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RESONANT ATTENUATION OF AN ELECTROMAGNETIC WAVE IN A BOUNDED, HIGH DENSITY, HOT ELECTRON PLASMA

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A B S T R A C T

Measurements of the attenuation of an electromagnetic wave in a cylindrical plasma show resonances at particular values of frequency and electron density. These are explained in terms of the time taken for hot electrons to traverse the plasma compared to the wave period, and agreement is obtained with a theory based on plane slab geometry.

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C O N T E N T S

1. INTRODUCTION

In our previous report [1] we discussed measurements of the attenuation of an electromagnetic wave in cylindrical mercury arc plasmas which had an electron temperature of 2·2 eV and various electron densities up to 10^{11} cm^{-3}. We found that the results agreed well with cold plasma theory in spite of the fact that the electron mean free path exceeded the tube radius and the thermal velocity was high. Weibel's theory of electromagnetic wave attenuation in a hot electron, semi-infinite plasma was used to show that this was not unreasonable at the small values of electron density used. At the highest value of electron density investigated, $9{\cdot}8 \times 10^{10}$ cm^{-3}, a small increase in the attenuation (R), defined by the modulus of the ratio of the magnetic field amplitude outside the plasma to the magnetic field amplitude at the plasma axis, was detected in the collisionless region. The increase was at first surprising, since simple theory of anomalous skin depth shows that thermal effects should decrease the attenuation, and it was suspected that the result was caused by a resonance between the wave and hot electrons oscillating between surfaces of the bounded plasma. Here we report on an extension of the experimental work to higher electron densities where we expected an increased resonance effect, and compare the results to a hot electron plane slab model which approximates more closely to the cylindrical geometry of the experiment than Weibel's semi-infinite model.

2. THEORY

The attenuation of electromagnetic waves in a plasma is calculated from Maxwell's equations and Boltzmann's plasma equation. For a wave angular frequency (ω) much less than the plasma frequency, Maxwell's equations give (electromagnetic units)

$$\nabla^2 \underline{B} = \frac{4\pi\ i\omega\sigma}{c^2}\ \underline{B} \qquad \ldots (1)$$

Where $\underline{B} = \hat{\underline{B}}\ \exp\ (i\omega t)$ is the wave magnetic field. The conductivity (σ) is often derived from Boltzmann's equation assuming cold electrons so that the $\underline{u}\cdot\nabla f_1$ term may be neglected ($\underline{u}$ is the electron velocity and f_1 the perturbed electron distribution function). This is equivalent to defining the conductivity as the current density divided by the local electric field and gives the Lorentz equation [2]

$$\sigma = \frac{n e^2}{m}\cdot\left(\frac{1}{\nu + i\omega}\right) \qquad \ldots (2)$$

where n, e, m and ν are the electron density, charge, mass and collision rate for momentum transfer respectively.

The solution of Eqs.(1) and (2) for a cold electron plasma are well known. For semi-infinite plane plasma with the electromagnetic field in the y,z plane, the magnetic field at a distance x from the surface is

$$B(x) = \hat{B}(o)\ \exp(i\omega t)\cdot\exp\left(-\frac{x}{\delta}\sqrt{\frac{i\omega}{\nu + i\omega}}\right) \qquad \ldots (3)$$

where $\hat{B}(o)$ is the magnetic field amplitude at the origin. The attenuation is exponential with a characteristic distance $\delta = (mc^2/4\pi ne^2)^{1/2}$ when $\omega \gg \nu$ (the collisionless skin depth) and $\delta_c = \delta\ (2\nu/\omega)^{1/2}$ when $\omega \ll \nu$ (the collisional skin depth). For a plane slab plasma extending between $x = -a$ to $x = +a$ with electromagnetic field in the y,z plane produced by current sheets at $x > +a$ and $x < -a$ so that B is symmetric and the electric field anti-symmetric about the origin,

$$B(x) = \hat{B}(o)\ \exp\ (i\omega t)\cdot\cosh\left(\frac{x}{\delta}\sqrt{\frac{i\omega}{\nu + i\omega}}\right) \qquad \ldots (4)$$

For a cylindrical plasma with $B(r)$ parallel to the axis and the electric field in the circumferential direction

$$B(r) = \hat{B}(o)\ \exp(i\omega t)\cdot I_o\left(\frac{r}{\delta}\sqrt{\frac{i\omega}{\nu + i\omega}}\right) \qquad \ldots (5)$$

In both the plane slab and cylindrical geometries the attenuation is not exponential.

