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EQUILIBRIUM DIFFUSION RATE IN A TOROIDAL PLASMA AT INTERMEDIATE COLLISION FREQUENCIES

T. E. STRINGER

Culham Laboratory
Abingdon Berkshire

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by

T.E. STRINGER

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ABSTRACT

It is shown that a consistent equilibrium must include an azimuthal electric field. The ambipolar diffusion rate differs in several respects from earlier estimates, though these differences are not primarily due to the azimuthal field. The new rate tends to be lower. In a stellarator field an ambipolar condition may occur in which the electron diffusion due to the toroidal field variation balances the ion diffusion in the helical variation.

U.K.A.E.A. Research Group,
Culham Laboratory,
Nr. Abingdon,
Berks.

C O N T E N T S

I. INTRODUCTION

This paper analysis the equilibrium of an axi-symmetric toroidal plasma. The range of collision frequencies considered, which will be referred to as the intermediate collisional regime, is bounded above by the resistive regime, and below by the weak collisional regime where some particles are trapped in the toroidal variation in magnetic field strength.

This problem was first investigated by Galeev and Sagdeev[1]. They used the guiding center kinetic equation with a collision term. When they included an arbitrary radial electric field in their equilibrium they found the ion and electron diffusion rates to be generally unequal. They concluded that the radial electric field would quickly build up to the ambipolar distribution, for which the diffusion rates are equal at all radii. This result has been challenged by Kovrizhnikh[2], who claims that when a more exact collision integral is used the ion and electron diffusion rates are always equal and are explicitly independent of the radial electric field.

The above authors neglect any electrostatic potential variation over magnetic surfaces. Their solution for the azimuthal variation in the ion and electron densities violates charge neutrality. As will be shown in Sec.III a consistent equilibrium requires the existence of an azimuthal electric field, which is uniquely determined by the quasi-neutrality condition. Its inclusion leads to a significant modification of the diffusion rate obtained by Galeev and Sagdeev. The results of the analysis have already been outlined in reference (3). This paper gives the detailed analysis, makes one correction to the earlier results, and examines the ambipolar diffusion rates more fully.

The treatment starts from the guiding center kinetic equation without a collision term. The fact that the expressions derived earlier for the diffusion rates in the intermediate collision regime are independent of the collision frequency at first appeared surprising. In fact, the following treatment shows that collisions can be dropped from the Galeev and Sagdeev analysis without affecting the diffusion. The diffusion results from Landau-type terms arising from integrals over the velocity distribution. These integrals have a form similar to those entering the standard theory of waves in a collisionless plasma. As will be explained in Sec.VII the particles responsible for the diffusion may be regarded as resonant particles in the azimuthally varying equilibrium.

In a later paper the solution of the guiding center kinetic equation will be given including both azimuthal electric fields and collisions. The transition from the intermediate collisional regime to the resistive/viscous regime considered in reference (4), which occurs when the dominant source of dissipation changes from Landau damping to parallel viscosity, will be discussed. The analysis will confirm that collisions do not affect the diffusion rates within the intermediate collision range.

Section II defines the coordinate system, the ordering assumed, and the role played by collisions in maintaining a local Maxwellian velocity distribution. Treating the radial distribution of density and electrostatic potential as specified, the azimuthal variation in the potential and velocity distribution is derived in Sec.III. The radial diffusion rate, which is a second order effect, is evaluated in Sec.IV. The values of the electric field at which the ion and electron diffusion rates balance, which is a necessary condition for

steady equilibrium, is derived in Sec.V, together with the corresponding ambipolar diffusion rate. The time taken for the space charge potential to build up to the ambipolar distribution is discussed in Sec.VI. Sec.VII gives a physical explanation of the most important features of the analytic solution. An attempt is made to explain why collisions, although they affect the drift of individual particles, do not influence the overall diffusion. The parameter range of greatest experimental interest is stated in Sec.VIII, although detailed experimental comparisons are deferred until after the general treatment covering the transition between the intermediate collision regime and the resistive/viscous regime. The extension of the analysis to include the helical variation in the magnetic field of a stellarator is outlined in Appendix A.

II. THE MODEL

The co-ordinates and magnetic field are the same as those described in reference (5). r, θ are polar co-ordinates centred on the magnetic axis, and φ measures angular distance along the axis, as illustrated in Fig.1. The magnetic field is taken to be

$$\underline{B} = \frac{R_O}{R}\left[0, \; B_{O\theta}(r), \; B_{O\varphi} \right] \qquad \ldots (1)$$

where R_O is the radius of the magnetic axis, $R = R_O(1+\varepsilon\cos\theta)$, and $\varepsilon = r/R_O$. The plasma current required to produce this field flows everywhere in the φ direction[5]. The magnetic surfaces are $r = $ constant. In a real toroidal field the magnetic surfaces are displaced relative to the axis. To take this into account would increase the analytic complexity, without appearing to affect the results significantly.

For a large aspect ratio torus the equilibrium velocity distribution function f_j and electrostatic potential Φ may be expanded as a power series in ε, as in reference (4).

$$f_j = f_{0j}(r, v_\parallel, p_\perp) + f_{1j}(r, \theta, v_\parallel, p_\perp) + \ldots, \quad \Phi(r, \theta) = \Phi_0(r) + \Phi_1(r, \theta) + \ldots$$

$$\ldots \ (2)$$

where $\Phi_s(r, \theta)$ contains all terms of order $\varepsilon^s \Phi_0$. Here $v_\parallel$ is the particle velocity parallel to the magnetic field, $p_\perp$ is the perpendicular momentum, and (r, θ) is the position of the guiding center. The temperature will be assumed uniform.

The basic equation is the guiding center kinetic equation.

$$\frac{\partial f_j}{\partial t} + (\underline{v}_j \cdot \nabla) f_j + \frac{\partial f_j}{\partial v_\parallel} \frac{dv_\parallel}{dt} + \frac{\partial f_j}{\partial p_\perp} \frac{dp_\perp}{dt} = 0 \qquad \ldots \ (3)$$

This may be derived from the continuity equation for the distribution function in guiding center space[6]. Since we are investigating equilibria, $\partial/\partial t = 0$. $\underline{v}_j$ is the guiding center velocity of a particle of the jth species, which may be written in the form

$$\underline{v}_j = v_\parallel \, \underline{B}/|B| + \underline{v}_{bj} + \underline{v}_0 + \underline{v}_1 + \ldots \qquad \ldots \ (4)$$

where

$$\underline{v}_{bj} = - \frac{m_j v_\parallel^2 + p_\perp^2/2m_j}{e_j B^2 R} (B_\varphi \underline{e}_z - B_z \underline{e}_\varphi)$$

$$\underline{v}_0 = - \frac{\nabla \Phi_0 \times \underline{B}}{B^2}, \quad \underline{v}_1 = - \frac{\nabla \Phi_1 \times \underline{B}}{B^2}$$

$\underline{v}_{bj}$ is the sum of the curvature and magnetic field gradient drifts. $\underline{e}_z$ and $\underline{e}_\varphi$ are unit vectors in the vertical and axial directions. Finite Larmor radius corrections to the guiding center drift have been neglected, as their effect is small.

