of the analyser above the injected beam energy, then the abrupt onset of current locates with fair precision the value of $\frac{\omega - \omega_c}{\omega}$ at which energy deviation begins. Fig.3 exhibits examples of this current onset, for two bias values, as a function of current in the magnetic field coils. Over the range of v/c applicable to the experiment the onset of detectable energy deviation is shown in Fig.4 as a function of $\frac{\omega - \omega_c}{\omega}$. Deviation occurs at $\frac{\omega - \omega_c}{\omega} \approx 3\ v/c$ at the lower values of velocity but relaxes somewhat to $\approx 2\ v/c$ for the larger values of velocity.

Under the conditions of multiple reflections discussed in the next Section detrimental effects upon confinement become evident when $\frac{\omega - \omega_c}{\omega} \gtrsim 10\ v/c$.

Fig.5 shows, for the beam energies employed, the r.f. potential ψ required for beam cutoff as calculated from the cavity Q and the measured power P. At low injection energies, cutoff could be obtained over considerable range of $\frac{\omega - \omega_c}{\omega}$ extending up to about 0·6. The largest values are set by the upper limit in the product PQ and the lower end is set by the onset of non-adiabatic effects discussed above. The mean values of ψ obtained from measurements over the full range of $\frac{\omega - \omega_c}{\omega}$ attainable are in quite good agreement with the predicted barrier heights. At the higher values of electron energy the range of $\frac{\omega - \omega_c}{\omega}$ for which cutoff could be achieved decreases and the barrier height found experimentally is less than expected from theory, i.e. a larger measured PQ is required than expected. The accuracy here is, of course, reduced because of the more stringent demands upon the stability and calibration of the magnetic field and it is likely that the deviation between the experimental and theoretical values of ψ lie within the accuracy of the experiment.

Examination of the barrier height as a function of $\frac{\omega - \omega_c}{\omega}$ appears in Fig.6 which shows the ratio of particle energy ε to the measured barrier height ψ required for cutoff. ε/ψ is generally somewhat less than the theoretically expected value of 1 but the deviation is slightly worse at the smaller values of $\frac{\omega - \omega_c}{\omega}$.

It is quite clear that most present day confinement devices employ magnetic fields at which the electron cyclotron frequency is too high for large steady state power and simple standing wave modes in cavities of size appropriate to the scale of the experiment. It would be much more convenient, for example, if quasi-potentials of respectable magnitude could be achieved at sub-harmonics of the electron cyclotron frequency. With this in mind, a search was made to obtain beam cutoff at $\omega \approx \omega_c/2$. The same cavity and microwave power source was employed but the magnetic field was doubled. Beam cutoff could not be achieved even at 10 eV suggesting that the coupling is very weak and that any potential barrier must be sufficiently narrow about $\frac{\omega_c}{2}$ as to escape detection. The ripple on the magnetic field is less than 0·1% suggesting an upper limit to the resonance width.

Considerations of r.f. supplemented confinement under reactor conditions envisage r.f. applied near the ion cyclotron frequency[1]. For the projected size of the vacuum vessel and magnetic field of thermonuclear reactors, the scale length L_ω and L_{ω_c} are automatically $\gtrsim c/\omega$ or c/ω_c. For present day devices, however, the scale length conditions are nearly impossible to meet at the ion frequency. Just how far these conditions can be relaxed while retaining adiabatic particle motion is not clear, but indications are[5] that scaling to present day beam injection mirror machines retains the particle adiabaticity at least to order v/c.

Operating near the ion frequency and using parallel plates to generate the r.f. field, partial cutoff of a 6 eV Li^+ beam was achieved with the existing coil arrangement ($B \approx 900$ gauss). The r.f. potential barrier is of the order expected; however, the scale lengths L_ω and L_{ω_c} are barely larger than the cyclotron radius for Li^+ and fail by orders of magnitude to surpass the vacuum wavelength of the r.f. employed. Results with protons at $\omega_c \approx 10^8$/sec are needed for serious measurements of the adiabaticity of ion motion.

