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TRIGGERING REQUIREMENTS FOR PULSED FUSION REACTORS

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1. INTRODUCTION

The only man-made fusion reactors at present are hydrogen bombs. It has frequently been suggested that economic power could be produced from a reactor running as a succession of controlled explosions. As a recent example, Winterberg [1] has published a calculation of the energy required to ignite a small thermonuclear explosion in a solid D-T target and he has suggested that the energy could be delivered by electrons, ions or micro-particles from an intense field emission discharge.

In this paper we consider the economics of power production based on this concept and show that the trigger energy requirement of a system competitive with fission reactors is many orders of magnitude higher than that achievable with current devices. The economics of a system based on an explosively driven liner producing megagauss confinement fields have recently been published [2].

2. ECONOMICS OF ANY EXPLOSIVE SYSTEM

The size of a reactor is governed by the permissible wall loading. In a continuous running fusion reactor, as described by Carruthers et al. [3] the main heating of the walls is due to the neutrons. A reactor producing 2000 MW(e) burning the equivalent of $\sim 0{\cdot}1$ cm^3 of solid D-T per second has to have a wall area of $\sim 5 \times 10^6$ cm^2.

In a pulsed reactor the Bremsstrahlung flash can raise the walls to above their melting point. 1 cm^3 of D-T at a density of 4×10^{22} cm^{-3} and temperature 10^8 deg radiates $\sim 2 \times 10^{15}$ W. Spalding [4] shows that for a pulse lasting 10^{-8} sec the wall area required to avoid excessive damage is $\gtrsim = 2 \times 10^8$ cm^2, corresponding to a sphere 80 m in diameter. To keep to a reasonable size for the tritium breeding blanket, therefore, we must shield the walls from the Bremsstrahlung flash. A very convenient solution would be to place the liquid lithium neutron absorber within the pressure vessel.

If we assume that the walls of the chamber are protected from the direct Bremsstrahlung flash we can use the generating cost figures given by Carruthers et al. [3] to calculate the maximum allowed cost of each explosion. The only major saving in the pulsed system is the magnet cost and this accounts for $\sim 25\%$ of the generating costs in Carruthers' study, or $\sim 0 \cdot 06$d/kW hr (240d = £1 sterling, 1d = 1c (US)). Savings due to the simpler vacuum requirements might increase this figure to $\sim 0 \cdot 08$d/kW,hr, but no allowance has been made for the capital cost of the energy storage system. Table I shows the total "component cost" calculated for a range of average power outputs, assuming an overall efficiency of 40% and assuming that 10% of the output is required to produce the triggering energy.

TABLE I

E_{out}(ergs)	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}	10^{19}
TNT equivalent	2 lbs	20 lbs	200 lbs	1 ton	10 ton	100 ton
kWhr produced (electrical)	0·84	8·4	84	840	8,400	84,000
Power* (MW)	3	30	300	3,000	3×10^4	3×10^5
Max. Component Cost	0·07d	0·7d	7d	72d	720d	7,200d

*assuming 1 shot/second.

Thus for a 3000 MW(e) system the component cost of 72d must cover repair of the damage to the pressure vessel and the source of triggering energy (such as the cathode of the field emission discharge) due to an explosion equivalent to 1 ton TNT. (See also Ref.5).

3. CRITICAL SIZE OF A D-T SPHERE FOR ENERGY GAIN

We now calculate the critical size required for a sphere of solid D-T heated instantaneously to a temperature T to produce a net energy gain from thermonuclear reactions.

