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THE EFFECT OF BOUNDARY CONDITIONS ON THE STABILITY OF A NON-UNIFORM PLASMA IN A MAGNETIC FIELD

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THE EFFECT OF BOUNDARY CONDITIONS ON THE STABILITY OF A NON-UNIFORM PLASMA IN A MAGNETIC FIELD

by

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ABSTRACT

The propagation of high frequency electrostatic waves is considered in a plasma in which there is a zero order temperature gradient perpendicular to the uniform magnetic field. The frequency range is such that the ions do not respond to the perturbed fields ($\omega \gg \omega_{ci}$, $\omega \gg \omega_{pi}$ where ω_{ci} and ω_{pi} are the ion cyclotron and ion plasma frequencies respectively). For $\omega \ll \omega_{ce}$, $\lambda_{\perp} \gg \rho_e$ and a specific form of the temperature gradient the differential equation for φ is reduced to an elementary form where ω, ω_{ce} are the wave and electron cyclotron frequencies and $\lambda_{\perp}$ and ρ_e the wavelength perpendicular to the uniform magnetic field and the electron Larmor radius respectively. φ is the electrostatic potential. For $\lambda_{\perp} \ll a$ where a is the scale length of the temperature gradient the exact solution is very close to the local solution of Mikhailovskii and Pashitskii(3) which neglects the effects of the boundaries. However, for $\lambda_{\perp} \gtrsim a$ the plasma is unstable to shorter axial wavelengths than predicted by the local theory. It is shown that the instability is due to the interaction of a positive energy wave with a negative energy wave. When the phase velocities of the two waves are different the plasma is stable. However, when the non-uniform plasma is adjacent to a cold resistive plasma, instability may again result. This is analogous to the resistive wall amplifier of Birdsall et al(10). The relevance of these results to the stability of low frequency waves in a non-uniform plasma is pointed out.

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CONTENTS

1. Introduction

There is a class of plasma instabilities which occur only in a non-uniform plasma in a magnetic field. These instabilities are usually referred to as drift instabilities[1] and are of great importance for research on thermonuclear fusion. If such instabilities are confined to the interior of the plasma due to the unstable wave growing to non-linear proportions faster than it can propagate a scale length in the direction of the non-uniformity[1] then the effect of the boundaries will be unimportant. The condition for this to be the case is that the wavelength transverse to the magnetic field be much less than the scale length of the non-uniformity[1]. Waves in a cylindrical plasma, of low azimuthal mode number, do not satisfy this condition and Chen[2] has shown that the growth rates obtained from a non-local solution (i.e. solution of boundary value problem) are markedly different from the growth rates obtained from the local solution.

In this paper we consider a high frequency drift instability discovered by Mikhailovskii and Pashitskii[3] in which only the electrons respond to the wave fields. Whereas Mikhailovskii and Pashitskii made use of the local approximation[1] we obtain the non-local dispersion relation taking into account the effect of the boundaries.

In section 2 we derive the dispersion relation for semi-infinite slab geometry and obtain a generalization of reference 3. The instability is shown to result from an interaction between a positive energy wave and a negative energy wave. In section 3 we consider the effect of a cold plasma adjoining the non-uniform plasma. Finally, in section 4 we summarize the main effect of the boundaries on this instability and the probable effect on other instabilities.

2. The Non-Local Dispersion Relation

We consider a slab model in which the non-uniformity is taken to be in the x-direction and the steady uniform magnetic field $\underset{\sim}{B}_o$ points along the z-axis. The plasma extends to infinity in the y and z directions. The situation we wish to consider is such that $\nabla T/T \gg \nabla n_o/n_o$ where T is the equilibrium electron temperature and n_o the equilibrium density. For the sake of simplicity we neglect the small density gradient and assume a uniform density. For such large temperature gradients the corresponding drift frequency $\omega_* = \frac{k_\perp \kappa T}{qB_o} \frac{1}{\alpha}$ satisfies the conditions $\omega_* \gg \omega_{ci}$, $\omega_* \gg \omega_{pi}$ where ω_{ci} and ω_{pi} are respectively the ion cyclotron and ion plasma frequencies. α is the scale length of the temperature gradient, $k_\perp$ is the wave vector perpendicular to $\underset{\sim}{B}_o$, q is the electron charge and κ is Boltzmann's constant. Since we shall be interested in frequencies $\omega \sim \omega_*$ we assume that the ions do not respond to the wave field i.e. they just provide a uniform background of positive charge. We also assume that $\omega \ll \omega_{ce}$ and that the steady magnetic field is sufficiently strong such that $\omega_{ce} \gg \omega_{pe}$ where ω_{ce} and ω_{pe} are the electron cyclotron and electron plasma frequencies respectively. If we are concerned only with wavelengths perpendicular to $\underset{\sim}{B}_o$ much greater than the electron Larmor radius then we can describe the motion of the electrons by means of the Vlasov equation in drift space[4]. The distribution function for the electrons is expressed as follows