3. MULTIPLE REFLECTIONS

With the addition of a second cavity located in the magnetic mirror formerly occupied by the electron gun one has the possibility of trapping electrons between the two cavities. In the multiple reflection schematic shown in Fig.1 the cavity directly in front of the electron gun is made of brass wave guide, has a $Q \approx 4000$ and a buildup time of r.f. energy $Q/\omega \approx 3 \times 10^{-7}$ sec. This time is of the order of the transit time for electrons to leave the gun, reflect from cavity No.1 and reach cavity No.2. The gate can be closed then, so to speak, in a time comparable to the loss time thus trapping part of an injected pulse of electrons. The timing sequence is shown in Fig.1. Cavity No.1 is operated in a steady state, except as noted below, and has its r.f. potential at a value above the beam energy. The electron gun is pulsed on via its grid for about 2 μsec. At near the center of this pulse, cavity No.2 is pulsed on which traps a part of the electrons spread between the two cavities. In most of the measurements reported about 10^7 electrons were thus trapped in a volume of a few cm^3.

To determine the number of electrons trapped and their decay in time, cavity No.1 is momentarily pulsed off allowing the trapped electron to spill out through the mirror to the energy analyser and charge collector. By varying the time of this dumping pulse, the electron decay can, on a shot-to-shot basis, be traced out.

In the case of multiple reflections and long electron path lengths, the most obvious cause of departure from adiabaticity is collisions with background gas molecules. From computer orbit calculations[5] we know with reasonable accuracy the paths of electrons in the r.f. field appropriate to the present experiment.

As the electron enters the cavity the total energy begins to increase. There is first a region in which scattering through any angle leads to irreversible gain or loss in $v_{\parallel}$ but without accompanying loss from the system. Deeper within the cavity the total energy continues to rise and at the normal reflection plane, has increased to about 7 times the injected value. In this region scattering, of course, continues to increase the dispersion in $v_{\parallel}$ but there is a restricted class of scatterings which can now lead to particle loss. Because of the unfortunately low r.f. potential (~ 30 eV) attainable with cavity No.2, whose coupling was far from ideal, scattering into an escape cone about the axis, such that $\frac{1}{2} m v_{\parallel}^2 > 30$ eV, leads to particle loss. There are thus two aspects of scattering which should relate to observation; the growth of a tail to the parallel energy distribution and particle loss from the system.

Specifically, if we consider the case for an electron injected at 20 eV, we know from orbit calculations[(5)] that the total energy rises to ~ 140 eV at the plane of reflection and the mean velocity during its excursion within the cavity is 5-6 × 10^8 cm/sec. As the scattering gas admitted was N_2 for which total scattering cross sections are known[(7)] and are not strongly energy dependent in the range 10-100 eV, we can calculate the collision rate of electrons with N_2 gas as an estimate for the expected rate of growth of a tail to the energy distribution.

At base pressure of 5 × 10^{-8} torr, we assume that in the presence of the LN_2 trap, the background is relatively hydrocarbon free and take the effective scattering to be 'nitrogen like' in the energy range of interest. With an elastic scattering cross section taken as 7 × 10^{-16} cm^2 and the mean electron velocity in the cavity of

$5\text{-}6 \times 10^8$ cm/sec, the collision frequency is ~ 800/sec. Since the electron spends only ¼ of its time in the r.f. fields, the effective collision frequency is reduced to 200/sec. Thus in 10^{-4} sec about 2% of the captured electrons will have experienced collisions which could lead to the growth of a tail to the energy distribution.

Fig.7 shows the growth of the group of electrons with energy in excess of 25 eV for 20 eV injection energy. Initially the growth is linear in time but later shows evidence of saturation. The flattening may be due to leakage of those particles with energies approaching 30 eV through the r.f. barrier, a feature noted in the orbit calculations(5), but this fails to account for the saturation level being pressure dependent as indicated in Fig.7. The growths observed at base pressure (5×10^{-8} torr) and at $1{\cdot}2 \times 10^{-7}$ torr of N_2 additive agree to better than a factor 2 with the admittedly rather rough calculation above. At 3×10^{-7} torr, the growth is a factor 2 lower than expected from the calculation.

