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THE FORMATION OF AN AXISYMMETRIC BULGED REGION IN A LONG UNIFORM PLASMA COLUMN

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THE FORMATION OF AN AXISYMMETRIC BULGED REGION IN A LONG UNIFORM PLASMA COLUMN

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ABSTRACT

The formation of an axisymmetric bulge located in the midplane of an 8 metre long plasma column is described. The bulge is formed by reducing the external magnetic field, B_2, in the central region with respect to the field, B_1, elsewhere. The experimental results are analysed to give average values of beta and plasma radius which are compared with the predictions of an analytic model and a time dependent one-dimensional MHD calculation. It is shown theoretically that no simple equilibrium is possible for $\frac{B_2}{B_1} \leqslant \sqrt{\beta}$; experimentally in such cases bulges were found to grow continuously over the time of observation.

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C O N T E N T S

1. INTRODUCTION

The study of bulged theta-pinch is currently of interest in relation to plasma confinement in a toroidal geometry of the M & S type[1-5]. The stability of such a plasma has been studied theoretically[6-12] and experimentally[12-16]; the predicted $m = 1$ instability has been observed and its measured growth rate found to agree with theory for a collisional plasma[12,14]. However, little work has been done on the formation of such configurations and in this paper we report an experimental and theoretical study of the growth of a bulge in a collisional plasma column. The axisymmetric bulge is located in the midplane of a coil 8 metres long; the magnetic field over a 25 cm length of the coil is reduced with respect to that elsewhere and the plasma flows axially into this region to form a bulge.

The growth of the bulged region was studied by means of a calibrated image converter camera, which yielded information on the axial and temporal variation of the radial density distribution. The results are analysed to give values of the average beta and plasma radius for comparison with the predictions of an analytic model, and with the results of a time dependent one-dimensional MHD computation. It is shown theoretically that even for modest reductions of the magnetic field in the bulged region no simple equilibrium is possible; experimentally in such cases bulges were found to grow continuously over the time of observation. Regions of high-beta develop in the midplane of the bulged region as expected theoretically.

2. EXPERIMENTAL DETAILS

The coil is 771 cms long with an 11 cm bore. For the experiments reported here the bank voltage was 34 kV giving a peak field of 20 kG rising in 5·5 μsec and decaying with an e-folding time of 180 μsec, when the coil was short-circuited at peak field. The quartz tube, of bore 8·3 cm, was filled with deuterium gas at an initial pressure of 20 mTorr and was pre-ionized to more than 50% by an axial current of 8 kA.

The plasma was observed stereoscopically in the midplane of the bulged region (viewed 45° above and below the horizontal collector plate) with a calibrated image converter camera. The radial density distribution was determined as a function of time from an Abel inversion of the intensity profiles obtained from streak photographs. This method has been described previously(14,17) where the results were found to give good agreement with those obtained from Thomson scattering, the latter also yielding the temperature distribution.

The plasma properties on the axis at peak field were $n_e = 3 \pm 0{\cdot}5 \times 10^{16}$ cm^{-3}, $T_e = T_i = 120 \pm 15$ eV, $\beta = 0{\cdot}7 \pm {\cdot}2$. As shown previously(14,17), the plasma in the midplane was uninfluenced by end effects for 25 μsec in these conditions and all measurements reported here were taken before that time. The radial density distribution showed a Gaussian shape and the radial temperature distribution was uniform(17).

3. GENERATION OF THE BULGE

In order to retain axial and cylindrical symmetry during the implosion stage the generation of the bulge was delayed for some microseconds. The method is shown schematically in Fig.1. The coil section encircling the region where the bulge was to be formed, known as the field shaping coil, was short-circuited (at a time $t = t_1$) earlier than the rest of the coil, which was short-circuited at field maximum ($t = t_2$). The length of the field shaping coil is 25 cm and determines the length, L, of the bulged region. The magnetic fields were measured at the inner surface of the midplane of the field shaping coil, and at the inner surface of the midplane of the neighbouring coil; they are denoted by B_2 and B_1 respectively. The vacuum field configuration was obtained using a low voltage full scale model bank and coil[18] by means of magnetic probes.

The ratio B_2/B_1 could be varied continuously between 0·5 and 1 by altering the time interval $\Delta t = (t_2 - t_1)$, between short-circuiting the field shaping and uniform regions of the coil, as seen in Fig.2 where the ratio B_2/B_1 is plotted as a function of Δt.

Figure 1 also shows a typical oscillogram of the magnetic field in the field shaping coil and elsewhere. The discontinuity in the waveform of the shaped region can be seen when it was short-circuited at 2·5 μsec. Thereafter the field in the shaped region rises more slowly than that elsewhere and the bulge is formed. It should be noted that the oscillations on the magnetic field after short-circuiting are very nearly in phase for both waveforms.