$$f = f(\underset{\sim}{r}_G, \mu, u_z, t)$$

where $\underset{\sim}{r}_G$ is the position vector of the electron guiding centre, μ is the magnetic moment $(\mu = \frac{1}{2} \frac{m u_\perp^2}{B_o})$ and u_z and $u_\perp$ the velocity of the guiding centre along $\underset{\sim}{B}_o$ and perpendicular to $\underset{\sim}{B}_o$ and t is the time. (f has been assumed independent of the azimuthal velocity co-ordinate.) If we split f into an unperturbed part f_o and a perturbed part f_1 the linearized Vlasov equation becomes:

$$\frac{\partial f_1}{\partial t} + \frac{\partial f_o}{\partial \underset{\sim}{r}_G} \cdot \frac{\underset{\sim}{E}_1 \times \underset{\sim}{B}_o}{B_o^2} + \frac{\partial f_1}{\partial z} u_z + \frac{\partial f_o}{\partial u_z} \frac{q}{m} E_z = 0 \qquad \ldots (1)$$

where we have used the fact that

$$\frac{d\mu}{dt} = 0\left(\frac{\omega}{\omega_{ce}}\right)$$

$$\frac{d\underset{\sim}{r}_G}{dt} = \frac{\underset{\sim}{E} \times \underset{\sim}{B}_o}{B_o^2} + u_z \ \hat{\imath}_z$$

and

$$\frac{du_z}{dt} = \frac{q}{m} E_z$$

where $\hat{\imath}_z$ is a unit vector in the z-direction and where we have assumed that the electric field is derivable from a scalar potential $\varphi_1 (\underset{\sim}{E}_1 = -\underset{\sim}{\nabla} \varphi_1)$. The conditions for the validity of the electrostatic approximation in this frequency range are

$$\frac{\omega_{pe}^2}{\omega_{ce}^2} \ll \frac{c^2 k^2}{\omega\omega_{ce}} , \quad \omega_{pe}^2 \ll c^2 k^2$$

and $\beta \ll 1$ [5] where β is the ratio of the electron pressure to the magnetic pressure. A subscript o indicates an unperturbed quantity and a subscript 1 a perturbed quantity. Looking for solutions of the form $\varphi_1 = \varphi_1(x) e^{i(k_y y + k_z z - \omega t)}$, we solve equation (1) for the perturbed part of the distribution function f_1 and hence calculate the perturbed electron density from

$$n_1 = \int_{-\infty}^{\infty} f_1 \, du_z \qquad \ldots (2)$$

giving

$$n_1 = - q \frac{k_z}{m} \varphi_1 \int \frac{\partial f_o}{\partial u_z} \frac{du_z}{(\omega - k_z u_z)}$$

$$+ \frac{iq}{m\omega_{ce}} \int (\underset{\sim}{\nabla} f_o \times \underset{\sim}{\nabla} \varphi_1)_z \frac{du_z}{(\omega - k_z u_z)} \qquad \ldots (3)$$

Substituting this expression for n_1 into Poissons equation we obtain the equation for φ_1

$$\nabla_\perp^2 \varphi_1 - k_z^2 \varphi_1 \left[1 + \frac{q^2}{\varepsilon_o m} \frac{1}{k_z} \int \frac{\partial f_o}{\partial u_z} \frac{du_z}{(\omega - k_z u_z)} \right]$$

$$+ \frac{iq^2}{\varepsilon_o m \omega_{ce}} \int (\underset{\sim}{\nabla} f_o \times \underset{\sim}{\nabla} \varphi_1)_z \frac{du_z}{(\omega - k_z u_z)} = 0 \qquad \ldots (4)$$

which is the same as that given by Mikhailovskii and Pashitskii.