The ordering assumed is the same as in reference (4). The zero order electric drift is assumed of the same order as the diamagnetic velocities U_{jn}, where

$$U_{jn} = \frac{\kappa T_j}{e_j B} \frac{1}{n_o} \frac{dn_o}{dr} \ .$$

This implies

$$e \frac{d\Phi_o}{dr} \sim \frac{\kappa T}{n_o} \frac{dn_o}{dr} \ , \quad v_1 \sim v_{bj} \sim \varepsilon U_{jn}. \qquad \dots \ (5)$$

The rotational transform ι will be assumed of order unity, i.e.

$$\Theta = \frac{B_\theta}{B_\varphi} = \frac{\varepsilon \iota}{2\pi} = O(\varepsilon) \ .$$

Hence

$$B = B_\varphi \left[1 + O(\varepsilon^2) \right] = B_{o\varphi} \left[1 - \varepsilon \ \cos\theta + O(\varepsilon^2) \right] \ .$$

The zero order distribution function is assumed to be locally Maxwellian. It is convenient to write it in the form

$$f_{oj}(r,v_\parallel,p_\perp) = \frac{1}{2m_j \kappa T_j} e^{-p_\perp^2/2m_j \kappa T_j} F_{oj}(v_\parallel) \qquad \dots \ (6)$$

where

$$F_{oj}(v_\parallel) = \frac{n_o(r)}{\sqrt{\pi} \ c_j} e^{-v_\parallel^2/c_j^2} \ , \quad c_j^2 = \frac{2\kappa T_j}{m_j}, \quad n = \iint f dv_\parallel dp_\perp^2 \ .$$

This assumes that collisions are sufficiently frequent to maintain a Maxwellian distribution in spite of the preferential loss of a certain class of particles. The condition for this is the same as the condition that no particles are trapped in the toroidal field variation, i.e. that the collision time for scattering out of the trapped velocity band is less than the transit time between reflection points[1]. It is this which imposes the lower limit on the intermediate collision

range[1], i.e. $\pi\lambda_{mfp} < L_c \varepsilon^{-3/2}$ where λ_{mfp} is the mean free path and $L_c = 2\pi\, r\, B/B_\theta$ is the connection length measured along a field line as it rotates once around the magnetic axis. The upper limit, above which the collision term can no longer be neglected in the kinetic equation, is of order of magnitude $\pi\lambda_{mfp} \sim L_c$. A more precise evaluation of this limit will be given in the later paper which includes a collision term.

III. SOLUTION OF THE EQUILIBRIUM EQUATIONS

The azimuthal variation in the distribution function will now be derived from Eq.(3). The variation of $p_\perp$ may readily be obtained from conservation of the magnetic moment $\mu = p_\perp^2/2mB$.

$$\frac{1}{p_\perp^2}\frac{dp_\perp^2}{dt} = \frac{1}{B}\frac{dB}{dt} = \left(\frac{v_{o\theta}}{r}\frac{\partial}{\partial\theta} + v_{\|}\frac{\partial}{\partial s}\right)B = \left(\frac{v_{o\theta}+\Theta v_{\|}}{R}\right)\sin\theta \qquad \dots (7)$$

where

$$\frac{\partial}{\partial s} = \frac{1}{B}(\underline{B}\cdot\nabla) = \frac{B_\theta}{rB}\frac{\partial}{\partial\theta} = \frac{\Theta}{r}\frac{\partial}{\partial\theta}\,.$$

Since $v_{o\theta} = v_o[1+O(\varepsilon^2)]$, the subscript θ can be dropped. We obtain $dv_{\|}/dt$ from the conservation of particle energy.

$$v_{\|} = \left[(E - e_j\Phi - \mu B)\,2/m_j\right]^{1/2}$$

$$\frac{dv_{\|}}{dt} = -\frac{1}{m_j v_{\|}}(\underline{v}\cdot\nabla)(e_j\Phi+\mu B) = -\frac{e_j\Theta}{m_j r}\frac{\partial\Phi}{\partial\theta} - \frac{\varepsilon}{r}\left(\frac{p_\perp^2\Theta}{2m_j^2} - v_o v_{\|}\right)\sin\theta$$

$$\dots (8)$$

where we have used

$$v_r = -\frac{1}{rB}\frac{\partial \Phi_1}{\partial \theta} - \frac{(p_\perp^2/2m_j + m_j v_{||}^2)}{e_j BR}\sin\theta, \quad v_\theta = v_o + \Theta v_{||} + O(\varepsilon).$$

$$\dots (9)$$

Linearising Eq.(3) with respect to ε, and integrating with respect to θ, gives

$$(v_o + \Theta v_{||})f_{1j} = \left[\Phi_1 - \frac{\varepsilon}{e_j}\left(m_j v_{||}^2 + \frac{p_\perp^2}{2m_j}\right)\cos\theta\right]\frac{1}{B}\frac{\partial f_{oj}}{\partial r}$$

$$-\frac{\varepsilon p_\perp^2}{2m_j \kappa T_j}(v_o + \Theta v_{||})\cos\theta\, f_{oj} + \left[\frac{e_j}{m_j}\Theta\Phi_1 - \varepsilon\left(\frac{p_\perp^2 \Theta}{2m_j^2} - v_o v_{||}\right)\cos\theta\right]\frac{\partial f_{oj}}{\partial v_{||}}$$

$$\dots (10)$$

We must now integrate f_{1j} over velocity to obtain the density variation. The integration over $p_\perp^2$ is trivial. Before integrating over $v_{||}$ it is convenient to express the θ variation in exponential form, i.e. replace $\cos\theta$ by $\exp(i\theta)$ with the understanding that only the real parts of the equations have physical significance. The integrals over $v_{||}$ have the form familiar from micro-instability theory

$$\frac{1}{n_o}\int_{-\infty}^{\infty}\frac{F_{oj}(v_{||})v_{||}^s}{v_{||} - W}\,dv_{||} = K_s\left(\frac{W}{c_j}\right). \qquad \dots (11)$$

