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ESTIMATE OF A TOROIDAL MHD-STABLE HIGH-β DYNAMIC EQUILIBRIUM

by

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A B S T R A C T

An MHD-stable, toroidal dynamic equilibrium for a high-β plasma can be obtained by superimposing an oscillating bumpy magnetic field upon a purely toroidal static main field.

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The MHD stability of toroidal high-β configurations is of interest, for example to confine a theta-pinch plasma in a closed system. Among the schemes considered thus far, stability can be obtained when the effect of a conducting wall, e.g. the coil surface, is included, but only for small compression ratios and/or restrictive conditions on the value of β. Dynamic methods have been suggested[1-6] to stabilize the MHD instabilities of toroidal equilibrium configurations containing highly compressed theta-pinch plasmas, such as for instance the M & S configuration[7-11]. This paper describes a method whereby not only the stability but also the equilibrium of a toroidal high-β configuration is achieved through the dynamics of the system.

In order to study the MHD stability of static toroidal high-β equilibria, the topological features typical of those configurations were investigated for the simplified case of no net toroidal curvature, i.e. a linear geometry. However, in experiments with such linear configurations it can be difficult to separate the problems of stability from those of equilibrium. In particular it is useful to know how closely the required equilibrium is approached, in order to reach conclusions about its stability.

Considering, for instance, a bulged linear theta-pinch[12-15] plasma, (the analogue of the toroidal M & S configuration), the dynamics of setting up the bulges[16] interfere with the investigation of their stability[18] and/or dynamic stabilization[19]; this difficulty is even more serious in closed toroidal configurations, where the lateral equilibrium is determined by the shape of the bulges.

It has been shown theoretically that the MHD instabilities of an axisymmetric high-β plasma ($\beta \leq 1$) with a bulged region can be dynamically stabilized by superimposing an oscillating, standing wave-like, bumpy magnetic field configuration[5,20], provided the growth rates of the instabilities are slower than the oscillation frequency of the superimposed field. Although experimentally this method of stabilization has been found to be effective when applied to a linear theta-pinch[19], the above mentioned difficulties complicate the quantitative comparison with theory.

However, since the standing wave scheme to be discussed here gives positive stability against modes of low m numbers, the superimposition of a small constant lateral force, F, leads only to a shift ξ_e of the original equilibrium position to a new one such that the (averaged) restoring force due to the standing wave balances F. It should be noted that the so-called 'inverted pendulum', when placed horizontally, represents a mechanical analogue to our system. In what follows we give an estimate of the relationship between the force F (e.g. due to a net toroidal curvature) and the parameters of the standing wave system, which establish an MHD-stable dynamic equilibrium position under conditions where no equilibrium would otherwise exist.

A linear high-β plasma column with a square profile of radius R_0 has been considered, upon which a standing wave-like oscillation of frequency ω_s, wavelength L and amplitude εR_0 is superimposed[5,20].

The resulting time dependent plasma surface can be described by:

$$R(z,t) = R_0 \left\{ 1 + \varepsilon \sin\omega_s t \cos \frac{2\pi z}{L} \right\} . \qquad \ldots (1)$$

Now we consider the potential change δW for a lateral $(m=1)$ displacement ξ of the oscillating configuration as a relevant quantity to describe the restoring effect of the oscillation. From the results presented elsewhere[20] we obtain:

$$\delta\bar{W} \geq \frac{\varepsilon^2\pi}{2} [Y^2 + 1] \left(\frac{2\pi R_o}{L}\right)^2 \frac{\rho_o C_s^2}{\gamma\beta} |\xi|^2 , \qquad \dots (2)$$

where

$$Y = \frac{x_o J_o(x_o)}{J_1(x_o)} \left[1 - \beta_o + \frac{\gamma\beta_o}{2} \frac{1}{1 - a^2}\right], \qquad \dots (3)$$

and

$$x_o = \frac{\omega_s R_o}{C_s} \left[\frac{1 - a^2 b}{b + \frac{1}{1 - a^2}}\right]^{1/2} , \quad a = \frac{2\pi C_s}{L\,\omega_s} , \quad b = \frac{2}{\gamma} \frac{1 - \beta_o}{\beta_o} . \qquad \dots (4)$$

J_o and J_1 are the zero and first order Bessel functions, C_s is the sound speed, ρ_o the mass density, β_o the ratio of the plasma pressure and the external magnetic field 'pressure'; all these quantities are evaluated in leading order. Finally γ is the adiabatic exponent.