ε_o and m are the dielectric constant of free space (MKS units) and the electronic mass respectively. f_o is the unperturbed electron distribution function and k_z is the z-component of the wave vector $\underset{\sim}{k}$. We assume that there is no equilibrium electric field and take $f_o(u_z, x)$ to be

$$f_o(u_z, x) = n_o \left\{ \frac{m}{2\pi\kappa T(x)} \right\}^{1/2} e^{-mu_z^2/2\kappa T(x)} \qquad \ldots (5)$$

where n_o is the equilibrium density, and $T(x)$ is the equilibrium electron temperature which is assumed to depend only on the co-ordinate x, i.e.,

$$\underset{\sim}{\nabla} f_o = (\frac{df_o}{dx}, 0, 0)$$

For the equilibrium distribution function f_o given by equation (5) the integrals over velocity space occurring in equations (3) and (4) can be expressed in terms of the Plasma Dispersion function[6]. We look for solutions such that $\omega/k_z \gg \left(\frac{\kappa T}{m}\right)^{\frac{1}{2}}$ and therefore make use of the asymptopic form of the Dispersion function[6]. Assuming a linear form for the dependence of T on x, i.e.

$$T = T_o\ (1 - \frac{x}{L}) \qquad \ldots (6)$$

equation (4) can be written

$$\frac{d^2\varphi_1}{dx^2} - k^2\ \varphi_1 + k_z^2\ \frac{\omega_{pe}^2}{\omega^2}\ \varphi_1 - k_z^2\ \frac{\omega_{pe}^2\omega_*}{\omega^3}\ \varphi_1 = 0 \qquad \ldots (7)$$

where
$$k^2 = k_y^2 + k_z^2$$

and
$$\omega_* = -\ k_y\ \frac{\kappa T_o}{\omega_{ce} m L}$$

The dispersion relation is now obtained by solving the boundary value problem specified by equation (7) and the values of φ_1 at $x = 0$ and $x = L$. Assuming the non-uniform plasma is bounded by a perfect conductor at $x = 0$ and $x = L$ the boundary conditions are

$$\varphi_1 = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = L \qquad \ldots (8)$$

The general solution of equation (7) is

$$\varphi_1 = A \sin \beta x + B \cos \beta x \qquad \ldots (9)$$

where
$$\beta^2 = k_z^2\ \frac{\omega_{pe}^2}{\omega^2} - k_z^2\ \frac{\omega_{pe}^2\omega_*}{\omega^3} - k^2$$

Applying the boundary conditions (8) we obtain $B = 0$ and $\beta L = n\pi$ resulting in the dispersion relation

$$\omega^3\ (1 + \frac{n^2\pi^2}{k^2L^2}) - \omega_{pe}^2\ \frac{k_z^2}{k^2}\ \omega + \frac{k_z^2}{k^2}\ \omega_{pe}^2\omega_* = 0 \qquad \ldots (10)$$

This equation has three sets of roots two of which correspond to the electron plasma wave and the third $\sim\omega_*$ which we refer to as the drift wave(7). The condition for this equation to have complex roots is

$$k_z < \left(1 + \frac{n^2\pi^2}{k^2L^2}\right)^{\frac{1}{2}} \frac{3\sqrt{3}}{2} \frac{\omega_*}{\omega_{pe}} k = k_z^c \qquad \text{... (11)}$$

For short wavelengths i.e., $\dfrac{n^2\pi^2}{k^2L^2} \ll 1$

condition (11) tends to that given in reference 3. The local solution gives a critical axial wavelength below which the plasma is stable. However, for $\dfrac{n^2\pi^2}{k^2L^2} \gtrsim 1$, instability can occur for shorter wavelengths. The finite geometry introduces an infinite set of unstable harmonics of decreasing axial wavelength. The conditions for the validity of these solutions are similar to those given in reference 3, namely

$$k_y\, d \ll 1/\left(1 + \frac{n^2\pi^2}{k^2L^2}\right)^{\frac{1}{2}} \qquad \text{... (12)}$$

where d is the electron Debye length and $k_y \gg k_z$ and

$$k_z\, L \ll \left(1 + \frac{n^2\pi^2}{k^2L^2}\right)^{\frac{1}{2}} \frac{\omega_{pe}}{\omega_{ce}} \qquad \text{... (13)}$$

