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D. C. ROBINSON
M. G. RUSBRIDGE

Culham Laboratory
Abingdon Berkshire

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TURBULENT DENSITY FLUCTUATIONS IN ZETA

by

D.C. ROBINSON
M.G. RUSBRIDGE*

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A B S T R A C T

Observations on the fluctuations from a double Langmuir probe are presented. It is shown that these can be interpreted in terms of fluctuations of the density, temperature, their gradients and the electric field. The measurements are consistent with the theory, and it is found that the fluctuations are adiabatic at high pressures and isothermal at low pressures. The properties of the fluctuations are compared with the predictions of a theory of hydromagnetic turbulence, which allows us to consider the origin of the observed density fluctuations. It is concluded that they arise from flow along the magnetic field lines. Measurements of correlation functions both in space and time are described and their significance in terms of a containment time and wave motion is discussed.

*University of Manchester, England.

U.K.A.E.A. Research Group,
Culham Laboratory,
Abingdon,
Berks.

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CONTENTS

1. INTRODUCTION

Some observations on the electric and magnetic field fluctuations in the ZETA plasma have previously been described (RUSBRIDGE et al., 1962; ROBINSON and RUSBRIDGE, 1964; ROBINSON and RUSBRIDGE, 1966; ROBINSON, 1966). We have attempted to interpret these fluctuations in terms of hydromagnetic turbulence, and a result of a simple theory has been given (ROBINSON and RUSBRIDGE, 1964). This theory can be used to determine certain properties of the density fluctuations which will be described here, though a fuller account of the theory is given elsewhere (ROBINSON, 1966; ROBINSON et al., 1968). A comparison of the results of this theory and the experimental observattions will be made in this paper.

The fluctuations have been measured using a double Langmuir probe. Conventional double probe theory (JOHNSON and MALTER, 1950) is incorrect in strong magnetic fields but good probe characteristics are observed experimentally. We interpret these as if the magnetic field had no effect because of the turbulence, which sweeps fresh plasma across the probe and thus effectively increases the perpendicular diffusion of plasma to the probe. An independent check on the behaviour of the probe has been made using a combined Langmuir-microwave probe (RUSBRIDGE and WORT, 1967), with which the transmission of 2 mm microwaves over a short path of 5 mm could be observed simultaneously with the Langmuir probe signal. A detailed comparison of the two signals has been made and this yields a good correlation.

In the next Section we outline the theory for the density fluctuations and give the principal results. Section 3 gives the theory of the current fluctuations to a Langmuir probe. These fluctuations can be analysed to give information about fluctuations of density and temperature, their gradients and the electric field. In Section 4 we give the experimental results confirming the theory of the current fluctuations, which are predominantly due to density fluctuations. The observed density correlation function and its significance is described in Section 5 while in Section 6 measurements of wave motion associated with the density fluctuations are given. Finally, the origin of the density fluctuations in the light of the presented theory and other theories is discussed.

2. THEORY FOR THE DENSITY FLUCTUATIONS

The compressible magnetohydrodynamic equations in the presence of an applied field are used to determine the density fluctuations by a method based upon supposing that random stationary fields are set up in response to a fluctuation force representing the effect of plasma instability. A full non-linear theory (ROBINSON, 1966, EDWARDS 1964; EDWARDS 1965) shows that it is possible to obtain estimates of magnitudes by performing only a linear analysis and using 'turbulent transport coefficients'. Results are derived in a form which is independent of the spectrum of the force so that it is taken in its most convenient form -white noise. The linearised set of equations that we solve (MARSHALL, 1958) is, in Fourier notation for a particular mode $\underline{K}$

$$\frac{1}{\rho_o}\frac{\partial \rho_{\underline{K}}}{\partial t} = -i\,K^{\alpha}u^{\alpha}_{\underline{K}}$$

$$\frac{\partial u^{\alpha}_{\underline{K}}}{\partial t} + \frac{\mu K^2}{\rho_o}u^{\alpha}_{\underline{K}} = -\frac{iK^{\alpha}}{\rho_o}\left(P_{\underline{K}} + \frac{\mu}{3\rho_o C_o^2}\frac{dP_{\underline{K}}}{dt}\right) + i\,H_o K_3\,h^{\alpha}_{\underline{K}} - i\,K^{\alpha}\,H_o h^3_{\underline{K}} + F^{\alpha}_{K}$$

$$\frac{\partial h^{\alpha}_{\underline{K}}}{\partial t} + \nu_m\,K^2 h^{\alpha}_{\underline{K}} = i\,H_o K_3 u^{\alpha}_{\underline{K}} - i\,H_o\,\varepsilon^{\alpha 3}\,K^{\lambda}u^{\lambda}_{\underline{K}}$$

$$\frac{\partial \rho_{\underline{K}}}{\partial t} = \frac{1}{C_o}\frac{dP_{\underline{K}}}{dt}$$

where: $u_{\underline{K}}$ fluid velocity, $h_{\underline{K}}$ magnetic field strength in velocity units $(B/\sqrt{4\pi\rho_o})$, H_o the applied field in the 3-direction (again in velocity units), C_o the sound speed, $\rho_{\underline{K}}$ the density, $\nu_m = \frac{C^2}{4\pi\sigma}$ where σ is the conductivity, $F_{\underline{K}}$ is the random force, $P_{\underline{K}}$ the pressure and μ/ρ_o is the kinematic viscosity (ν), which for a plasma is given in (MARSHALL, 1958). The approximation we are making here is that the integral of the basic density correlation function

$$\int \langle \rho_K \, \rho_{-K} \rangle \, dK$$

can be replaced by $\overline{\rho^2} K_{typical} \approx \overline{\rho^2}/\Lambda$ where $\overline{\rho^2}$ is the r.m.s. density fluctuation and $K_{typical}$ is a typical wave number associated with this fluctuation; more precisely, we take $K_{typical} = 1/\Lambda$ where Λ is the integral scale. Our observations are only concerned with this integral. In addition the transport coefficients can be replaced by effective values to take account of the non-linearities.

Solving these equations in the steady state limit for the case of no magnetic field gives the result for the density correlation

$$\frac{\langle \rho_{\underline{K}} \, \rho_{-\underline{K}} \rangle}{\rho_o^2} = \frac{K^\alpha \, Q_{\underline{K}}^{\alpha\beta} \, K^\beta}{C_o^2 K^2} \qquad \ldots (2)$$

in terms of the velocity correlation

$$Q_{\underline{K}}^{\alpha\beta} = \langle u_{\underline{K}}^\alpha \, u_{-\underline{K}}^\beta \rangle .$$

Upon integrating over K and using the approximation given above we find

$$\frac{\overline{\rho^2}}{\rho_o^2} \approx \frac{\overline{u^2}}{C_o^2} .$$

Note we have not assumed anything about the symmetry of the fluctuations, but that they are stationary and homogeneous. The introduction of the magnetic field makes the equations intractable so it was assumed that the system was close to two-dimensional in the sense that

$$H_o^2 \; K_3^2 \ll \nu\nu_m \; K^4 \qquad \ldots (3)$$

in which limit the transverse components of the magnetic field go to zero (ROBINSON, 1968). The final expression for the density fluctuations is

$$\frac{\langle \rho_{\underline{K}} \, \rho_{-\underline{K}} \rangle}{\rho_o^2} = \frac{\left(C_o^2 + \tilde{\nu}_m \, (\frac{4}{3}\tilde{\nu} + \tilde{\nu}_m) \, K^2\right) Q_{\underline{K}}^{\alpha\beta} \, K^\alpha \, K^\beta}{C_o^2 \left[H_o^2 + C_o^2 + \tilde{\nu}_m \, K^2 \left(\frac{4}{3}\tilde{\nu} + \tilde{\nu}_m\right)\right] K^2} \qquad \ldots (4)$$

which in the ordering

$$H_o^2 \gg C_o^2 \gg \tilde{\nu}_m (\tfrac{4}{3}\tilde{\nu} + \tilde{\nu}_m) \; K^2$$

gives

$$\frac{\overline{\rho^2}}{\rho_o^2} \approx \frac{\overline{u^2}}{H_o^2} .$$

An estimate of the turbulent viscosity is $\tilde{\nu} \approx u\Lambda_\perp$, where $\Lambda_\perp \sim K^{-1}$ is the transverse correlation length. In the above approximation we also obtain the result that

$$\frac{Q_{\underline{K}}^{\alpha\alpha}}{\langle h_{\underline{K}}^3 \, h_{-\underline{K}}^3 \rangle} = \frac{H_o^2 + C_o^2 + \tilde{\nu}_m \, (\frac{4}{3}\tilde{\nu} + \tilde{\nu}_m) \; K^2}{H_o^2} \qquad \ldots (5)$$

which gives equipartition of energy (energy in velocity field is the same as that in the magnetic field) in the limit of large Alfvén speed. In ZETA $C_o^2 \ll H_o^2$ so the bounds on the density fluctuations are

$$\frac{\overline{u^2}}{H_o^2} < \frac{\overline{\rho^2}}{\rho_o^2} < \frac{\overline{u^2}}{C_o^2} \qquad \ldots (6)$$

depending on the value of the dissipative terms. Experiments to determine the parallel correlation length $\Lambda_\parallel \sim K_3^{-1}$ show that it is large ($\gtrsim$ 100 cm) but nevertheless equation (3) may not be satisfied. The result in this case is complicated but for the fluctuations arising from the perpendicular part of u we have an additional term

$$H_o^2 K_3^2 \left(\frac{\tilde{\nu}}{3} + C_o^2/\tilde{\nu}_m K^2\right) \Big/ (\tilde{\nu} + \tilde{\nu}_m) K^2$$

in the brackets in both numerator and denominator in equation (4). An appreciable $H_o K_3$ can then give rise to density fluctuations which are closer to equation (2) than equation (4); however equation (5) is essentially unaltered by the now non-negligible $H_o K_3$ terms. The results for ρ^2/ρ_o^2 in the various limits are summarised in Table I.

Inserting known experimental values (BURTON et al., 1962) for the temperatures, density, velocity and field strength, we obtain the density fluctuations as a function of filling pressure in the discharge. The electron temperature and density is determined from Langmuir probes and microwaves; however the ion temperature is somewhat uncertain and is estimated from energy balance calculations and some spectroscopic measurements. The two bounds as given in equation (6) are shown together with a curve for $K_3 = 0{\cdot}02$ in Fig. 1. A turbulent viscosity and a classical resistivity was used to obtain this curve. The value of u is obtained from electric field probes (RUSBRIDGE et al., 1962; ROBINSON and RUSBRIDGE, 1964), spectroscopic measurements of Doppler broadening (JONES and WILSON, 1962), and microwave scattering (WORT 1965). We have thus studied a system where Alfven waves can propagate, and also fast and slow magneto-sonic waves (THOMPSON 1962). The results for $K_3 \to 0$ arise from damped fast waves alone whereas the latter more complicated results arise from the effects of all three damped waves. From equation (4) it is evident that the fast wave gives rise to fluctuations of order $\overline{u^2}/(H_o^2 + C_o^2)$ and the slow and fast wave to $\overline{u^2}/C_o^2$ if we assume no damping.

The above model for the fluctuations can also be extended to obtain expressions for the temperature fluctuations in certain limits, by using the additional equation

$$\frac{\partial T_K}{\partial t} = - \frac{\kappa K^2}{\rho_o k} T_K - \frac{2}{3} T \, i K^\alpha u_K^\alpha$$

where κ is the thermal conductivity and T the temperature. This leads to estimates of the temperature fluctuations and to the cross correlation between density and temperature. For example in the case of no magnetic field we find

$$\frac{\langle T_K T_{-K} \rangle}{T^2} = \frac{4}{9K^2} \frac{K^\alpha Q_K^{\alpha\beta} K^\beta}{\left[\frac{5}{3} kT + \frac{\kappa}{\rho_o k} \left(\frac{\kappa}{\rho_o k} K^2 + \nu K^2\right)\right]} .$$

It is also possible to use the model to investigate the effect of inhomogeneities, provided that $L \gg K^{-1}$ where L is a typical scale of the inhomogeneity and K is the minimum effective wave number, essentially the large scale length of the turbulence.