With regard to particle loss, a more restricted group of electrons can, after acquiring sufficient total energy from the r.f. field, scatter into an escape cone and cross the r.f. potential barrier. As the differential scattering cross sections should be reasonably flat at these electron velocities, we can estimate the effective collision frequency for such events to be at most ~ 25% of the 200/sec calculated above. Of more consequence is the fact that collision anywhere along the electron trajectory can build up a class of particles that are mirror confined which cannot reach the charge collector when cavity No.1 is pulsed off. As the mirror ratio R is 1·5, about half of the collisions will lead to $\frac{v_\perp}{v_\parallel} \geqslant \sin\theta$ where $\sin\theta = \sqrt{\frac{1}{R}}$.

Thus in the region between the cavities the effective collision frequency is approximately $n \sigma v \left[1 - \int_{0}^{\theta} \sin \theta \, d \theta \right]$ multiplied by the fraction of time spent in this region. At 5×10^{-8} torr, this yields ~ 200/sec. For the cavity region about 100/sec is obtained, so that the total loss rate $\dot{N}/N$ should be 300-400/sec.

The experimental decays shown in Fig.8, where the e folding times appear in parenthesis, are about a factor 5 faster than the estimates made above. That the pressure recordings are off by any such factor seems most unlikely, moreover, the dispersion developing in the energy distribution appears to agree with the pressures recorded.

It would be appropriate to vary the mirror ratio, in particular to reduce it to ~ 1 to see if the expected improvement in confinement results. As this was not practicable, a negative voltage pulse was applied to the LN_2 chamber to see if magnetically trapped electrons could be driven out into the collector. Evidence of such particles was not obtained, however. Currents to the grids of the analzer were recorded along with the collector signals but no evidence was found to suggest that erroneous decay measurements resulted.

In spite of the discrepancy between the observed decay and the calculations, the linearity of the decay rate as a function of N_2 pressure (Fig.9) supports our belief that losses are entirely collisional in nature and do not arise from inherent non adiabaticities of the particle motion in the r.f. and magnetic fields. The observation that the particle motion remains conservative for ~ 1000 transists in an r.f. field near the cyclotron frequency is an important result which points out the distinct difference from particle behaviour at resonance. The magnitude of the r.f. potential and the onset of non-adiabatic effects upon particle energy are found to be in agreement with theory within the accuracy of the experiment.

4. ACKNOWLEDGEMENTS

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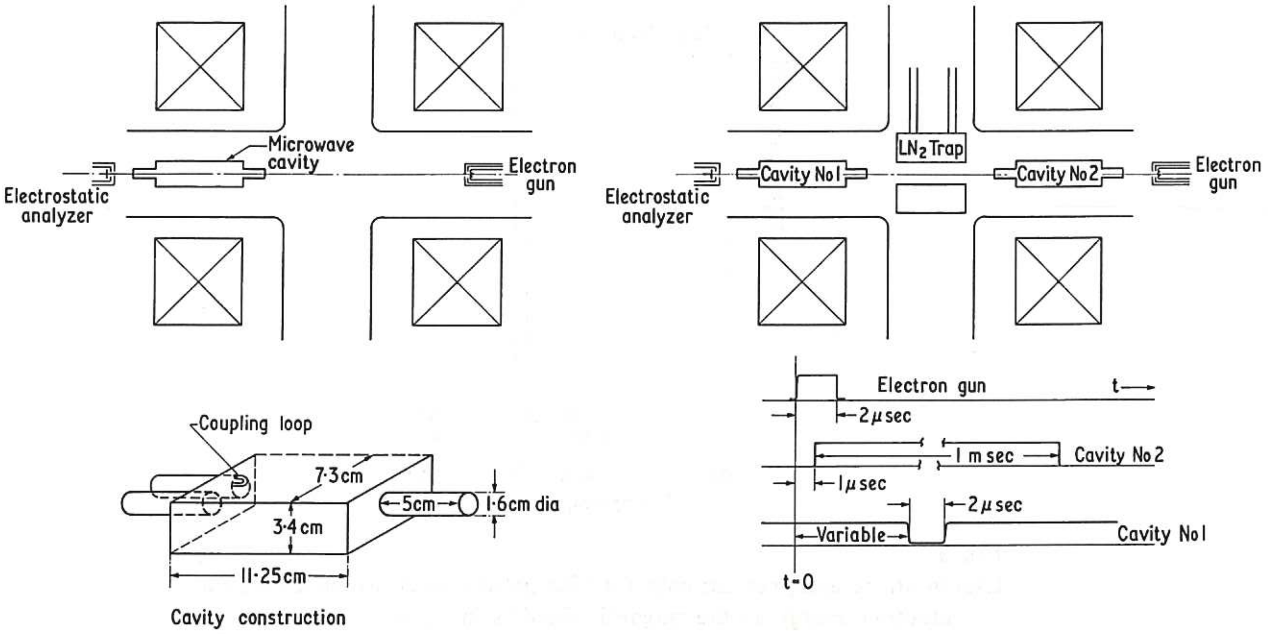