These integrals may readily be expressed in terms of the plasma dispersion function $I(z)$ using the recurrence relation

$$K_s\left(\frac{W}{c_j}\right) = WK_{s-1}\left(\frac{W}{c_j}\right) + J_s$$

where $J_s = (s-2)(s-4)\dots 1\,(c_j^2/2)^{(s-1)/2}$ when s is odd, $J_s = 0$ when s is even,

$$K_o\left(\frac{W}{c_j}\right) = \frac{1}{W}\left[I\left(\frac{W}{c_j}\right) - 1\right], \quad K_1\left(\frac{W}{c_j}\right) = I\left(\frac{W}{c_j}\right),$$

and

$$I(z) = 1 - 2z\, e^{-z^2} \int_0^z e^{t^2}\, dt + i\sqrt{\pi}\, z\, e^{-z^2}. \qquad \dots (12)$$

In this way one obtains

$$\frac{n_{1j}}{n_0} = \frac{e_j \Phi_1}{\kappa T_j} \left[\frac{U_{jn}}{v_0}(1-I_j) - I_j \right] + \varepsilon\, e^{i\theta} \left(1 + \frac{U_{jn}}{v_0} \right) \left[I_j(1+2z_j^2) - 1 \right] \qquad \dots (13)$$

where

$$z_j = -\frac{v_0}{c_j\Theta}, \quad I_j = I(z_j) \ .$$

Invoking quasi-neutrality gives Φ_1

$$(F+iL)\Phi_1 = \frac{\varepsilon\kappa T_e}{e}\, e^{i\theta} \left[\left(1 + \frac{U_{in}}{v_0} \right)(1 + 2z_i^2)\, I_i - \left(1 + \frac{U_{en}}{v_0} \right)(1 + 2z_e^2)\, I_e \right.$$

$$\left. + \left(\frac{U_{en}-U_{in}}{v_0} \right) \right] \qquad \dots (14)$$

where $F+iL = (1+U_{en}/v_0)I_e + \tau(1+U_{in}/v_0)I_i$, and $\tau = T_e/T_i$. Splitting I_j into its real and imaginary parts

$$I_{oj} = 1 - 2z_j\, e^{-z_j^2} \int_0^{z_j} e^{t^2}\, dt, \quad J_j = \sqrt{\pi}\, z_j\, e^{-z_j^2},$$

$$F = (1+U_{en}/v_0)I_{oe} + \tau(1+U_{in}/v_0)I_{oi} \qquad \dots (15)$$

$$L = -\frac{\sqrt{\pi}}{c_e\Theta}(v_0+U_{en})\, e^{-z_e^2} \quad -\frac{\sqrt{\pi}}{c_i\Theta}\tau(v_0+U_{in})e^{-z_i^2} \qquad \dots (16)$$

IV. EVALUATION OF THE DIFFUSION

The net outward flux across a magnetic surface may be obtained by integrating the radial velocity multiplied by the distribution

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function over $v_{\|}$, $p_\perp^2$ and θ. The surface element is[5]

$$dS = rd\theta \; R_o(1+\varepsilon\cos\theta)d\varphi$$

$$nv_{dj} = -\frac{1}{2\pi B_o r} \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dv_{\|} \int_0^{\infty} dp_\perp^2 \; (f_{oj}+f_{1j}) \left[\frac{\partial \Phi_1}{\partial \theta} + \frac{\varepsilon}{e_j}\left(m_j v_{\|}^2 \right. \right.$$

$$\left. \left. + \frac{p_\perp^2}{2m_j} \right) \sin\theta \right] \cdot (1+\varepsilon\cos\theta)^2$$

$$= -\frac{1}{2\pi B_o r} \int_0^{2\pi} d\theta \left[\frac{\partial \Phi_1}{\partial \theta} \; (2\varepsilon n_o\cos\theta+n_1) + \frac{\varepsilon}{e_j} \sin\theta \int dv_{\|} \int dp_\perp^2 \left(m_j v_{\|}^2 \right. \right.$$

$$\left. \left. + \frac{p_\perp^2}{2m_j} \right) f_{1j} \right] \qquad\qquad \dots (17)$$

The required moments of f_{1j} may be obtained from Eq.(10)

$$\frac{1}{n_o} \int dv_{\|} \int dp_\perp^2 \; v_{\|}^2 \; f_{1j} = -\frac{e_j\Phi_1}{m_j} \left[1 + 2z_j^2 \left(1 + \frac{U_{jn}}{v_o} \right) I_j \right]$$

$$+ \frac{\varepsilon\kappa T_j}{m_j} \left(1 + \frac{U_{jn}}{v_o} \right) 2z_j^2 \left[1 + \left(1+2z_j^2 \right) I_j \right] e^{i\theta} \qquad \dots (18)$$

$$\frac{1}{n_o} \int dv_{\|} \int dp_\perp^2 \; p_\perp^2 \; f_{1j} = 2m_j\kappa T_j \left[\frac{n_1}{n_o} - \varepsilon e^{i\theta} \left(1 + \frac{U_{jn}}{v_o} \right) (1 - I_j) \right] \qquad \dots (19)$$

Before substituting into Eq.(17) one must, of course, take the real parts of the expressions for Φ_1, n_1, and the moments in Eq.(17) and (18). The result of performing the integration over θ can be expressed in the concise form

$$v_{dj} = \frac{\varepsilon^2 \kappa T_j}{2re_j B} \left(1 + \frac{U_{jn}}{v_o} \right) J_j \left[1 + \frac{[F(1+2z_j^2)-\beta_j P]^2+[L(1+2z_j^2)-\beta_j Q]^2}{F^2 + L^2} \right] \qquad \dots (20)$$

where

$$\beta_j = (e_j/e) \; (T_e/T_j) = \tau \text{ for ions and } -1 \text{ for electrons.}$$

P and Q are proportional to the real and imaginary parts of the

right hand side of Eq.(14), defined by

$$\Phi_1(r,\theta) = \frac{\varepsilon\kappa T_e}{e}\left[\frac{P+iQ}{F+iL}\right] e^{i\theta} \, . \qquad \ldots (21)$$

In all conditions of experimental interest $z_e = O(U_{jn}/c_e\Theta) \ll 1$, and hence $I_{oe} \approx 1$. The expressions in Eq.(20) can then be expressed as follows

$$(1 + 2z_i^2) \, F - \tau P = 1 + \tau + 2z_i^2 \, (1 + U_{en}/v_o) = S$$

$$F + P = (1 + U_{in}/v_o)(1 + \tau + 2z_i^2) \, I_{oi} - (1 + \tau) \, U_{in}/v_o = G$$
$$\ldots (22)$$

$$(1 + 2z_i^2) \, L - \tau Q = - \sqrt{\pi} \, (1 + \tau + 2z_i^2)(v_o + U_{en})/c_e\Theta$$

$$L + Q = - \sqrt{\pi} \, (1 + \tau + 2z_i^2)(v_o + U_{in}) \, e^{-z_i^2}/c_i\Theta.$$