If we consider equations (1) in the incompressible case but with a finite density gradient transverse to the mean field then in the limit that $\nu_m \to 0$ (if the plasma is not frozen to the lines of force we find the fluctuations are no longer stationary, unless we introduce a source term) it is possible to obtain an expression for the density fluctuations of the form (ROBINSON, 1966).

$$\frac{\langle \rho_K \rho_{-K} \rangle}{\rho_o^2} = \frac{\langle u_K u_{-K} \rangle}{H_o^2 K_3^2} \frac{1}{\rho_o^2} \left(\frac{\partial \rho}{\partial x}\right)^2 J' \qquad \ldots (7)$$

where J' is an angular factor of order unity. Thus density fluctuations from this origin should possess a 'hole' at the centre of symmetry of the mean density. In this case, if we use a non-white noise input, the correlation $\langle \rho_K u_{-K}^\alpha \rangle$ is finite and gives the rate at which matter flows down the density gradient. In the above case we obtain

$$\langle \rho_K u^{\alpha}_{-K} \rangle = \frac{\langle \rho_K \rho_{-K} \rangle}{\rho_o^2} \cdot \frac{\rho_o^2}{\partial\rho_o/\partial x} \cdot \frac{H_o^2 K_3^2}{\nu K^2} J^{\alpha} \qquad \ldots (7a)$$

where J^{α} is an angular factor. This is of a similar form to that quoted earlier (CHEN 1965) when considering "anomalous diffusion," except that $J^{\alpha} H_o^2 K_3^2/\nu K^2$ is replaced by $I_m(\omega)$ which is the inverse correlation time of the fluctuations in his case. In the above case the correlation time of the fluctuations is $\nu K^2/H_o^2 K_3^2$ provided $\nu K^2 > H_o K_3$. If we use this approach on the drift wave equations (KADOMTSEV, 1965) and assume $\omega < \Omega = \frac{eB_o}{mc}$ and $u < C_o$ then we obtain a diffusion coefficient similar to the Bohm one, namely $\overline{n^2}/n_o^2 \cdot \frac{KT_e}{eB_o}$.

3. FLUCTUATIONS IN THE CURRENT TO A LANGMUIR PROBE

The current drawn by an equal area double Langmuir probe immersed in a homogeneous plasma and biassed to a voltage V is given by (JOHNSON and MALTER, 1950)

$$i = \alpha A n e \left(\frac{2KT_e}{m_i}\right)^{\frac{1}{2}} \tanh \frac{e(V + V_c)}{2KT_e} \qquad \ldots (8)$$

where A is the area of the probe, V_c the potential difference arising within the plasma and α is of order unity (values of 0·4 → 1·0 have been obtained by various authors). In a turbulent plasma fluctations of n, T_e, V_c will give rise to probe current fluctuations, while second order effects which do not vanish when averaged over time may affect the mean current (see also DEMETRIADES and DOUGHMAN, 1965). The mean square probe current fluctuations will be given by an expression of the form

$$\overline{\delta i^2} = f_1^2\,\overline{\delta n^2} + f_2^2\,\overline{\delta T^2} + f_3^2\,\overline{\delta E^2} + 2f_1f_2\,\overline{\delta n \delta T} + 2f_1f_3\,\overline{\delta n \delta E} + 2f_2f_3\,\overline{\delta T \delta E} \qquad \ldots (9)$$

where

$$f_1 = \left(\frac{\partial i}{\partial n}\right)_{T,E} \quad , \quad f_2 = \left(\frac{\partial i}{\partial t}\right)_{n,E} \quad , \quad f_3 = \left(\frac{\partial i}{\partial E}\right)_{n,T}$$

and E is the electric field associated with gradients of the plasma potential V_c. Similarly the first order effect on the mean current is given by

$$2 \langle \Delta i \rangle = f_4 \overline{\delta n^2} + f_5 \overline{\delta E^2} + f_6 \overline{\delta T^2} + 2f_7 \overline{\delta n \delta E} + 2f_8 \overline{\delta n \delta T} + 2f_9 \overline{\delta E \delta T} \quad \ldots (10)$$

where

$$f_4 = \left(\frac{\partial^2 i}{\partial n^2}\right)_{T,E} \quad , \quad f_5 = \left(\frac{\partial^2 i}{\partial E^2}\right)_{n,T} \quad , \quad f_6 = \left(\frac{\partial^2 i}{\partial T^2}\right)_{n,E}$$

$$f_7 = \left(\frac{\partial^2 i}{\partial n \partial E}\right)_T \quad , \quad f_8 = \left(\frac{\partial^2 i}{\partial n \partial T}\right)_E \quad , \quad f_9 = \left(\frac{\partial^2 i}{\partial E \partial T}\right)_n .$$

The last two terms in equation (9) are antisymmetric with respect to positive and negative values of the applied bias voltage and thus can be removed by symmetrising. (There would be no antisymmetric terms in a homogeneous plasma.) The non-dimensional forms of the f's are

$$f_1 = \frac{2}{1 + e^{-x}} - 1 \quad , \quad f_3 = \frac{2e^{-x}}{(1 + e^{-x})^2} \quad , \quad f_2 = \frac{f_1}{2} - xf_3 \quad , \quad f_4 = 0$$

$$f_5 = -f_3 + \frac{4e^{-2x}}{(1+e^{-x})^3} \quad , \quad f_6 = -\frac{f_1}{4} + xf_3 - x^2 f_3 + \frac{4x^2 e^{-2x}}{(1+e^{-x})^3} \quad , \quad f_7 = f_3 \quad ,$$

$$f_8 = \frac{f_1}{2} - xf_3 \quad , \quad f_9 = -\frac{f_3}{2} + xf_3 - \frac{4xe^{-2x}}{(1 + e^{-x})^3}$$

where

$$x = \frac{eV}{KT_e}$$

and

$$\delta n \equiv \frac{\delta n}{n} \quad , \quad \delta i \equiv \frac{\delta i}{i_{satn}} \quad , \quad \delta T \equiv \frac{\delta T}{T} \quad , \quad \delta E \equiv \frac{\delta V_c e}{kT} .$$

Thus for any curve of $\overline{\delta i^2}$ against x we can attempt to obtain the four coefficients in equation (9) but this may not be easy due to the limited accuracy of $\overline{\delta i^2}$. $\overline{\delta E^2}$ is immediate from the value of

$\overline{\delta i^2}$ at $X = 0$ as $f_{1,2,6,8} \to 0$ as $X \to 0$. As f_2 has a zero at $X \approx 2{\cdot}2$ then we could determine $\overline{\delta n^2}$ from this point and as $X \to \infty$, $\overline{\delta i^2} \to \overline{\delta n^2} + \frac{1}{4}\,\overline{\delta T^2} + \overline{\delta n \delta T}$ which gives a further relation to determine the remaining two parameters.

The temperature is determined from the mean probe bias curve by determining the slope for small bias voltages but this is affected by the fluctuations through equation (10), as is the consequent determination of n from the bias curve for large V. We find from equation (10) that

$$i(V \to \infty) = \alpha A n e \left(\frac{2KT_e}{m_i}\right)^{\frac{1}{2}} \left[1 - \frac{1}{8}\,\frac{\overline{\delta T_e^2}}{T_e^2} + \frac{1}{2}\frac{\overline{\delta n \delta T_e}}{nT_e}\right]$$

$$i(V \to 0) = \alpha \frac{Ane}{2} \left(\frac{2KT_e}{m_i}\right)^{\frac{1}{2}} \left[\frac{\overline{\delta n \delta V_c}}{nKT_e}\, e - \frac{1}{2}\frac{\overline{\delta V_c \delta T_e}}{KT_e^2}\right]. \qquad \ldots (11)$$

The bias at which the slope for small bias intercepts $i(V \to \infty)$ is now

$$V = 2T_e \frac{\left(1 - \frac{1}{8}\frac{\overline{\delta T_e^2}}{T_e^2} + \frac{1}{2}\frac{\overline{\delta n \delta T_e}}{nT_e} - \frac{1}{2}\frac{\overline{\delta n \delta V_c}}{nKT_e} + \frac{1}{4}\frac{\overline{\delta V_c \delta T_e}}{KT_e^2}\right)}{\left(1 + \frac{3}{8}\frac{\overline{\delta T_e^2}}{T_e^2} - \frac{1}{4}\frac{\overline{\delta V_c^2}\, e^2}{(KT_e)^2} - \frac{1}{4}\frac{\overline{\delta n \delta T_e}}{nT_e}\right)} \qquad \ldots (11a)$$

Note that none of the fluctuation effects on the mean current depends strongly on the density fluctuations and we shall show below that the probe current fluctuations are principally due to density fluctuations thus there will only be small effects on the temperature and density determination.

We have simplified the fitting of equation (9) to our experimental results by writing

$$\delta T_e = \delta T_e^c + \delta T_e^u$$

where the two parts are defined by

$$\overline{\delta T_e^u \, \delta n} = 0$$

$$\overline{\delta n \delta T_e^c} = (\overline{\delta n^2})^{\frac{1}{2}} \, (\overline{\delta T_e^{c\,2}})^{\frac{1}{2}}$$

then necessarily $\overline{\delta T_e^c \, \delta T_e^u} = 0$ and we define Υ by

$$\frac{\delta T_e}{T_e} = (\Upsilon - 1)\frac{\delta n}{n} .$$

The results are much more sensitive to δT_e^c than δT_e^u, hence unless the latter is very large it cannot easily be detected. We have therefore assumed $\delta T_e^u \equiv 0$ in fitting our numerical results and make this assumption in the rest of this paper.

Υ is the usual ratio of specific heats; however we can also consider $\Upsilon - 1$ as the quantity $\frac{d \log T}{d \log n}$. With this simplification equation (9) becomes

$$\overline{\delta i^2} = (f_1 + f_2 \, (\Upsilon-1) \,)^2 \, \overline{\delta n^2} + f_3{}^2 \, \overline{\delta E^2} + 2 \, f_3 \, (f_1 + (\Upsilon - 1) \, f_2) \times \overline{\delta n \delta E} \qquad \ldots (12)$$

An example of the variation of $(\overline{\delta i^2})^{\frac{1}{2}}$ as a function of the bias is shown in Fig.2 for $\overline{\delta E^2} = 0$ and $\Upsilon = \frac{5}{3}$, 1 - the adiabatic and isothermal case.

The plasma will not in general be homogeneous and the density and temperature may differ at the two probes by Δn and ΔT_e respectively, which are also fluctuating quantities. We assume, and ensure practically, that $\overline{\Delta n}, \overline{\Delta T_e} = 0$ and so $\overline{\Delta n^2} \approx \overline{\delta n^2}$ (probe separation/Λ)^2, which is a small quantity. Provided $\frac{\Delta n}{n} \ll 1$ and $\Delta T_e/T_e \ll 1$ we can rederive equation (8) to give the modified expression for the probe current (see Appendix).

$$i = \alpha A n e \left(\frac{2KT_e}{m_i}\right)^{\frac{1}{2}} \left(\frac{2}{1 + (1 + \frac{\Delta n}{n} + \frac{\Delta T_e}{2T_e}) \, e^{\varphi}} - \left(1 - \frac{\Delta n}{2n} - \frac{\Delta T_e}{4T_e}\right)\right) \qquad \ldots (13)$$

where

$$\varphi = -\frac{e}{KT_e}\left(V + V_c + V_s\frac{\Delta T_e}{T_e} + \frac{K\Delta T_e}{e}\,\ell n\cosh\frac{eV}{2KT_e}\right) \quad ,$$

and Vs is the floating potential of the plasma. As E, Δn, ΔT_e are all proportional to probe separation then they all contribute to $\overline{\delta i^2}$ at $V = 0$. In the central regions of the discharge these corrections are negligible, except possibly for the term in φ involving the plasma potential which will also give rise to a mean probe current with no bias on the probe (c.f. equation (11)). As V, $V_c \to 0$ the bracketed part of equation (13) becomes $\frac{eV_s}{KT_e} \cdot \frac{\Delta T_e}{T_e}$ which is of order $\Delta T_e/T_e$ and again a small quantity. For large bias voltages the departures are of order $\frac{\Delta n}{2n} + \frac{\Delta T_e}{4T_e}$. A similar expression to equation (9) can now be obtained from equation (13), in this case involving 15 coefficients.

Our experiment thus consists of measuring i and its root mean square $(\overline{\delta i^2})^{\frac{1}{2}}$ as functions of the bias voltage. From the mean measurements we determine T_e (little affected by the fluctuations, equation (11a)) and an estimate of n and use the value of T_e in determining $\overline{\delta n^2}$, $\overline{\delta E^2}$ and Υ from the fluctuating results.

