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BINARY COLLISION LOSSES IN STELLARATORS

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ABSTRACT

There can exist a class of particles in stellarator fields for which the cancellation of the toroidal drift is incomplete. These localised particles, reflected in the gradients of the helical field, can drift through the separatrix of particular stellarators no matter how strong the field; they form a loss region in velocity space. The loss of plasma due to binary collision scattering into this loss region, in the absence of electric fields, is calculated. The possibility of confining a plasma with reactor parameters in the presence of this loss is examined.

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CONTENTS

		Page
1.	INTRODUCTION	1
2.	DIFFUSION CAUSED BY LOCALISATION	2
3.	NUMERICAL MODEL FOR THE DIFFUSION	5
4.	COMPUTED RESULTS	8
5.	THE EFFECT OF IMPOSING A LIMIT ON THE DENSITY GRADIENT	10
6.	CONCLUSIONS	11
7.	ACKNOWLEDGEMENT	13
8.	REFERENCES	13

1. INTRODUCTION

The rate of loss of plasma from axisymmetric toroidal confinement systems has been calculated by PFIRSCH and SCHLUTTER (1962) in the high density case where orbit effects are insignificant, and by GALEEV and SAGDEEV (1967) in the low density case where orbit effects are dominant. The orbits considered by Galeev and Sagdeev are those of passing and blocked particles (GIBSON and TAYLOR, 1967) and these are the only types present in axisymmetric systems. However in stellarator systems, because of the asymmetry, a further group of particles occurs (GIBSON and TAYLOR, 1967). These localised particles, reflected in the gradients of the helical field, are much more difficult to contain and can drift through the separatrix of particular stellarators no matter how strong the field; they form a loss region in velocity space. The three types of orbit which can occur in stellarator fields are compared in Fig.1.

This paper examines the loss which arises because binary Coulomb collisions scatter particles into the loss region. The form of the self-consistent electric field resulting from these processes in stellarators is not known and consequently its effect on the orbits is not included, however in each process the effective loss rate is taken to be the slowest of the ion and electron rates.

Diffusion coefficients due to the existence of localised particles are estimated in Section 2, and in Section 3 are combined with the results of GALEEV and SAGDEEV (1967) to produce a diffusion equation which can be solved numerically. This equation is used to evaluate the loss rate in a high shear $\ell = 3$ stellarator approximating to the reactor parameters proposed by CARRUTHERS (1967) and the results are compared with the LAWSON (1957) criteria in Section 4. In Section 5 the effect of introducing a limiting density gradient is examined. The work described in this paper has been reported at an American Physical Society Meeting (GIBSON, 1967a).

2. DIFFUSION CAUSED BY LOCALISATION

In a stellarator with a sufficiently large number of field periods even the localised particles can remain within the separatrix (GIBSON and TAYLOR, 1967) although they may still have large displacements from the magnetic surfaces. However for most configurations, including those discussed here, the localised particle orbits will intersect the separatrix; we shall assume that if a particle remains in a localised region of velocity space, then it is lost.

Consider first the very low density case when localised particles drift freely out through the separatrix, ions and electrons drifting at the same rate. The localised region of velocity space thus tends to become depleted, and is replenished by particles scattered in by small angle collisions. Since the localised region in velocity space is a disc of small width centred on $v_{\parallel} = 0$; the time for a particle, on average, to be scattered into this region is of order t_{90}, the time for small angle collisions to rotate the velocity vector by 90^{o}. At small enough densities this time will be longer than the time required for a particle to drift through the separatrix and will be the effective loss time. In this regime the ions are lost $\sqrt{Mi/Me}$ times more slowly than the electrons.