Eq.(20) may be expressed in a form similar to earlier expressions for the diffusion.

$$v_{di} = - \frac{\sqrt{\pi}\varepsilon^2 a_i^2 c_i}{8r\Theta}\left(1 + \frac{v_o}{U_{in}}\right) e^{-z_i^2} \frac{1}{n}\frac{dn}{dr}\left[1 + \frac{S^2 + J_e^2(1+U_{en}/v_o)^2(1+\tau+2z_i^2)^2}{F^2 + L^2}\right]$$
$$\ldots (23)$$

$$v_{de} = - \frac{\sqrt{\pi}\varepsilon^2 a_e^2 c_e}{8r\Theta}\left(1 + \frac{v_o}{U_{en}}\right) e^{-z_e^2} \frac{1}{n}\frac{dn}{dr}\left[1 + \frac{G^2 + J_i^2(1+U_{in}/v_o)^2(1+\tau+2z_i^2)}{F^2 + L^2}\right]$$
$$\ldots (24)$$

where $a_j = c_j/\Omega_j$ is the Larmor radius. Equation (24) differs in one respect from the corresponding result given in reference (3). In deriving that result the $dp_\perp/dt$ term was omitted from Eq.(3)[7]. As a result a term proportional to $(1 + v_o/U_{in})J_i$ incorrectly appeared in the electron diffusion equation.

The result which would have been obtained if the azimuthal electric field had been neglected in the foregoing analysis may be found by putting $P = Q = 0$. As may be seen from Eq.(20), the

expressions in the square brackets then equal $1 + (1 + 2z_j^2)^2$. This gives a diffusion rate for each species which is exactly one quarter of that obtained by Galeev and Sagdeev[1] if $z_j < 1$ (which seems to have been tacitly assumed in reference (1)). The reason for this numerical discrepancy is not understood.

V. THE AMBIPOLAR CONDITION

As may be seen from Eqs.(23) and (24), for arbitrary values of v_o the ion and electron diffusion rates are unequal, their ratio being roughly $(m_i/m_e)^{1/2} \tau^{-3/2} \exp[- (v_o/c_i\Theta)^2]$. The differing diffusion rates may be expected to increase the space charge potential until an ambipolar distribution is reached, defined by the condition $v_{di}(v_o) = v_{de}(v_o)$ at all radii. This ambipolar condition may be written in the form

$$\left(1 + \frac{v_o}{U_{en}}\right) = \left(1 + \frac{v_o}{U_{in}}\right) e^{-z_i^2} \left(\frac{m_i}{m_e}\right)^{1/2} \tau^{-3/2} \left[\frac{F^2+L^2+G^2+J_i^2(1+U_{in}/v_o)^2(1+\tau+2z_i^2)^2}{F^2+L^2+S^2+J_e^2(1+U_{en}/v_o)^2(1+\tau+2z_i^2)^2}\right]$$

$$\ldots (25)$$

The ratio in the square brackets in Eq.(25) will generally be of order unity. Thus the most obvious solution is either $v_o \approx - U_{in}$ or $- U_{en}$, depending on whether $(m_i/m_e)^{1/2} \exp(-z_i^2)$ is greater than or less than unity. For a hydrogen plasma this corresponds to z_i less than or greater than 2 or, since $U_{in}/c_i\Theta = a_i/2r_n\Theta$, $a_i/r_n\Theta$ less than or greater than 4.

We will first consider the case $a_i/r_n\Theta > 4$, and hence $v_o \approx U_{en}$ and $z_i > 2\tau$. In this parameter range the ions tend to have the slower diffusion, and so they determine the ambipolar rate. This will now be estimated. Since $I(z)$ falls off for large z

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as $-1/2z^2$, $F + iL \approx (1 + U_{en}/v_o)I_e$. Thus for the ambipolar condition $v_o \approx - U_{en}$, both F and L are small. It might be expected that the ambipolar diffusion rate would be correspondingly large. However, when the numerator in Eq.(23) is evaluated, one finds that in leading order it also is proportional to $(1 + U_{en}/v_o)^2$. In fact, the ambipolar diffusion rate

$$v_{da} = - \frac{\sqrt{\pi} \; \varepsilon^2 (1+\tau) a_i^2 c_i}{2r_n \Theta} \left(\frac{a_i}{2r_n \Theta} \right)^4 \exp \left[- \left(\frac{a_i}{2r_n \Theta} \right)^2 \right] \frac{1}{n} \frac{dn}{dr}$$

$$\ldots (26)$$

shows no significant enhancement as a result of the resonance with the electron drift wave.

We now enquire whether there are any other solutions to the ambipolar condition when $a_i/r_n\Theta > 4$. It may readily be confirmed that solutions exist near $v_o = \pm \; 2c_i\Theta < U_{jn}$. For these solutions, $|z_i| \approx 2$ and hence $J_i \approx J_e$. The corresponding ambipolar diffusion rate is

$$v_{da} = - \frac{\sqrt{\pi} \; \varepsilon^2 a_i^2 c_i}{8r\Theta} \left(\frac{m_e}{m_i \tau} \right)^{\frac{1}{2}} \left[\tau^2 + (2+\tau)^2 \right] \frac{1}{n} \frac{dn}{dr} \qquad \ldots (27)$$

That there are no other ambipolar fields can be seen from Fig.2, which illustrates the variation in the diffusion rates with v_o, when $a_i/r_n\Theta > 4$. This shows only the qualitative behaviour. The difference in magnitude between the two rates will generally be much greater than shown in Fig.2.

We will now consider the case $a_i/r_n\Theta < 4$. One solution of Eq.(25) is $v_o \approx - U_{in}$, for which $z_i < 2$. The evaluation of the ambipolar diffusion rate is rather easier if we assume $z_i \ll 1$, although the result given below is, in fact, valid for $z_i \sim 1$.

Then $F \approx 1 + \tau$, $L \ll 1$, $S \approx G \approx 1 + \tau$. Substituting into Eq.(24) gives

$$v_{da} = - \frac{\sqrt{\pi} \; \varepsilon^2 a_i^2 c_i \tau^{\frac{1}{2}}(1+\tau)}{4r\Theta} \left(\frac{m_e}{m_i} \right)^{\frac{1}{2}} \frac{1}{n} \frac{dn}{dr} . \qquad \dots (28)$$

There are no other obvious solutions to the ambipolar equation when $a_i/r_n \Theta < 4$.