4. EXPERIMENTAL RESULTS

Our experiments have been performed in the ZETA discharge (BUTT et al., 1958; JONES and WILSON, 1962; BUTT et al., 1965; ROBINSON et al., 1967) at low power, at a current of 150 kA, applied magnetic field of 370 G and filling pressures in the range 0·5 to 5 mtorr D_2. The probe bodies were of quartz 11 mm in diameter; the electrodes were of platinum, 2 mm in diameter and 3 or 6 mm separation. The platinum pins are enclosed in quartz sheaths 1 mm thick which project 5 mm from the end of the body, so as to reduce the amount of quartz in the immediate neighbourhood of the probe.

Before each experiment the platinum electrodes are conditioned by applying a high bias voltage (in the region of 60-100 volts) in each sense for about 100 discharges. Initially arcs are formed between the electrodes during the discharge, but these disappear as the probe cleans up. There is no difficulty in distinguishing arcs from genuine probe signal. A long period integrator (LEES and RUSBRIDGE, 1964) has been used to measure the mean and variance of the probe current and the correlation of the signal from two probes. In the case of fluctuations it measures $\langle |\delta i| \rangle$ rather than the variance, but it can be shown that $\langle |\delta i| \rangle = 0{\cdot}769 \langle \delta i^2 \rangle^{\frac{1}{2}}$ provided the probability distribution of δi is gaussian. A filter passing frequencies above 10 kHz was used as it is known that this is sufficient to exclude the large scale motions of the entire discharge, leaving only the more localised turbulence. The signals were gated, the gate being open for some 400 μs centred around peak gas current for twenty successive discharges. Using the autocorrelation function for the fluctuations and the known gate length we can estimate the 'effective number or readings' per discharge (RUSBRIDGE 1962) and we then adjust the number of discharges necessary to achieve a given accuracy ($\lesssim$ 10%).

Before describing the experiments relating to equations (9) and (12) a number of preliminary experiments were performed to determine the radial variation of various quantities. As many of our relations involve the variation with filling pressure we first verified that the mean density varied approximately linearily with the filling pressure. If the voltage V developed across a 1Ω resistor due to the probe current is measured for large bias voltages then at saturation we have

$$\left(\frac{V}{\pi a^2}\right)^2 = \alpha^2 n^2 e^2 \frac{2T_e}{m_i} \qquad \ldots (14)$$

where a is the radius of the probe. At a particular pressure measurements of T_e from a probe plot, Fig.6, have been made and using the above relation we have deduced n. The variation is essentially linear except at the highest pressures where ionisation is probably incomplete.

Measurements of $\overline{\delta i^2}$ at these large bias voltages have also been made and these can be directly related to $\overline{\delta n^2}$ by equation (12) if $\Upsilon = 1$. The results, shown in Fig.3, have been corrected for the actual values of Υ obtained from results shown below. We see that the percentage density fluctuations range from 24 → 6%.

The radial variation of T_e has been obtained from a number of Langmuir probe plots and using equation (14) the radial variation of the mean density has also been measured. The results at 5 m torr of deuterium are shown in Fig.4. The variation of the r.m.s. density fluctuations has also been measured as is shown in Fig.5. This variation is very similar to that shown by the radial magnetic field fluctuations (ROBINSON, 1966; ROBINSON et al., 1968) and suggests a central 'core' region in which the turbulence is approximately homogeneous out to some 35 cm where we enter an 'edge' region. The relative level of the density fluctuations increases as one goes outwards and in the outer 10 cm of the discharge, the probe current traces give the impression of isolated plasma bunches moving over the probe.

A probe bias curve is shown in Fig.6 obtained at 0·5 mtorr; the displacement of the zero is a regular feature of our probe plots and its origin may be associated with the effects mentioned in equations (11) and (13). If we measure the fluctuations at the same time then we find that their minimum coincides with the bias at which the probe current vanishes.

Measurements of the liner voltage on the torus enables us to determine the discharge resistance at the time when our temperature measurements are made. Using the Spitzer formulae (SPITZER, 1956) and allowing for the mean field configuration we can estimate the average electron temperature. At 5 mtorr for example, we obtain 7 eV whereas a Langmuir probe plot gives 5·5 eV; thus the probe measurements are in fair agreement with the observed value of the resistance.

The experimental curves of $(\overline{\delta i^2})^{1/2}$ as a function of bias voltage have been obtained at various pressures and in different frequency bands. The results were analysed by first setting the saturation level from the mean of points with bias greater than $4\ T_e$, where saturation has been achieved to within 3%, and 10% for the isothermal and adiabatic cases respectively - thus a back correction could be made to the level. The r.m.s. electric field fluctuations were obtained from the points near zero bias then a fit was made to the remaining points by varying γ. A computer programme was also used to give a least squares fit for the three parameters and their errors.

Fig.7 shows the r.m.s. current fluctuations as a function of bias voltage for fluctuations in the range 10-25 kHz. The temperature of 5·5 eV was obtained from the probe plot and a fit of the experimental points for $\gamma = 5/3$, 1 is shown. The results definitely indicate that the density fluctuations are adiabatic at 5 mtorr, leading to temperature fluctuations of about 4%. The fluctuations > 25 kHz have also been studied and Fig.8 shows that we again obtain a good fit for the adiabatic assumption.

The results of 1·5 mtorr are shown in Figs.9 and 10 and least squares analysis in this case gives $\gamma = 0{\cdot}98 \pm 0{\cdot}05$ (the error on the temperature is about 10%), so the fluctuations are isothermal at this pressure. A similar curve for fluctuations > 25 kHz was also obtained with the result that $\gamma = 1{\cdot}06 \pm 0{\cdot}09$. Fig.10 shows a curve obtained with a 1 cm separation probe instead of a 3 mm one, and the effective electric field level should increase by a factor ~ 3. There may also be contributions from the Δn and ΔT_e terms which will modify this factor. The figure shows it to increase by a factor 2·5, verifying that the major part of the signal at zero bias voltage does behave as a genuine electric field signal. All these results confirm the theory presented in the previous section for the probe current fluctuations.

At ½ mtorr we obtain the results shown in Fig.11(a) but in this case to obtain a good fit we require the density and temperature to be negatively correlated, then we find $\gamma = 0{\cdot}83$. Under much higher current conditions and pressures of about 2 mtorr experiments have been made (ROBINSON et al., 1967) on the Thomson scattering of light from a giant pulse laser, which gives values for n_e and T_e over a small region in space and essentially at a particular time (pulse width 20 ns). The values are found to be negatively correlated, i.e. a local peak in density is cold. For example, four successive discharges yielded a Υ of $0{\cdot}3$, and the normalised density-temperature correlation was $-0{\cdot}9$. Thus this negative correlation is also observed by a different method, and may be associated with a differential heating mechanism. Our results for Υ as a function of I^2/ρ are summarised in Fig. 11(b).

The asymmetry term in equation (12) associated with a non zero $\overline{\delta n \delta E}$ correlation can be associated with the loss or injection processes in the plasma as $\delta E/B_0 \sim u$ (ROBINSON et al., 1968) - equation (7a). We may therefore expect an appreciable $\overline{\delta n \delta E}$ correlation in the region of maximum density gradient and arrive at an estimate for the containment time of the plasma. The effect of the electric field on a fluctuation-bias curve as given by equation (12) is shown in Fig. 12, where the cross correlation has been taken as $0{\cdot}3$ and $\frac{\overline{\delta n^2}}{n^2} = \frac{\overline{\delta V_c^2}\, e^2}{(kT_e)^2}$. Accordingly a fluctuation curve was obtained at $0{\cdot}5$ mtorr some 40 cm out from the geometric centre. Two curves were obtained, one with the pins aligned along the mean field which should show no asymmetry, and one with the pins transverse to the field. The first curve is shown in Fig. 13 which gives $\Upsilon = 0{\cdot}87$, and neither curve shows any sign of asymmetry. A full analysis of the two curves gave $\overline{\delta n \delta E}/(\overline{\delta n^2})^{1/2}\,(\overline{\delta E^2})^{1/2} \lesssim 0.1$. A value of about 0.1 does give a containment time which is of the same order as that measured by other methods (BURTON and WILSON, 1961).

These measurements show that the observed current fluctuations at high bias voltages are principally due to density fluctuations, though at the higher pressures the temperature fluctuations are not negligible.

5. DENSITY CORRELATION FUNCTION

Measurements of the radial density correlation function

$$J(r) = \langle n(\underline{x})\ n(\underline{x} + r) \rangle / \langle n^2(x) \rangle \qquad \dots (15)$$

have been made using two Langmuir probes each biassed to about 40 volts. The results are shown in Fig.14 for filling pressures of 5 mtorr and 1·5 mtorr. These results show that the integral scale L, where $L = \int_0^\infty J(r)\,dr$, is somewhat smaller than that observed for the magnetic field fluctuations (ROBINSON et al., to be published). The correlation function appears to possess no 'tail' but ends rather abruptly, and there is no evidence for a negative correlation at the longer separations.

If the density fluctuations arise from flow across the magnetic field lines then by conservation of particles in this two-dimensional plane

$$0 = \int_0^X \int_0^X \int_0^Y \int_0^Y \langle n(x_1 y_1)\ n(x_2 y_2) \rangle\, dx_1\, dx_2\, dy_1\, dy_2$$

$$= 4 \int_0^X \int_0^Y (x - \rho)\ (y - \xi)\ J(\rho,\xi)\ d\rho d\xi$$

if the system is isotropic and homogeneous. As $X, Y \to \infty$ the dominant term, after transformation to polar coordinates, gives $\int_0^R rJ\,(r)dr \sim 2\ L^3/R$ and so

$$\lim_{R \to \infty} \int_0^R rJ(r)dr \propto \frac{1}{R} \to 0 \qquad \dots (16)$$

thus the correlation $J(r)$ should change sign at large r. Our results do not support this and suggest instead that density fluctuations arise from motions along the lines of force.

Measurements made in the direction of the magnetic field indicate that the correlation length in this direction is at least 30 cm as we would expect from previous measurements on the electric and magnetic fields (ROBINSON et al., to be published).

The correlation curves in Fig. 14 do not exhibit perfect correlation at small separations. This may be due to some small scale density effects with lengths in the region of some millimetres or to some possible probe interference effects at small separations.

An independent check on the density correlation has been made (KING, to be published) by measuring the correlation function of the emitted visible light, which is a function of density and temperature. If the light, $l = f(n,T)$ then $\delta l = A\ \delta n + B\ \delta T^{c} + B\ \delta T^{u}$ and as $\delta T^{c} \propto \delta n$, $\overline{(\delta l)^2} = A_1^2\ \overline{\delta n^2} + B\ \overline{\delta T^{u2}}$, hence if δT^{u} is negligible (the laser results on $\overline{\delta n \delta T}$ suggest that it is) then δl reproduces the properties of δn. It can be shown in the simplest case that if $I(r)$ is the measured intensity correlation function then

$$I(r) = \int_r^\infty J(\rho)d\rho \Big/ \int_0^\infty J(\rho)d\rho$$

and also

$$\int_0^\infty I(r)dr = \int_0^\infty \rho J(\rho)d\rho \Big/ \int_0^\infty J(\rho)d\rho \ .$$

Thus equation (16) implies $\int_0^\infty I(r)dr = 0$ so $I(r)$ must also change sign at large r, and because of the weighting with r in equation (16), observations of $I(r)$ provide a more sensitive test of equation (16) than direct measurements of $J(r)$ itself. The results indicate very strongly that $J(r)$ does not change sign.

The intensity fluctuations are related to the density fluctuations by

$$\frac{\overline{\delta I^2}}{I^2} = \frac{2L}{D} \frac{\overline{\delta n^2}}{n^2} \qquad \ldots (17)$$

where D is the length of plasma emitting the light. The observed I(r) curve is in reasonable agreement with our measured J(r) in particular L for the intensity fluctuations is 4 cm at 5 mtorr which is a little larger than that given by Fig. 14.