4. EXPERIMENTAL RESULTS

Figure 3(a) shows a series of streak photographs (one view from the stereoscopic records) taken at the centre of the bulged region for various values of B_2/B_1. It was found from an analysis of the stereoscopic photographs that the bulged column retained a high degree of rotational symmetry as long as the $m = 1$ motion did not drive the plasma too close to the walls. Stereoscopic framing photographs, taken through perforated coils, show the axial distribution of luminosity (see Fig.3(b)) for weakly and strongly bulged discharges. A marked decrease in the light intensity near the ends of the bulged region is apparent in the latter case.

Figure 4 shows two radial electron density distributions at 9·8 μsec for a discharge with no field shaping and with $\frac{B_2}{B_1} = 0{\cdot}75$. Without field shaping there is the Gaussian-like distribution observed previously[14,17] and typical of most low density theta-pinches. In contrast, for a bulged plasma the distribution is no longer of Gaussian form but tends to be more square-shaped.

5. DETERMINATION OF AVERAGE BETA, PLASMA RADIUS AND BULGE AMPLITUDE

In order to compare the results with a one-dimensional axial theory it is necessary to assume a square density distribution; thus, for the diffuse density profile shown in Fig.4 an effective plasma radius, r_p, and average value of beta, $\langle\beta\rangle$, have to be introduced where $\beta = \frac{nkT}{B^2/2\mu_0}$, $T = T_e + T_i$ and B is the external magnetic field. Since the observed radial temperature distribution is uniform these quantities may be defined by replacing the diffuse density distribution by a square one of equal temperature which has the same line density and excludes the same amount of magnetic flux, i.e. has the same diamagnetism. These assumptions lead to the following equations:

Equal line density: $$\langle\beta\rangle\ r_p^2 = 2\int_0^\infty \beta(r)\ r\,dr \qquad \ldots (1)$$

Equal diamagnetism:

$$[1 - \sqrt{1 - \langle\beta\rangle}]\ r_p^2 = 2\int_0^\infty [1 - \sqrt{1 - \beta(r)}]r dr \qquad \ldots (2)$$

Since $\beta(r) = \beta(o)\,\frac{n(r)}{n_o}$ for a constant radial temperature distribution, $\langle\beta\rangle$ and r_p may be calculated, using the measured values of β_o and $\frac{n(r)}{n_o}$. As the temperature in the bulge was not measured β_o was not derived directly. However, due to the large electron thermal conductivity and short electron-ion energy equipartition time the temperature was assumed axially uniform (c.f. Sec.7). Hence β_o was calculated from the known values of B_e, n_o in the bulged region together with the value of T_e measured previously[14,17]. This procedure is justified as it was found that the calculated value of r_p was insensitive to the value of β_o for the shape of density distributions observed. The strength of the bulge is defined as:

$$\delta = \tfrac{1}{2}\left(\frac{r_{p_2} - r_{p_1}}{r_{p_1}}\right) \qquad \ldots (3)$$

where r_{p_2} is the radius of the plasma in the midplane of the bulge and r_{p_1} its radius in the unperturbed region. In Fig.5 the value of δ derived from the measured values of r_{p_1} and r_{p_2} is shown as a function of time; also plotted are the results from 1-D axial computation (c.f. Sec.7). It can be seen that the bulges grow almost linearly with time over a wide range of $\frac{B_2}{B_1}$. Only for $\frac{B_2}{B_1} \geqslant 0{\cdot}75$ is a plateau of δ reached before the $m = 1$ instability makes further measurements of δ impossible.

6. ANALYTIC MODEL - COMPARISON WITH EXPERIMENT

Consider the configuration shown in Fig.6. We define B_i and B as the internal and external field strengths; suffixes 1 and 2 refer to the unshaped region and the midplane of the shaped region respectively. It should be noted that the external fields were measured between the tube and the coil (see Fig.1) and thus the experimental values are only equal to B_1 and B_2 when the field line curvature is negligible, which is a good approximation for the configuration used in the experiments discussed here since $\left(r_{p_2} - r_{p_1}\right)/L \ll 1$. A square radial pressure distribution is assumed and also that the internal axial flux is conserved. For equilibrium, neglecting field line curvature, pressure balance gives:

$$\frac{B^2}{2\mu_o} = p + \frac{B_i^2}{2\mu_o}$$

$$\frac{\partial p}{\partial z} = 0 \qquad \ldots (4)$$

where p is the plasma pressure. Conservation of flux yields:

$$\frac{\partial}{\partial z}(r_p^2 B_i) = 0 \qquad \ldots (5)$$

Solving the above equations we have:

$$\delta = \tfrac{1}{2}\left[\left\{\left(\frac{B_1}{B_2}\right)^2 \frac{(1-\beta_1)}{1-\beta_1\left(\frac{B_1}{B_2}\right)^2}\right\}^{1/4} - 1\right] \qquad \ldots (6)$$

$$\beta_2 = \beta_1\left(\frac{\beta_1}{\beta_2}\right)^2 \qquad \ldots (7)$$

It can be seen that $\delta \to \infty$ and $\beta_2 \to 1$ as $\frac{B_2}{B_1} \to \sqrt{\beta_1}$; thus no simple equilibrium is possible when

$$\frac{B_2}{B_1} \leqslant \sqrt{\beta_1} \qquad \ldots (8)$$

i.e. the bulge increases in amplitude until the curvature of the field lines becomes dominant. When comparing the experimental results with theory the values of B_2 and B_1 are measured with the plasma present and thus its influence on the external fields is taken into account - this effect is small for the bulges considered as the plasma only occupies a small fraction of the total inductive volume of the coil. It follows, as was found experimentally, that the external fields in the presence of the bulged plasma closely approximate to the vacuum values.

An indentical analysis may be applied to a diffuse distribution utilizing the same assumptions as previously. Defining r_1 as the position of a flux surface in the unperturbed region and r_2 as its position in the midplane of the bulge we may solve the equations given previously to define a value of δ for each flux surface as follows:

$$\delta(r_1) = \tfrac{1}{2}\left[\sqrt{\frac{B_1}{B_2}}\left\{2\int_0^1\left\{\frac{1-\beta_1(r')}{1-\beta_1(r')\left(\frac{B_1}{B_2}\right)^2}\right\}^{\frac{1}{2}} r'\,dr'\right\}^{\frac{1}{2}} - 1\right] \quad \ldots (9)$$

where $r' = r/r_1$ and $\beta(r')$ is a function of radius.

It can be seen that $\delta(r_1) \to \infty$ if a region where $\beta_1(r)\left(\frac{B_1}{B_2}\right)^2 \to 1$ exists anywhere within $r \leqslant r_1$. For the profiles considered $\beta_1(r)$ reaches a maximum at $r = 0$ and hence no simple equilibrium is possible for:

$$\frac{B_2}{B_1} \leqslant \sqrt{\beta_1(0)} \quad \ldots (10)$$

This condition shows that for a diffuse distribution the equilibrium is determined by the value of beta on the axis rather than the average value. It can be seen from measured density profile (Fig.4) that preferential expansion has taken place near the axis which is in agreement with Eq.(9), i.e. the growth rate of the bulge is dominated by the central regions when $\frac{B_2}{B_1} \approx \sqrt{\beta_1(0)}$.

In Fig.7 δ is shown as a function of β_1 calculated assuming a square profile for different ratios of $\frac{B_2}{B_1}$, together with the experimentally observed values. Comparison could only be made for $\frac{B_2}{B_1} \geqslant 0{\cdot}75$ since only in these cases had the bulge reached an equilibirum, during the time of observation. The best fit was obtained with $\beta_1 = 0{\cdot}5$ which was rather higher than the independently determined average value of $\beta_1 = 0{\cdot}35$ and less than $\beta_1(0) = 0{\cdot}7$. This discrepancy is believed to be due to the preferential expansion of the profile near the axis as discussed above, which leads to an effective value of beta between $\beta_1(0)$ and β_1. The critical value of $\frac{B_2}{B_1}$ for the bulge to grow continuously, given by Eq.(8), is $0{\cdot}7$ for $\beta_1 = 0{\cdot}5$ and is consistent with the lack of equilibrium experimentally observed for $\frac{B_2}{B_1} < 0{\cdot}75$ (c.f. Fig.5). It was also found that the measured value of β_2 on the axis was close to unity and since the radial distributions of pressure and beta are the same for the observed uniform temperature distribution, it may be deduced from Fig.4 that in the bulged region the plasma consists of a central region with $\beta \approx 1$ surrounded by a sheath whose thickness is of the same order as in the uniform column.