The ambipolar diffusion rate in Eq.(28) differs from that derived by Galeev and Sagdeev[1] only by the factor $\frac{1}{4}$ already referred to. Their tacit assumption that $v_o/c_i \Theta < 1$ for the ambipolar v_o is valid only when $a_i/r_n \Theta < 4$. The more common condition in stellarator experiments is $a_i/r_n \Theta > 4$. The ambipolar diffusion rates for this case, Eq.(26) and (27) differ from the Galeev and Sagdeev result mainly because $v_o/c_i \Theta > 1$, rather than the effect of the azimuthal electric field. Although a consistent treatment demands an azimuthal electric field, and in spite of the fact that it can markedly change the electron diffusion rate over a range of v_o close to $- U_{en}$, it does not change any of the ambipolar diffusion rates by a large factor.

VI. <u>BUILD-UP TIME FOR THE AMBIPOLAR FIELD</u>

The initial radial space charge field will depend on the plasma formation process, and will not in general be ambipolar. The differing diffusion rates of the ions and electrons will cause the space charge to build up towards an ambipolar distribution.

An estimate of the build-up time and the diffusion during the build-up phase may readily be obtained if the rate of change in the equilibrium distribution is assumed slow, such that $\frac{\partial f_{1j}}{\partial t} \lesssim O(\varepsilon \underline{v}_j . \nabla) f_{1j}$. The omission of $\partial f_j/\partial t$ from the linearized form of eq.(3) is then still valid. The time variation introduces an additional term into eq.(4)

for the guiding center velocity

$$\underline{v}_{tj} = \frac{1}{\Omega_j B} \frac{dE}{dt} = -\frac{1}{\Omega_j} \frac{\partial v_o}{\partial t} \underline{e}_r - \frac{1}{\Omega_j B} \frac{\partial}{\partial t} (\underline{v}_1 \times \underline{B}) + \qquad \ldots (29)$$

where $\Omega_j = e_j B/m_j$ and $\underline{e}_r$ is a unit vector in the radial direction. The assumption of a slow rate of change on the equilibrium quantities implies that $\partial v_o/\partial t < 0 \ (\varepsilon \Omega_i v_o)$, e.g. if $\partial/\partial t \sim \varepsilon v_o/r$ and $v_o \sim U_{jn}$, then $\partial v_o/\partial t \sim \varepsilon \Omega_i v_o a_i^2/r^2$. Thus the inertial correction v_{tj} does not contribute to the first order quantities retained in eq.(10). Hence the first order expressions for the azimuthal variation in potential and velocity distribution functions derived in Section III are un-affected by a slow time variation.

Since the radial flux vanishes to first order when integrated over a magnetic surface, v_{tj} must be retained in the evaluation of the diffusion. In lowest order v_{tj} is independent of θ and contributes a flux $-\frac{n_o}{\Omega_j} \frac{\partial v_o}{\partial t}$. The other components of the radial flux are still correctly described by the expressions in Sections III and IV. Thus the diffusion in a slowly varying equilibrium, denoted by v_{Dj}, can be expressed in terms of the diffusion expression derived assuming a steady equilibrium, v_{dj}, as follows

$$v_{Dj} = v_{dj} - \frac{1}{\Omega_j} \frac{\partial v_o}{\partial t} \cdot \qquad \ldots (30)$$

Quasi-neutrality requires that $v_{Di} = v_{De}$ at all times i.e.

$$\frac{\partial v_o}{\partial t} = \Omega_i (v_{di} - v_{de}) \cdot \qquad \ldots (31)$$

From eq.(30) the time variation is seen to have a negligible effect on the electron diffusion rate. Thus it is given by eq.(24) even during the build up phase. The inertial correction to the ion rate is such as to bring it to equality with the electron rate.

As may be seen from the sign of $v_{di} - v_{de}$ in Fig.2, the ambipolar condition $v_o \approx -2c_i\Theta$ is unstable in the sense that, if v_o is initially close to this value, it will steadily move further away. The ambipolar conditions $v_o \approx 2c_i\Theta$ and $-U_{en}$ are stable. If initially $v_o > -2c_i\Theta$ the radial field will move towards the condition $v_o \approx 2c_i\Theta$. If initially $v_o < -2c_i\Theta$ it will move towards $v_o \approx -U_{en}$. In the first case, $v_{di} \gg v_{de}$ over most of the non-ambipolar phase. Hence

$$\frac{\partial v_o}{\partial t} \approx -\frac{\sqrt{\pi}}{4} \frac{\varepsilon^2 a_i c_i^2}{r\Theta} \frac{1}{n} \frac{dn}{dr} . \qquad \ldots (32)$$

The time to build up to $v_o \approx 2c_i\Theta$ is $t_a \sim (8rr_n/\sqrt{\pi}a_i c_i)(\iota/2\pi)^2$. In typical experimental conditions this is of the order of tens of microseconds.

A weakness in the above analysis is that the initial assumption

$$\frac{\partial f_{1j}}{\partial t} \lesssim 0\left[\varepsilon \ (v_o + \Theta v_\parallel) \frac{1}{r} \frac{\partial f_{1j}}{\partial \theta}\right]$$

is invalid for the resonant particles $v_\parallel \approx -v_o/\Theta$. If we replace $\partial f_{1j}/\partial t$ by γf_{1j}, the effect on f_{1j} of a slow variation is formally similar to the effect of weak collisions. These are known not to significantly affect the microscopic behaviour, which gives some confidence that the above analysis should provide a valid estimate of the diffusion process. Eq.(32) should be checked for consistency with the initial assumption of slow rate of change in the equilibrium parameters. Assuming $v_o \approx c_i\Theta$, the condition $\partial v_o/\partial t \lesssim \varepsilon v_o^2/r$ requires $\Theta^2 > a_i/r_n$.

VII. PHYSICAL EXPLANATION

It will first be shown how the diffusion process can be interpreted as a resonant particle phenomenon. It is therefore not surprising that

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it results analytically from Landau-type terms. A heuristic derivation will be given showing how collisions, although they affect the motion of individual particles, do not influence the net flux. This is not intended as a substitute for a rigorous treatment, which will be given in a later paper. Finally the physical origin of the denominator $F^2 + L^2$ will be demonstrated.

The origin of the diffusion is the magnetic drift in the toroidal field. For the field considered, this drift is approximately vertically downwards for ions, and upwards for electrons. The radial component of this drift is $v_{bj} \sin \Theta$. The guiding center of a typical particle rotates around the magnetic axis with angular velocity $d\theta/dt = (v_0 + \Theta v_{||})/r$, due to the combination of electric drift and parallel motion along a field line. Thus the magnetic drift alternates between outwards and inwards. The displacement from a magnetic surface is limited, $\delta r = r v_b \cos \theta /(v_0 + \Theta v_{||})$. The exception is the class of particles for which $v_0 + \Theta v_{||}$ is almost zero. As will be discussed later, the azimuthal variation in the equilibrium may equally well be regarded as an $m=1$ forced oscillation, which happens to be at rest in the laboratory frame. A resonant particle is one which travels approximately in phase with a wave, so by this definition those particles for which $v_0 + \Theta v_{||} \approx 0$ are in resonance with the toroidal variation.