We now apply the lateral force F (independent of position and constant in time). The potential energy associated with this force can be expressed as

$$\delta W_F = -\xi F \qquad \dots (5)$$

per unit length in the z-direction.

Superimposing the two potentials (2) and (5) leads to a new potential δW_D. The new 'dynamic equilibrium position' is given by $\xi = \xi_e$, where ξ_e is determined by

$$\frac{\partial}{\partial\xi} \delta W_D = 0 . \qquad \dots (6)$$

From Eqs.(2), (5) and (6) we obtain:

$$\xi_e \leqslant \frac{F}{A} , \qquad \dots (7)$$

where F is given in Eq.(5) and

$$A = \varepsilon^2\pi [Y^2 + 1] \left(\frac{2\pi R_o}{L}\right)^2 \frac{\rho_o}{\gamma\beta_o} C_s^2 . \qquad \dots (8)$$

The potentials $\delta\bar{W}$, δW_F and δW_D as a function of ξ are schematically shown on Fig.1, where the minimum of δW_D is located at $\xi = \xi_e$.

Bending the system described by Eq.(1) to a torus with major radius R_T, results in a force F_T[21-24], where

$$F_T = \frac{2\pi}{\gamma} \frac{C_s^2 \rho_o}{R_T} R_o^2 \quad , \qquad \ldots (9)$$

which due to the net toroidal curvature of the main field acts on the plasma column in the direction of the radius of the torus. The potential energy associated with this force is obtained from Eq.(5). In order now to estimate the shift ξ_e of the equilibrium position away from the magnetic axis of the oscillating field (i.e. the axis of symmetry of the oscillating field coils), we assume that going over to a toroidal arrangement of the oscillating field coils themselves does not discontinuously change the restoring effect of the oscillation as described by Eq.(2). One can expect the restoring force obtained for the linear system to be the zero order contribution from an expansion in the aspect ratio, R_o/R_T, of the toroidal system; a possible reduction of the restoring force will at least not alter the order of magnitude of the estimated value of ξ_e. It is plausible, therefore, that for a weak toroidal curvature, i.e. a small aspect ratio, Eq.(7) may still be used, with the quantity F_T taken from Eq.(9). This provides an estimate of ξ_e:

$$\xi_e \leq \frac{R_o}{R_T} \left(\frac{L}{2\pi R_o}\right)^2 \frac{2\beta_o R_o}{\varepsilon^2 [Y^2 + 1]} \quad . \qquad \ldots (10)$$

Note that these results are derived from a theory[5] which is based on small perturbation analysis and linearisation which put limits to the validity range. Furthermore, dynamic stabilization will be effective only on phenomena with a characteristic time much longer than the period of the forced oscillations. This requires in the case of the toroidal force given by Eq.(9) that :

$$\frac{C_s}{\omega_s R_o}\sqrt{\frac{2\,R_o^2}{\xi_e R_T}} \ll 1 \quad . \qquad \ldots (11)$$

It is of interest to put experimentally relevant parameter values into Eqs.(2)-(11). By choosing:

$$a^2 \approx \frac{1}{10} \quad , \quad b \approx 1 \quad , \quad \frac{\omega_s R_o}{C_s} \approx 1 \ ,$$

it follows from Eqs.(3) and (4) that $Y \approx 2$, and from Eq.(10) that

$$\xi_e \leqslant 2 \quad \frac{R_o}{\varepsilon^2 R_T} R_o \ . \qquad \ldots (12)$$

With $\varepsilon = 0{\cdot}2$ and $\frac{R_o}{R_T} = 3 \times 10^{-3}$ one obtains $\xi_e \leqslant 0{\cdot}15\ R_o$. With the same parameter values the lefthand side of the inequality (11) takes the value 0·2 .