We assume that an isolated sphere of radius r has an effective containment time τ given by $\frac{r}{v_{th}}$ where v_{th} is the ion thermal velocity $\sqrt{\frac{3kT}{M}}$. We also assume that the total thermonuclear yield W_n is given by

$$W_n = \frac{4}{3}\pi r^3 \tau \frac{1}{4} n^2 \langle \sigma v \rangle Q_T \qquad \dots (1)$$

where $\langle \sigma v \rangle$ is the reaction parameter calculated for the initial temperature and Q_T is the energy released in a single reaction. Linhart [5] has calculated the thermonuclear yield per unit length from a freely expanding cylinder allowing for the reduction in density and temperature during the expansion and also for the reactions that will still be taking place after the sound speed transit time τ. For a D-T cylinder 1 cm long and 1 cm diameter at an initial density 4×10^{22} cm^{-3} and temperature 10^8 deg Linhart's formula (A9) gives

$$W_n = 3 \cdot 5 \times 10^8 \text{ J}$$

and the approximate formula (1) for a sphere of 1 cm diameter gives adequate agreement

$$W_n = 2 \cdot 9 \times 10^8 \text{ J}$$

If we require a net gain A from the system we have

$$E_{out} = A\,E_{in}$$

where $E_{in} = 3nkT$

and $E_{out} = 3nkT + P_n\tau$

where $P_n = \frac{1}{4}n^2\langle\sigma v\rangle Q_T$

and $Q_T = 3{\cdot}6 \times 10^{-5}$ erg.

Therefore we require

$$\tau = \frac{(A-1)\ 3\ nkT}{P_n}$$

$$= \frac{(A-1)\ 3kT}{\frac{1}{4}n\langle\sigma v\rangle Q_T}$$

Since we assume that the containment time is given by r/v_{th} the pellet size for a given value of A scales as n^{-1}. The volume of the reacting material varies as n^{-3}, and the energies involved scale as n^{-2}. The smallest systems will thus use solid material. The value of n can be increased marginally by compression, but the value $4 \times 10^{22}\ cm^{-3}$ we use corresponds to close packing of the fuel molecules. Since we assume that the containment time is given by r/v_{th} we can plot a minimum value for r as a function of A and T, (Fig.1). We also have a maximum value for r for a given E_{out} and A as a function of T, so that from the graph we can readily pick out the range of conditions for which these two limits on r are compatible. For a gain of 10, the minimum size is 1 cm radius, producing $\sim 10^{17}$ ergs (~ 1 ton TNT equivalent) per shot, heated to ~ 20 keV with containment time $\tau \sim 10^{-8}$ sec. The input energy required is 10^{16} ergs or 10^3 MJ in a time $\sim 10^{-9}$ sec. The component cost is 6/- per shot. A wall area of $\sim 10^7\ cm^2$ is needed to keep the mean energy flux to $1\ kW/cm^2$, i.e. the chamber could be a reasonable size ~ 9 metres radius.

We have ignored Bremsstrahlung losses because for this example the plasma radiates 2×10^{23} ergs sec^{-1} or $\sim 2 \times 10^{15}$ ergs in the 10^{-8} sec shot, small compared with the other energies involved. The radius at which Bremsstrahlung losses exceed expansion losses is plotted as a dotted line in Fig.1.

4. IMPROVEMENT OF CONFINEMENT BY TAMPING

The confinement time of the plasma might be increased by surrounding it with a high Z material, as suggested by Winterberg. A hole must be left in the confining material for the triggering energy to reach the D-T so the confinement time will be limited by the plasma streaming out through that hole. In addition the confinement time may be limited by thermal conduction and Bremsstrahlung radiation losses.

We can define characteristic times for the various loss processes:

(1) Free expansion

$$\tau_{exp} = r/v_{th}$$

or ... (2)

$$\tau_{exp} = 10^{-4}\ r\ T^{-\frac{1}{2}}$$

(where r is in cm, T in deg and τ in sec)

(2) Bremsstrahlung radiation

$$\tau_{rad} = \frac{\text{Plasma energy}}{\text{Radiated Power}}$$

$$= \frac{4\pi r^3 nkT}{4/3\pi r^3 n^2 T^{\frac{1}{2}} \times 1\cdot 4 \times 10^{-27}}$$

or

$$\tau_{rad} = \frac{2\cdot 9 \times 10^{11} T^{\frac{1}{2}}}{n}$$

(3) Thermal

$$\tau_{cond} = \frac{\text{Plasma energy}}{\text{Conducted Power}}$$

Artsimovich [7] gives conducted power $Q = \frac{8\pi}{7}\, \alpha\, r\, T^{7/2}$ where α is the coefficient in the expression for thermal conductivity K

$$K = \alpha T^{5/2}$$

Using Spitzer [8] and putting $\ell n\Lambda = 10$

$$\alpha = 2{\cdot}1 \times 10^{-6}$$

therefore

$$\tau_{cond} = 2{\cdot}3 \times 10^{-10} \frac{r^2 n}{T^{5/2}} \qquad \ldots (4)$$