Fig. 1 (CLM-P 170)
Schematic arrangement of apparatus for single and multiple reflections

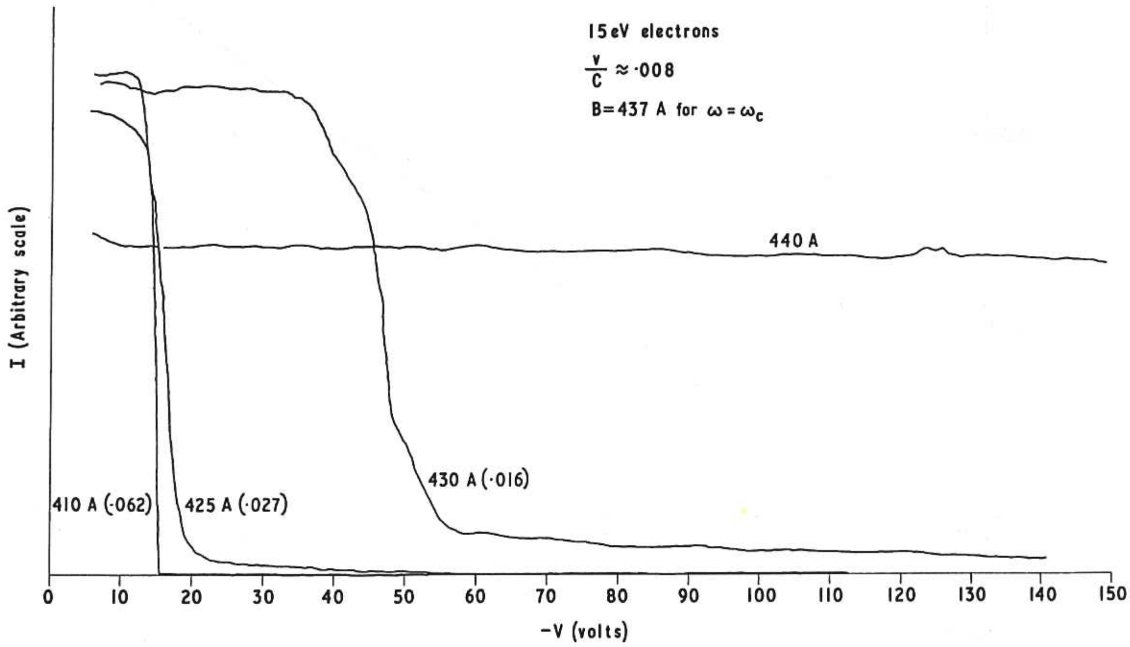

Fig. 2 (CLM-P 170)
Recorder tracings of current to electrostatic analyser vs retarding potential for different magnetic field currents in amperes. Values of $(\omega - \omega_c / \omega$ are given in parenthesis and show the distortion of the monoenergetic distribution as $\omega - \omega_c / \omega \rightarrow v/c$