At higher densities the localised particles will be lost less rapidly than the t_{90} rate because there will be a good chance that the particle will be scattered out of the localised region before it drifts to the separatrix. The particle will then be scattered in velocity space until it again enters the localised region at approximately the same radius. Thus for a given particle there will be periods when it is passing or blocked and has compensation of its toroidal drift; alternating with periods when it is localised and has an uncompensated drift. The drift in radius will be randomly in and out according to whether the particle is trapped above or below the median plane. The step length for this random walk process is:

$$\delta r \sim V_D \cdot t_L$$

where t_L is the time for which the particle is localised and V_D is the toroidal drift velocity:

$$V_D = \left(\frac{V^2}{2R_o}\right)\left(\frac{mc}{ZeB}\right)$$

where

R_o = major radius

V = thermal speed

B = field strength

m = particle mass

and the other symbols have their usual meaning. We define α such that the condition for a particle to be localised is

$$\left(\frac{V_{\parallel}}{V_{\perp}}\right) \leqslant \alpha$$

where $V_{\parallel}$ and $V_{\perp}$ are velocity components parallel to and perpendicular to the magnetic field, t_L is then:

$$t_L \sim \left(\frac{2\alpha}{\pi}\right)^2 t_{90}$$

If the frequency with which a particle enters the localised region is f_L then the effective diffusion coefficient is D_L:

$$D_L = \langle f_L \cdot \overline{\delta_r^{\,2}} \rangle$$

where $\langle \ \rangle$ indicates an average over all velocity and $\overline{\delta_r^{\,2}}$ is the mean square steplength. Now

$$\overline{f_L \cdot t_L} = a$$

where the bar indicates that the product is averaged over all the occasions when the particle enters the loss region. Evaluating the average gives:

$$D_L \approx a^3 \langle t_{90} \cdot V_D^{\,2} \rangle$$

Since, for a given particle, $t_{90} \cdot V_D^{\,2}$ varies like V^7 the more energetic particles diffuse most rapidly. Averaging over a Maxwellian distribution yields

$$D_L \approx 10\, a^3 \left(\frac{3c\ kT}{2e\ R_o} \right)^2 \cdot t_{90} \qquad \ldots (1)$$

where T is the temperature in ^oK.

In this case, as opposed to the low density case, the ions diffuse $\sqrt{Mi/Me}$ times faster than the electrons, and the diffusion coefficient <u>decreases</u> with increasing density. The dependence on a, implies that D_L will increase steeply with radius.

The value of a can be estimated from approximate theory (GIBSON and TAYLOR, 1967). It is given by

$$a^2 \approx 2(\ell-1) \left(\frac{\iota_o}{\ell\pi} \right)^{3/2} \left(\frac{p r_m}{R_o} \right)^2 \left(\frac{r}{r_m} \right)^{3\ell-4} \qquad \ldots (2)$$

where

ℓ = number of pairs of conductors in the helical winding

p = number of field periods on the torus

r = minor radius

r_m = separatrix radius, taken to be equal to the plasma radius

It has been assumed that the rotational transform per field period (ι_K) is given by

$$\iota_K = \iota_o \left(\frac{r}{r_m} \right)^{2\ell-4}$$

When $\ell = 3$ and $\iota_o = \pi/6$ (GIBSON, 1967b) equation (2) can be written

$$\alpha \sim 0{\cdot}2 \left(\frac{pr_m}{R_o} \right) . \left(\frac{r}{r_m} \right)^{5/2} \qquad \ldots (3)$$

and in this case numerical computation (GIBSON and TAYLOR, 1967) for a toroidal stellarator shows equation (3) to be a good approximation.

At still higher densities particles are scattered out of the localised region before completing even one transit between mirrors and localised diffusion stops. This situation will arise for $n > n_c$ where

$$n_c = \frac{7{\cdot}2 \times 10^3}{\log \Lambda} . \frac{\alpha^3 \, pT^2}{R_o} \qquad \ldots (4)$$

where $\log \Lambda$ is the Debye screening factor.

3. NUMERICAL MODEL FOR THE DIFFUSION

To estimate the loss rate due to these processes we will integrate numerically the diffusion equation for a cylindrical system, but using the diffusion coefficients derived for the toroidal case. Consider such a system maintained in equilibrium by the injection of Q ions and electrons/per sec cm length on the axis, then:

$$\frac{dn}{d\rho} = - \frac{Q}{2\pi\rho D(n,\rho)} \qquad \ldots (5)$$

where

$$\rho = r/r_m .$$

The diffusion coefficient will be taken to be:

$$D = D_L + D_G \qquad \ldots (6)$$

where D_L represents the diffusion due to localised particles, D_G that due to passing and blocked particles. The low density behaviour of the localised diffusion coefficient in this model, is somewhat different from that described in Section 2. Thus as the density falls with increasing radius so the time for which a particle is localised increases, and the time required for it to drift to the wall decreases. Ultimately the localised time will exceed the drift time and at this point localised diffusion stops and is replaced by a unidirectional drift of localised particles through the separatrix and out of the system. We will take the radius at which this change-over occurs to give the boundary condition for integrating equation (5). The radius (r_b) and the density (n_b) at this radius can be obtained by assuming that, in this outer region, the loss due to localised particles will greatly exceed the other loss process. Thus:

$$Q \sim \alpha n_b \, r_b \, V_D \qquad \ldots (7)$$

where αn_b is the number density of localised particles and we have taken half the localised particles to be drifting outwards at the boundary. The condition for the localised time and drift time to be equal is:

$$\left(\frac{2\alpha}{\pi} \right)^2 \cdot t_{90} = \frac{(r_m - r_b)}{V_D} \qquad \ldots (8)$$

From (7) and (8) we see that r_b and n_b are determined by:

$$r_b \left[1 + 46A^{\frac{1}{2}} \, T^{\frac{3}{2}} \, \alpha_b{}^3 \, V_D{}^2/QZ^4 \log \Lambda \right] = r_m \qquad \ldots (9)$$

and

$$n_b = Q/\alpha V_D \, r_b \qquad \ldots (10)$$

where A is the atomic weight of the diffusing particle and α_b is defined in terms of r_b by (2).

For the case of $\ell = 3$ and $\iota_o = \pi/6$ the expressions of Section 2 can be written as:

$$D_L = \frac{3\cdot5 \times 10^7}{Z^4 \log \Lambda} \left(\frac{pr_m}{R_o}\right)^3 \frac{T_e^{3\cdot5} \, \rho^{7\cdot5}}{n(\rho)\ R_o^2 B^2} \quad \text{for} \quad n < n_c \qquad \ldots (11)$$

and

$$D_L = 0 \quad \text{for} \quad n > n_c$$

where (11) is evaluated for electrons (i.e. the more slowly diffusing particles) and correspondingly A in (9) is taken as the electron atomic weight.

Galeev and Sagdeev's results for the ambipolar diffusion of passing and blocked particles can be written .

$$D_G = D_c \quad \text{for} \quad D_c > D_{G1} \qquad \ldots (12)$$

$$D_G = D_{G1} \quad \text{for} \quad D_{G2} > D_{G1} > D_c \qquad \ldots (13)$$

and

$$D_G = D_{G2} \quad \text{for} \quad D_{G2} < D_{G1} \qquad \ldots (14)$$

where

$$D_c = \frac{\beta\eta}{8\pi}\left[1 + \frac{8\pi^2}{\iota^2}\right] \qquad \ldots (15)$$

$$D_{G1} = \frac{4\pi^{3/2}}{\iota}\left(\frac{r_{ce}}{R_o}\right)\left(\frac{ckTe}{eB}\right) \qquad \ldots (16)$$

$$D_{G2} = 3\cdot6\ (R/r)^{3/2}\ D_c \qquad \ldots (17)$$

and where η is the plasma resistivity, ι is the rotational transform in going once around the machine and r_{ce} is the electron Larmor radius. For systems with $\ell > 2$ D_G tends to infinity at small radius.

4. COMPUTED RESULTS

The density profile can be obtained by integrating equation (5) from the boundary (10) inwards to the axis. The dependence of D_G on ι (equation (15)) ensures that the use of a line sources does not lead to a singularity for $\ell > 2$. Equation (5) has been integrated numerically, in this way, for the parameters in Table 1, which approximate to those given by CARRUTHERS (1967). An example with $Q = 10^{18}$ cm^{-1} is shown in Fig.2, the boundary condition imposed by (9) and (10) is in this case $n_b = 2{\cdot}6 \times 10^{12}$ at $\rho_b = 0{\cdot}94$. Fig.2 shows that the diffusion coefficient is small at intermediate radii but large at both small and large radius. At small radius in this $\ell = 3$ field there is poor compensation of the toroidal drift so that particles make large excursions from the magnetic surfaces; collisions, by changing $(V_{\parallel}/V_{\perp})$, cause large displacements of the drift surfaces for a given particle and hence lead to large diffusion. At large radius the inhomogeneity of the helical field increases leading to a rapid increase in the number of localised particles and in the associated loss.