The Taylor microscales λ_n of the density fluctuations defined as $\frac{1}{\lambda_n^2} = \left(\frac{\partial^2 J}{\partial r^2}\right)_{r \to 0}$, may be small (ROBINSON et al., to be published) and our usual method of measurement of this quantity fails in this instance because of the imperfect correlation at small separations. This lead us to consider shadow-graph techniques (UBEROI and KOVANZNAY, 1955; TATARSKI, 1961) which are sensitive to the double derivative of the density. It can be shown that the resultant intensity fluctuations in this case possess an integral scale which is essentially the microscale of the density fluctuations. If diffraction effects are avoided then the intensity fluctuations are related to the size of the density fluctuations and the spectral index of these fluctuations through a combination of the large and small scale lengths. Measurements have been made (GONDHALEKAR, private communication) with a laser beam working at 10 μ and give values of λ_n comparable with L at high pressures as we would expect (ROBINSON et al., to be published), though significantly smaller at lower pressures.

The auto-correlation function of the density fluctuation – $\langle n(x,t)n(x,t+\tau) \rangle$ has been measured for a delay of up to 40 μs and is shown in Fig. 15. The large scale oscillation is due to the 10 kHz filter used. By studying a simple frequency spectrum which is cut off at 10 kHZ and otherwise resembles the measured spectrum we can calculate the auto-correlation function which we compare with the experimental curve to obtain the auto-correlation time, $\tau_c = \int_0^\infty J(x,\tau)d\tau$. This gives a time of about 14 µs. Using this time scale and our known integral scale of 3 cm at 1·5 mtorr, we can construct a velocity, which gives 2×10^5 cm s^{-1}. This figure is smaller than the figure obtained from direct measurements of the electric field and constructing a velocity using the mean field strength. A comparable calculation with the magnetic field auto-correlation function does give a velocity directly comparable with the electric field one.

Delayed correlations of the type $\langle n(x,t)\ n(x + r,\ t + \tau) \rangle$ have been measured for τ = 10 and 30 µs and the results at 1·5 mtorr are shown in Fig.16. If the density fluctuations arise from across magnetic field lines then the integral scale of the delayed correlation should be greater than that of the ordinary radial correlation and given by an expression of the form

$$L_\tau^2 = L_0^2 + 2D_r\tau \ ,$$

where L_τ is the integral scale of the delayed correlation, L_0 the usual integral scale, D_r is the radial diffusion coefficient and τ the delay, which must be greater than the correlation time for the expression to be valid. The results show no significant broadening in contrast to the results obtained for the radial magnetic field fluctuations where a significant broadening is observed and D_r can be calculated (ROBINSON, 1966; ROBINSON et al., 1968).

Measurements of the fluctuating magnetic field parallel to the mean field have been made. These show that the ratio of the r.m.s. fluctuations to the mean field is small (~ 0·3%) and much less than the relative density fluctuations. From the pressure balance equation we must have

$$\frac{\gamma(\overline{n^2})^{\frac{1}{2}}\beta}{n_o} = \frac{2(\overline{b_{11}{}^2})^{\frac{1}{2}}}{B}$$

where γ is the ratio of specific heats and β is the ratio of plasma pressure to magnetic pressure. This relation is well satisfied by the measured fluctuation quantities, as is demonstrated in Table II.

Estimates of density fluctuations and scale lengths based on microwave transmission (WORT, 1966) agree reasonably well with our results.

6. WAVE MOTION

Measurements have been made to detect any wave motion shown by the density fluctuations. Drift waves (BOL, 1964; GALEEV et al., 1964) may be expected in the 'edge' region of the discharge where the mean density gradient is important. Initially two probes were inserted 15 cm in from the outer wall with a separation of 6 cm and the correlation measured as a function of the time delay; however it is apparent that one could also fix the time delay and measure the correlation as a function of position. Both methods will give rise to a velocity which will only be identical if we have a pure wave propagating and not superposed on some 'background' turbulence. If there is no propagation then the first velocity will be infinite and the second zero. A similar situation has been examined by Briggs (BRIGGS et al., 1950) when interpreting the reflection of radio waves from the ionosphere. If it is assumed that the correlation contours in the $\underline{r}$ - t plane are ellipses then if V' is the first velocity and V the second then $VV' = V_c^2$ where V_c is the velocity derived from the basic correlation length and time. V is the required true velocity i.e. the velocity of an observer who has adjusted his motion so that in comparing two signals τ_1 apart, has maximised $J(\underset{\sim}{\xi},\tau)$ where $\underset{\sim}{\xi}$ is the distance apart.

If we calculate the drift velocity $\frac{KT_e}{eB}\frac{1}{n}\frac{dn}{dx}$ using the results of Fig.4, we obtain values in the region $0{\cdot}3 - 1{\cdot}0 \times 10^5$ cm s^{-1} and the direction of propagation should be in the electron drift direction. The results obtained for the fixed spatial separation are shown in Fig.17 and indicate a velocity $V' \gtrsim 1{\cdot}5 \times 10^6$ cm s^{-1}. If this is combined with an estimate of V_c as mentioned in the previous section then $V \lesssim 5 \times 10^4$ cm s^{-1}. The measurements for a fixed time delay of 5 μs are shown in Fig.18 and give a velocity $V \lesssim 1 \times 10^5$ cm s^{-1} which is in the electron drift direction. These measurements are not meaningful until corrected for a velocity arising from the radial electric field in the plasma. This is in the same direction as the electron drift and of magnitude $\lesssim 7 \times 10^4$ cm s^{-1}. Consequently without a further improvement in accuracy of the experiment it is not possible to say that we have detected such waves.

Such a wave has its electric field vector 90° out of phase with the density perturbation and thus the cross correlation should be zero. We have measured this cross correlation directly with two probes placed close together and obtained a value of $0{\cdot}32 \pm 0{\cdot}04$. We can then use this directly to estimate the containment time of the plasma using the data of Fig.4 and the following reasoning. The radial loss current is

$$j_r = \langle \rho u_r \rangle \sim \frac{\langle \rho Ez \rangle}{B_\theta} = \alpha \, (\overline{\rho^2})^{\frac{1}{2}} \frac{(\overline{E_z{}^2})^{\frac{1}{2}}}{B_\theta}$$

where α is the measured correlation. Identifying this with $D \frac{\partial\rho}{\partial x}$ where D is the diffusion coefficient and then using $a^2 = 2D\tau$

where τ is the containment time and a is the distance from the central core region of the discharge to the wall (~ 15 cm), we can estimate τ. Values in the region of 50-150 μs are obtained and are consistent with values obtained by other methods (BURTON and WILSON, 1961).

Note that the value of the cross correlation obtained here is somewhat greater than the upper limit obtained from the asymmetry determination of Fig.13. This may be due to the fact that $\Delta n/n$ is not negligible in this region and fluctuating density gradient terms contribute significantly to the probe current fluctuations.

7. DISCUSSION

We can distinguish three ways in which the density fluctuations may arise:-

(1) By convection in the presence of a density gradient, e.g. equation (7) or $\delta\rho \approx \underline{\xi} \cdot \nabla\rho_o$ where ξ is the plasma displacement and ρ_o the mean density. Following the usual mixing length theories (KADOMTSEV, 1965) we might identify ξ with such a length or the transverse correlation length. Such an expression does not fit our results since $\delta\rho$ exhibits no minimum at the discharge centre where $\nabla\rho_o$ vanishes. However, we cannot exclude the possibility that $\underline{\xi}$ should be a Lagrangian correlation length which could be long; which roughly means that the density within a 'turbulent element' remains equal to the mean density at the point where it was formed, even though it may wander through most of the discharge in its subsequent life. Experiments on turbulent gas <u>flow</u> in a discharge (GRANATSTEIN and BUCHSBAUM, 1965) do agree with an expression such as equation (7) for the energy containing eddies but not for the smaller scale

eddies where the observations are similar to ours.

(2) From the divergence of the flow normal to the magnetic field. As we have already seen this case can be distinguished experimentally by the form of the correlation function normal to the field lines - equation (16). The results showed that there was no change in sign. In addition, our theoretical model for $K_3 = 0$ could only give rise to small density fluctuations, equation (4), unless the value of ν_m is anomalously high, as might be the case if plasma motion across the field lines were due to Bohm diffusion rather than classical finite resistivity. Thus it seems probable that this motion is not the origin of the density fluctuations.

(3) By flow along the magnetic field, for example, by acoustic waves. We have examined such a situation with our theoretical model with $K_3 \neq 0$ and found that density fluctuations of a magnitude similar to those observed can be obtained for values of K_3 (~ 0.02) which are not too small and classical values of ν_m. If K_3 is very much smaller than this then the model can only produce density fluctuations of the correct magnitude by again involving an anomalously high ν_m (or turbulent conductivity). Pure acoustic waves should be isothermal - which agrees with our results at lower pressures but not at high pressures where the discharge is radiation cooled. As $Ti \gtrsim Te$ these waves are very heavily damped and may be difficult to detect as acoustic waves. Note that the theoretical model does give a filling pressure variation which is similar to that observed experimentally.

8. CONCLUSIONS

Fluctuations in the current to a Langmuir probe in a turbulent plasma have been measured for various filling pressures, frequency bands, probe separations and positions in the discharge. The measurements were found to be quite consistent with the probe theory. At high bias the fluctuations are predominantly due to plasma density fluctuations. Information about the temperature fluctuations was also obtained, notably that the fluctuations are adiabatic at high pressures and at low pressures it is apparent that the density and temperature become negatively correlated, requiring a differential heating mechanism.

The origin of the fluctuations has been considered and from results on the radial correlation function, the radial variation of the density fluctuations and the variation with filling pressure it was concluded that they arise from flow along the magnetic field. A theoretical model was able to predict the correct size and pressure variation for these fluctuations, though it may be necessary to assume Bohm diffusion rather than finite resistivity. Convection due to a mean density gradient and flow across the magnetic field lines do not account for the observed fluctuations.

Measurements of the fluctuating magnetic field in the mean field direction confirm the pressure balance relation. Confirmation of the radial density correlation function was also obtained from two other independent measurements.

Attempts at detecting drift waves in the edge regions of the discharge by measuring the delayed correlation function resulted in no conclusive evidence for a drift wave though a motion with the right sort of velocity and direction was observed. The cross correlation function between the density and electric field was measured at the edge of the discharge to yield an estimate of the containment time, which was found to be consistent with that obtained by other methods.

9. ACKNOWLEDGEMENTS

We are grateful to Mr. R.E. King and Mr. R.B.E. Sturch who assisted in the probe measurements.

TABLE I

Density fluctuations in various limits

Field limit	$H_0^2 \gg C_0{}^2 \gg \nu m.(\frac{4}{3}\nu + \nu m)k^4$ strong field	$C_0{}^2 \gg H_0^2 \gg \nu m.(\frac{4}{3}\nu + \nu m)k^4$	dissipative $\nu m(\frac{4}{3}\nu + \nu m)K^4 \gg C_0{}^2$, H_0^2
$H_0 \to 0$	--	--	$u^2/C_0{}^2$
$H_0^2\ k_3^2 \ll \nu\nu_m k^4$ two-dimensional	u^2/H_0^2	$u^2/C_0{}^2$	$u^2/C_0{}^2$
$H_0^2\ k_3^2 \gg (\nu + \nu m)\nu mk^4$ 1. $C_0^2\ k_3^2 \gg (\nu + \nu m)\nu mk^4$	$u^2/C_0{}^2$	$u^2/C_0{}^2$	$u^2/C_0{}^2$
2. $C_0^2\ k_3^2 \ll (\nu + \nu m)\nu mk^4$	$\frac{u^2k_3^2}{\nu m(\nu + \nu m)k^4}$	$u^2/C_0{}^2$	$u^2/C_0{}^2$

TABLE II

Pressure mtorr	Υ	$\delta n/n$	$\beta = \frac{2b_{\parallel}}{Bo}\ \frac{n}{\Upsilon\delta n}$ %	Te + Ti (eV)	$\beta = \frac{nK\ (Te + Ti)}{B_0^2\ 8\pi}$ %
½	0·83	0· 24	2·6	42	2·7
1	0· 9	0· 12	1·5	12	1.7
1½	0·98	0· 07	2·4	10	2·3
5	1·67	0·065	3·4	6	2·7

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APPENDIX

As the total positive ion current must equal the total electron current

$$i_{p_1} + i_{p_2} = i_{e_1} + i_{e_2} = A\ j_1 e^{-\varphi_1 V_1} + A\ j_2 e^{-\varphi_2 V_2}$$

and

$$V_d = V_1 - V_2 + V_c\ .$$

A is the area of the probe and j, its respective random electron current, V_1 is the potential across the sheath of probe 1, V_c is a potential in the plasma, V_d the probe bias and φ_1 is $\frac{e}{KT_{e_1}}$ for probe 1. The net probe current, in terms of the two saturation currents, is then

$$id = \frac{i_{s_1} + i_{s_2}}{1 + \frac{j_1}{j_2} e^{\varphi_1 V_c + (\varphi_2 - \varphi_1) V_2 - \varphi_1 V_d}} - i_{s_2}\ .$$