Thus, by using values of ε and ω_s , which have been already achieved technically[19], the scheme proposed here leads to aspect ratios amenable to an experimental study. Such an experiment might be useful, for instance, to investigate the validity range of the linearised theory, the problem of parametric resonances, and the influence of the oscillating field on the plasma diffusion. In particular, the simplicity of our method should allow some of the above-mentioned problems connected with static bulged high-β plasmas to be avoided.

Considering the confinement of thermonuclear plasmas, the application of dynamic methods will become an economic question. A rough estimate shows, that the proposed method is not necessarily less economic than for instance dynamically stabilizing an M & S equilibrium configuration.

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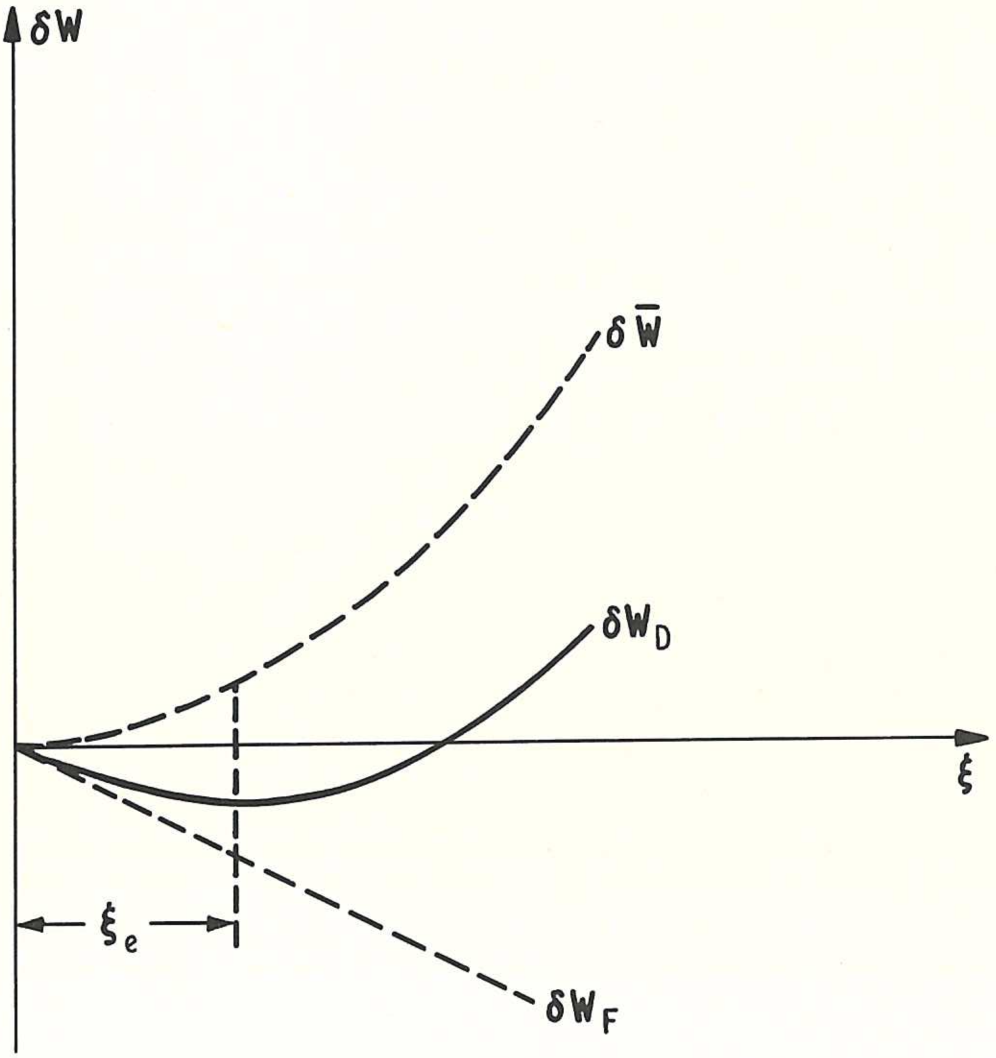

Fig. 1 (CLM-P200)
Schematical representation of the potentials $\delta\overline{W}$, δW_F and the combined potential δW_D