The significance of the binary collision loss may be assessed by plotting the LAWSON (1957) factor (L_F):

$$L_F = 10^{-14} \cdot \langle nt_c \rangle = 10^{-14} \cdot \frac{2\pi a^2}{Q} \int n^2 \rho d\rho$$

against the average thermonuclear power generated in a D-T plasma with $T_i = 20$ keV:

$$P = 8 \times 10^{-28} \int n^2 \rho d\rho \quad \text{watts/cc.}$$

Such a plot is shown in Fig.3. A self sustaining reactor (in which it is assumed that cold input material is heated directly by the reactor products) requires L_F to exceed 1 and in injection systems (KOFOED-HANSEN, 1960) L_F should exceed about 10. For a reactor to be economically feasible Carruthers concludes that P should exceed at least 10 watts/cc. The cross hatched region, (Fig.3) indicates on this basis, the confinement necessary for an economic reactor. The maximum value of β in the plasma will be limited by equilibrium considerations, to some critical value say β_E, the point at which β first reaches 1% and 10% is indicated by the bars in Fig.3. The dashed curves show PFIRSCH and SCHLUTER'S (1962) and GALEEV and SAGDEEV'S (1967) results for comparison*. It will be seen that the margin over the Lawson criteria which is 2×10^5 for Pfirsch and Schluter's calculation is reduced to ~ 400 when passing and blocked orbit effects are included and to ~ 7 (for $\beta_E = 10\%$) when the effects of localised particles are included.

A crude estimate for β_E for this configuration can be obtained by requiring the shift of magnetic axis due to the field of the secondary currents to be less than half the plasma radius; it is: $\beta_E = 3\%$. In order to obtain higher $\beta_E \sim 10\%$ and so obtain P ~ 10 watt/cc it is usual to consider adding: a transverse field, scallops, or an $\ell = 2$ winding. All of these may introduce more localised particles and so make the collisional diffusion worse. However we have made a calculation for a pure $\ell = 2$ winding with similar parameters to those of Table I, and having 6 field periods giving sufficient transform for $\beta_E \sim 10\%$. The containment is better than the $\ell = 3$ system

*For these cases the boundary condition is set by supposing that plasma flows out of the separatrix with sonic speed, along lines of force which make an angle ψ (~ 30°) with its surface and which connect to the wall. The density at the separatrix $(r = r_m)$ is then:

$$n_s = Q/(2\pi\ r_m c_s .\ \sin\psi)$$

where c_s is the sound speed.

giving a margin over Lawson of a factor of 20. Thus adding an $\ell = 2$ component to an $\ell = 3$ system may well increase β_E without increasing the diffusion due to localised particles.

The results shown in Fig.3 are not especially sensitive to the exact form of the boundary condition or the precise value of the diffusion coefficients. Thus if we change equation (8) so that the boundary occurs where the localised time is three times the drift time, then the maximum Lawson factor only changes by 10%. Similarly decreasing the localised diffusion coefficient by a factor of four, increases the maximum Lawson factor by 15%. On the other hand increasing the magnetic field by a factor 2 to 200 kG produces a substantial improvement. The Lawson factor at a given density is increased by a factor 3·5 and for the same β_E a larger n is permissible leading to a further increase. The results for this increased field are shown as the chain dotted curve in Fig.3; the margin over Lawson is a factor of 150 (for $\beta_E = 10\%$).

5. THE EFFECT OF IMPOSING A LIMIT ON THE DENSITY GRADIENT

It has been assumed so far that the density gradient can be arbitrarily steep. Let us now limit the gradient by assuming an enhanced diffusion takes place if

$$\theta \leqslant \varepsilon \sqrt{\frac{M_e}{M_i}} = \theta_c \qquad \ldots (18)$$

where the shear parameter θ is given by:

$$\theta = \frac{nr}{L_K} \left[\frac{d\iota_K/dr}{dn/dr} \right] \qquad \ldots (19)$$

where L_K is the length of a field period, and ε is a constant. We shall assume the rate of enhanced diffusion approaches that predicted by