The attenuation of an electromagnetic wave in a hot electron plasma is more complicated when the electron thermal velocity is sufficient for electrons to move through regions of changing electric field during the wave period because the current density is determined by the integral of the electromagnetic forces over the electron path. The conductivity then involves the electron thermal velocity and the spatially varying electric field, requiring the $\underline{u}\cdot\nabla f_1$ term in the Boltzmann equation to be retained. The condition for this is that both the electron mean free path and the distance moved by the electron during a wave period must be greater than the characteristic distance for the electric field to change. For a semi-infinite plane plasma with a surface that perfectly reflects electrons, Weibel [3] found that in general the attenuation is not monotonic and it was only possible to describe the attenuation by a characteristic distance near the surface, defined by $2dx/d\ (\log EE^*)$ at $x = o$ where E is the wave electric field. This gave an attenuation length $\frac{8}{9\pi^{1/4}}\left(\frac{V}{\omega\delta^2}\right)^{1/3}$ where V is the most probable electron speed $\left(2kT/m\right)^{1/2}$ (T is the electron temperature). Apart from the numerical constant, this length is the same as the anomalous skin depth in metals, previously investigated by Pippard [4] and by Reuter and Sondheimer [5], which predicts an enhanced penetration compared to the cold electron case. Deep in the plasma, however, Weibel found that electrons which carry momentum acquired near the surface can be in or out of phase with the field, leading to interference effects.

A closer approximation to the cylindrical geometry of most experiments is the plane slab, since the plasma is finite and the electric field can be chosen to be zero at the origin. One would expect the attenuation to exhibit resonances greater than the interference effects of semi-infinite geometry because electrons can oscillate from side to side of the plasma (reflected by sheath electric fields) successively sampling the regions of high electric field. The resonant frequency should therefore depend on the slab thickness, the first resonance occurring when the electron transit time is equal to the wave half period provided the mean free path exceeds the slab thickness. In other words, if $\varphi_1 \equiv 2a\omega/\pi V$ and $\varphi_2 = 2a\nu/\pi V$, the resonance occurs when $\varphi_1 \approx 1$ provided $\varphi_2 < \pi^{-1}$.

These simple ideas are, however, complicated by the electron velocity distribution and collisions. While some groups of the velocity distribution are in phase others must be out of phase and the overall effect depends on the number of electrons in each group, their phase and time spent in the region of high electric field. The condition that this time must be appreciable compared to the wave quarter period for significant resonance effects (i.e. $a/\delta \sim 2\varphi_1$) leads to a resonance of the attenuation as a function of electron density as well as frequency. We have developed [6] a plane slab theory which allows for these effects by using a Maxwellian distribution of electron velocities. The magnetic field at a position x, expressed in terms of a Fourier expansion, is

$$B(x) = B(a)\exp(i\omega t)\cdot\frac{4}{\pi}\sum_n \frac{\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\cos\left(\frac{n\pi x}{2a}\right)}{1+\left(\frac{2a}{n\pi\delta}\right)^2\frac{\varphi_1}{n}Z(\zeta)} \quad \ldots (6)$$

where $Z(\xi)$ is the plasma dispersion function of the argument $\xi = \frac{1}{n}(-\varphi_1 + i\varphi_2)$. The numerical results are conveniently presented as contour plots of the attenuation in the a/δ, φ_1 plane. **Fig.1** shows plots for three values of the dimensionless collision rate, $\varphi_2 = 0$ (the collisionless case), $\varphi_2 = 0{\cdot}3$ and $\varphi_2 = 1{\cdot}0$ (corresponding to π collisions across the slab). It can be seen that the resonances, represented by the contour knolls, are very sharp and rise to at least an order of magnitude above the cold plasma theory. Their magnitude does not change significantly with φ_2 up to $\varphi_2 = 1$ and the resonant coordinates of a/δ and φ_1 increase only slowly with φ_2. The first resonance occurs in the region expected on elementary grounds. For example with $\varphi_2 = 0{\cdot}3$ the attenuation rose to ~ 300 at $\varphi_1 = 1{\cdot}46$ and $a/\delta = 3{\cdot}43$. However, a surprising result was the absence of the subsequence resonances at $\varphi_1 = 3,5$ and 7, the next resonance not developing until $\varphi_1 = 9{\cdot}1$; we have not found a simple physical explanation for this although it is likely to be connected with the velocity distirbution. The plot for $\varphi_2 = 0{\cdot}3$ approximates to experimental conditions (described later) where the attenuation was measured both as a function of frequency for a constant electron density (i.e. a/δ constant), and as a function of electron density at a constant frequency. These cases correspond to horizontal and vertical scans of the Fig.1 contour plots respectively. Fig.2 shows the horizontal scan for the plane slab, hot electron theory (Eq.(6)) compared to the plane slab cold electron theory (Eq.(4)) using $\varphi_2 = 0{\cdot}3$ and $a/\delta = 1{\cdot}0$, $2{\cdot}0$ and $2{\cdot}5$ (the region below the first resonance at $a/\delta = 3{\cdot}43$). It can be seen that the resonant peak rises rapidly above the cold plasma theory curve as a/δ is increased. At low a/δ the hot and cold plasma theories differ only at very low frequencies, the finite electron temperature causing a decrease in the attenuation compared to the cold plasma value.