The radial drift of such a particle may be limited either because its slow angular velocity carries it into the other half of the cross section, or because its angular velocity is increased markedly by collisions. The second mechanism is more important for 'resonant particles'. We first estimate the range of $\Delta v = v_{||} + v_0/\Theta$ over which the fractional change in Δv due to small angle scattering is

small during one rotation around the magnetic axis. This requires

$$\text{Scattering Time} = \frac{1}{\nu_j}\left(\frac{\Delta v}{c_j}\right)^2 > \text{Transit Time} = \frac{r}{\Theta \Delta v} \ . \qquad \ldots \ (33)$$

Hence if $\Delta v > w_j = (r\nu_j c_j^2/\Theta)^{1/3}$, we can neglect collisions. Such particles, which will be referred to as passing particles, describe circular orbits about the magnetic axis whose displacement from a magnetic surface is $\delta r = r v_b \cos\theta/\Theta\Delta v$. If $\Delta v < w_j$ the particle rotational transform is so small that it may be considered to remain at a constant θ until small angle scattering changes it into a passing particle. This takes an average time $\tau_s = w_j^2/\nu_j c_j^2$, during which the particle suffers a radial displacement $v_{bj}\tau_s \sin\theta$. The flux of particles across a magnetic surface is

$$n v_{dj} \approx \frac{1}{2\pi} \int dv \int_0^{2\pi} d\theta \ f_0 \ (r-\delta r) \ v_{bj}\sin\theta$$

$$= \frac{1}{2\pi} \int_{v_0/\Theta-w_j}^{v_0/\Theta+w_j} dv_\| \int_0^{2\pi} \frac{\partial F_0}{\partial r} \frac{\tau_s}{2} v_{bj}^2 \ \sin^2\theta$$

$$\sim \tfrac{1}{2} \ \tau_s w_j \ v_{bj}^2 \left(\frac{\partial F_0}{\partial r}\right)_{v_\| = -v_0/\Theta} \ .$$

Substituting a mean value for v_{bj} from Eq.(4), and for $\tau_s w_j$, gives a diffusion rate which differs from Galeev and Sagdeev's[1] by only a numerical factor. The essential feature is that, as may be seen from Eq.(31), the time spent in the steady drift condition is such that $\tau_s w_j = r/\Theta$, independent of the collision frequency. The analogous effect in the propagation of a small amplitude plasma wave is that Landau damping obeys the collisionless theory even when the time for collisional scattering out of the resonant velocity range may be less than the growth time.

The explanation of the factor $F + iL$ in Eq.(14) for Φ_1 is the same as that given for the factor D in the resistive/viscous plasma[3,4]. If we transform to the plasma frame, the toroidal magnetic field and the resulting separation currents appear as an $m=1$ variation, whose wavelength along the field lines corresponds to a wave number $k_\| = \Theta/r$, rotating with angular frequency $-v_o/r$. The response of the plasma to such an externally driven charge separation may be expected to vary inversely as its dielectric constant. The factor $F + iL$ is identical with the dielectric constant of a stationary inhomogeneous collisionless plasma for electrostatic excitation with this wavenumber and frequency. Since the diffusion rate contains products of azimuthally varying quantities, it must include terms proportional to $|F+iL|^{-2}$.

For most values of the doppler-shifted frequency, $|F+iL|$ is of order unity and the above effect does not markedly affect the diffusion. The exception is if $-v_o/r$ is close to the frequency of a natural mode in the plasma rest frame, and if this mode is weakly damped. Then $|F+iL| \ll 1$, the azimuthal variation in density and potential is large, and the diffusion rate may be enhanced.

When $U_{jn} \gg c_i k_\|$ the natural modes are the electron drift wave, $\omega = m U_{en}/r$, and the slow ion wave $\omega = -c_s^2 k_\|^2 r/m U_{en}$. The electron drift mode is marginally stable when finite Larmor radius effects are neglected and the temperature is uniform. This is why the electron diffusion rate is large when $a_i/r_n \Theta > 4$ and $v_o \approx -U_{en}$, corresponding to resonant excitation of a drift wave (see Fig.2). No comparable amplification occurs when $-v_o/r$ coincides with the slow ion wave frequency, because there is heavy ion Landau damping of this mode.

VIII. RELATION TO EXPERIMENT

From the plot of $a_i/r_n\Theta$ against $2\pi\lambda_{mfp}/L_c$ for typical operating conditions in Fig.2 of reference (4), it may be seen that the most common condition is $a_i/r_n\Theta \geqslant 4$, $2\pi\lambda_{mfp}/L_c < 10$. In all the $\ell=3$ stellarator experiments the plasma is in the resistive/viscous condition at smaller radii, where the connection length is long. Before the diffusion rates derived here and in reference (4) can be used to predict the overall confinement time, the transition range between the two regimes must be established more precisely than $L_c = 0 \ (2\pi\lambda_{mfp})$. This transition will be investigated in a later paper.

IX. CONCLUSIONS

It has been shown that the guiding center drifts in a toroidal magnetic field inevitably produce an azimuthal electric field. This was neglected in earlier treatments of the intermediate collision range[1,2]. The diffusion rates differ in three respects from those derived by Galeev and Sagdeev[1]. Firstly, the coefficient of both the ion and electron rates are less than Galeev and Sagdeev's by a factor of 1/4. This difference would seem to be of analytic origin. Secondly, Galeev and Sagdeev appear to tacitly assume that $v_o/c_i\Theta < 1$, although the reverse inequality more commonly applies to present experiments when v_o has the ambipolar value. Higher powers of $z_i = -v_o/c_i\Theta$, absent from the Galeev and Sagdeev expressions, then become dominant. Thirdly the inclusion of the azimuthal electric field introduces a factor which is inversely proportional to the square of the plasma dielectric constant, evaluated at the doppler frequency $-v_o/r$. In general the dielectric constant is of order unity and this factor does

not significantly affect the diffusion. The exception is when $-v_0/r$ is close to the frequency of a natural plasma wave in the plasma rest frame, in which case the dielectric constant is small and the diffusion may be rapid.