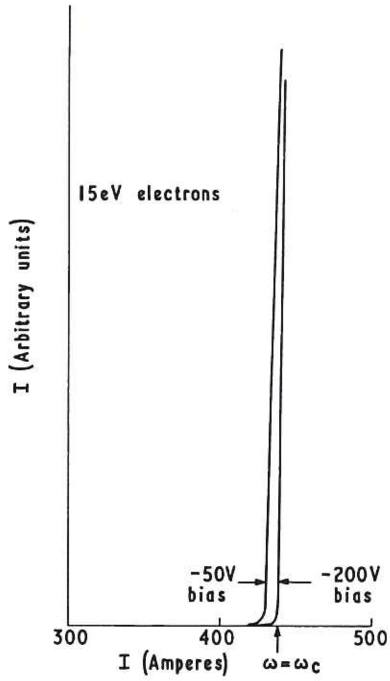

Fig. 3 (CLM-P 170)
Electrostatic analyser currents for bias greater than injected electron electron energy as the magnetic field is increased so that $\omega_c \to \omega$

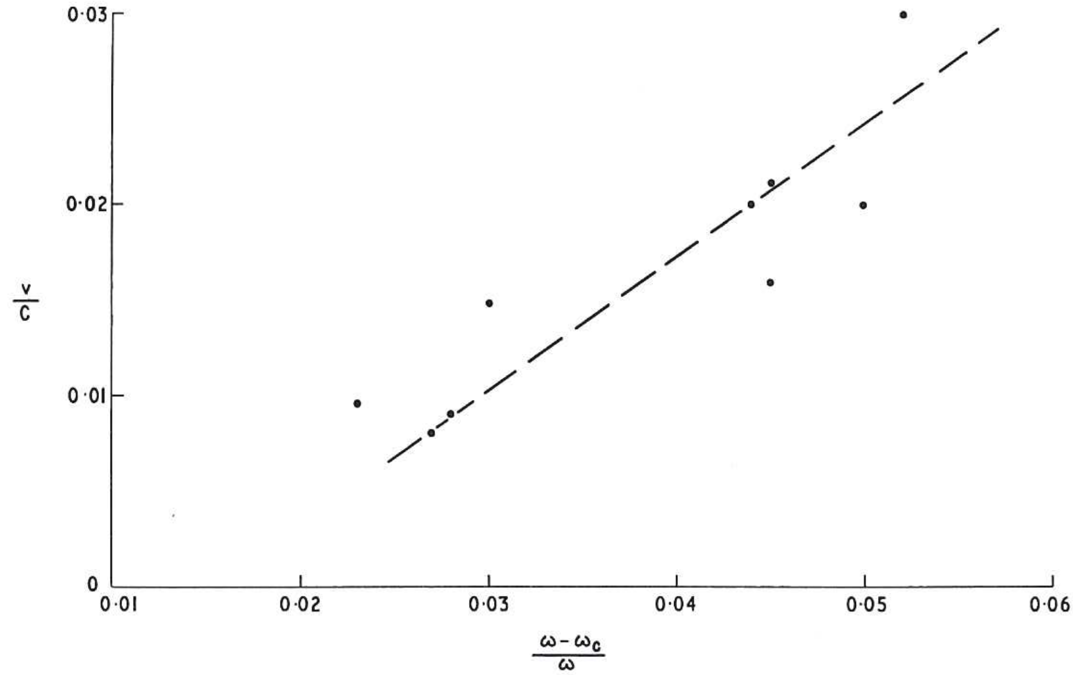

Fig. 4 (CLM-P 170)
Detectable onset of non adiabatic effects in single transmission through cavity for various electron energies given in terms of v/c as a function of $(\omega - \omega_c)/\omega$