The attenuation of an electromagnetic wave in a hot cylindrical plasma has not been calculated. One would expect the result to approximate to the plane slab calculation if the slab thickness is about equal to the plasma diameter because in both cases the electrons can oscillate between two regions of high electric field near the plasma surface, and at the axis the electric field is zero. An obvious difference is that in cylindrical geometry integration should be made over the angular distribution of electrons leaving the sheath.

In this report we present experimental results demonstrating the existence of resonant attenuation of an electromagnetic wave in a hot electron cylindrical plasma. The agreement with hot electron plane slab theory expressed by Eq.(6) is good considering the geometry difference. Previous investigations [7- 10] were made using plasmas maintained by the wave itself, leading to doubts about the constancy of the electron temperature and density, and the influence of the wave magnetic field on the electron motion. In the present work the plasma was maintained by a low pressure arc, and the wave power was negligible compared to the arc power so that the wave did not perturb the plasma. Both the wave magnetic field and the magnetic field produced by the arc current were so small that the electron motion was unaffected (i.e. the electron cyclotron radius was $> a$ and the electron cyclotron frequency was $< \omega$).

3. APPARATUS

The plasma was a direct current mercury arc contained in a glass tube 8·1 cm diameter and 150 cm long. The apparatus was similar to that used for our previous experiments [1] except that the cooling was improved to allow a maximum arc current of 26 A, which gave an electron density of $1{\cdot}42 \times 10^{11}$ cm^{-3} ($a/\delta = 2{\cdot}87$). The electron collision frequency (measured from the electric field in the plasma associated with the arc current) was 1·2 MHz, neutral atom density $1{\cdot}0 \times 10^{13}$ cm^{-3} and electron temperature 2·2 eV.

Higher electron densities, up to $\sim 1{\cdot}4 \times 10^{12}$ cm^{-3} ($a/\delta \sim 9$), were obtained by momentarily increasing the current through a 13·7 A direct current arc. A 20 μsec square current pulse of 250 A was generated by a 12 μF, 3 kV capacitor discharged via a 12 Ω resistor. The current was switched into the plasma by one thyratron, then diverted to a dump circuit 20 μsec later by another thyratron. The electron density increase during the pulse then decayed after the pulse in a time ~ 200 μsec. The electron temperature of the afterglow was maintained by the direct current. Measurements were made after the current pulse was diverted when the magnetic field due to the pulse was < 0·2 G to avoid perturbations of the electron trajectories.

The electromagnetic wave was generated by an 80 cm long electrostatically screened solenoid coaxial with the plasma. The attenuation of the wave magnetic field was measured by two electrostatically screen coils, each 0·6 cm diameter and 50 turns, mounted in a glass tube. One was placed along the plasma axis and the other just outside the plasma. Both steady and pulsed plasma densities were measured by a Langmuir probe mounted at the discharge tube wall [1]. For the pulsed plasma, the probe potential was maintained by a capacitor, and transients from the current pulse generator were clipped by a diode to prevent overloading the oscilloscope.

4. RESULTS

Fig.3 shows the measured attenuation (R) as a function of frequency for the direct current arcs; the magnitude of a/δ was selected by adjusting the arc current. The theoretical curves for a cold cylindrical plasma (Eq.5) using δ and ν obtained from Langmuir probe measurements are shown for comparison. The results confirm and extend our previous measurements [1] which showed that up to an electron density of about 10^{11} cm^{-3} the cold plasma theory applies, whereas above this density the attenuation can exceed the cold plasma value. The difference between the measured field ratio and the cold plasma theory rises to a maximum in the collisionless region near $\omega/\nu \sim 5$ and falls to zero at higher frequencies. The difference increases rapidly with increasing electron density (i.e. a/δ).

These measurements may be compared to the curves of Fig.2 computed from plane slab theory using $\varphi_2 = 0{\cdot}3$ (note that although the experimental value of φ_2 was $0{\cdot}22$, the difference should not affect our general conclusions because the theoretical curves shown in Fig.1 do not change rapidly with φ_2). It can be seen that there is good qualitative agreement in that the experimental peak occurs near the predicted frequency, it rises rapidly from the cold plasma theory as a/δ increases and agrees with cold plasma theory at high frequencies. However, the experimental peak is broader than theory suggests, the magnitude of the attenuation at a particular a/δ is not the same, and there was little evidence to suggest that at very low frequencies the attenuation is less than cold electron plasma theory predicts, as indicated by the hot electron theory. These differences may be due to geometrical differences between theory and experiment.