If we then write

$$\varphi_1 = \varphi + \frac{\Delta\varphi}{2}, \quad \varphi_2 = \varphi - \frac{\Delta\varphi}{2}, \quad n_1 = n + \frac{\Delta n}{2}, \quad T_1 = T + \frac{\Delta T}{2}$$

assume $\frac{\Delta n}{n}, \frac{\Delta T}{T} \ll 1$, and use the usual expressions for i_s we obtain

$$id = \alpha A n e \left(\frac{2KT_e}{m_i}\right)^{1/2} \left\{ \frac{2}{1 + \left(1 + \frac{\Delta n}{n} + \frac{\Delta T_e}{2T_e}\right) e^{-\Delta\varphi\ (V_1 + V_2)/2 - \varphi(V_D - V_c)}} - \left(1 - \frac{\Delta n}{2n} - \frac{\Delta T_e}{4T_e}\right) \right\}$$

As we need only the mean value of $\frac{V_1 + V_2}{2}$ we neglect V_c, $\Delta\varphi$ etc. and obtain

$$\frac{V_1 + V_2}{2} = \frac{V_d}{2} - \frac{1}{\varphi} \ell n \frac{5}{1 + e^{-\varphi V_d}} + V_S$$

where V_S is the plasma potential Hence the exponential factor is

$$e^{-\frac{e}{KT_e}\left(V_D - V_c + V_S \frac{\Delta T_e}{T_e} + \frac{K\Delta T_e}{e} \ell n \cosh \frac{eV_D}{2KT_e}\right)}$$

which gives equation (13).

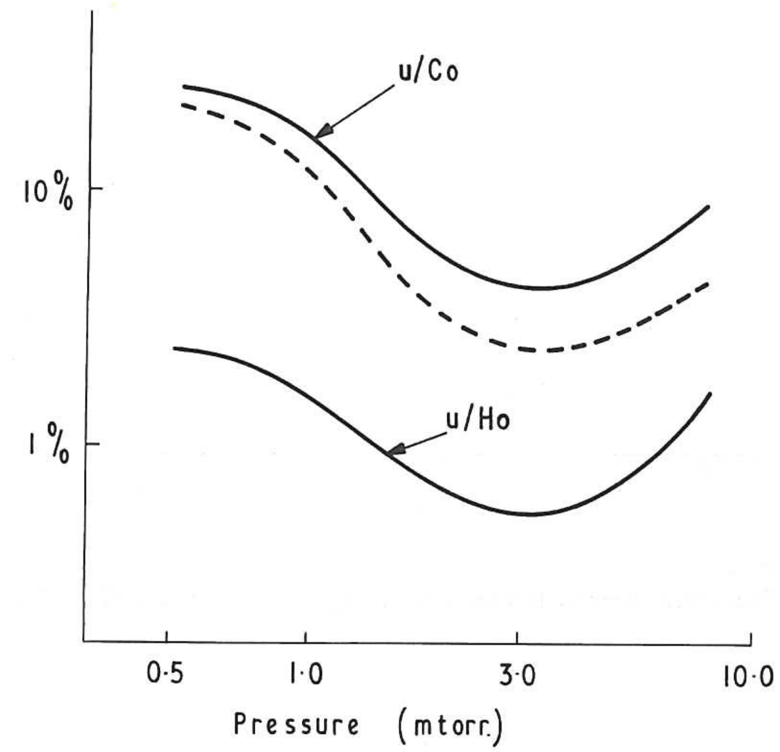

Fig. 1 (CLM-P 178)
Percentage density fluctuations as a function of filling pressure .... $K_3 = 0.02$ for a gas current of 150 kA axial field 370 G

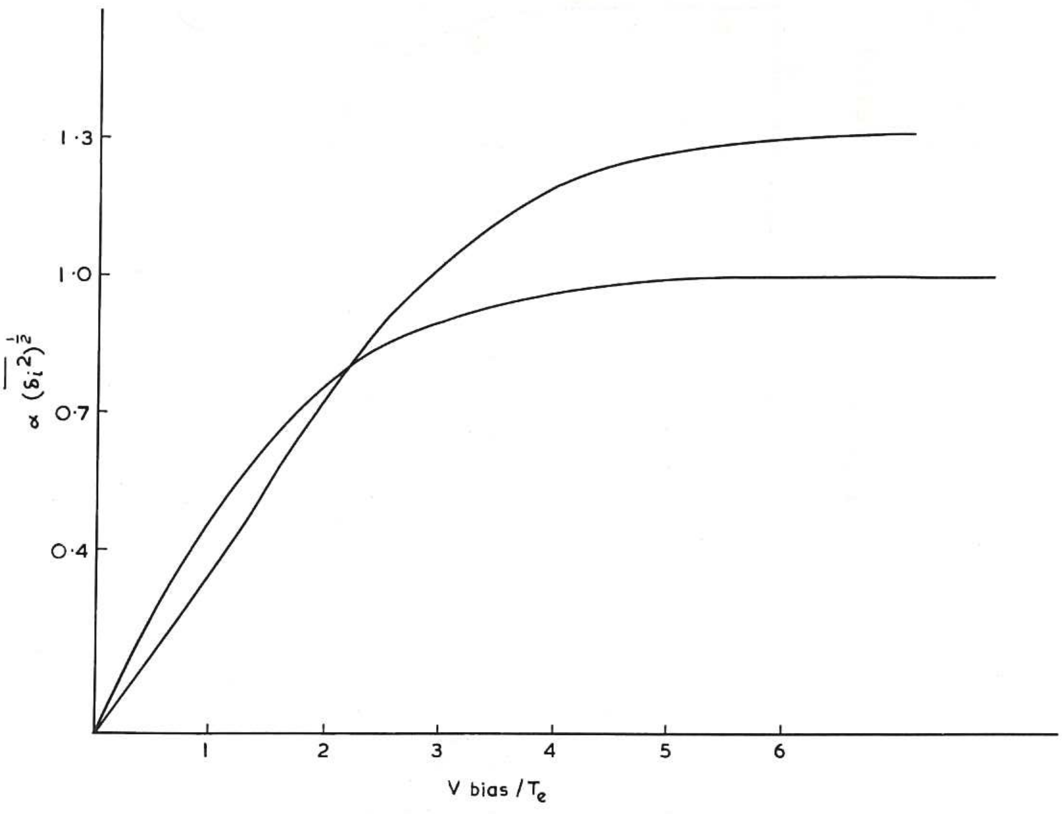

Fig. 2 (CLM-P 178)
Fluctuations in probe current as a function of bias voltage
Upper curve $\gamma = 5/3$, lower $\gamma = 1$, isothermal, $\overline{\delta E^2} = 0$

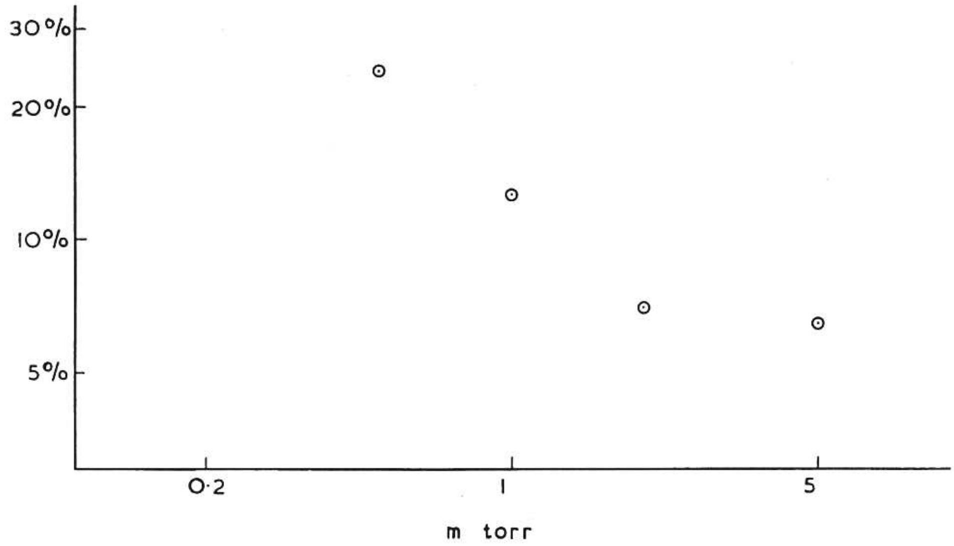

Fig. 3 (CLM-P 178)
Percentage density fluctuations in D_2, 150 kA, axial field 320 G

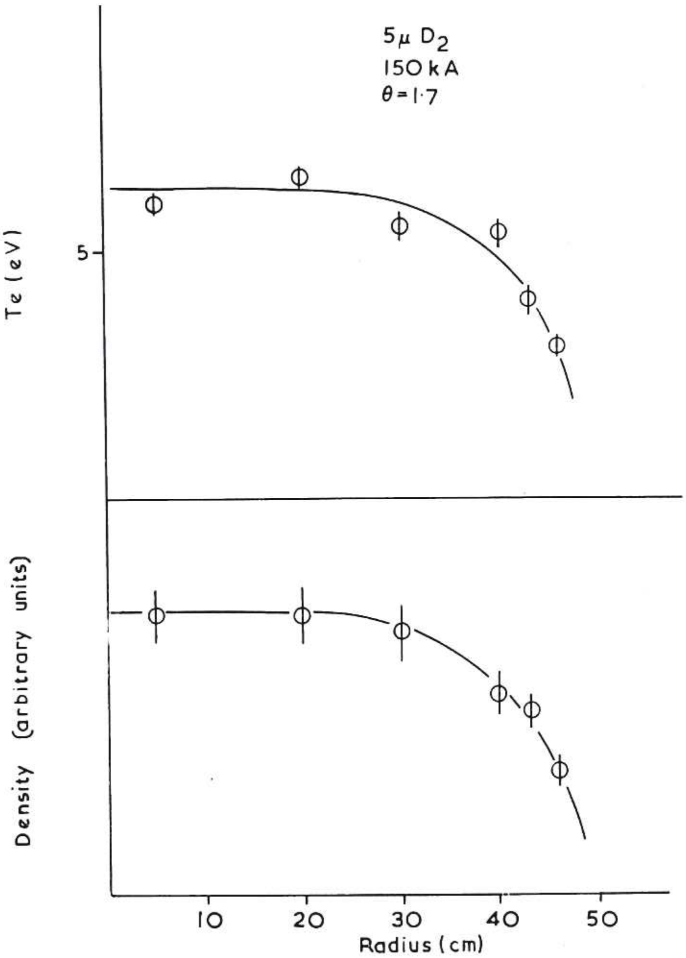

Fig. 4 Radial variation of electron temperature and density (CLM-P 178)

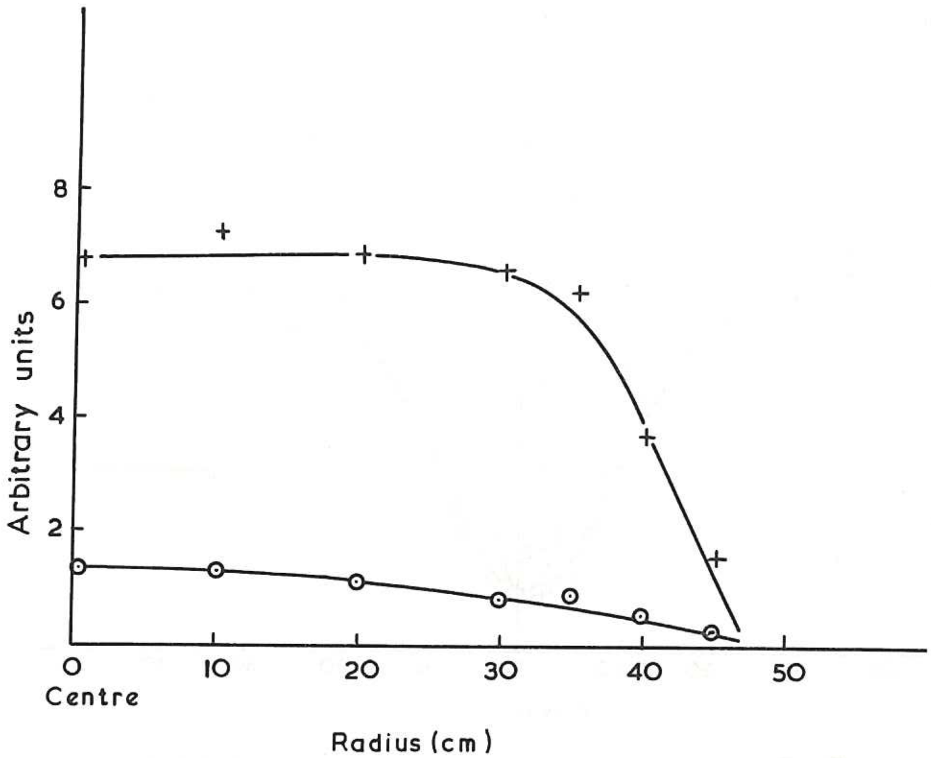

Fig. 5 (CLM-P 178)
Probe current fluctuations as a function of radius. + 30 volts bias (essentially density fluctuations), 5 mtorr D_2, 150 kA axial field 370 G. ⊙ O volts bias, electric field