Hasegawa(8) has recently given a method for obtaining the condition for instability in terms of the longitudinal conductivity defined as

$$\sigma^\ell = \frac{i\omega q n_1}{-\nabla^2\varphi_1} \qquad \text{... (14)}$$

where n_1 is the perturbed electron density. The conductivity obtained here is

$$\sigma^\ell = \frac{i\omega\varepsilon_0 \dfrac{\omega_{pe}^2}{\omega^2} k_z^2 \left(1 - \dfrac{\omega_*}{\omega}\right)}{\left(\dfrac{n^2\pi^2}{L^2} + k^2\right)} \qquad \text{... (15)}$$

For instability to be possible σ^{ℓ} must be active, as defined by Hasegawa. The σ^{ℓ} in equation (15) is purely reactive and the condition for this to be active is[8]

$$\frac{\partial}{\partial\omega}\,(\mathrm{Im}\,\sigma^{\ell}) \;>\; 0$$

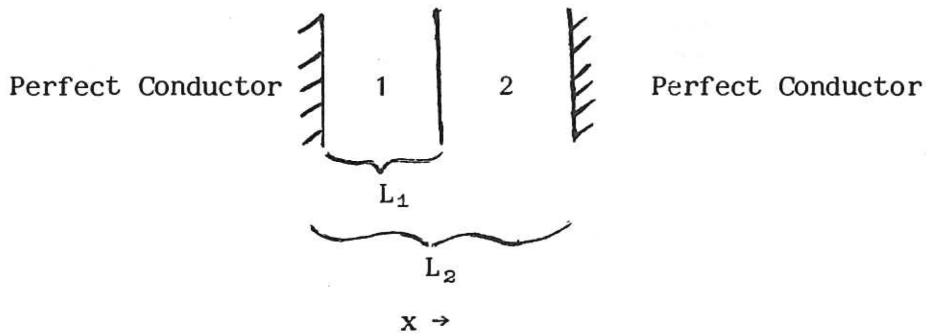
for some real ω. This gives the condition

$$\omega \;<\; 2\omega_*$$

as a necessary condition. The occurrence of a reactively active conductivity corresponds to a negative energy wave[8] and such a wave can only be unstable when there is some sink for it to lose energy. For the instability given here the sink is provided by a positive energy plasma wave which for the condition (11) is in resonance with the negative energy wave i.e. the phase velocities are equal and the interaction of the two waves is analogous to the travelling wave tube[9]. The occurrence of the roots as a complex conjugate pair is characteristic of this interaction. For axial wavelengths which do not satisfy condition (11) the plasma wave goes out of synchronism with the negative energy wave and the three roots are real even though the conductivity is still reactively active.

3. Non-Uniform Plasma Bounded by a Uniform Cold Plasma

We next consider the configuration shown in the diagram



where region 1 denotes the hot, non-uniform plasma and region 2 denotes the cold uniform plasma. As before, the non-uniform parameter in region 1 is the electron temperature. The electron density is taken to be uniform in both regions and the density in region 1 is assumed to be equal to the density in region 2. As before the system is infinite in the y and z directions and the steady magnetic field again points along the z-axis. The equation for the perturbed electrostatic potential in region 1 is given by equation (7). The dependence of T on x is again given by equation (6) except that L is replaced by L_1. Since the plasma in region 2 is assumed to be cold we can obtain the equation for φ_1 from a fluid description. For this we use the following equations

$$\frac{d\underset{\sim}{v}_1}{dt} + \nu\underset{\sim}{v}_1 = \frac{q}{m}\underset{\sim}{E}_1 + \frac{q}{m}\underset{\sim}{v}_1 \times \underset{\sim}{B}_0 \qquad \ldots (16)$$

$$q\frac{\partial n_1}{\partial t} + \underset{\sim}{\nabla}\cdot\underset{\sim}{J}_1 = 0 \qquad \ldots (17)$$

$$\underset{\sim}{\nabla}\cdot\underset{\sim}{E}_1 = \frac{n_1 q}{\varepsilon_0} \qquad \ldots (18)$$

which are respectively, the equations of motion, continuity and Poisson. $\underset{\sim}{v}_1$ is the fluid velocity, $\underset{\sim}{J}_1$ the perturbed current density and the effect of collisions has been included through the collision frequency ν. With the aid of equations (16)-(18) and the electrostatic assumption we obtain the equation for φ_1 in region 2

$$\frac{d^2\varphi_1}{dx^2} - k^2\varphi_1 + k_z^2\frac{\omega_{pe}^2}{\omega(\omega+i\nu)}\varphi_1 = 0 \qquad \ldots (19)$$