The attenuation for higher electron densities (up to 10^{12} cm^{-3}) was measured using the afterglow of the pulsed plasma. The density decreased with time after the pulse was terminated, so measurement of the wave magnetic field amplitude as a function of time gave the attenuation as a function of decreasing a/δ. Fig.4 shows examples of the oscilloscope traces of the amplitude of the wave magnetic field at the plasma axis as a function of time for the fixed frequencies 3•0, 4•5 and 6•0 MHz. It can be seen that the amplitude (which is proportional to R^{-1} since the solenoid producing the wave was loosely coupled to the plasma so the wave magnetic field amplitude outside the plasma was practically constant) showed a sharp minimum at both a particular time of 70 μsec (corresponding to $a/\delta = 3{\cdot}7$) and a particular frequency of 4•5 MHz; this is in qualitative agreement with hot electron plane slab theory. An amplified trace at the resonance (Fig.(8a)) showed that out-of-phase harmonics are associated with the development of the resonance. Fig.5 shows contour plots of constant attenuation in the a/δ vs frequency plane for comparison to the hot electron plane slab theory plots shown in Fig.1. Up to about 9 MHz the general form of the results is in good agreement with the theory: the experimental value of a/δ at the resonance is 3•7 compared to the theoretical value of 3•43 and the experimental resonant frequency of 5 MHz corresponded to $\varphi_1 = 0{\cdot}923$ compared to the theoretical value of 1•46. The differences may again be geometrical effects, or errors in interpreting Langmuir probe measurements of electron density, or the assumption that the electron temperature in the afterglow of the pulsed plasma was maintained by the direct arc current at exactly the same value as prior to the pulse. Although we did not measure the electron temperature as a function of time because of difficulties in interpreting the Langmuir probe under pulsed conditions, we did find that if no direct arc current was passed to maintain the electron temperature during the afterglow, no resonances were observed.

A more quantitative comparison between theory and experiment is obtained if we compare horizontal and vertical scans through the resonant points. This reduces the effect of errors in the absolute values of a/δ and φ_1. The results are shown in Figs.6(a) and (b) in which we have also plotted curves calculated using cold electron plane slab theory. It can be seen that quite good agreement is obtained. An important point demonstrated in Fig.6(a) is that the experimental value of the attenuation at low frequencies lies below the cold plasma theory. We attempted to detect this effect at lower a/δ in the direct current arc plasmas (Fig.3), but could not find convincing evidence (within the experimental scatter). Plots of the attenuation vs frequency at lower a/δ using the pulsed plasma results also showed little evidence for this effect, which is in agreement with the direct current arc results.

Since the first resonance occurs when the electron transit time across the plasma is near the wave half period (i.e. $2a/_V \sim \pi/_\omega$ or $\varphi_1 \sim 1$), one would expect that for a constant thermal velocity the resonant frequency should be inversely proportional to plasma radius. The influence of the plasma radius on the measured critical frequency was found by restricting the plasma by inserting glass tubes of various radii down to 2·33 cm. The wave attenuation as a function of time was qualitatively similar to Fig.4, but the resonant frequency was changed. Fig.7 shows that a plot of the resonant frequency as a function of a^{-1} is linear, in agreement with the simple picture, but it does not extrapolate to zero as one may expect on the simple picture.

Above 9 MHz the experimental results (Fig.5) showed a second resonance at ~ 13 to 15 MHz ($\varphi_1 \sim 2{\cdot}6$) extending over a large range of a/δ centred around 5. This is about 3 times the frequency of the first resonance, in agreement with elementary considerations of monoenergetic electrons, but contrasting with hot electron plane slab theory which predicted the next resonance as a sharp peak near $\varphi_1 \sim 10$, $a/\delta \sim 8$. The waveform at the second resonance was much more complicated than at the first resonance, and a typical example is shown in Fig.8(b) for a frequency of 15 MHz. Again it seems not unlikely that the deviation of the results from plane slab theory is connected with the cylindrical geometry.

5. CONCLUSIONS

We have demonstrated that the attenuation of an electromagnetic wave propagated radially in a cylindrical plasma exhibits sharp resonances at particular values of electron density, wave frequency and plasma radius. The first resonance agrees well with predictions of the plane slab theory assuming a Maxwellian distribution of electron velocities. A second resonance, detected at about three times the frequency of the first resonance, was expected on elementary consideration of monoenergetic electrons but not predicted by the plane slab theory. This will be the subject of further investigation.