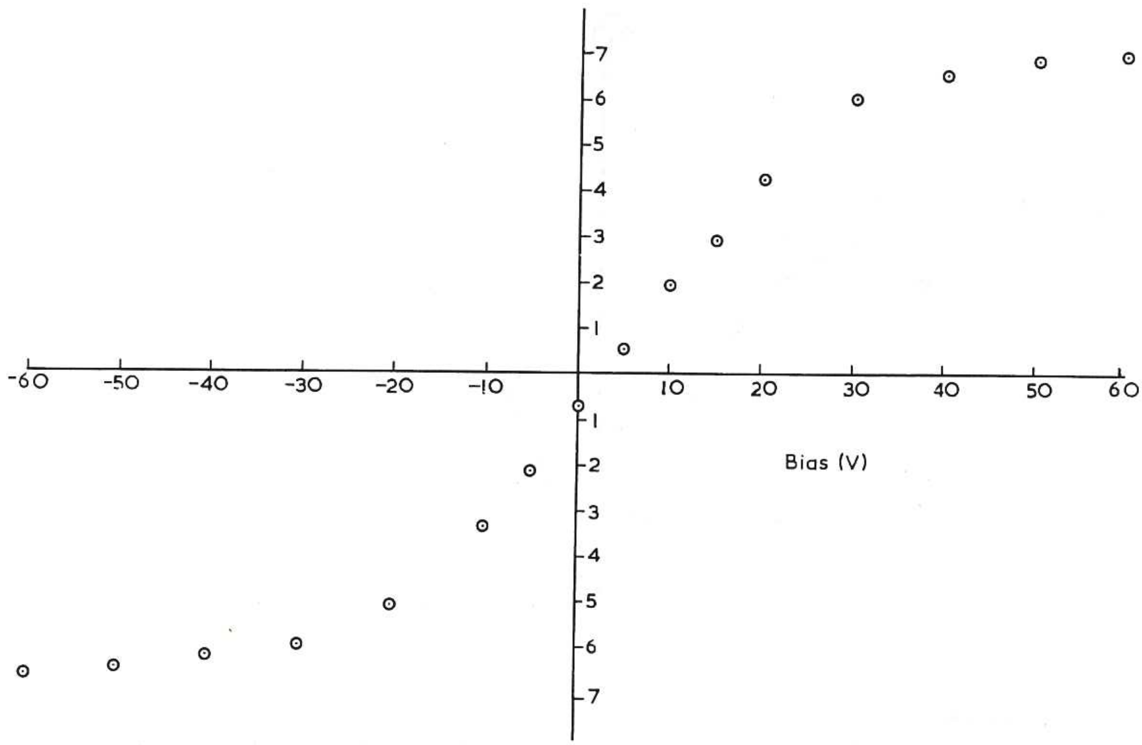

Fig. 6 (CLM-P 178)
Langmuir double probe characteristic, 0.5 mtorr D_2, 150 kA, axial field 370 G, T_e = 11.5 eV

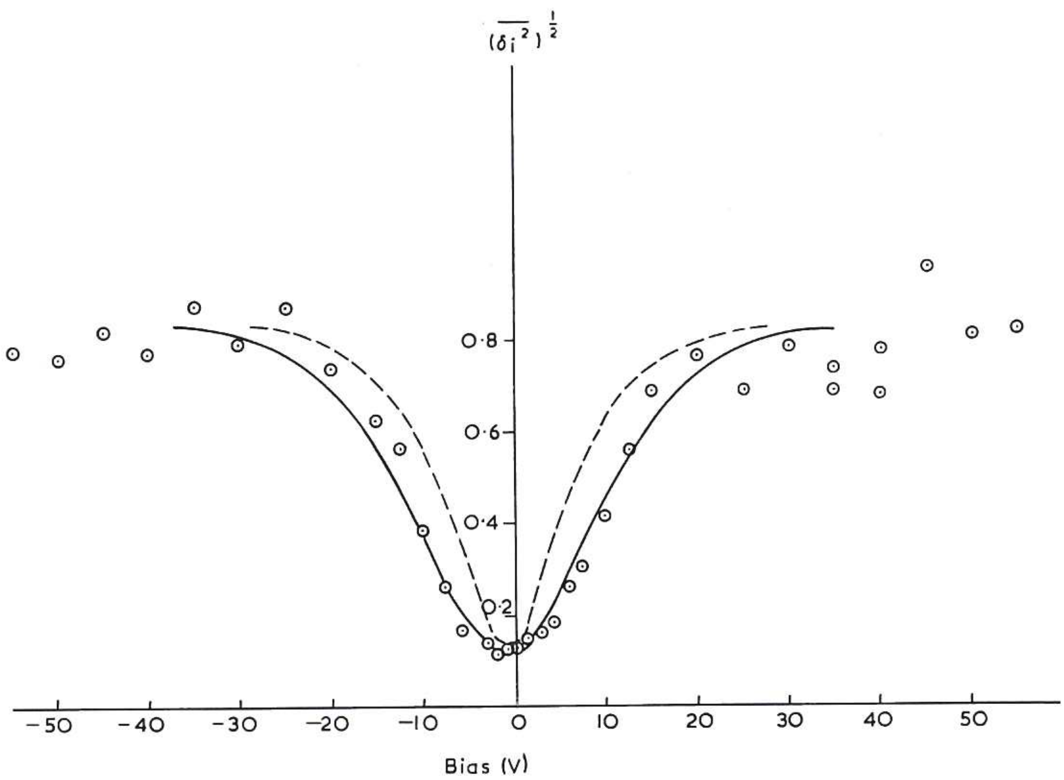

Fig. 7 (CLM-P 178)
r.m.s. probe current as a function of bias voltage, 5 mtorr D_2, 150 kA, axial field 370 G, 3 mm probe separation, $T_e = 5.5$ eV, ——— $\gamma = 5/3$, – – – – isothermal

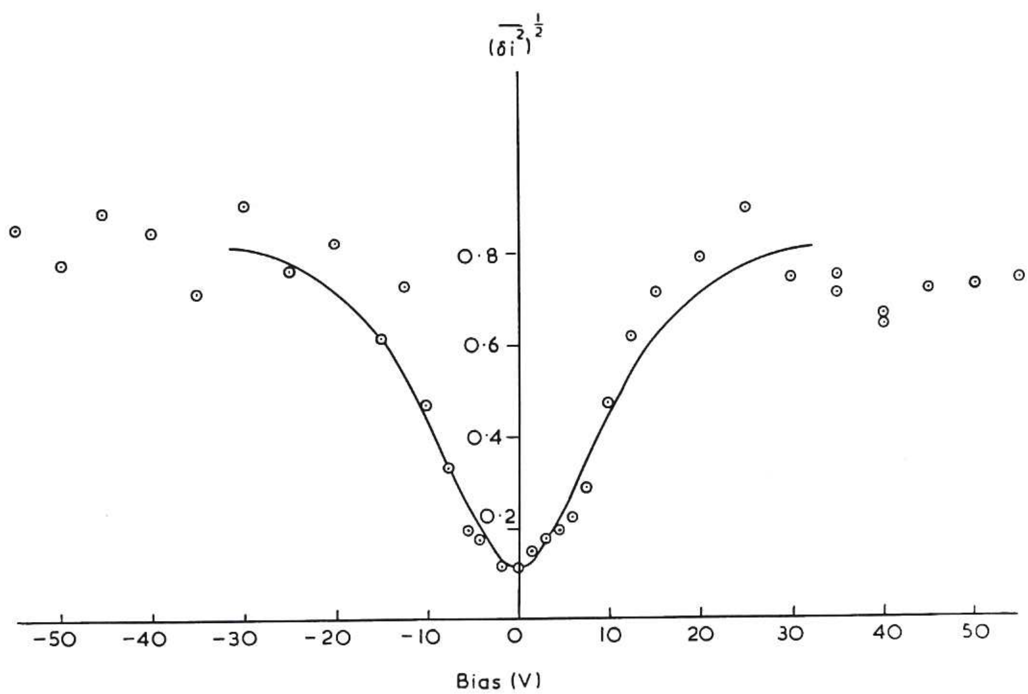

Fig. 8 (CLM-P 178)
r.m.s. probe current as a function of bias voltage, 5 mtorr D_2, 150 kA, axial field 370 G, > 25 kHz, 3 mm probe, ——— $\gamma = 1.5$, $T_e = 5.5$ eV

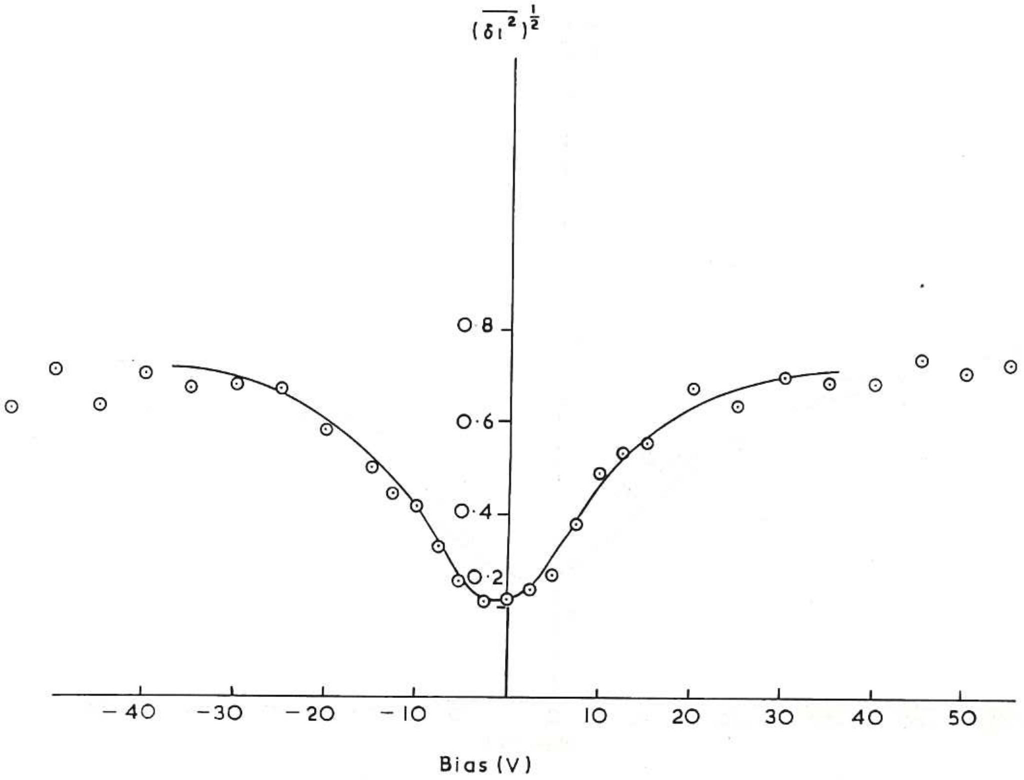

Fig. 9

(CLM-P 178)

r.m.s. probe current as a function of bias voltage, 1.5 mtorr D_2, 150 kA, axial field 370 G, 3 mm probe, $T_e = 7.6$ eV, —— isothermal