Limits on the usefulness of tamping to improve the confinement are set by considering the ratios between the various loss times:

$$\frac{\tau_{cond}}{\tau_{rad}} = 8 \times 10^{-22} \frac{r^2 n^2}{T^3}$$

and for a given temperature and density this gives a maximum plasma radius beyond which Bremsstrahlung losses dominate over conductivity losses.

Similarly

$$\frac{\tau_{cond}}{\tau_{exp}} = 2{\cdot}3 \times 10^{-6} \frac{rn}{T^2}$$

and for a given temperature and density this gives a minimum radius below which thermal conductivity losses are greater than those due to free expansion.

In Section 2 we calculated a minimum radius for producing net gain A at a temperature T by equating the time τ required for the thermonuclear energy to be released with the free expansion containment time

$$\tau_{exp} = r/v_{th}$$

We now calculate the minimum radius assuming the losses are due to thermal conduction by equating τ with the thermal conduction containment time

$$\tau_{cond} = 2{\cdot}3 \times 10^{-10} \frac{r^2 n}{T^{5/2}} .$$

The results are plotted as full curves in Fig.2 for the same values of A in Fig.1. The curves of Fig.1 for minimum radii are re-plotted as dashed curves on Fig.2 and it is clear that a considerable reduction in minimum radius for a given A can be achieved by tamping.

The containment time is limited both by thermal conduction and by loss of the plasma through the hole left after triggering. The degree of tamping required is shown by the lines $f = 1, 3, 10$ etc. ($f = \tau_{cond}/\tau_{exp}$). High values of f require that the trigger hole be a small fraction of the target sphere (to a first approximation the hole size is such that $f \approx$ area of trigger hole/surface area of D-T target). For $A = 10$, $f = 3$ is the useful limit, an increase to $f = 10$ only reducing the minimum radius by about 15%. With $f = 30$ one could achieve $A = 100$ with a $1{\cdot}25$ cm radius sphere at $\sim 6{\cdot}3 \times 10^7$ deg, instead of the 10 cm radius at $\sim 2 \times 10^8$ deg that would be required if free expansion were allowed. If such a high tamping factor were possible Bremsstrahlung would be the dominant loss process and these losses would have to be made good by α particle heating. This is possible without invoking co-operative effects as the classical α particle range at these low operating temperatures is $\sim 0{\cdot}5$ cm.

To achieve as high a tamping factor as 30 the solid D-T must be almost entirely surrounded by the confining block and heated without leaving a large escape hole for the plasma. In addition, the simple model used does not allow for the inevitable temperature drop across the D-T sphere and the excess radiation from impurities in the edge region. The actual energy gain will thus be reduced as some of the trigger energy will be used in heating material outside the central fusing region.

5. ALPHA PARTICLE HEATING

So far we have ignored the effect of the fusion reactions on the plasma itself. The alpha particle energy ($\sim 3{\cdot}5$ MeV) will be thermalised if the plasma is large enough. At temperatures around 5 keV the classical alpha range is $\sim 0{\cdot}5$ cm, [9] but at high temperatures (~ 80 keV) the classical range is as much as 30 cm. Collective effects must be invoked if a large fraction of the alpha particles' energy is to be used to heat the plasma.

There are two possibilities:

A. that a small region of plasma is ignited and then a thermonuclear detonation wave propagates through and heats up the rest of the D-T block.