The solution φ_1 in region 1 is

$$\varphi_1^{I} = A\cos\beta_1 x + B\sin\beta_1 x \qquad \ldots (20)$$

and in region 2

$$\varphi_1^{II} = C\cos\beta_2 x + D\sin\beta_2 x \qquad \ldots (21)$$

where

$$\beta_1^2 = k_z^2 \frac{\omega_{pe}^2}{\omega^2} - k_z^2 \frac{\omega_{pe}^2}{\omega^2} \frac{\omega_*}{\omega} - k^2 \qquad \ldots (22)$$

and

$$\beta_2^2 = k_z^2 \frac{\omega_{pe}^2}{\omega(\omega+i\nu)} - k^2 \qquad \ldots (23)$$

A variable with a superscript I or II indicates that the corresponding quantity refers to region 1 or 2 respectively. The dispersion relation is obtained by matching the solutions (20) and (21) to the boundary conditions at $x = 0$, $x = L_1$ and $x = L_2$. The boundary conditions at $x = 0$ and $x = L_2$ are

$$\varphi_1^{I} = 0 \quad \text{at} \quad x = 0 \qquad \ldots (24)$$

$$\varphi_1^{II} = 0 \quad \text{at} \quad x = L_2 \qquad \ldots (25)$$

The remaining two boundary conditions are

$$\left.\begin{aligned} \varphi_1^{I} &= \varphi_1^{II} \quad \text{at} \quad x = L_1 \\ \frac{\partial \varphi_1^{I}}{\partial x} &= \frac{\partial \varphi_1^{II}}{\partial x} \quad \text{at} \quad x = L_1 \end{aligned}\right) \qquad \ldots (26)$$

Applying these conditions we obtain the dispersion relation

$$\tan \beta_1 L_1 + \frac{\beta_1}{\beta_2} \tan \beta_2 (L_2 - L_1) = 0 \qquad \ldots (27)$$

Equation (27) reduces to equation (10) when $L_2 = L_1$.

In the example in section 2 (equation (11)) we found a critical value of k_z such that for $k_z > k_z^c$ the positive energy plasma wave went out of synchronism with the negative energy wave and stability resulted. For $\nu = 0$ and $k_z > k_z^c$ we can find real solutions to equation (27) such that $\omega < 2\omega_*$. These solutions represent negative energy waves but are stable since they are out of synchronism with the positive energy wave solutions. However, if some dissipation is present in the system due to a finite collision frequency in the cold plasma in region 2, then these negative energy wave solutions will lose energy and hence become unstable. This is a plasma analogue of the resistive wall amplifier discovered by Birdsall et al (10). Figure 1 shows just such a case where $\mathrm{Im}\,\omega/\omega_*$ is plotted against ν/ω_* for $k_z > k^c$. For $\nu = 0$, $\mathrm{Im}\omega = 0$ but when ν has a non-zero value $\mathrm{Im}\omega > 0$ indicating instability. We observe that $\mathrm{Im}\omega$, after its initial rapid increase, is almost independent of ν.

It was pointed out by Hasegawa[8] that when the vacuum conductivity is included in the expression for σ^ℓ the resulting condition for a reactively active conductivity is the same as the condition for the small signal energy to be negative[11,12]. In order to see under what conditions a non-zero collision frequency in the adjoining cold plasma will give rise to instability we must calculate the longitudinal conductivity for the total system, including both the vacuum and cold plasma conductivities. Clearly, it is the hot electrons in the non-uniform plasma which contribute to negative small signal energy whereas the electrons in the cold plasma and the electric fields will give positive contributions.