7. MHD CODE CALCULATIONS

In order to calculate the temporal and spatial variation of the bulge the MHD equations were solved for the axial spatial dimension only. The measured spatial and temporal variation of the external field together with the initial plasma properties (at the onset of field shaping) where used as input data. The radial MHD code has been described previously[19] and the axial version used here is a simple adaptation of this code[20]. All quantities are a function of z only and the model is that of a square density profile of radius r_p, containing a uniform field B_i and a plasma with an electron temperature T_e and an isotropic ion temperature T_i, which is justified since the ion collision time is shorter than the bulge formation time. Separate electron and ion thermal conductivities are included and also electron-ion energy equipartition; the plasma is assumed to have infinite electrical conductivity, as the resistive field diffusion time is much longer than the bulge formation time. Typical results for strongly and weakly shaped bulges are shown in Fig.8. The main distinguishing feature is that in the former case (Fig.8(a)) the bulge does not reach an equilibrium and a strong localised M = 0 deformation occurs at the ends of the bulge, while plasma flows continuously into the bulged region and the axial flow velocity is a maximum at the ends of the bulge. In the latter case (Fig.8(b)) the bulge reaches an equilibrium and a weak $m = 0$ wave travels away from the bulged region down the column and the flow velocity peaks further away. The results of these calculations are in agreement with the experimentally observed behaviour for strong bulges (c.f. Fig.3) where a marked decrease in the light intensity between 12 and 15 cm from the midplane of the bulge was found, corresponding to a reduction in the line density in this region. It can be seen that in both cases the electron and ion temperatures are almost equal and the plasma is nearly axially isothermal; this behaviour is a consequence of the large axial thermal conductivity coupled with the short ion-electron equipartition time. Thus the temperature in the bulge is essentially determined by its value in the unperturbed region. In Fig.9 the temporal history of δ for values of β_1 (where β_i is defined at the onset of field shaping), is shown for large and small field ratios. It can be seen that for large ratios (above) the bulge reaches an equilibrium, while for $\beta_1 \approx \sqrt{\frac{B_2}{B_1}}$ (below) the rate of growth of the bulge is very sensitive to β_1 and as expected from the analytic model there is no equilibrium for $\beta_1 \gtrsim \sqrt{B_2/B_1}$. It is expected that for very large bulges other effects such as field curvature, finite geometry of the coil etc., will allow an equilibrium to be obtained before the plasma reaches the wall. Comparison of a series of such results with the experimentally determined temporal history of δ showed that the best fit over the whole range of $\frac{B_2}{B_1}$ was given by $\beta_1 = 0{\cdot}6$ and the results computed using this value are plotted in Fig.5 where reasonable agreement with experiment is seen. This value of β_1 is also in reasonable agreement with that derived previously from the simple analytic model (c.f.Sec.6).

8. CONCLUSION

The agreement between experiment and theory shows that the gross plasma behaviour during the formation of a bulged region and, in particular, the axial flow prcesses are adequately described by a one-dimensional MHD computation.

It is shown that for a given value of β_1 the rate of growth and amplitude of a bulged region in a collisional plasma is a sensitive function of $\frac{B_2}{B_1}$ for $\frac{B_2}{B_1} \approx \sqrt{\beta_1}$; furthermore for $\frac{B_2}{B_1} < \sqrt{\beta_1}$ no simple equilibrium is possible.

The consequence of this result are twofold. Firstly, in relation to the $m = 1$ instability(7,14) of such bulged configurations it follows that for larger bulges, i.e. $\frac{B_2}{B_1} \leqslant \sqrt{\beta}$, no axial equilibrium is reached and δ varies during the growth of the instability.

Secondly, these results have also some relevance to the setting up of an M & S configuration[1], although there the reservoir of plasma to fill the bulge is limited to the length of one period. For such a configuration it is apparent that careful control of both $\frac{B_2}{B_1}$ and β is required to set up a bulge of the required amplitude, given for instance by equilibrium requirements.

