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Preprint

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1969

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THE ANOMALOUS SKIN EFFECT IN BOUNDED PLASMAS

by

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(Submitted for publication in Phys. Rev. Letters)

A B S T R A C T

The penetration of an electromagnetic field into a cylindrical plasma has been measured under conditions where the electron mean free path is comparable to the plasma diameter. The experimental results are in qualitative agreement with a theoretical model in which electron thermal motion is included.

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January, 1969 (MEJ)

An electromagnetic wave incident on a bounded plasma with no static magnetic field present is strongly attenuated when $\omega \ll \omega_p$ (ω is the wave frequency, ω_p the plasma frequency). The skin depth is usually calculated using the complex conductivity $\sigma = Ne^2/m(i\omega+\nu)$ where N,e,m and ν are the electron density, charge, mass and collision frequency respectively. However, when electrons travel a distance comparable to the skin depth during a wave period (due to their thermal motion), a local relation between the electric field and the current density no longer provides a valid description of wave attenuation[1,2,3]. In this communication we describe an experiment which demonstrates the importance of electron thermal motion and compare the results with a simplified theory.

The plasma was produced by a pulsed discharge through low pressure mercury vapour in a cylindrical glass tube 150 cm long and 8·1 cm diameter. After a 20 μsec current pulse of 250 A, a steady current of 13·7 A was passed through the plasma, maintaining the electron temperature at about 2 eV. The electron-neutral collision frequency ν_{en} was $\sim 7 \times 10^6$ sec^{-1}. Wave penetration measurements were carried out during the decaying phase of the discharge with $10^{12} > N > 8 \times 10^{10}$ cm^{-3} as measured by Langmuir probes. An axial magnetic field, alternating at frequencies from 100 kHz to 10 MHz, was produced by a signal generator feeding into a 80 cm long screened solenoid. The amplitude of the magnetic field was $\sim 0{\cdot}5$ G and had negligible influence on the plasma parameters. These conditions differ from those described in earlier published work[4,5,6].

The ratio, R, of the amplitude of the alternating magnetic field at the plasma boundary to the amplitude at the plasma axis was measured using two screened magnetic probes, the central probe being

contained in a glass tube placed along the axis of the plasma. For each pulsed discharge the frequency of the field was constant, giving R as a function of electron density. The results were reproducible so that by changing the frequency after each pulse it was possible to determine R over the ranges of frequency and electron density indicated above.

For a uniform cylindrical plasma, the theory which neglects electron thermal motion gives $R = \left|I_0[(a/\delta_0)(i\omega/i\omega+\nu)^{\frac{1}{2}}]\right|$, where is a modified Bessel function; a, the plasma radius; $\delta_0 = c/\omega_p$, the collisionless skin depth; and c is the velocity of light. For a fixed frequency this expression gives values of R which decrease monotonically as the electron density decreases, even allowing for the dependence of ν on electron density. Our experimental results do not show this behaviour. In the frequency range 3-6 MHz R remains almost independent of electron density from 10^{12} to 4×10^{11} cm^{-3}, increases to a maximum at $N \sim 2{\cdot}5 \times 10^{11}$ cm^{-3} and then decreases steadily as the density decreases further. The maximum in R is particularly marked for a critical frequency near 4·5 MHz and Fig.1 shows experimental values of R as a function of $\ell n\ a/\delta_0$ (a/δ_0 is proportional to $N^{\frac{1}{2}}$) for this frequency.

This behaviour is attributed to the thermal motion of electrons, the critical frequency being related to the transit time for electrons crossing the tube. Resonance phenomena would be expected to become particularly noticeable when the electron transit time is of the same order as a half-period of the field. This view is confirmed by calculations of the field penetration into a uniform plasma slab.

Consider a plasma which extends from $x = -a$ to $x = +a$. In

the absence of plasma a magnetic field, represented as $Be^{i\omega t}$, exists in the z-direction. For specular reflection of electrons at $x = \pm a$ the induced electric field can be represented by

$$E_y(x) = \sum_n \alpha_n \sin\left(\frac{n\pi x}{2a}\right) e^{i\omega t}$$

and the linearised Boltzmann equation solved for the perturbed velocity distribution. The current density can then be calculated and related back to the electric field by means of Maxwell's equations so determining the coefficients α_n. For $\omega \ll \omega_p$ and a Maxwellian velocity distribution in the unperturbed state,

$$\alpha_n = \frac{-8ia\omega B(a)}{(n\pi)^2} \cdot \sin\left(\frac{n\pi}{2}\right) \cdot \left\{1 + 8(a/\delta_o)^2(a\omega/c_o)\, Z(\xi)/(n\pi)^3\right\},$$

where $B(a)$ is the amplitude of the magnetic field at $x = \pm a$, c_o is the most probable speed of electrons, $Z(\xi)$ is the plasma dispersion function[7], $\xi = 2a\nu(i - \omega/\nu)/\pi c_o n$. The electromagnetic field and the current distributions are then determined, leading to a value for R. In making these calculations we put $\nu = \nu_{en} + \nu_{ei}$, where the electron-ion collision frequency ν_{ei} was obtained from an expression for the resistivity of a fully ionized plasma given by Spitzer[8]. The theory confirms the presence of a critical frequency and density for which R is much greater than predicted by cold plasma theory. Theoretical values of R are shown in Fig.1 for a frequency near the critical value. Despite the plane geometry used in the theoretical treatment, the theory gives a good description of the observed field penetration. On the other hand the cold plasma theory in plane geometry gives

$$R = \left| \cosh\left[(a/\delta_o)(i\omega/i\omega + \nu)^{1/2}\right] \right|$$

which is plotted in Fig.1 for the same value of ω/ν_{en}. The results show a monotonic increase of R with increasing N (similar to the Bessel function expression given earlier) and do not agree with the experiment.