KADOMTSEV and POGUTSE (1966) for universal modes i.e.

$$\left.\begin{aligned} D_K &= D_B\, r_{Li}/r_m\theta \\ D_B &= 2\pi eB/ckT_e \end{aligned}\right\} \quad \ldots (20)$$

with an upper limit of D_B for small θ. The density gradient limit is imposed only inside the boundary (equation (10)); the region near the separatrix where an anisotropic velocity distribution will develop is not considered. When equation (5) is integrated with this additional term the margin over the Lawson criteria is reduced from a factor of 7 for the binary collision case to unity for $\varepsilon = 1$ and to a factor 3 for $\varepsilon = 1/4$. For $\varepsilon = 1/4$ the density and diffusion coefficient are shown in Fig.4, and the variation of the Lawson factor with power density is indicated in Fig.3.

6. CONCLUSIONS

The loss rate of plasma from a stellarator due to binary collisions has been calculated. The treatment is incomplete in that, in the absence of knowledge of the form of the self-consistent electric field, it has been assumed that the orbits are those predicted (GIBSON and TAYLOR, 1967) when no electric field is present. Some attempt is made to include ambipolar effects by assuming that, in each process, the loss rate is the slower of the electron and ion rates. Calculations are presented for a specific configuration and changing the parameters from those given will change the loss rates, thus increasing the field above 100 kG will reduce all the diffusion coefficients and increasing the dimensions and number of field periods will increase the rotational drift of the localised particles and improve their confinement.

Subject to these limitations in a particular case, corresponding to reactor parameters, localisation has a serious effect. The Lawson factor is reduced from about 2×10^5 calculated from Pfirsch and Schluter's formula, to about 7 (for B = 200 kG the corresponding reduction is from 8×10^5 to 150). This margin would be satisfactory if achieved in practice, but it is uncomfortably small in view of the idealised nature of the calculation. The poor margin occurs because, in the high shear system we have considered, the two main diffusion processes - localisation at large radius and Galeev diffusion at small radius - have a measure of overlap. The results can be expected to be sensitive to effects which modify the diffusion in the overlap region where the biggest density gradient occurs. In this respect it is encouraging that if instabilities do not cause enhanced diffusion except where the density gradient is so steep that $\theta \lesssim 1/4 . \sqrt{M_e/M_i}$; then they do not make the situation much worse.

TABLE 1

Number of pairs of helical conductors (ℓ)	3
Number of field periods (p)	32
Minor radius of helical winding	2·5 m
Separatrix radius	1·25 m
Major radius	25 m
Rotational transform (ι)	$= \left(\frac{p\pi}{6}\right) \rho^2$
Field strength (B)	10^5 G
Electron and ion temperatures	$T_e = T_i = 20$ keV