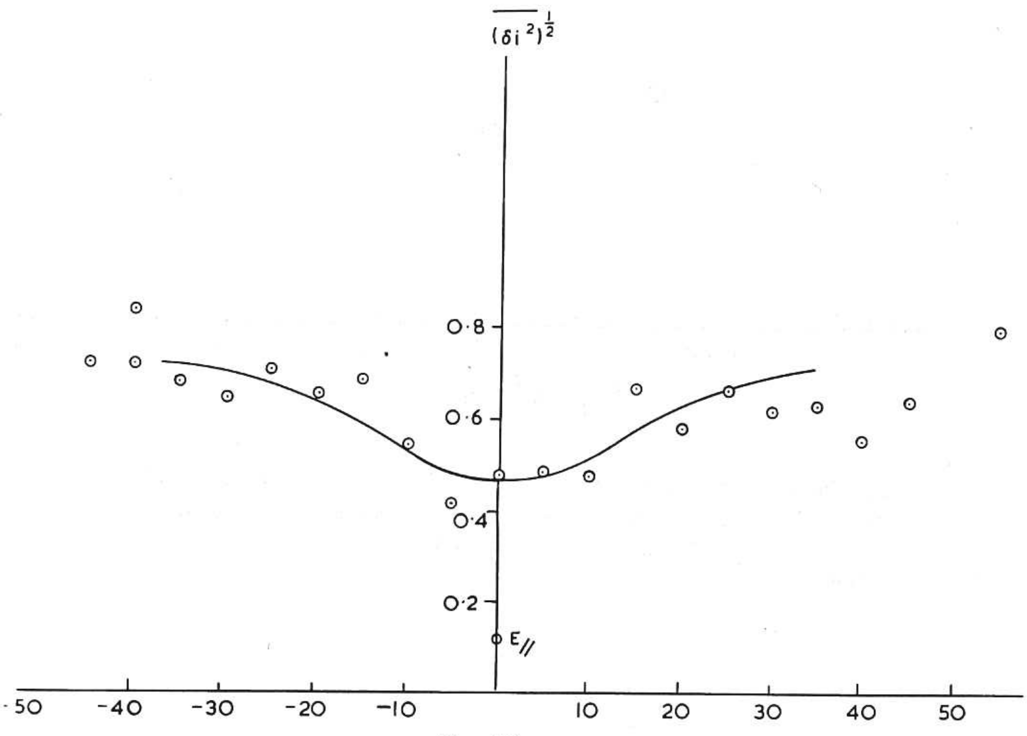

Fig. 10

(CLM-P 178)

Variation of the r.m.s. probe current as a function of bias voltage, 1.5 mtorr D_2, 150 kA, axial field 370 G, 1 cm probe, $T_e = 7.6$ eV, —— isothermal

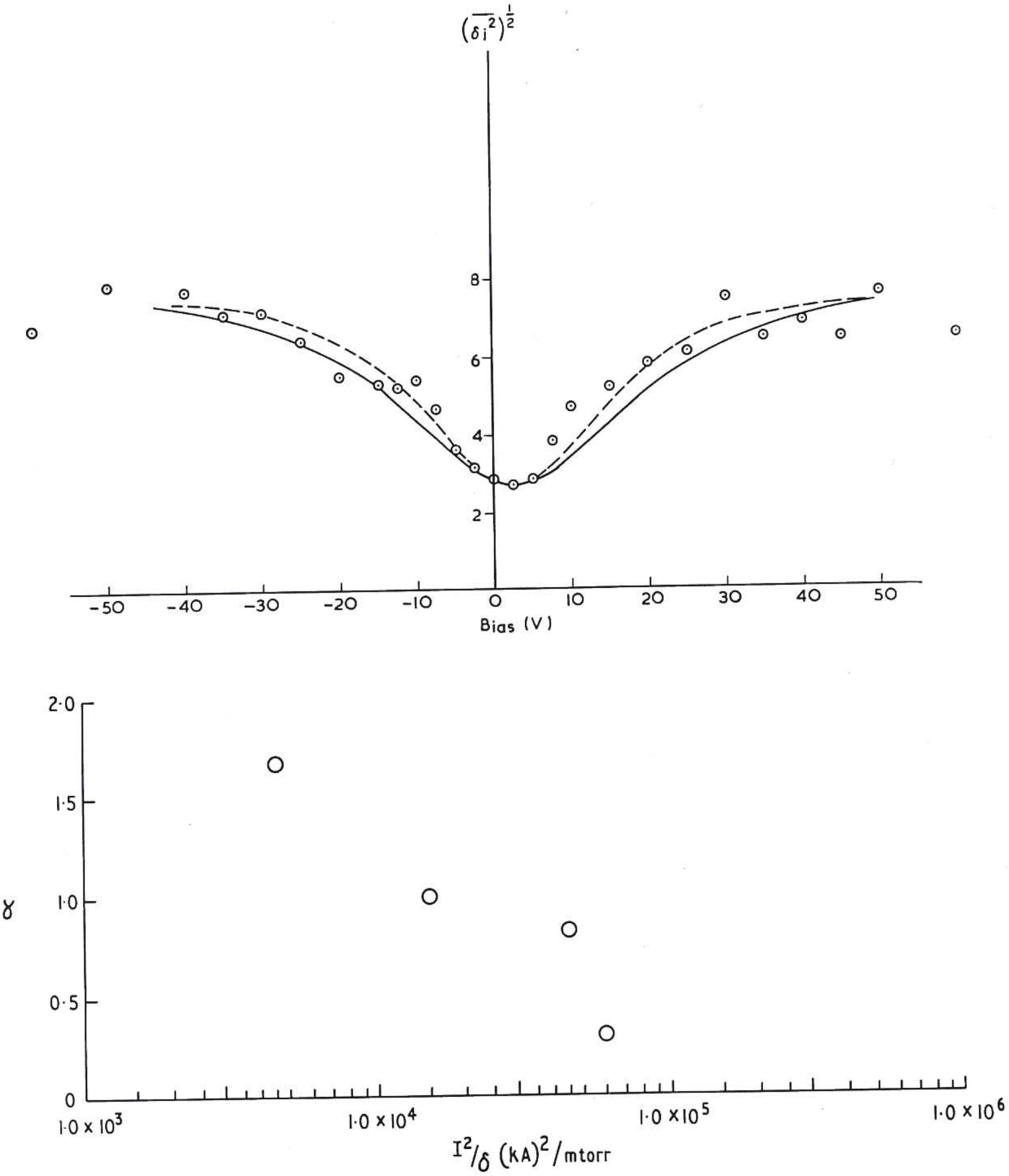

Fig. 11 (CLM-P 178)

(a) r.m.s. probe current as a function of bias voltage, ½ mtorr D_2, 150 kA, axial field 370 G, 10 – 25 kHz, 3 mm probe at centre, T_e = 11.0 eV, ——— isothermal, ----- $\gamma = 0.83$, $\langle nT \rangle / nT = -1$

(b) γ as a function of I^2/ρ

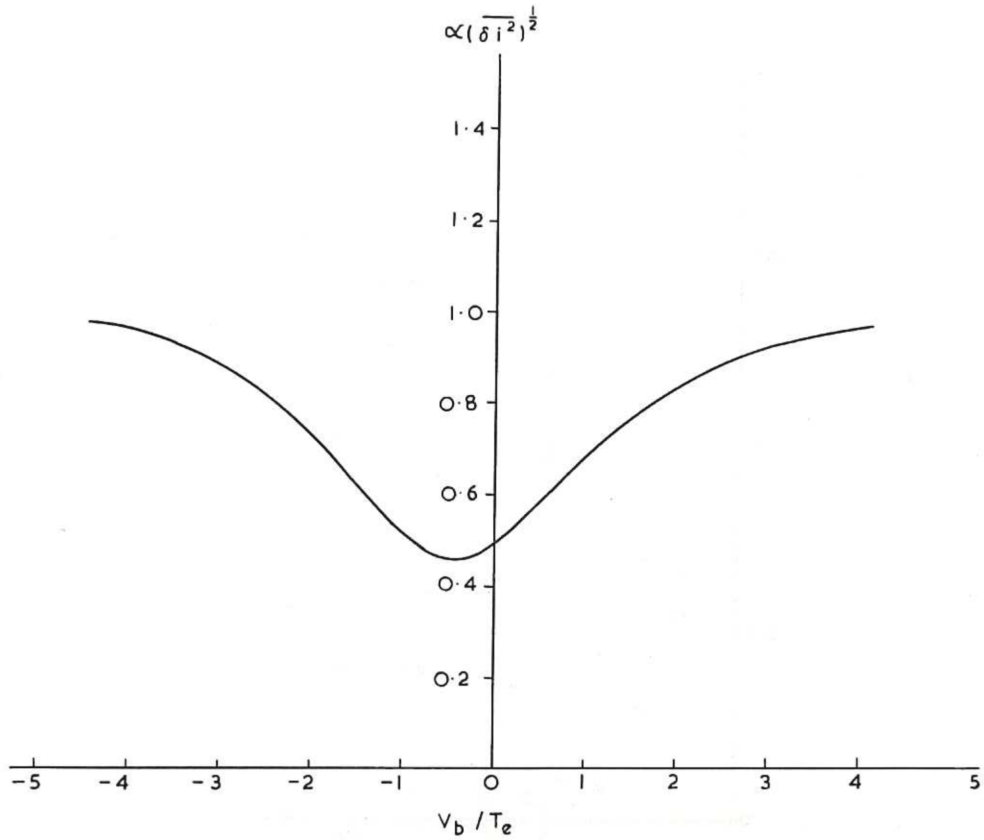

Fig. 12 (CLM-P 178)
Variation of r.m.s. probe current for $(\overline{\delta n^2})^{1/2} \equiv (\overline{\delta E^2})^{1/2}$, $\langle \delta n \delta E \rangle / nE = 0.3$, isothermal

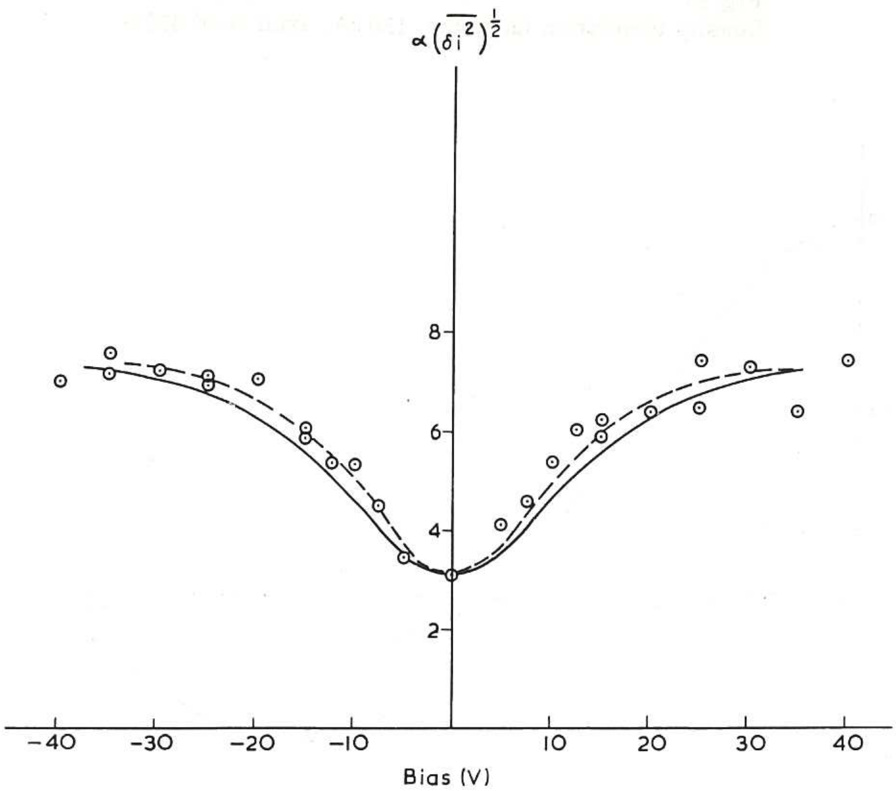

Fig. 13 (CLM-P 178)
r.m.s. probe current as a function of bias voltage, 0.5 mtorr D_2, axial field 370 G, 150 kA, > 10 kHz, 1 cm probe, T_e = 8.25 eV, 40 cm from centre, —— isothermal, ---- $\gamma = 0.87$, $\langle nT \rangle / nT = -1$