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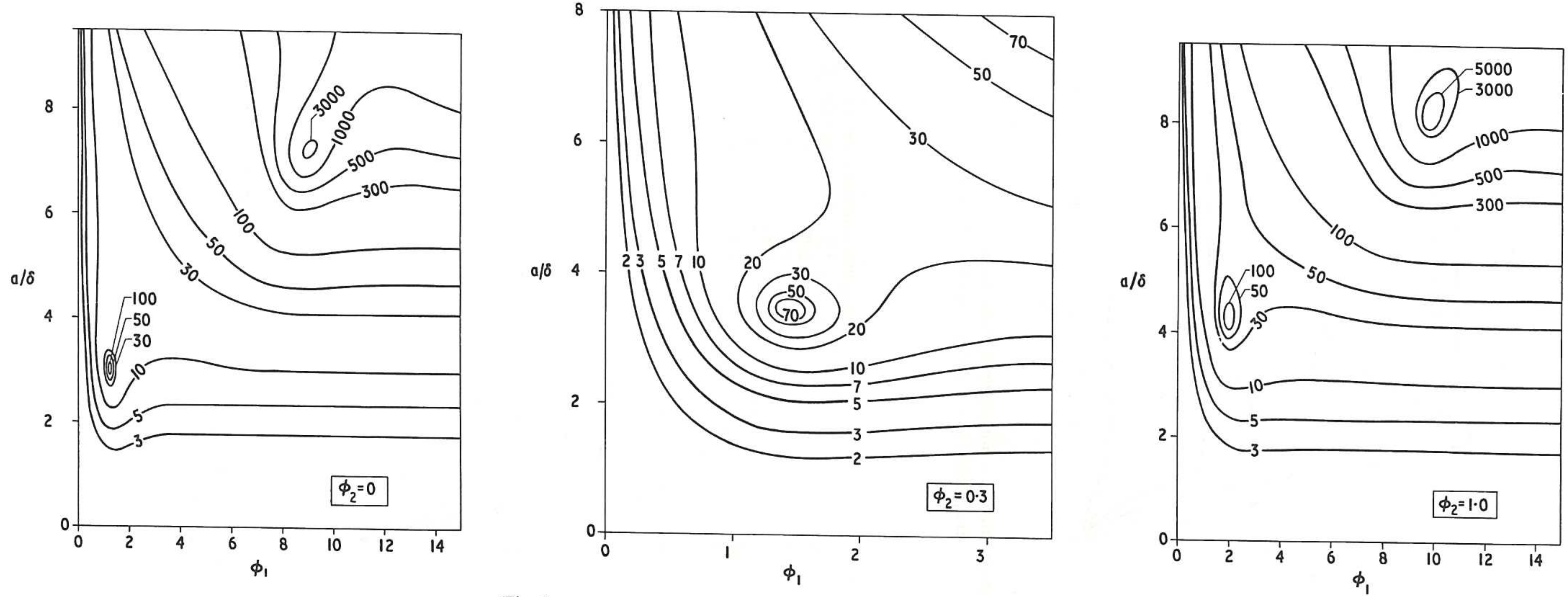

Fig. 1 (CLM-P 213)
Contours of the attenuation in a hot electron plane plasma slab in the a/δ, dimensionless frequency (ϕ_1) plane calculated from Eq. (6)

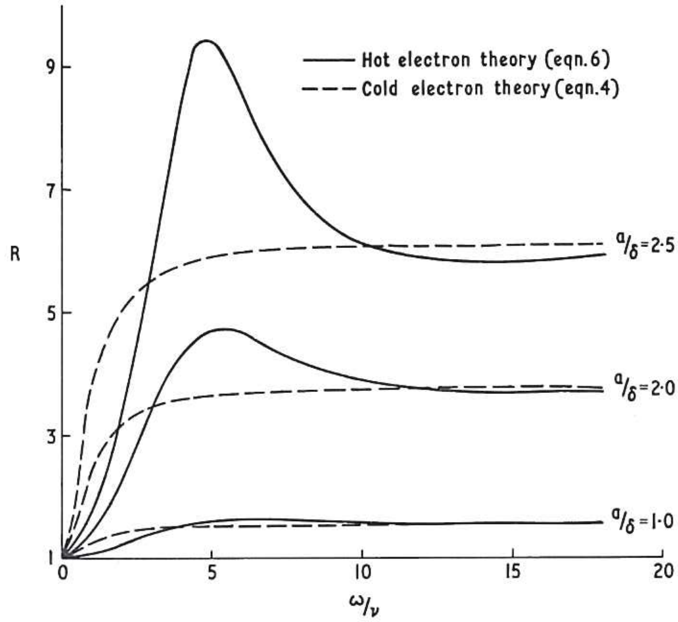

Fig. 2 (CLM-P 213)
The attenuation in a hot electron plane plasma slab vs the normalized frequency ($\omega/\nu = \phi_1/\phi_2$) calculated from Eq. (6)