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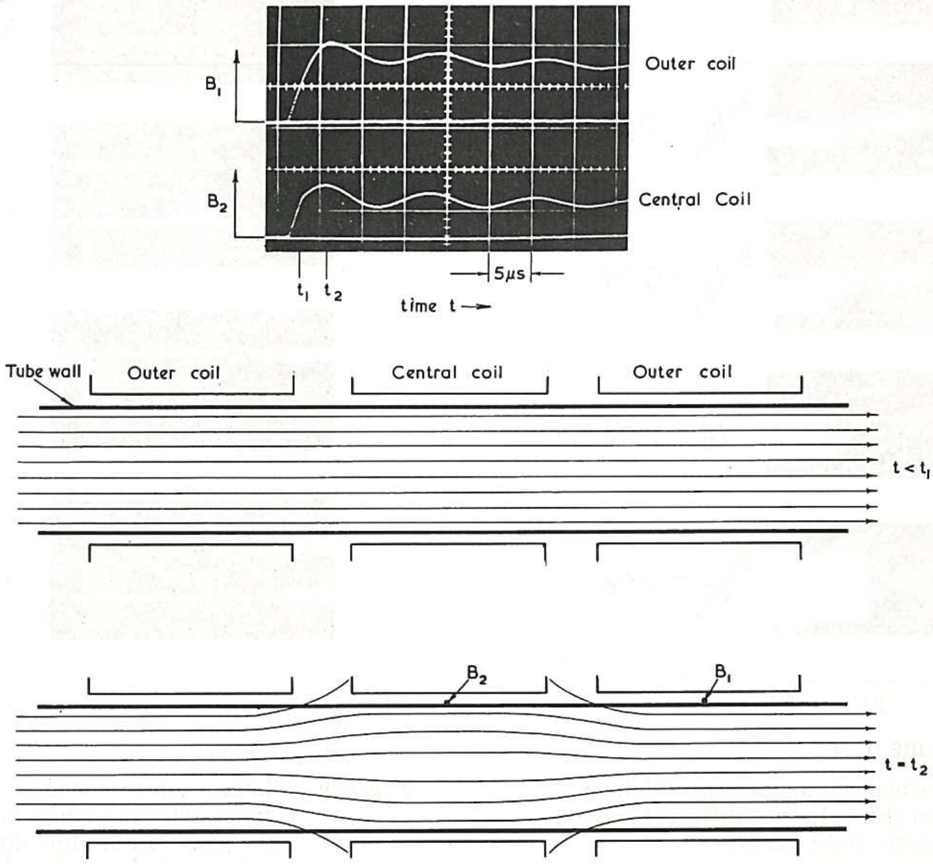


Fig.1 The generation of the bulge.

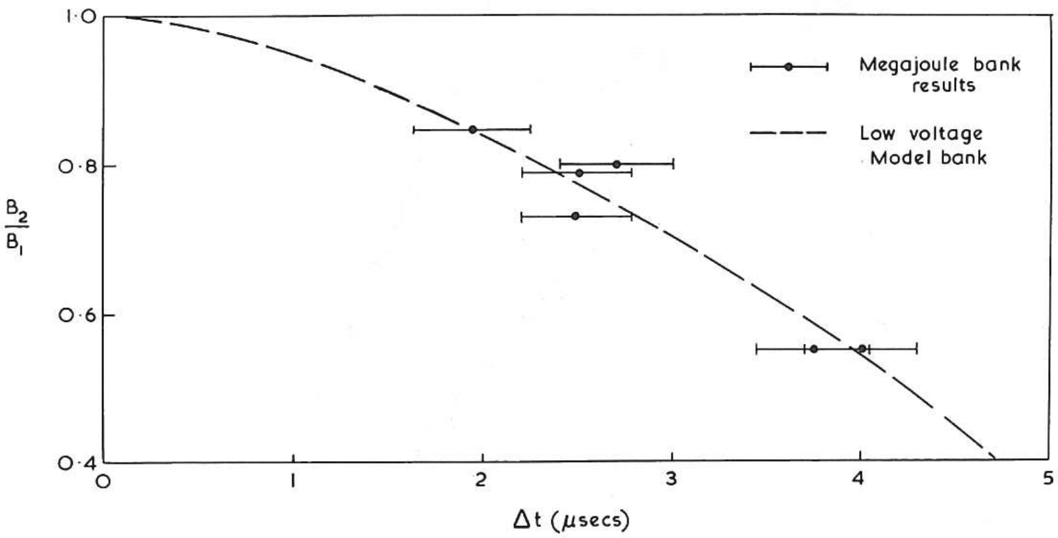


Fig.2 Field ratio $\frac{B_2}{B_1}$ as a function of crowbar timing. (CLM-P223)

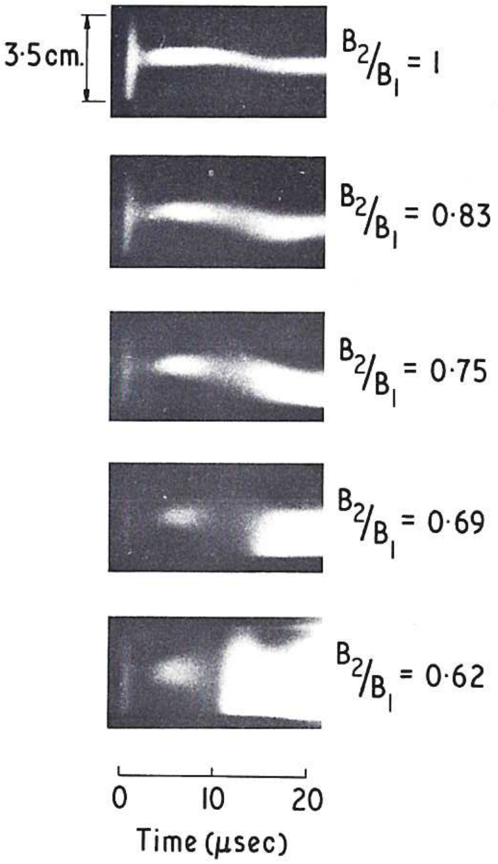


Fig.3(a) Streak photographs at midplane of the bulge for different magnetic field ratios.

