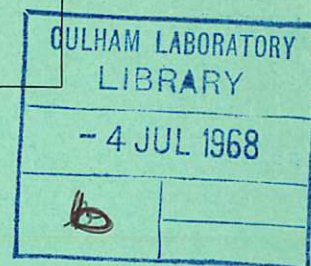


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ELECTRICAL CONDUCTIVITY OF A HIGHLY TURBULENT PLASMA

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1968

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ELECTRICAL CONDUCTIVITY OF A HIGHLY TURBULENT PLASMA

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(Submitted for publication in Physical Review Letters)

A B S T R A C T

The electrical conductivity of a highly turbulent plasma is measured in the electron density range $10^{10} - 10^{13} \text{ cm}^{-3}$, using hydrogen, argon and xenon. When the applied electric field is large ($\gtrsim 100 \text{ Vcm}^{-1}$) it is found that the conductivity may be described by the formula:

$$\sigma \approx \frac{1}{2} \left(\frac{M}{m} \right)^{\frac{1}{3}} \omega_{pe}$$

which was first suggested by Buneman in 1958.

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June, 1968

We have measured the electrical resistance of a plasma in which there is a high level of electrostatic turbulence. In this case the energy associated with the collective fields of the plasma waves represents a significant fraction of the whole energy of the plasma, and the scattering of electrons (and thus the conductivity) is determined by their interaction with these collective fields rather than with the fields of individual plasma particles. Experimentally the necessary conditions may be achieved by applying a sufficiently strong electric field that the electrons 'run-away'¹ and reach velocities v_d (with respect to the ions) equal to or greater than their thermal velocity $\bar{v}_e = (kT_e/m)^{1/2}$, where T_e is the electron temperature. In this event both theory² and computer experiments³ suggest that plasma waves with wave numbers around ω_{pe}/v_d should grow at a rate $\gamma \sim (m/M)^{1/3} \omega_{pe}$ (where m, M are the electron and ion masses, $\omega_{pe} = (4\pi ne^2/m)^{1/2}$ is the electron plasma frequency) and the drifting electrons are effectively scattered through 90° in a time $\tau_{eff} \sim (M/m)^{1/3}$ plasma periods. The conductivity associated with such a scattering time is therefore

$$\sigma_{(e.s.u.)} = \frac{\omega_{pe}^2}{4\pi} \tau_{eff} \sim \frac{1}{2} \left(\frac{M}{m} \right)^{1/3} \omega_{pe} \quad \dots (1)$$

This argument was first proposed by Buneman⁴ exactly ten years ago. The rapid wave growth is due to a form of the well-known two-stream instability. For rather lower applied electric fields, such that $v_e > v_d > (kT_e/M)^{1/2}$, the ion sound speed, an instability with a significantly slower growth rate develops² (provided $T_e \gg T_i$) and leads

to a weakly turbulent situation which is more amenable to theoretical treatment. Under these conditions Sagdeev⁵ suggests a formula:

$$\tau_{\text{eff}} \sim 10^2 \frac{T_i}{T_e} \frac{\bar{v}_e}{v_d} \frac{1}{\omega_{pe}} . \quad \dots (2)$$

These so-called anomalous resistance effects have already been utilized in a number of experiments to obtain plasma heating at much faster rates than are possible under quiescent conditions^{6,7}. They have also been invoked to explain certain types of collision-free shock⁸ and some aspects of solar flare behaviour⁹.

Our experiments were conducted in a silica torus of 5 cm minor and 32 cm major radii, in a plasma immersed in a longitudinal quasi-static field $\approx 2,500$ gauss, with a maximum rotational transform $t = 120^\circ$ provided by helically wound ($\ell = 3$) conductors. The working plasma was prepared by a conventional ohmic heating current pulse in various gases.

The plasma radius (determined by the magnetic field separatrix) was 4.5 cm. The gas filling pressure was varied between 5×10^{-6} and 10^{-3} torr. The large electric field used to generate the turbulence was electromagnetically induced at a predetermined electric density (during the decay following the pre-ionizing discharge) parallel to the longitudinal magnetic field by discharging in series four $0.28 \mu\text{F}$ low inductance capacitors, each charged to a maximum voltage of 45 kV, via spark gaps into circumferential windings around the torus⁶. The applied pulse was either a single half cycle at a frequency of 1 MHz, produced with the use of non-linear resistors (see Fig.1 inset) or a damped oscillatory waveform of similar frequency. The largest electric field had a maximum amplitude (with plasma present) approximately 500 Vcm^{-1} (100 kV per turn on the plasma

secondary).

The initial electron density was measured from the phase change suffered by a 2 mm wavelength microwave beam traversing a diameter of the torus perpendicular to the major axis. Although the gas is not fully ionized, ionization rate considerations show that no significant change in plasma density can occur within times of interest ($\leq 0.4 \mu\text{s}$) even under the worst conditions of these experiments. The plasma current I was measured by a self-integrating Rogowski coil (time constant $10 \mu\text{s}$), and the applied E-field by the flux change in a calibrated pick-up coil. Typical waveforms are shown inset in Fig.1. The resistance was found from the ratio E/I when $dI/dt = 0$, i.e. at peak current. No correction for time-varying inductance was necessary, since magnetic probe measurements have confirmed that the current fully penetrates the plasma well before peak current is reached (consistent with the low conductivity), and that no pinching of the current channel occurs. Intense unpolarized microwave emission was observed at times corresponding to peak current indicating strong electrostatic turbulence¹⁰.