7. ACKNOWLEDGEMENT

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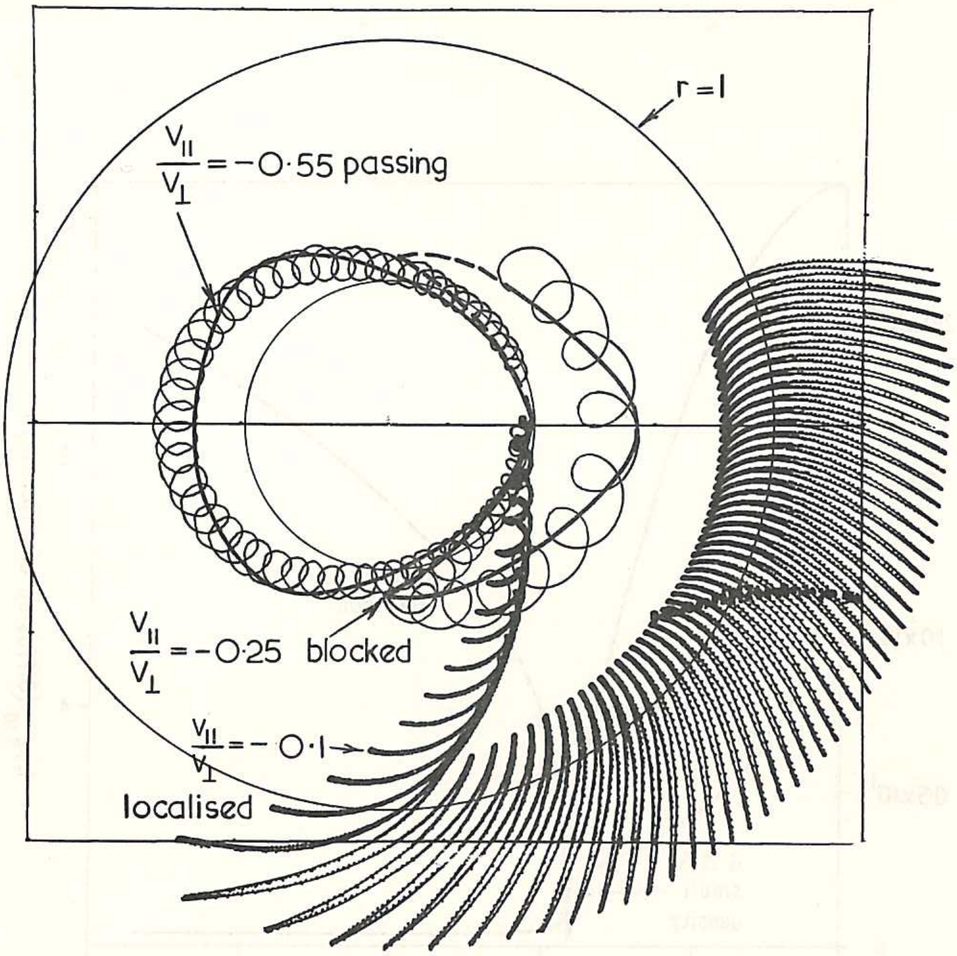

Fig. 1 (CLM-P 173)

The three types of orbit which occur in stellarator fields. The orbits are plotted in the R-Z plane of a cylindrical polar coordinate system (R, θ,Z) having its Z axis coincident with the major axis of the stellarator. The thick curves show the intersection with a plane θ = constant. The three particles originate at the same point in space and have the same initial Larmor radius (r_L), they differ only in the starting value of ($V_{II}/V_{\perp}$) the ratio of velocity parallel and perpendicular to the magnetic field. The outer circle (r = 1) represents the mean separatrix radius and the particles start at r = 0.36 with r_L^{-1} = 120, the inner circle is the maximum radius of the magnetic surface through the starting point