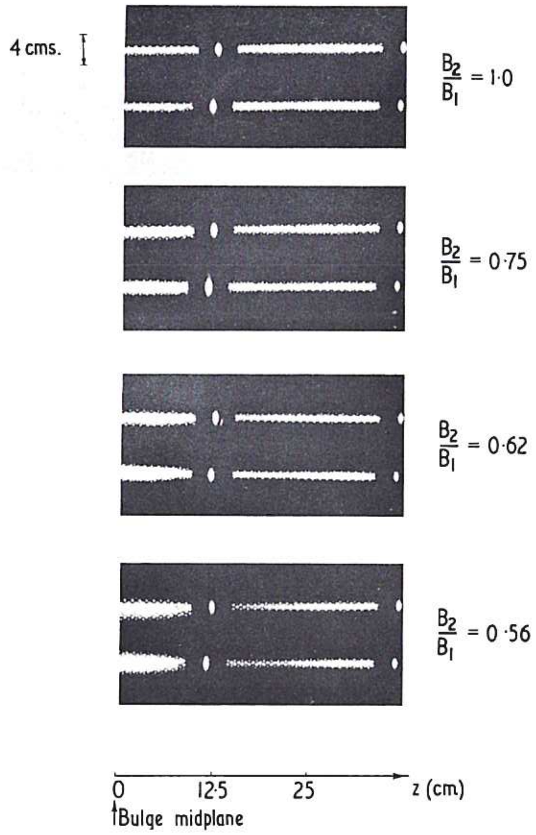


Fig.3(b) Stereoscopic framing photographs showing shape of bulge at peak field for different values of the field shaping ratio.

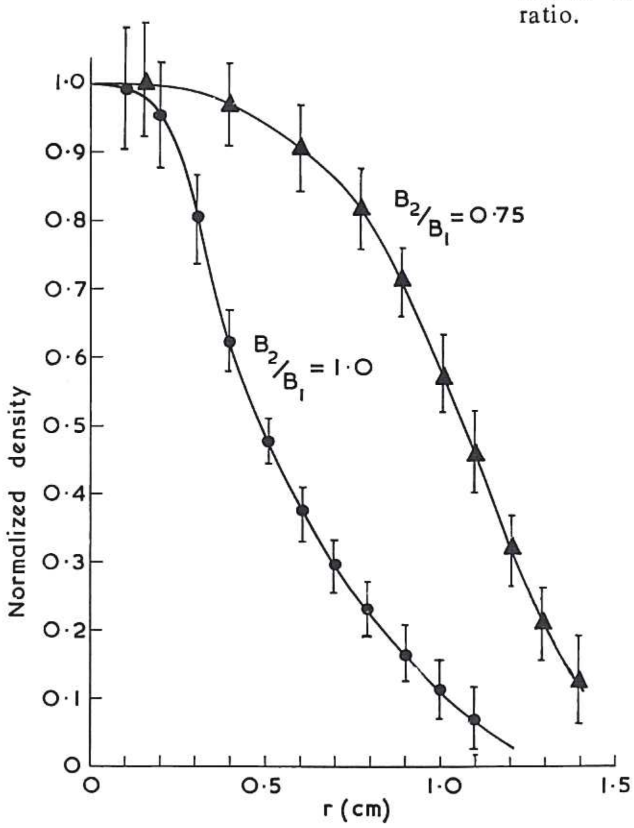


Fig.4 Experimental density profiles at t=9.8μsec in the midplane of the bulge for a field shaping ratio

$$\frac{B_2}{B_1} = 0.75,$$

and for $\frac{B_2}{B_1} = 1.0$.