For densities greater than that corresponding to the maximum of R the theory shows that in general $|B|$ does not decrease monotonically from the plasma surface to the central plane. This has been observed by Demirkhanov et al.[4]. The theory of Weibel[3] for the anomalous skin effect in a semi-infinite plasma also predicts analogous behaviour in that the amplitudes of the field vectors do not decrease steadily from the plasma boundary.

Other experiments have been carried out with plasmas of different radii and also with steady state plasmas for which the plasma parameters could be determined more accurately. These results will be described in a later publication.

We wish to thank Mr. A.G. Allender for computational work.

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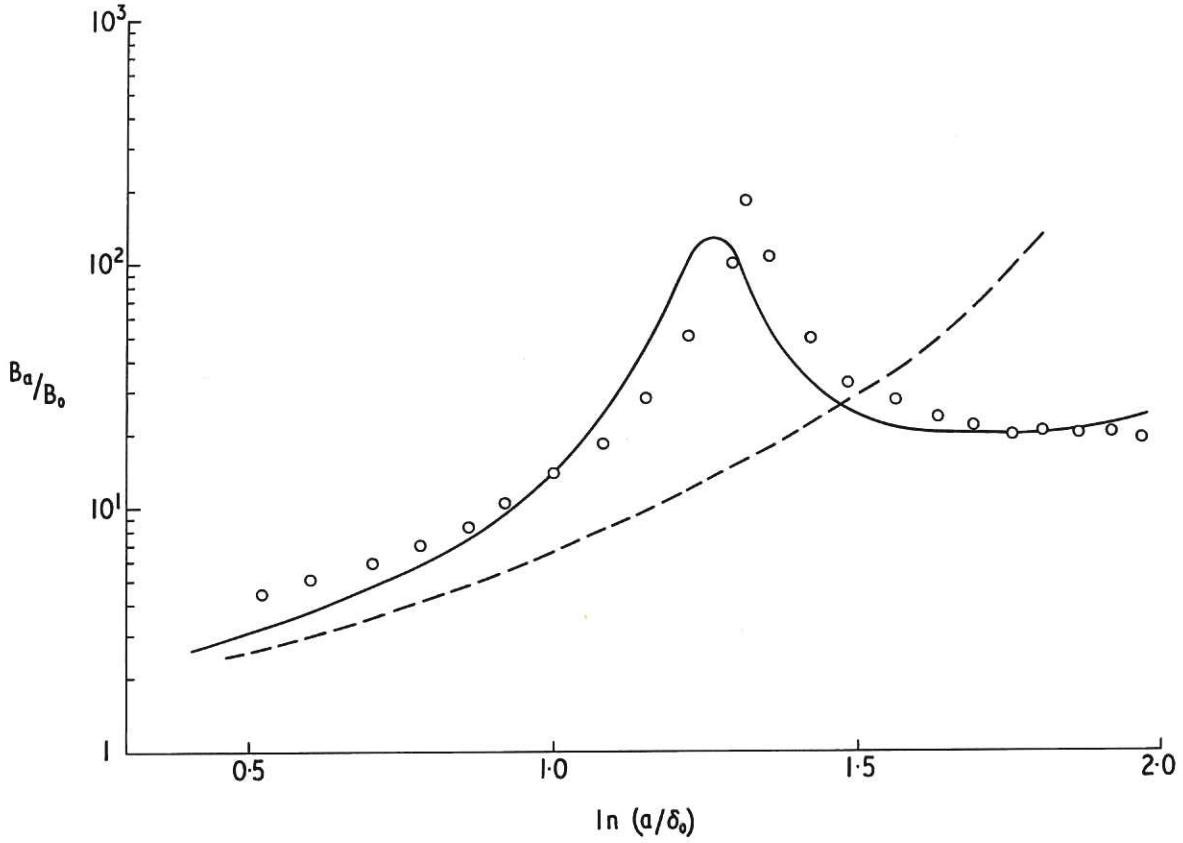

Fig. 1 (CLM-P 194)

Values of $R = B_a / B_o$, where B_a is the amplitude of the magnetic field at the plasma boundary and B_o is the amplitude at the plasma axis in cylindrical geometry or at the centre plane in plane geometry

O Experimental values of R for a cylindrical plasma, Wave frequency = 4.5 MHz

——— Hot plasma theory for a plane plasma slab, $\omega/\nu_{en} = 5.0$, $2a\nu_{en}/\pi c_o = 0.3$

– – – Cold plasma theory for a plane plasma slab, $\omega/\nu_{en} = 5.0$