Equating the ion and electron diffusion rates gives an equation for the ambipolar electric field. When $a_i/4r_n\Theta < 4$ there is only one solution, $v_0 = - U_{in}$. In the condition of most experimental interest, where $a_i/r_n\Theta > 4$, this equation has three roots, i.e. $v_0 \approx - U_{en}, - 2c_i\Theta, + 2c_i\Theta$. The middle one is unstable. If the initial potential distribution satisfies $\partial\Phi_0/\partial r > - 4\kappa T_i\Theta/ea_i$, the space charge distribution will move towards that defined by $v_0 \approx 2c_i\Theta$. If initially $\partial\Phi_0/\partial r < - 4\kappa T_i\Theta/ea_i$, it will move towards $v_0 \approx U_{en}$. This ambipolar electric drift is very close to that for resonant excitation of an electron drift wave in the plasma. Although the electron diffusion is enhanced over a range of v_0 close to $-U_{en}$, there is no enhancement of the ion rate. Thus the ambipolar rate, which in this condition is determined by the ions, is not significantly affected by the resonance.

When $a_i/r_n\Theta < 4$ the ambipolar diffusion rate, Eq.(28), agrees with the Galeev and Sagdeev rate, apart from the factor of $1/4$. When $a_i/r_n\Theta > 4$, the diffusion in the ambipolar condition $v_0 \sim 2c_i\Theta$, given by Eq.(27), roughly equals the Galeev and Sagdeev rate when $T_e = T_i$. The other possible ambipolar condition, $v_0 = - U_{en}$, has a diffusion rate, Eq.(28), which is much less than the ambipolar rate given by Galeev and Sagdeev. This is mainly because $z_i > 2$ and hence the ion diffusion rate is greatly reduced by the exponential factor.

The effect of the helical field component of a toroidal stellarator has been evaluated in Appendix A, still in the intermediate collision frequency regime. Although in present experiments the magnetic drift due to the helical field variation is generally small compared to the toroidal drift, the number of ions whose trajectories are in phase with the helical field may be much greater than those in phase with the toroidal variation (i.e. for which $d\theta/dt = 0$). The net diffusion due to the helical variation may thus exceed the toroidal diffusion. In the ambipolar condition the electron diffusion due to the toroidal field variation will then be balanced by the ion diffusion due to the helical variation. This would give a larger ambipolar diffusion rate than in a comparable axi-symmetric torus.

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7. I am grateful to J.W. Connor for pointing out this omission.

APPENDIX A

DIFFUSION IN A STELLARATOR FIELD

The equilibrium analysis will now be extended to include the helical field component of a toroidal stellarator. A vector potential describing such a field is given in reference (4). The field strength can be written, correct to first order in ε and δ, in the form

$$B \approx B_\varphi \approx B_0 \left[1 - \varepsilon\cos\theta - \delta\ell I_\ell(\ell ar/R)\cos\ell(\theta - \alpha\varphi) \right] \qquad \ldots \text{ (A1)}$$

where δ is a measure of the strength of the helical component. Assuming δ and ε to be small quantities of the same order, all the equilibrium quantities can be expanded in power series in ε and δ, e.g.

$$\Phi(r,\theta,\varphi) = \Phi_0(r) + \Phi_1^{(\varepsilon)}(r,\theta) + \Phi_1^{(\delta)}(r,\theta-\alpha\varphi) + \ldots$$

As we follow a field line $\Phi^{(\varepsilon)}(\theta)$ has a fast oscillatory variation due to the helical path of the field line, and a slow steady change due to the rotational transform ι. The fast component has small amplitude, of order δ, and can be averaged out to give

$$\frac{\partial\Phi_1^{(\varepsilon)}}{\partial s} \approx \frac{\varepsilon\iota}{2\pi r}\frac{\partial\Phi_1^{(\varepsilon)}}{\partial\theta} = \frac{\Theta}{r}\frac{\partial\Phi_1^{(\varepsilon)}}{\partial\theta}$$

where now $\Theta = \varepsilon\iota/2\pi$. On the other hand, $\Phi_1^{(\delta)}$ goes through one cycle during a helical field period. The variation in $\Phi_1^{(\delta)}$ due to the change in θ over a period is relatively small and can be neglected. Hence

$$\frac{\partial\Phi_1^{(\delta)}}{\partial s} \approx \frac{1}{R}\frac{\partial\Phi_1^{(\delta)}}{\partial\varphi} .$$

We now consider the effect of the helical field on the guiding center motion. The radial component of the guiding center drift is now

$$v_{jr} = - \frac{(m_j v_{\parallel}^2 + p_{\perp}^2/2m_j)}{e_j rB} \left[\varepsilon \sin\theta + \delta \ell^2 I_{\ell}(\ell \alpha r/R) \sin\ell(\theta - \alpha\varphi) \right] - \frac{1}{rB} \frac{\partial}{\partial\theta} (\Phi_1^{(\varepsilon)} + \Phi_1^{(\delta)}) .$$

The helical variation in B and Φ introduces additional terms into $dv_{\parallel}/dt$ and $dp_{\perp}/dt$, e.g.

$$\frac{dv_{\parallel}}{dt} = - \frac{e_j \Theta}{m_j r} \frac{\partial\Phi^{(\varepsilon)}}{\partial\theta} + \frac{e_j \alpha}{m_j R} \frac{\partial\Phi^{(\delta)}}{\partial\theta} - \frac{\varepsilon}{r} \left(\frac{p_{\perp}^2 \Theta}{2m_j^2} - v_o v_{\parallel} \right) \sin\theta$$

$$+ \frac{\delta \ell^2 I_{\ell}}{r} \left(\frac{p_{\perp}^2 \varepsilon \alpha}{2m_j^2} + v_o v_{\parallel} \right) \sin \ell(\theta - \alpha\varphi) .$$

These expressions may be substituted into Eq.(3), and the equation linearised in ε and δ. Because of their different θ dependence, terms which are functions of θ and those which are functions of $\theta - \alpha\varphi$ must separately vanish. The equation relating $\Phi_1^{(\varepsilon)}$ and $f_{1j}^{(\varepsilon)}$ is identical to Eq.(10). The corresponding equation relating $\Phi_1^{(\delta)}$ and $f_{1j}^{(\delta)}$ may be obtained from Eq.(10) by the substitutions

$$\Theta \to - \varepsilon\alpha, \quad \theta \to \ell(\theta - \alpha\varphi), \quad \varepsilon \to \delta\ell^2 I_{\ell}(\ell \alpha r/R). \qquad \text{... (A.2)}$$

When the radial flux is integrated over a magnetic surface, only products of terms having the same θ dependence survive. In addition to a toroidal component, which is still given by the equations in Sec. IV, the diffusion has a helical component which can be obtained from Eqs.(23) and (24) by making the substitutions set out in Eq.(A.2).

In evaluating the helical component z_j must be interpreted as $z_{jh} = v_o/c_j \varepsilon\alpha$. Comparing z_j for the two components

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$$\frac{z_{jh}}{z_{jt}} = -\frac{\iota}{2\pi a} = -\frac{\iota \ell}{2\pi N}$$

where N is the number of field periods around the major circumference. This ratio is typically of order $0\cdot 1$ for an $\ell=2$, or $0\cdot 1(r/r_0)^2$ for an $\ell=3$ stellarator. Whereas generally $z_{it} > 1$ for conditions of experimental interest, usually $z_{ih} < 1$.