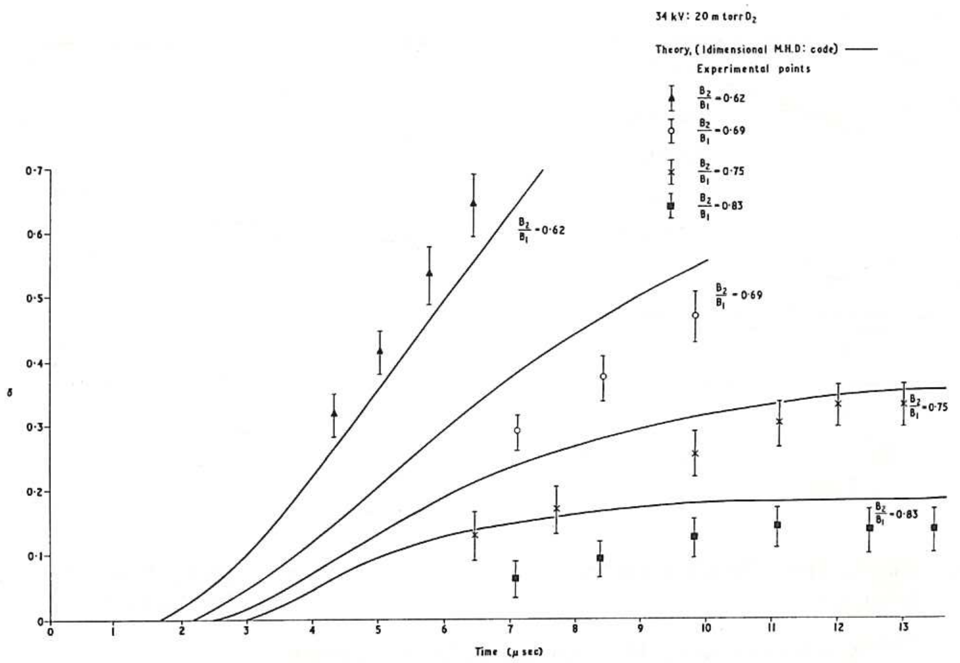


Fig.5 Bulge amplitude, δ, as a function of time – theory and experiment

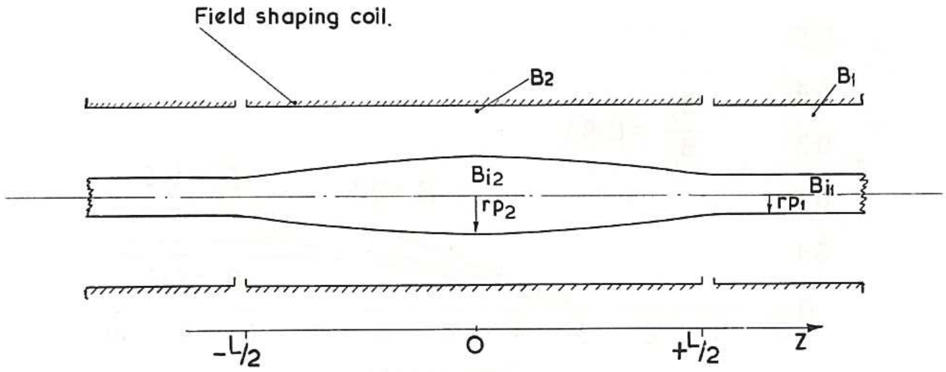


Fig.6 Equilibrium configuration of bulge, showing also the coil position. B and B_i are the external and internal magnetic fields respectively.

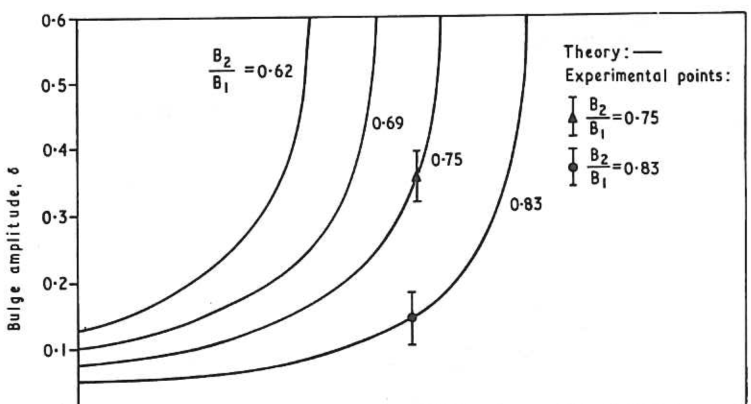
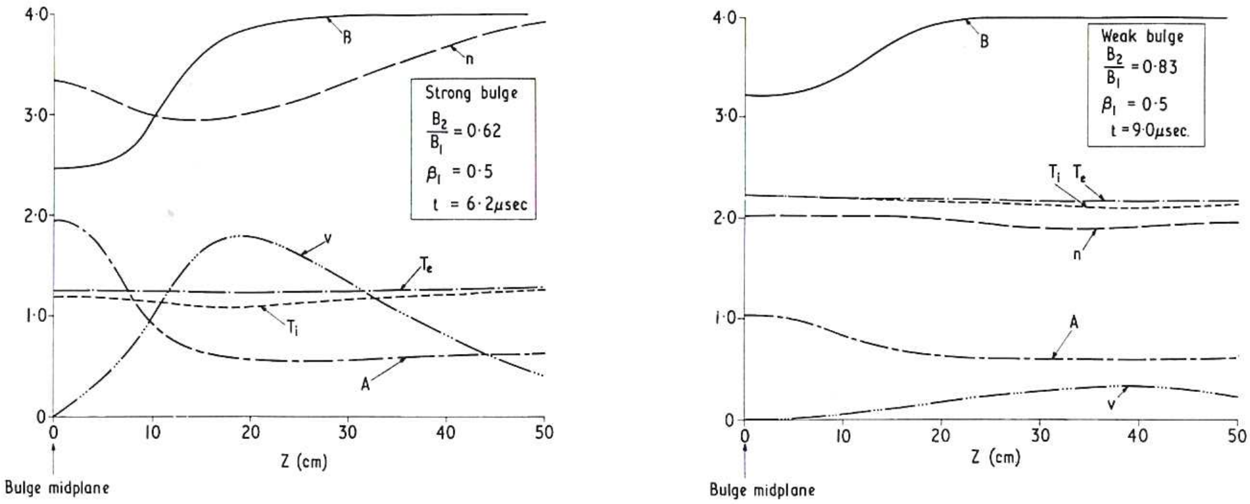