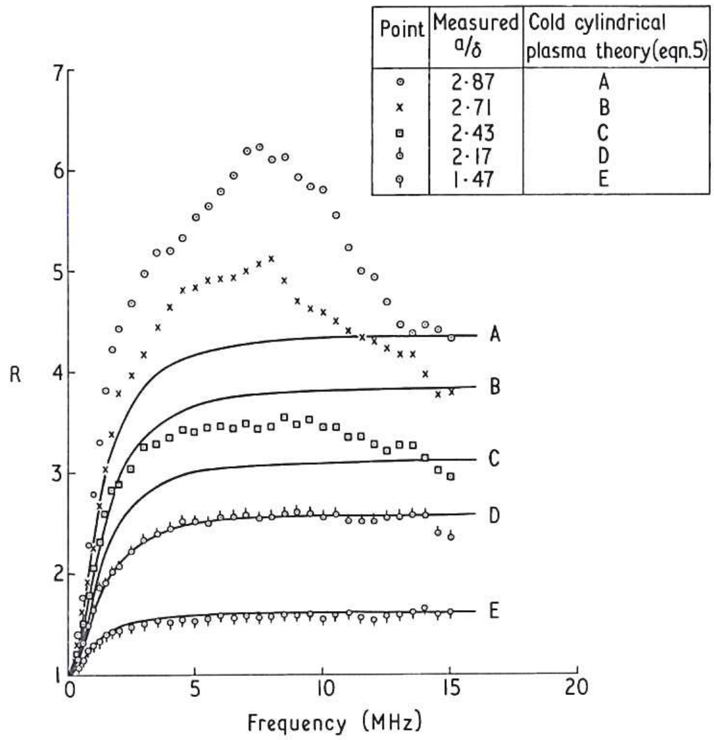

Fig. 3 (CLM-P 213)
The measured attenuation in a cylindrical plasma (T_e = 2.2 eV, $\nu/2\pi$ = 1.2 MHz) as a function of wave frequency. The curves are cold electron cylindrical plasma theory (Eq. (5)) calculated using a/δ and $\nu/2\pi$ measured by a Langmuir probe

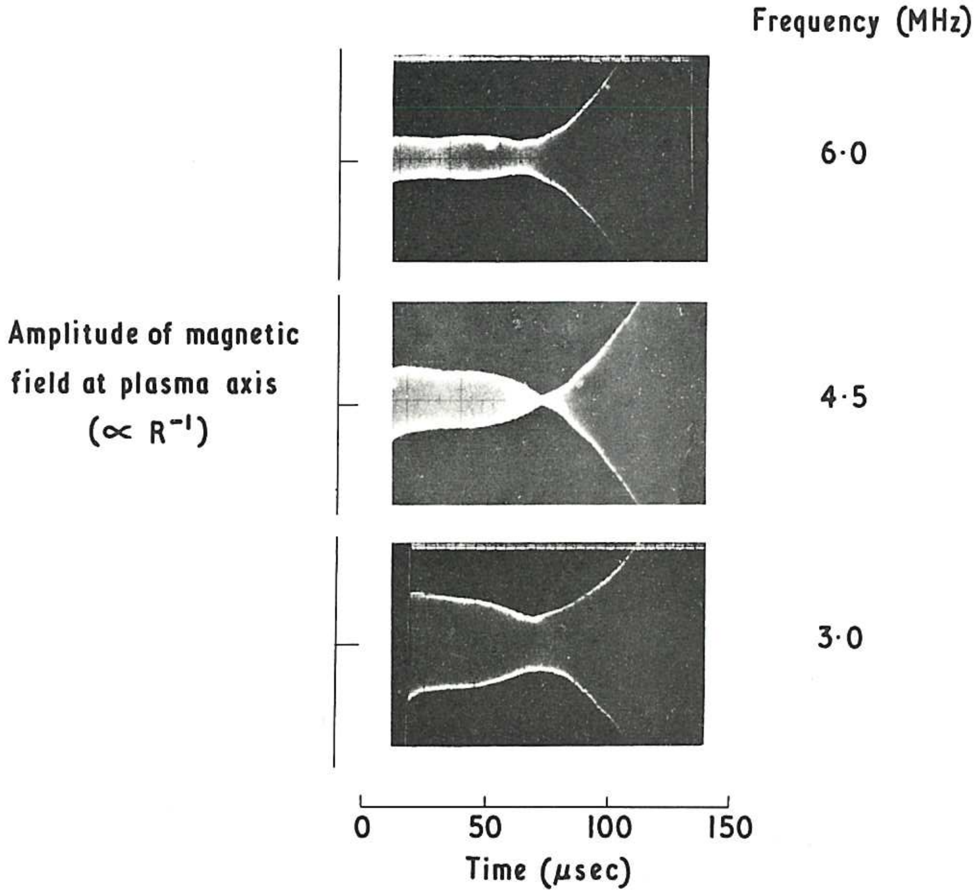

Fig. 4 (CLM-P 213)
Oscilloscope traces of the wave magnetic field amplitude at the plasma axis as a function of time after the end of the ionization current pulse