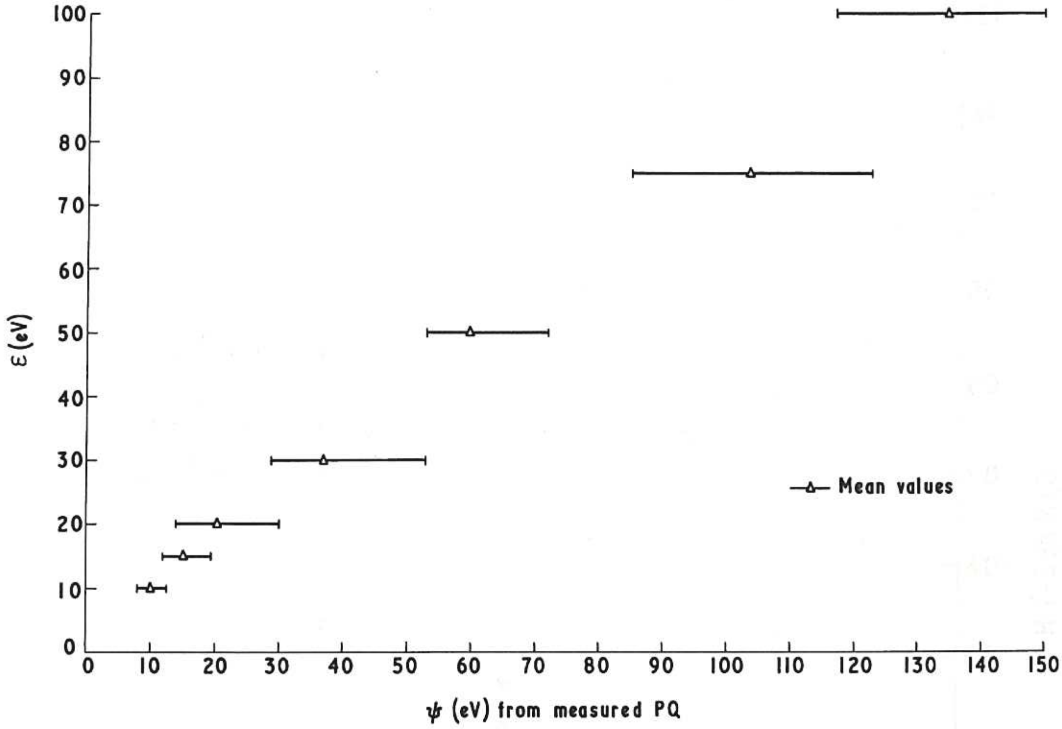

Fig. 5 (CLM-P170)

R.f. potentials ψ, calculated from measured Q of cavity and power required to just cutoff electron beam transmission as a function of beam energy ϵ. Horizontal error bars reflect scatter in 5—10 measurements with indicated mean values

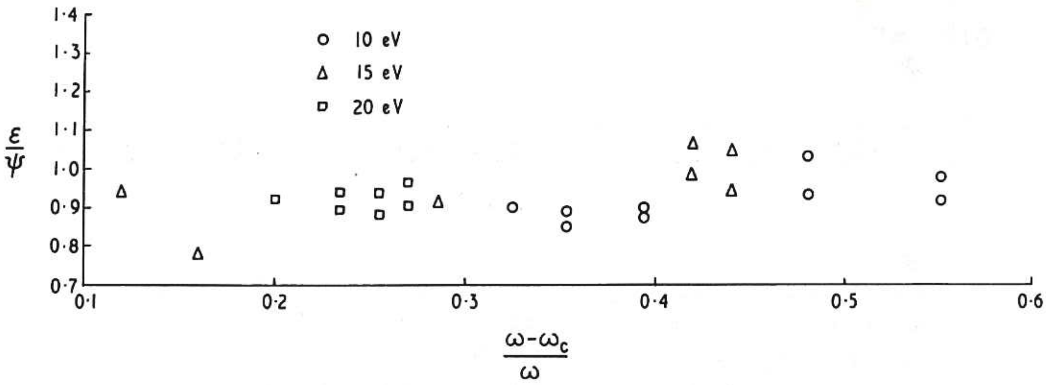

Fig. 6 (CLM-P170)