Fig.7 Equilibrium bulge amplitude δ as a function of β_1 for different values of

$$\frac{B_2}{B_1}$$



1 division = 100eV, 5cm^2, 5kG, 0.5×10^{16}cm^{-3}, 2cm μsec^{-1}

1 division = 50eV, 5cm^2, 5kG, 10^{16}cm^{-3}, 2 cm μsec^{-1}.

A – plasma area; B – external field; n = density;
V – axial flow velocity; T_e,T_i – electron and ion temperatures.

Fig.8 Computed axial variation of plasma parameters for strong and weak bulges

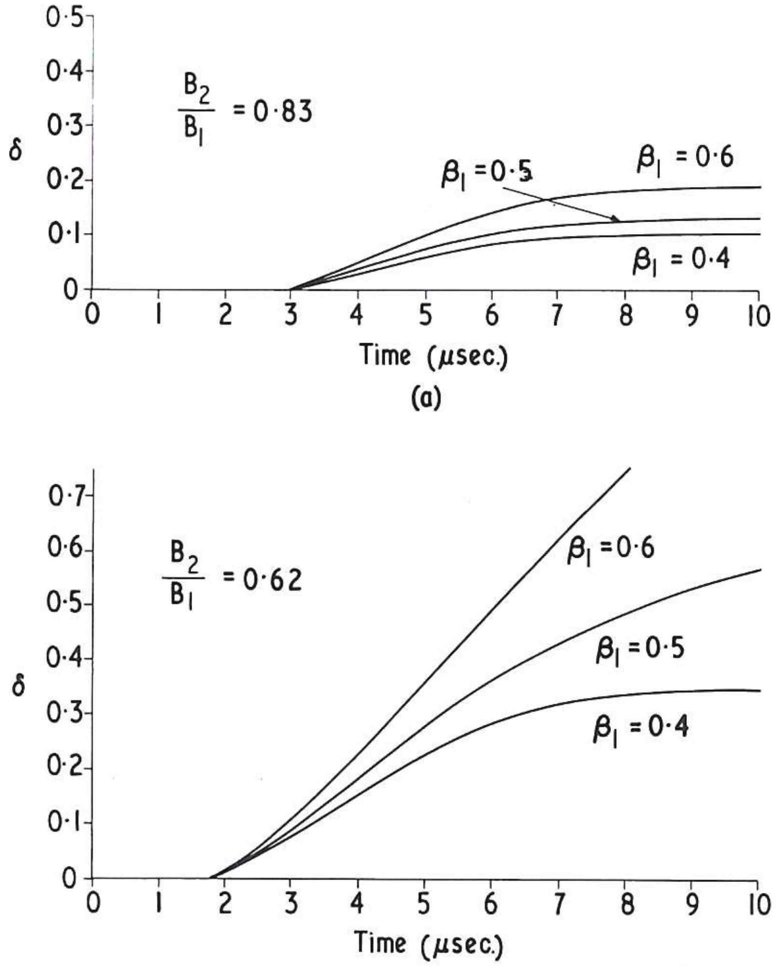


Fig.9 Computed time variation of bulge amplitude δ for various initial values of beta and for two different field ratios.