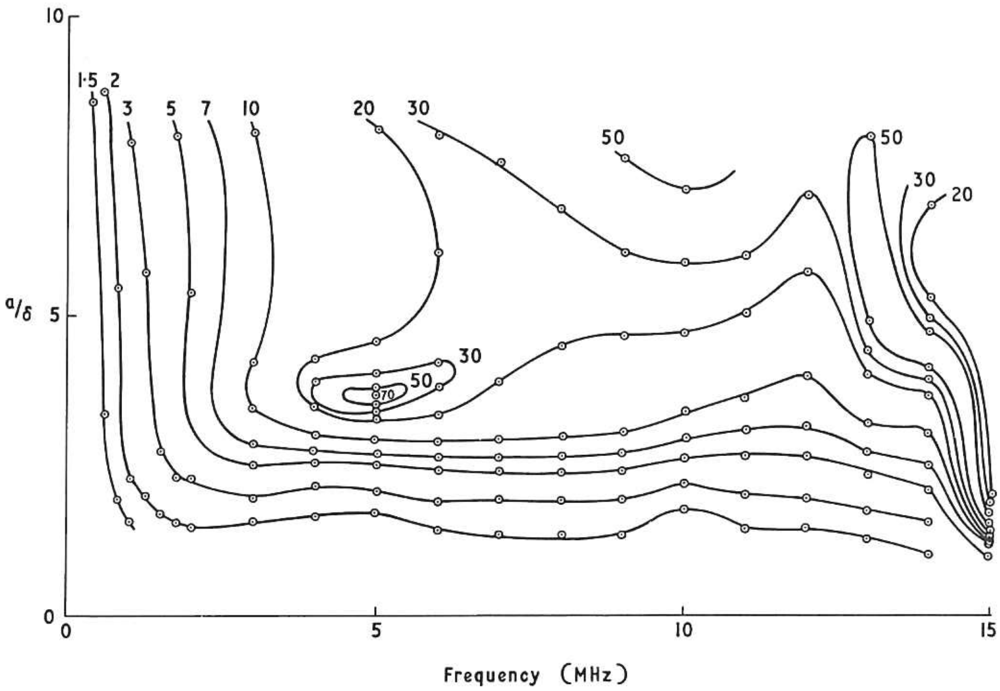

Fig. 5 (CLM-P 213)
Measured contours of constant attenuation in a cylindrical plasma in the a/δ, frequency plane

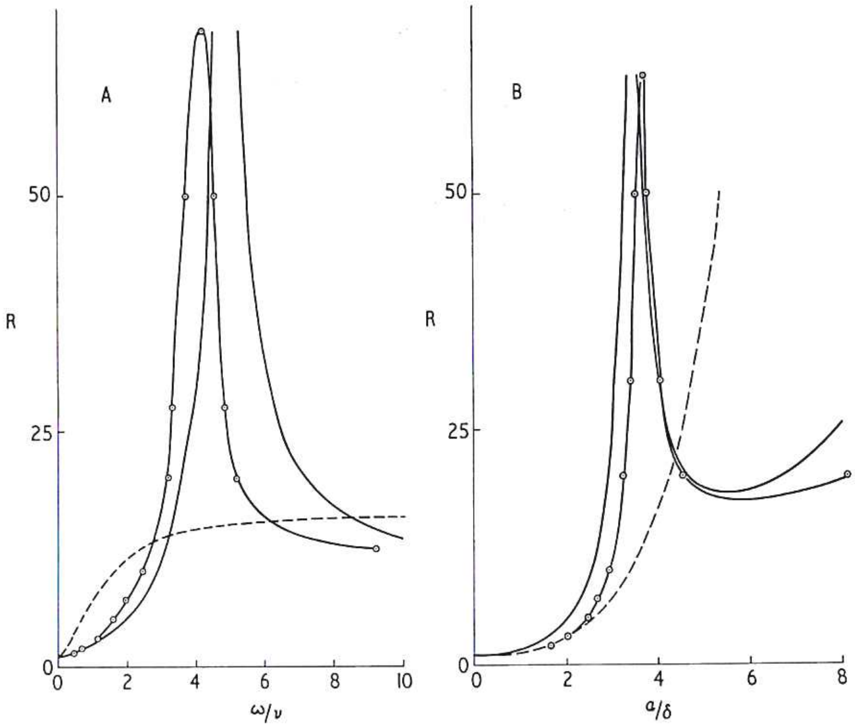

Fig. 6 (CLM-P 213)
Comparison of experimental measurements at the first observed resonance (from Fig. 5) to plane slab theory ($\phi_2 = 0.3$) at the first theoretical resonance (from Fig. 1) —⊙— experiment; ——— hot electron theory [Eq.(6)]
– – – cold electron theory [Eq.(4)]