Demidov et al¹¹, working in hydrogen, and using an arrangement essentially similar to ours, have shown that, for electron density $n = 10^{12} \text{ cm}^{-3}$, the conductivity falls rapidly for $E \gtrsim 0.6 \text{ V cm}^{-1}$, remains essentially constant for $1 < E < 15 \text{ V cm}^{-1}$ and then decreases again for $E \gtrsim 25 \text{ V cm}^{-1}$. Working at the same density but with larger electric fields we have extended their curve (Fig.1); our results show that there is another essentially field-strength independent region with lower conductivity for $E \gtrsim 100 \text{ V cm}^{-1}$. The two discontinuities would seem to correspond respectively with the onset of ion-sound wave and two-stream instability. Thus for large electric fields

($E \gtrsim 100 \text{ V cm}^{-1}$) we should be in the regime for which Buneman's arguments apply, and the conductivity given by equation (1).

To test this hypothesis we have varied the initial plasma density in the range $10^{10} - 10^{13} \text{ cm}^{-3}$, and the ion mass in the ratio 1 : 40 : 131 by using hydrogen, argon and xenon, and measured the plasma resistance under conditions for which the applied electric field exceeded 100 V cm^{-1} .

Fig.2 shows the measured dependence of resistance per unit length of plasma column R upon mean electron density \bar{n} for the three ion species. Each experimental point shown represents a local averaging of a number of observations, and the error bars are estimated from the experimental uncertainties. Since the resistance appears to follow the expected law $R \propto \bar{n}^{-1/2}$ (cf. eqn.(1)) we have computed a best fit curve for a large number of data points for each case, with the appropriate slope, also shown in Fig.2. The lines fit the experimental data with probable errors of $\pm 12\%$ (H), $\pm 15\%$ (A) and $\pm 20\%$ (Xe). From these we can find the effect of the ion mass on the average conductivity at a fixed density. Including the probable errors we obtain $\bar{\sigma}_H : \bar{\sigma}_A : \bar{\sigma}_{Xe} = 1 : 3.0 \pm 0.8 : 6.5 \pm 2.0$, Equation (1) predicts the ratio to vary as $(M/m)^{1/3}$ i.e. $\sigma_H : \sigma_A : \sigma_{Xe} = 1 : 3.4 : 5.1$. Hence the experimental data is in accordance with the law $\sigma = K(M/m)^{1/3} \omega_{pe}$, where K is a numerical constant.

Using the hydrogen data we can find a constant for the average conductivity $\bar{\sigma}$ in terms of the mean electron density \bar{n} :

$$\bar{\sigma} = 0.45 \pm 0.05 (M/m)^{1/3} \bar{\omega}_{pe} \quad \dots (3)$$

where

$$\bar{\omega}_{pe}^2 = \frac{4\pi \bar{n} e^2}{m}.$$

It is unlikely that any very serious errors are introduced by

using averaged quantities (compared with our experimental errors):

for example, if a parabolic form of radial density variation is assumed, we should obtain

$$\sigma = 0.53 (M/m)^{1/3} \omega_{pe} . \quad \dots (4)$$

Thus the measured conductivity agrees closely with the predicted value (equation (1)) in all respects. It is also clear that, as expected, equation (2) does not apply, neither in absolute magnitude nor in dependence on ion mass.

In these experiments the role of the magnetic field is simply to provide containment for the initial plasma, the stellarator geometry enabling us to work with a wide variety of plasma density. Both Demidov and his co-workers¹¹ and ourselves have found that, for a given density, the measured conductivity does not depend either on the magnitude or the shape of the magnetic field. Buneman¹² has in fact shown that the magnetic field would not be expected to alter significantly the growth rate of the type of instability considered in this work.

In conclusion we should point out that both direct and indirect measurements of plasma pressure and temperature show that the electrical energy dissipated appears as thermal energy in the plasma. For example, for $\bar{n} = 1.5 \times 10^{12} \text{ cm}^{-3}$, $E = 300 \text{ V cm}^{-1}$, a piezo-electric pressure probe showed a maximum particle energy density $\approx 2 \times 10^{16} \text{ eV cm}^{-3}$ in the plasma at $t = 0.2 \mu\text{s}$, in close agreement with the electrical energy dissipated in the current pulse. X-ray and neutral particle emission measurements have shown that the energy goes into both electrons and ions⁶.

The authors wish to thank Dr R.J. Bickerton for his encouragement and helpful criticism, and P. Richold and S. Turley for their experimental assistance in this work.

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Fig.1 Variation of plasma conductivity with electric field for hydrogen at electron density $\bar{n}_e \approx 10^{12} \text{ cm}^{-3}$. (Insert) Current and voltage waveforms for $\bar{n}_e = 1.5 \times 10^{11} \text{ cm}^{-3}$, capacitor voltage 45 kV.

Fig.2. Variation of measured resistance per cm of plasma column with density for various gases.