B. that the block as a whole is ignited and then the temperature throughout rises due to alpha particle heating.

Case A

We heat a small region - radius r - and then require that the alpha particles produced shall heat up the rest of the pellet. Each hot ion arriving at the edge of the fusing region is capable of ionizing many of the cold target atoms, so that the two species - cold fresh ions and penetrating energetic ions will intermingle at the boundary region. The high energy ions direct from the fusing region will ensure that the boundary region is expanding with at least their thermal speed (radiation may increase the ionization front velocity even further). If there is insufficient energy to heat the fresh cold ions as they are formed, the reaction is not self-sustaining, and the trigger energy, which was concentrated in a small region of the target, will be distributed throughout the entire target. The fusing region has, in effect, been tamped using an unsuitable material which, by becoming totally ionized, has been able to dilute the fusing plasma.

Consider a spherical shell of thickness dR at the edge of the fusing region of radius R. The rate at which heat is required to heat the shell and sustain the reaction is

$$4\pi R^2 \frac{dR}{dt} 3n\, kT$$

The rate at which α particle energy is produced in the fusing region is $4/3\ \pi R^3\ \frac{1}{4}\ n^2 \langle\sigma v\rangle\ Q_\alpha$

Putting $n = 4 \times 10^{22}\ \mathrm{cm}^{-3}$

$\frac{dR}{dt} = 10^8\ \mathrm{cm\ sec}^{-1}$, the thermal velocity of the ions at the ionization front,

and $Q_\alpha = 3{\cdot}5$ MeV,

one obtains the following minimum values of r for a sustained reaction

$T = 10$ keV , $r \sim 3$ cm

$T = 30$ keV , $r \sim 1{\cdot}5$ cm

$T = 80$ keV , $r \sim 2{\cdot}4$ cm

The minimum value of r requires a trigger energy of $\sim 8 \times 10^3$ MJ*.

These large values of minimum radius preclude this mechanism from increasing our value of A without inordinately large explosions.

Case B The alpha particle power is $\frac{1}{4}\ n^2\langle\sigma v\rangle Q_\alpha$ per unit volume, so that

*Linhardt (Private Communication) has pointed out that a strong shock travelling into the undisturbed solid will produce a local increase in density and thermonuclear reaction rate. This could reduce the trigger energy requirement provided that the alpha particle energy is deposited at the shock front.

the time τ_α for the temperature to rise by alpha particle heating is given by

$$\tfrac{1}{4}\, n^2 \langle \sigma v \rangle Q_\alpha \tau_\alpha = 6\ nkT \ ,$$

assuming $T_e = T_i$ and neglecting all losses. A D-T pellet with a temperature of 10 keV would take ~ 1 nsec to reach 11 keV by alpha particle heating. The time required for the rise from ~ 7 keV to an operating temperature of 20-30 keV will clearly be many nanoseconds. This mechanism will therefore be applicable only to large pellets for which the inertial confinement time is long enough to include both this slow heating phase and a sufficient period at the operating temperature. The trigger energies for such large pellets (2-3 cm in radius) even at the low initial temperatures would be 10^3-10^4 MJ, so that the system while allowing large energy gains does not result in a reduction in the triggering requirements.

6. CONCLUSION

The triggering energy required for a power producing reactor has been calculated for a freely expanding D-T target. A 10 MJ trigger would give an energy gain of 3 which could be just sufficient for a zero net energy device, assuming that output energy can be converted to trigger energy with an efficiency of 1/3. For the minimum useful gain of 10 the target radius is ~ 1 cm, requiring a trigger energy of 10^3 MJ.

Tamping reduces the energy requirement; with a tamping factor f = 4 a gain of 10 can be produced from a target with radius 0·4 cm at 10 keV. However, the output, assuming one shot per second would be only 64 MW and the component cost of each shot could not exceed

2d. If a tamping factor f = 30 were possible a gain of 100 could be achieved, but with a trigger energy of 850 MJ, and an explosion equivalent to ~ 10 tons of TNT. Alpha particle heating can be used to increase the gain, but only above a minimum radius of ~ 2 cm. Collective effects must be used to slow down the alpha particles. Intense electron beams have produced pulses containing $\sim 10^5$ J [10]. An increase of 3 orders of magnitude over this figure, together with a reasonable degree of tamping of the D-T target would enable net power production to be achieved, but the component cost would have to be less than 2d/shot. A further order of magnitude with trigger energies $\sim 10^3$ MJ would allow a system producing $\sim 10^5$ MJ per shot. The components to be replaced after this explosion, equivalent to ~ 10 tons of T.N.T. could cost no more than 600d.