In comparing the magnitudes of the helical and toroidal diffusion components, it will be assumed that $-v_0/r$ is not close to the frequency of a natural plasma mode having the toroidal wavenumber ($m=1$, $k_\| = 2\pi/L_c = \Theta/r$), nor is $-\ell v_0/r$ close to one having the helical wavenumber ($m=\ell$, $k_\| = 2\pi/$field period $= \ell a/R$). We shall take for the bracketed expressions in Eqs.(23) and (24) the value $1 + (1 + 2z_j^2)^2$ which one derives when azimuthal electric field is neglected. This gives the correct order of magnitude so long as v_0 is not near a resonance. To permit direct comparison with experiment, δ will be expressed in terms of rotational transform

$$(\delta \ell^2 I_\ell)^2 = (\iota/2\pi)\ \varepsilon^4 a^3 \ell^3 / (\ell-1)\ .$$

The ratio of the two components in the ion diffusion rate is then

$$\frac{(v_{Di})_h}{(v_{Di})_t} \sim \left(\frac{a\varepsilon\iota}{2\pi}\right)^2 \frac{\ell^3}{(\ell-1)} \left[\frac{1+(1+2z_{ih}^2)^2}{1+(1+2z_{it}^2)^2}\right] e^{z_{it}^2 - z_{ih}^2}\ .$$

Typically $a = N/\ell \sim 5$, $\iota/2\pi \sim 0\cdot 25$, $\varepsilon \sim 2 \times 10^{-2}$, giving $(a\varepsilon\iota/2\pi)^3\ \ell^3/(\ell-1) \sim 10^{-2}$. However, this small coefficient may be more than balanced by the large exponential factor, since $z_{it} > 1$ and $z_{ih} < 1$.

The physical explanation is as follows. A particle for which $v_\| = v_0/\varepsilon a = v_0 L_w/2\pi r$, where L_w is the winding period, will stay

exactly in phase with the helical winding as it moves under the combined effect of the azimuthal electric rotation and its parallel motion along a field line. The helical magnetic drift of such a particle has a constant radial component, until it is scattered by collisions. It is these resonant particles which are responsible for the helical component of diffusion. Typically $v_o L_w/2\pi r$ is less than the ion thermal velocity, so there is no shortage of such particles. Resonance with the toroidal field variation requires $v_\parallel = -v_o 2\pi/\iota\varepsilon$. This is typically greater than the ion thermal velocity, and the number of such ions is exponentially small. Thus although the magnetic drift due to the helical variation may be small, compared to the toroidal drift, the ion diffusion rate may be greater because the ions resonant with the helical variation are much more numerous.
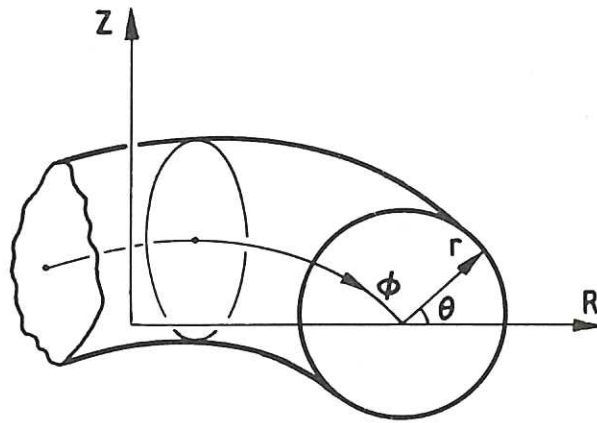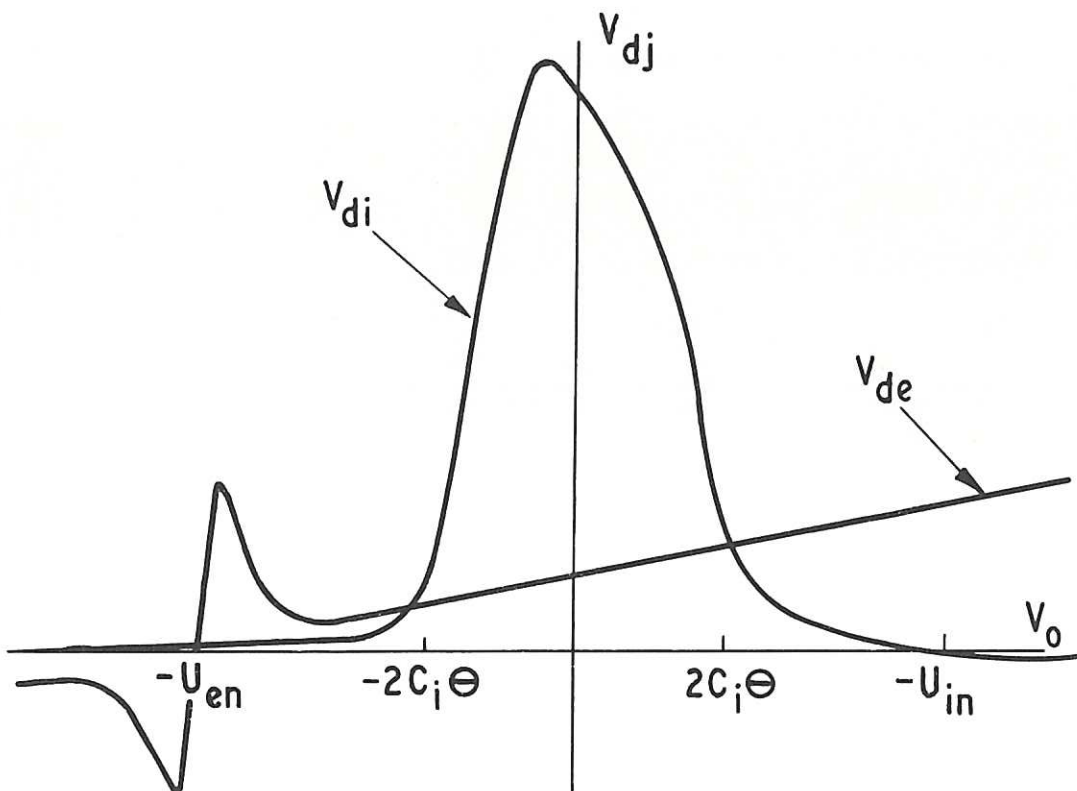
Fig. 1   The co-ordinates

Fig. 2   Diffusion rates v. electric drift velocity for $a_i / r_n \Theta > 4$

(CLM-P214)