Since we are now considering a sandwich configuration we must integrate over unit area in the y and z direction and from 0 to L_2 in the x direction.

$$\sigma^\ell = \frac{1}{L_2}\int_0^{L_1} \frac{i\omega q n_1^{\mathrm{I}}}{-\nabla^2 \varphi_1^{\mathrm{I}}}\,dx + \frac{1}{L_2}\int_{L_1}^{L_2} \frac{i\omega q n_1^{\mathrm{II}}}{-\nabla^2 \varphi_1^{\mathrm{II}}}\,dx - \frac{1}{L_2}\int_0^{L_2} i\omega\varepsilon_0\,dx \qquad \ldots (28)$$

n_1^{II} is obtained from equations (16) and (17) and is

$$n_1^{\mathrm{II}} = \frac{k_z^2}{\omega^2}\frac{q n_0}{m}\varphi_1^{\mathrm{II}} \qquad \ldots (29)$$

where we have put $\nu = 0$.

The expression for σ^{ℓ} becomes

$$\sigma^{\ell} = i\omega\varepsilon_0 \frac{\omega_{pe}^2}{\omega^2} k_z^2 \frac{(1 - \frac{\omega_*}{\omega})}{(\beta_1^2 + k^2)} \frac{L_1}{L_2} + i\omega\varepsilon_0 \frac{\frac{\omega_{pe}^2}{\omega^2} k_z^2}{(\beta_2^2 + k^2)} \frac{(L_2 - L_1)}{L_2}$$

$$- i\omega\varepsilon_0 \qquad \ldots (30)$$

where β_1 and β_2 have the values corresponding to the particular ω - solution and where the last term represents the effect of the free space conductivity. The condition for σ^{ℓ} to be an active conductivity (reactively active) is

$$\frac{\partial}{\partial \omega} (\mathrm{Im}\, \sigma^{\ell}) > 0$$

and this gives

$$\frac{(-1 + 2\frac{\omega_*}{\omega})}{(\beta_1^2 L_1^2 + k^2 L_1^2)} - \frac{(L_2 - L_1)}{L_1} \frac{1}{(\beta_2^2 L_1^2 + k^2 L_1^2)} - \frac{L_2}{L_1} \frac{\omega^2}{\omega_{pe}^2 k_z^2 L_1^2} > 0 \ldots (31)$$

For the parameters shown in figure 1 the quantity in equation (31) is positive hence the instability when a finite value of ν is included. However, for larger values of L_2 the expression in equation (31) becomes negative (the wave is now a positive energy wave) and the inclusion of collisions leads to damping. This is illustrated in figure 2.

4. Conclusion

We have considered the stability of a non-uniform plasma in a strong magnetic field solving the eigenvalue problem specified by a second order differential equation and the boundary conditions. This is in contrast to most treatments of similar problems where the effect of the boundaries is neglected and the solution to the differential equation is approximated by the WKB method (the local approximation). For wavelengths transverse to the steady magnetic field which are much shorter than the scale length of the non-uniformity the exact solution is very close to that obtained from the local approximation. Instability occurs for axial wavelengths longer than a certain critical value. However, for transverse wavelengths $\lambda_\perp \gtrsim a$ instability can occur at smaller axial wavelengths than that predicted by the local theory.

The instability results from an interaction between the electron plasma wave (which is a positive energy wave) and a negative energy wave which is referred to in this paper as a drift wave[13]. When the phase velocities of these two waves are in synchronism instability results, otherwise there is stability. In section 3 of the paper it was shown that one can find real solutions to the dispersion relation when the drift wave and plasma wave are non-synchronous but where the drift wave still retains its negative energy property since $\omega < 2\omega_*$. For this case it was shown that the presence of a cold resistive plasma adjacent to the hot non-uniform plasma could also result in instability and a condition was given for this to occur. This instability is a direct result of the boundary conditions.

Since the instability described here does not involve the motion of the ions and since also it requires large temperature gradients it is probably not a serious threat to plasma containment. However, the results obtained are of relevance to low frequency instabilities which are of importance to containment. The results obtained here indicate the importance of boundary conditions to stability, especially as the hot plasma of a fusion device would be surrounded by a colder plasma. For an instability due to the growth of a positive energy wave, one might expect that the presence of a cold resistive plasma bounding the hot plasma could result in stability, provided the cold plasma could absorb energy faster than the hot plasma could supply it. When a negative energy wave is present a resistive boundary may result in instability where previously there was stability. This instability will not occur if there is enough cold plasma to convert the negative energy wave into a positive energy wave.

5. Acknowledgements

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(13) It is not suggested that the low frequency drift waves are necessarily of the negative energy variety.