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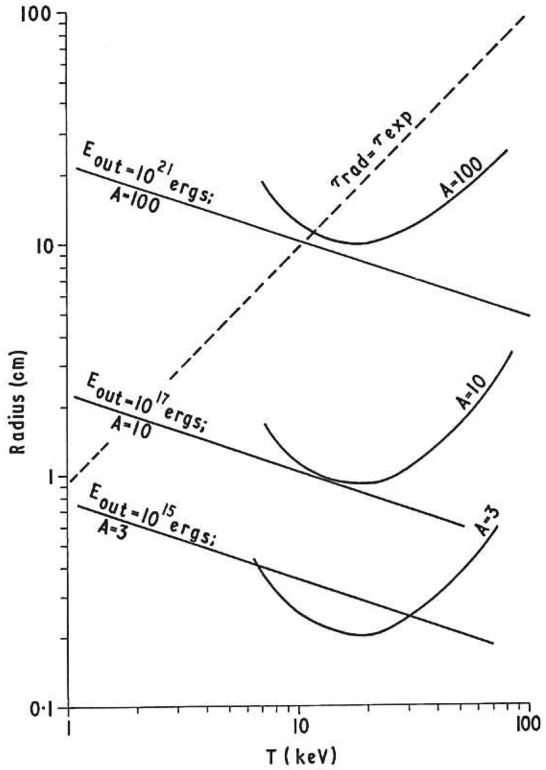

Fig.1 The solid lines show the radius of pellet for given E_{out} and A. The dotted line shows where Bremsstrahlung exceeds expansion loss. The curves show the minimum radius required to produce the given A by inertial containment, so that the reactor parameters are only feasible where the curve passes below the corresponding line. (CLM-P 226)

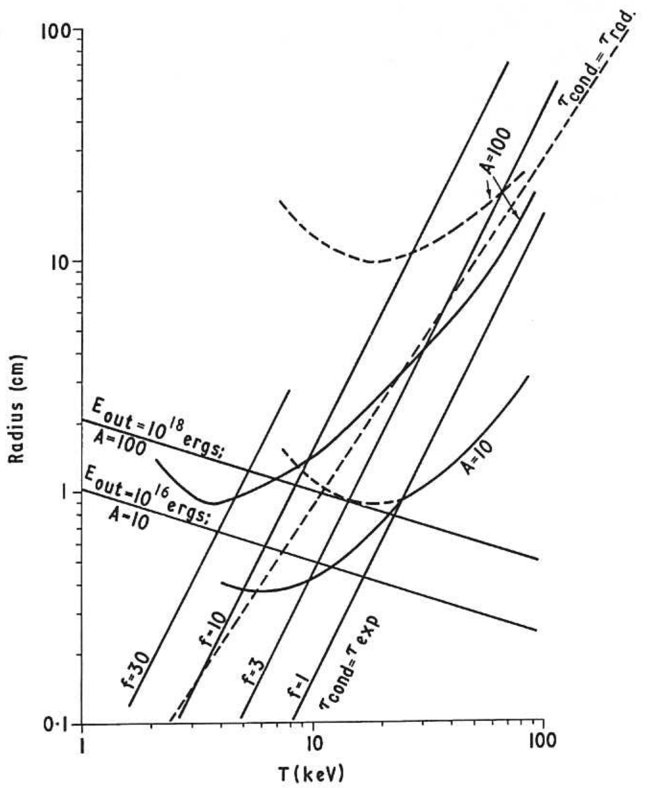

Fig.2 The minimum radius for a given A is shown by dotted curves without tamping, the solid curves with tamping. The solid lines show the minimum and maximum radii given by

$$\frac{\tau_{cond}}{\tau_{exp}} = 1 \qquad \text{and} \qquad \frac{\tau_{cond}}{\tau_{rad}} = 1$$

respectively. The dashed lines show the tamping factor f required. The higher values of f, requiring a small escape hole after triggering, would be difficult to achieve in practice. (CLM-P 226)