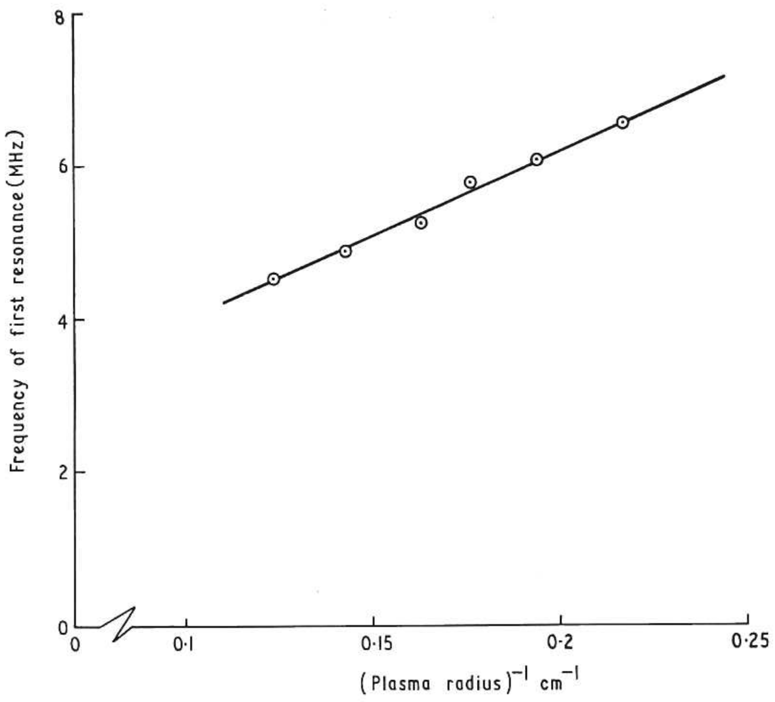

Fig. 7 (CLM-P 213)
Observed frequency of the first resonance vs (plasma radius)$^{-1}$

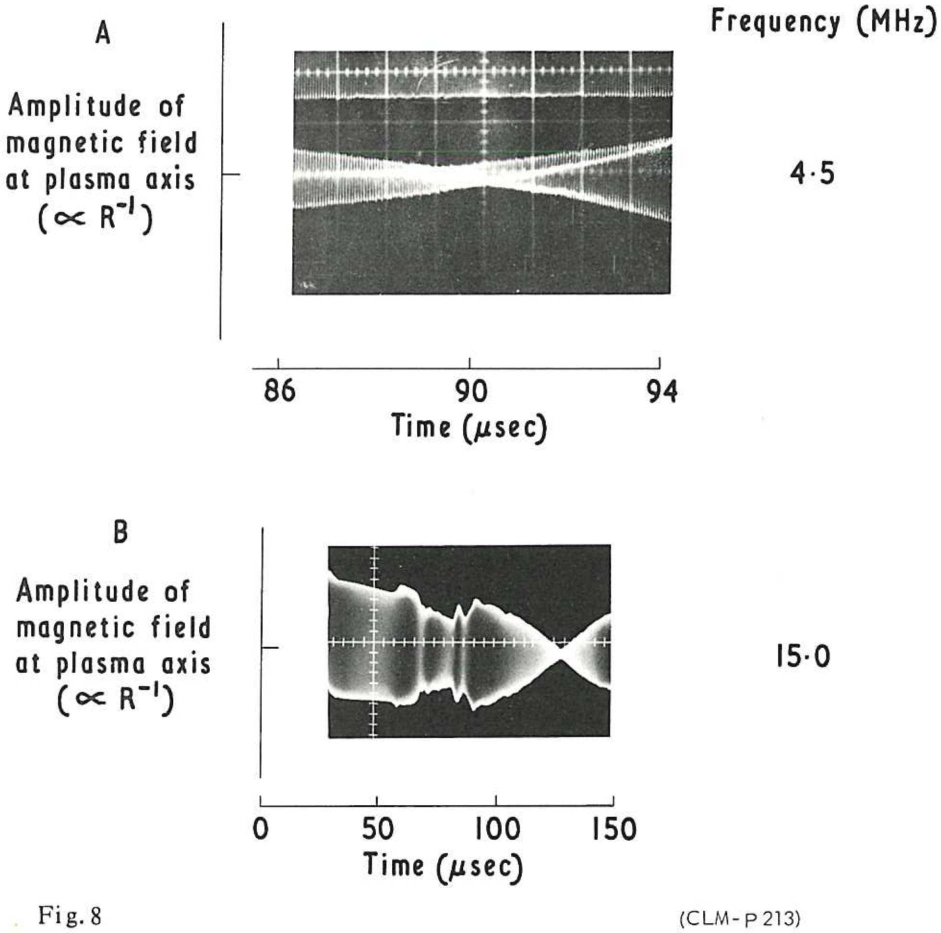

Fig. 8 (CLM-P 213)

(a) Magnetic field amplitude at the first resonance as a function of of time to show the harmonic content. (b) Magnetic field amplitude near the second resonance as a function of time