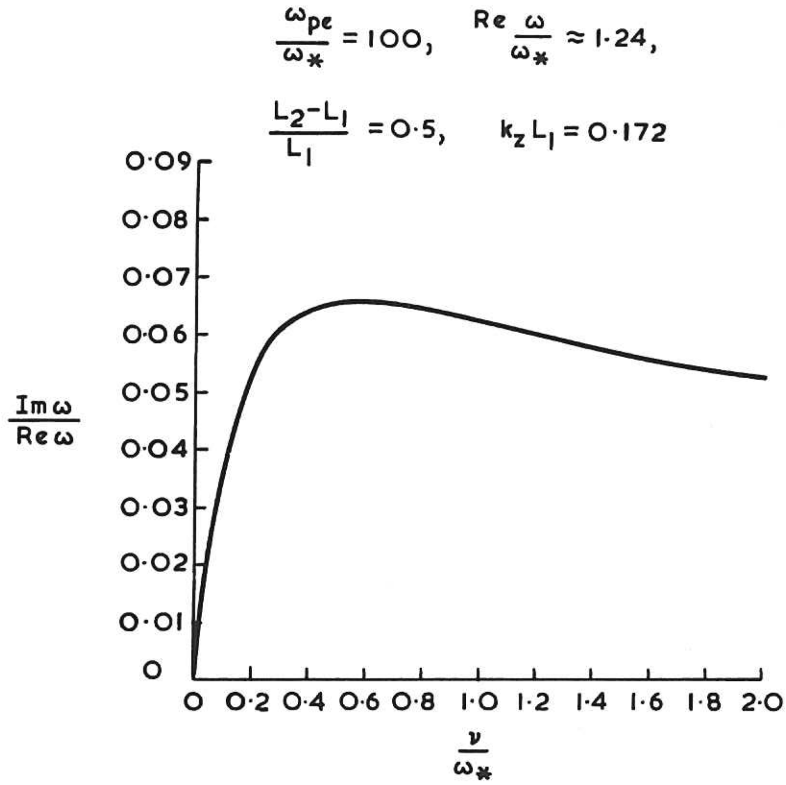

Fig. 1 (CLM-P 189)

Computed growth rate for hot non-uniform plasma adjacent to cold uniform plasma when σ^{ℓ} is active (see equation (31))

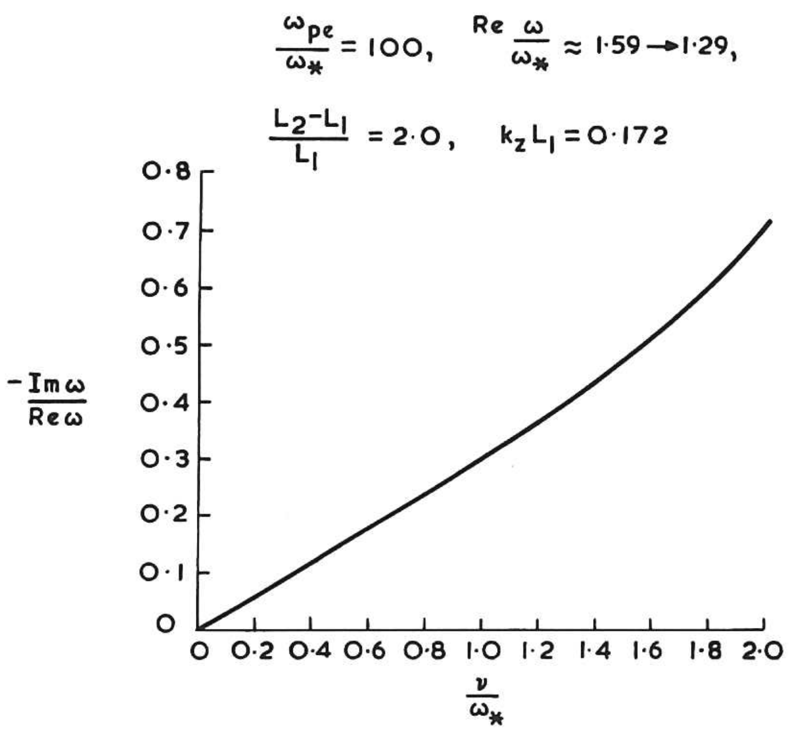

Fig. 2 (CLM-P 189)

Computed decay rate for hot non-uniform plasma adjacent to cold uniform plasma when σ^{ℓ} is passive (see equation (31))