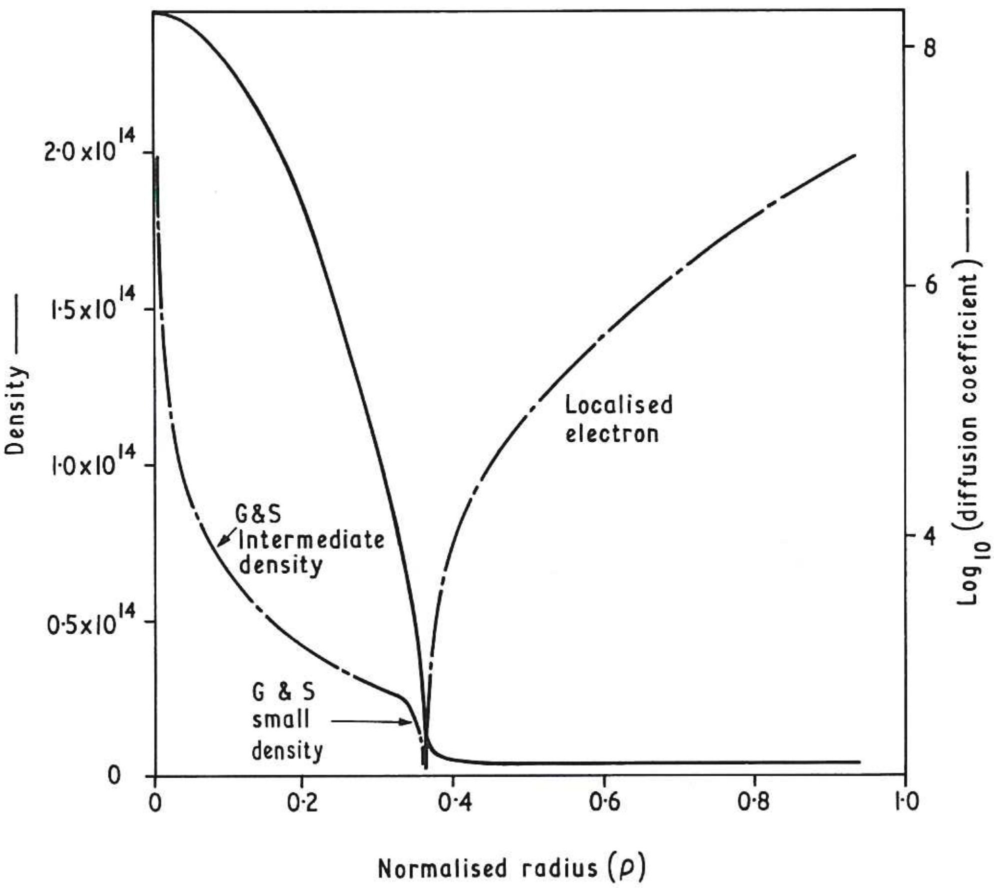

Fig. 2 (CLM-P173)

Density and diffusion coefficient due to binary collisions in an $\ell = 3$ stellarator