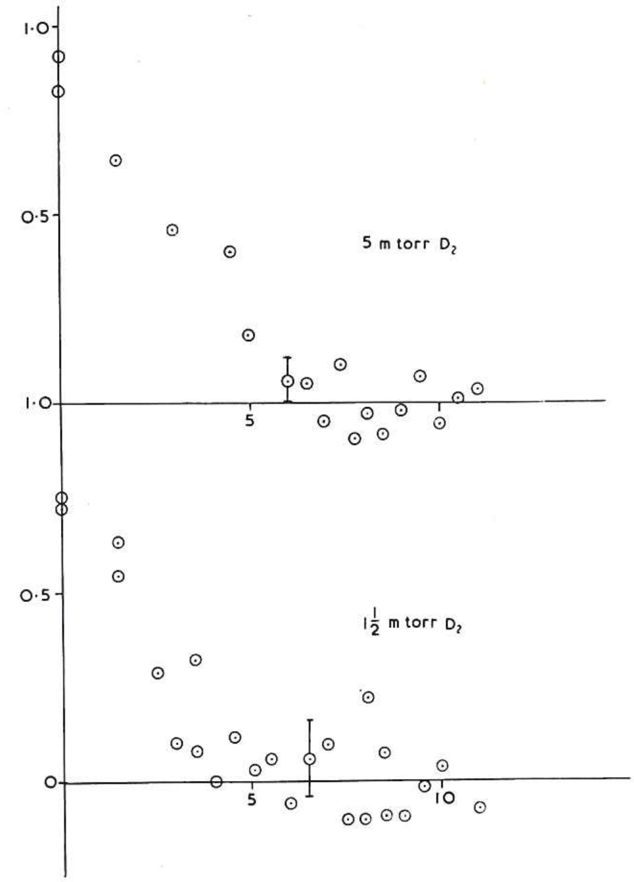

Fig. 14 (CLM-P 178)
Density correlation functions, 150 kA, axial field 370 G

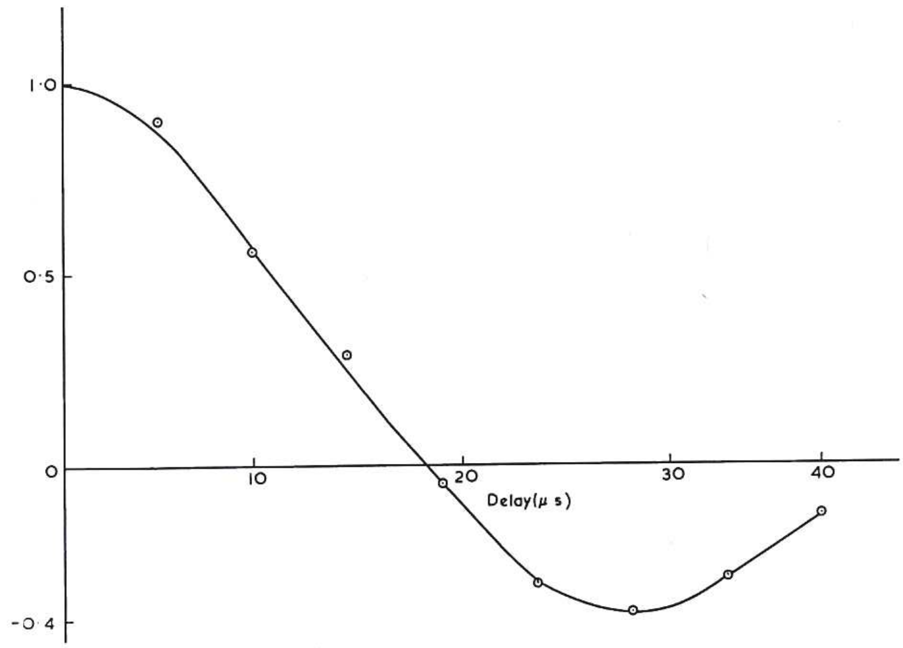

Fig. 15 (CLM-P 178)
Auto-correlation function for the density fluctuations,
1.5 mtorr D_2, 150 kA, axial field 370 G

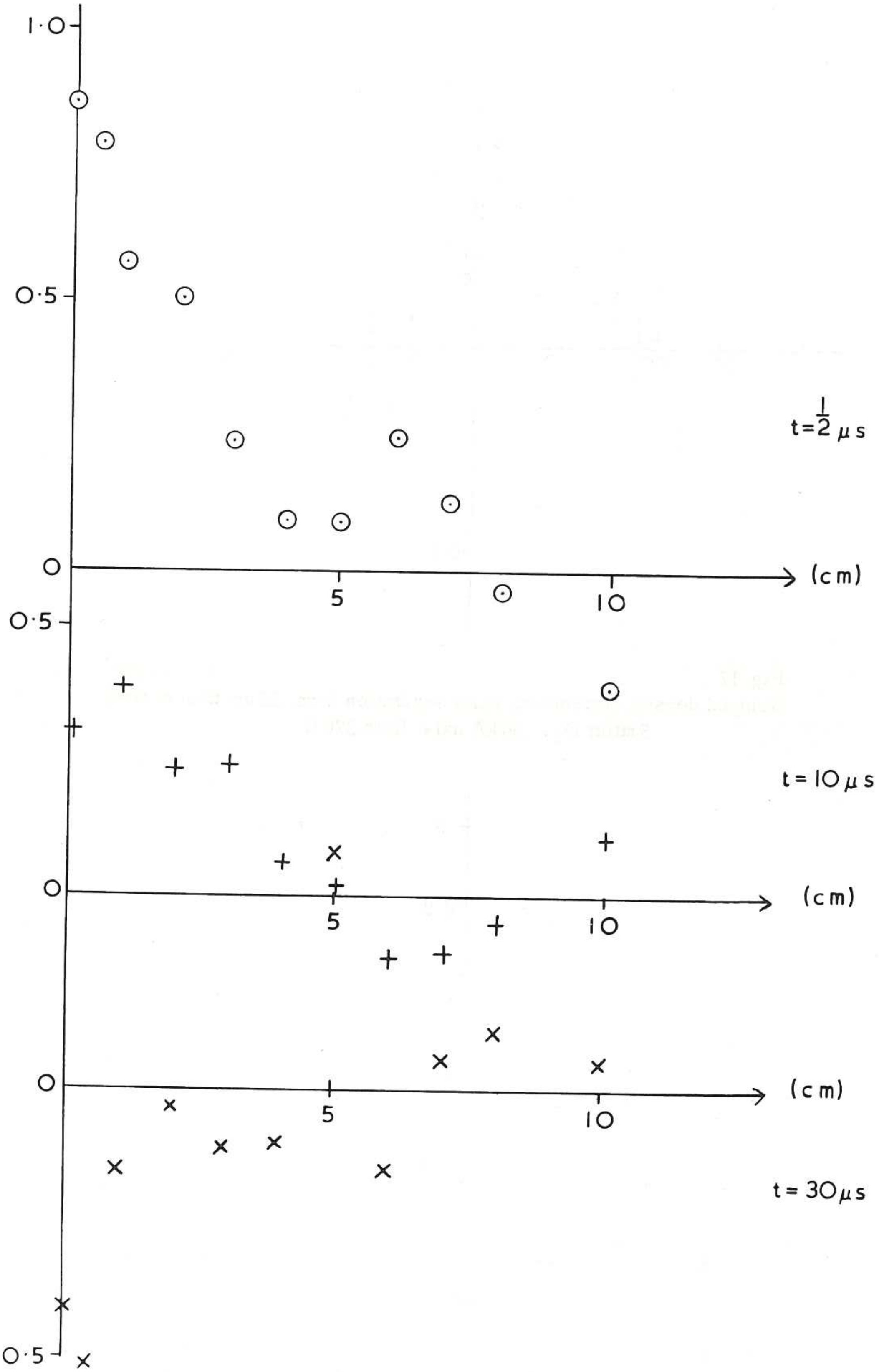

Fig. 16 (CLM-P 178)
Density correlation functions with time delay, 1.5 mtorr D_2, 150 kA, axial field 370 G

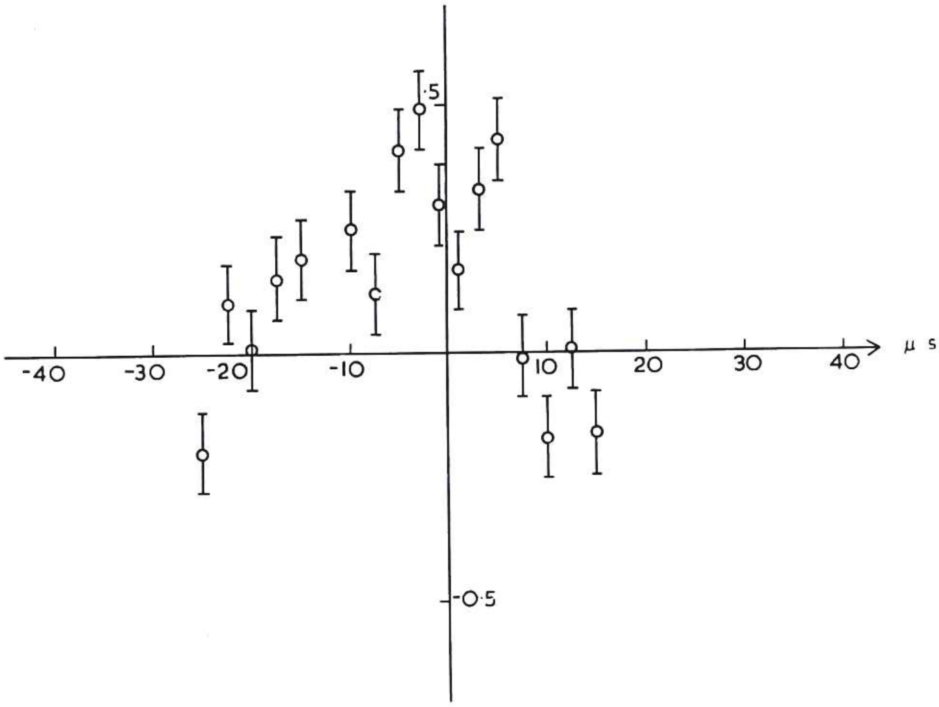

Fig. 17 (CLM-P 178)
Delayed density correlation, axial separation 6 cm, 35 cm from centre, 5 mtorr D_2, 150 kA axial field 370 G

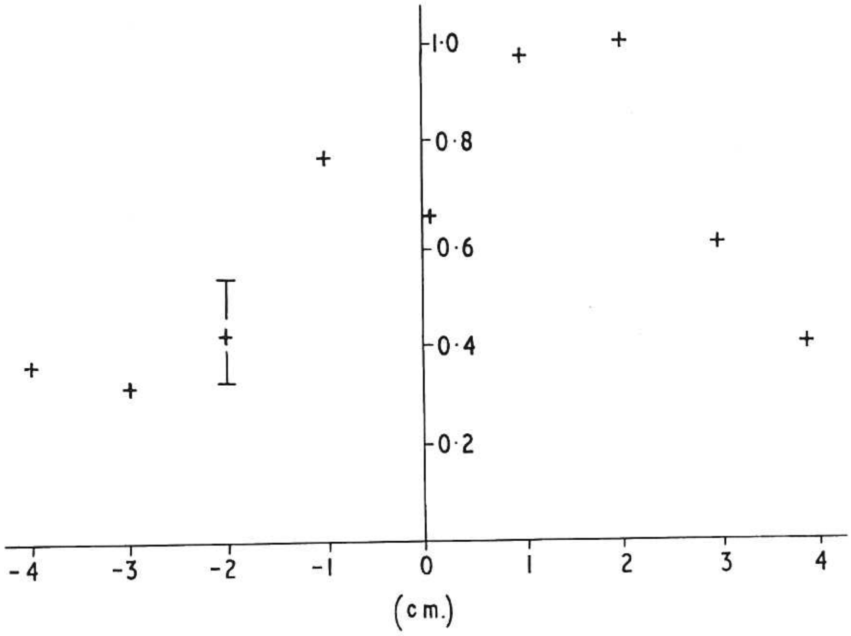

Fig. 18 (CLM-P 178)
Spatial density correlation, time delay 5 μs, 35 cm from centre, 2 mtorr D_2, 150 kA, axial field 370 G