Examinations of r.f. potential as a function of $(\omega - \omega_c)/\omega$. ϵ/ψ should equal 1 on basis of theory. Deviations are within experimental accuracy

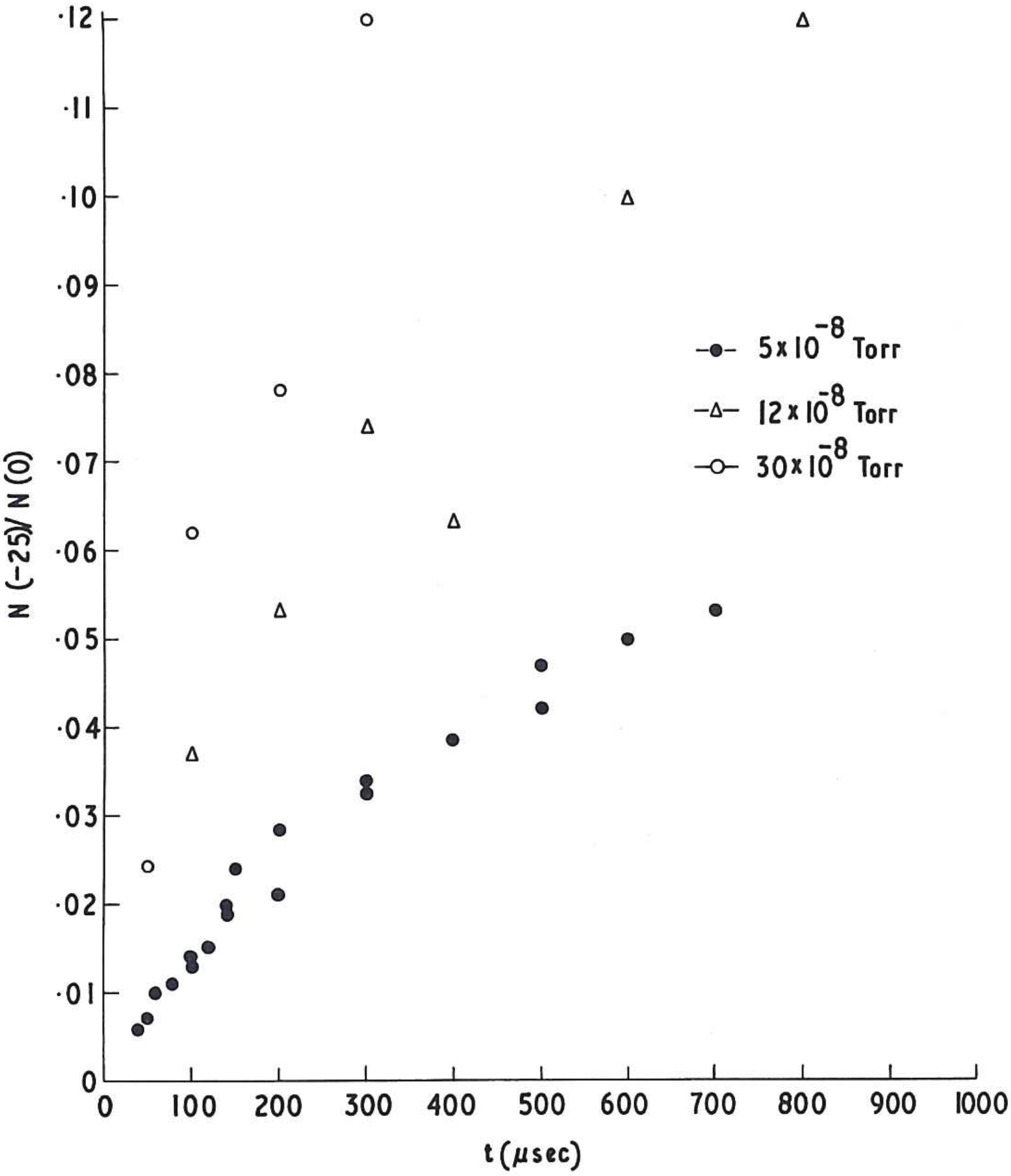

Fig. 7 (CLM-P 170)
Ratio of electrons with energies 25 eV to total number of electrons showing growth of energy tail (20 eV injection energy) for different pressures of N_2 gas. Assumed growth rate dependent only upon electron molecule scatterings agrees well with experiment