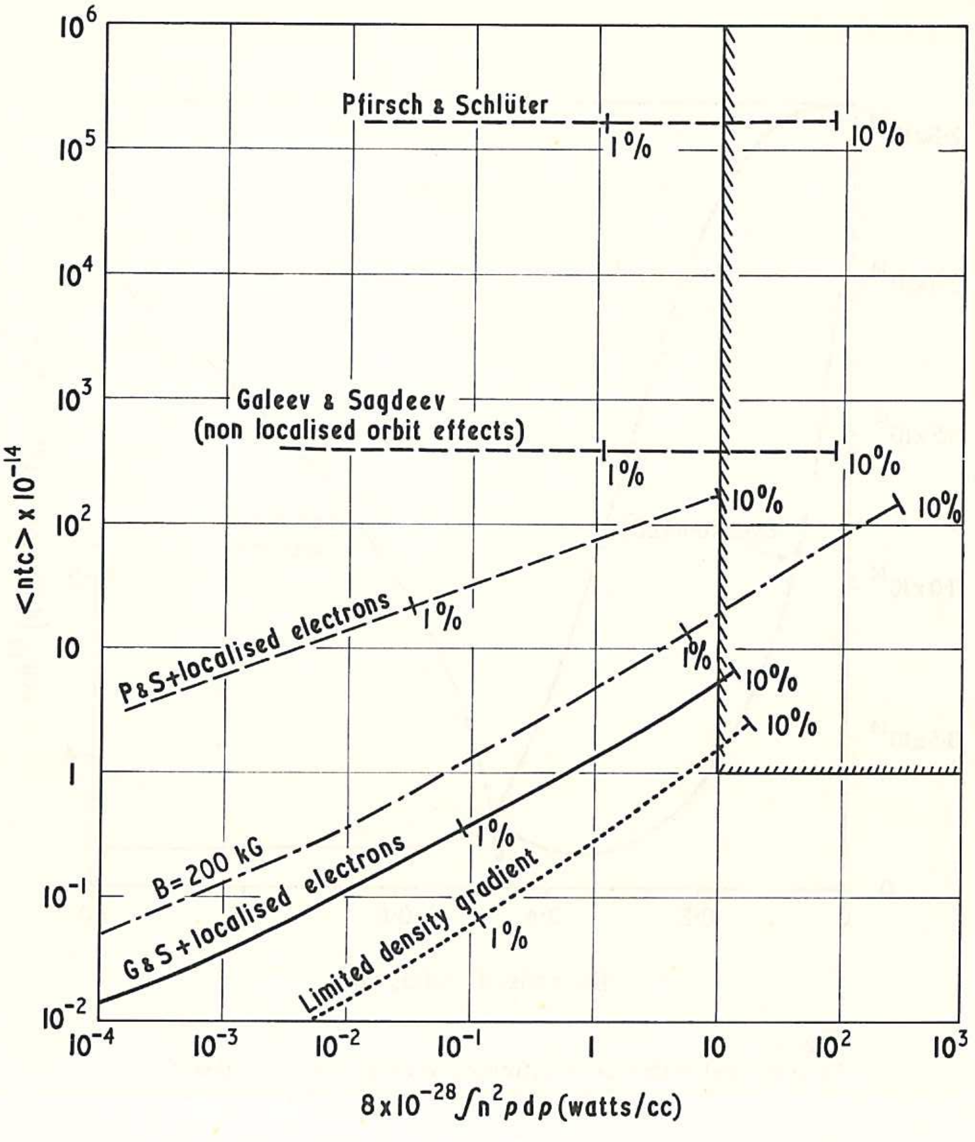

Fig. 3 (CLM-P 173)

The full curve shows reactor criteria for an $\ell = 3$ stellarator (Table I) when plasma containment is limited by binary collisions. The dashed curves show the results of previous calculations for comparison, and the chained curve shows the improvement when the field strength is increased to 200 kG. The dotted curve shows the effect of introducing a limit on the density gradient ($\varepsilon = 1/4$)

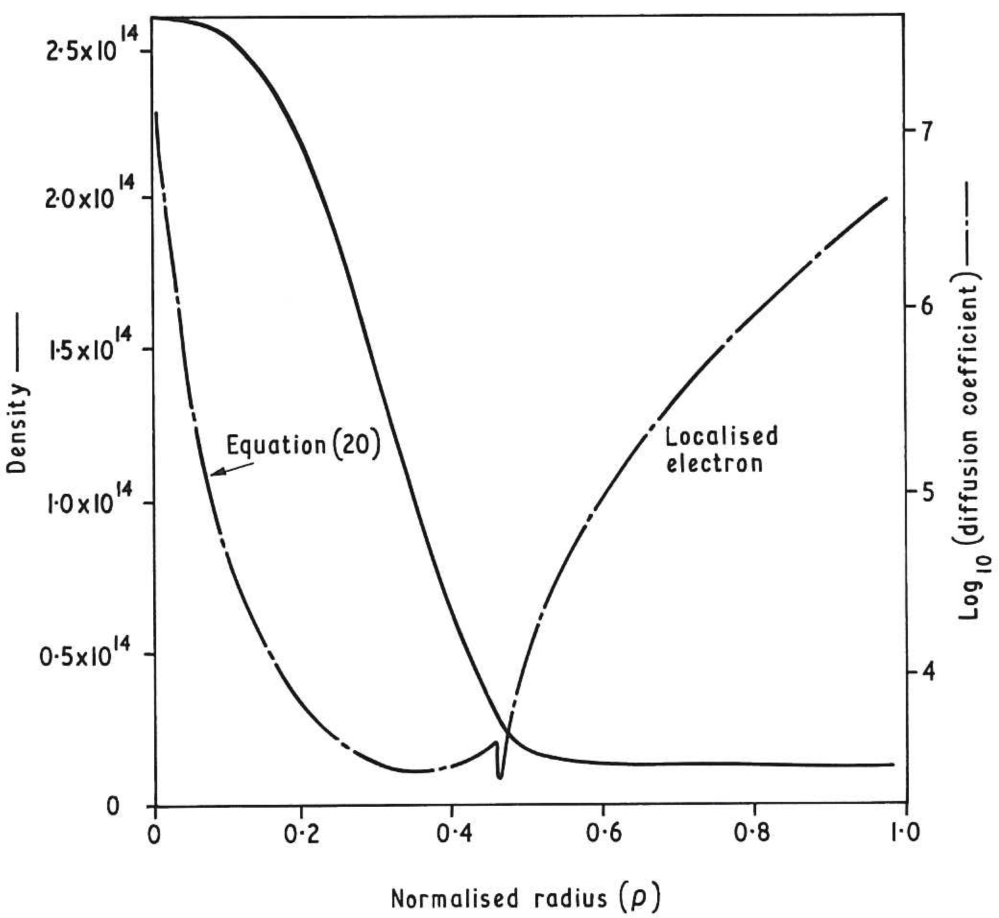

Fig. 4 (CLM-P 173)

Density and diffusion coefficient when a limit is imposed on the density gradient ($\varepsilon = 1/4$)