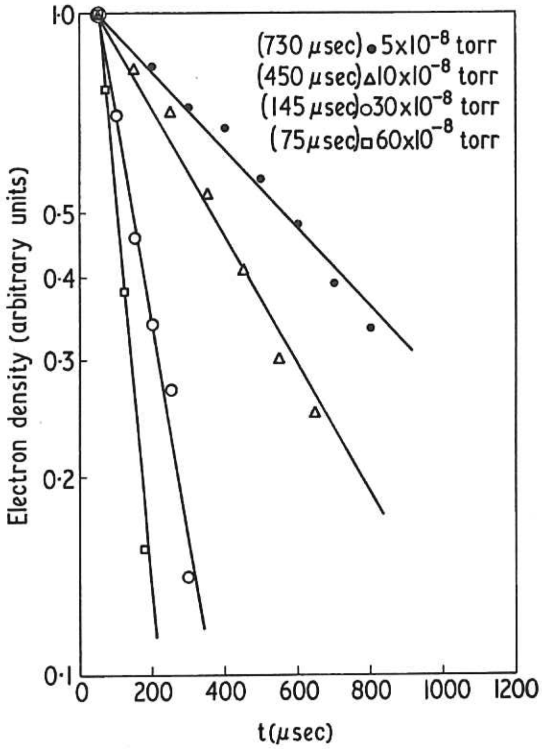

Fig. 8 (CLM-P 170)
Electron density decay curves for different background N_2 pressures, e-folding times given in parenthesis

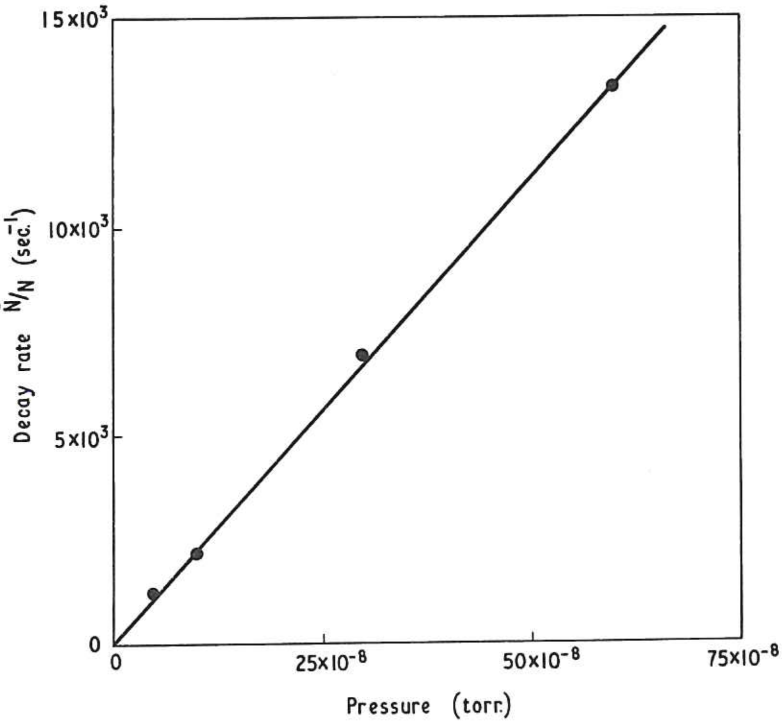

Fig. 9 Decay rates vs N_2 pressure (CLM-P 170)