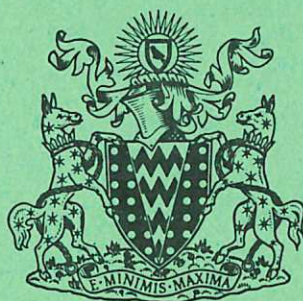


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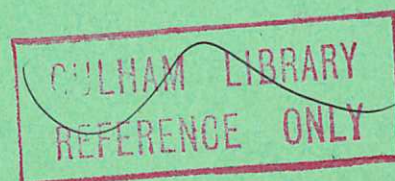
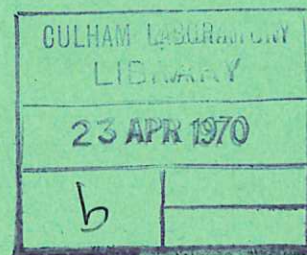


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# THE EFFECT OF IMPURITIES ON THE SPECTRUM OF LASER LIGHT SCATTERED BY A PLASMA

D. E. EVANS



Culham Laboratory  
Abingdon Berkshire

1969



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# THE EFFECT OF IMPURITIES ON THE SPECTRUM OF LASER LIGHT SCATTERED BY A PLASMA

by

D.E. EVANS

(Submitted for publication in Plasma Physics)

## A B S T R A C T

The expression for the frequency distribution of electron density fluctuations in a plasma containing more than one ionic species is used as the basis for numerical computation which investigates the effect of varying the charge, mass and abundance of impurity ions on the ion feature of the scattered light spectrum. The calculation is done for various values of the electron/ion temperature ratio and for correlation parameters  $\alpha = 1$  and  $\alpha = \infty$ . The effect of a temperature difference between the two ionic species in the plasma is also studied.

U.K.A.E.A. Research Group,  
Culham Laboratory,  
Abingdon,  
Berks.

December, 1969.

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## 1. INTRODUCTION

In the form in which it is generally used to interpret laser light scattering experiments, the theory of electron density fluctuations ignores the possibility that the plasma may contain ions of more than one kind. Yet it is not uncommon to find a few per cent of impurity ions in many laboratory plasmas, and their influence upon the electron fluctuations may occasionally be more drastic than their abundance might lead one to expect. Although most of the papers that derive the spectrum of electron density fluctuations do so in a way that can readily be extended to several types of ion, that of FEJER (1961) appears to be the only one that includes multiple ionic species explicitly. In the investigation to be described here, the expression for the scattered light spectrum is generalized to include any number of different ions of arbitrary charge and mass. No approximation, such as that of SALPETER (1960) has been employed; the exact analytic expression under a variety of different plasma conditions has been evaluated numerically. The effect of varying the charge, mass, and abundance of the impurity ions has been calculated for different values of the scattering scale length to Debye length ratio  $\alpha$ , and for different ion and electron temperature ratios.

## 2. GENERALIZED FREQUENCY SPECTRUM

The form factor for scattering,  $S(\underline{k}, \omega)$ , is related to the intensity of light scattered per unit solid angle and per unit frequency interval through the expression

$$I(\underline{k}, \omega) d\omega d\Omega = I_0 N \sigma_T S(\underline{k}, \omega) d\omega d\Omega$$

where  $I_0$  is the incident laser intensity,  $N$  the number of electrons per unit volume,  $\sigma_T = \left(\frac{e^2}{mc^2}\right)^2$  the Thomson cross section, and  $d\omega$

and  $d\Omega$  are the elements of frequency and solid angle respectively.  $\sigma_T S(\underline{k}, \omega)$  may be thought of as the scattering per electron. The vector  $\underline{k} = \underline{k}_0 - \underline{k}_s$ , where  $\underline{k}_0$  and  $\underline{k}_s$  are the propagation vectors of incident and scattered beams respectively.

The form factor for a plasma free from impurity can be written (e.g. EVANS and KATZENSTEIN, 1969)

$$S = \frac{|1 - G_i|^2 F_e + Z |G_e|^2 F_i}{|1 - G_e - G_i|^2} \quad (1)$$

where  $F_e$  and  $F_i$  are velocity distribution functions and  $G_e$  and  $G_i$  are screening integrals (which depend upon  $F_e$  and  $F_i$ ) for electrons and ions respectively.  $S$  was calculated by assuming that the electron and ion distribution functions each satisfied its own collisionless Boltzmann equation, the two being coupled through a Poisson equation. To extend  $S$  to include several ionic species, it was only necessary to assume a separate distribution function for each, together with a corresponding Boltzmann equation. Again, all the Boltzmann equations, one for the electrons and one for each ionic species, were coupled by a Poisson equation. The modified form factor which resulted was

$$S = \frac{|1 - \sum_j G_j|^2 F_e + |G_e|^2 \sum_j b_j F_j}{|1 - G_e - \sum_j G_j|^2} \quad (2)$$

where

$$b_j = \frac{Z_j^2 N_j}{N}$$

with

$$N = \sum_j N_j Z_j \cdot$$

In the foregoing,  $Z_j$  is the charge of the  $j$ th kind of ion and  $N_j$  is the number of such ions per unit volume. In either case (equation (1) or equation (2)) the form factor can be divided into two terms the first having frequency bandwidth of the order of  $kv_e$ ,  $v_e$  being the electron thermal speed, and called the electron term, the second having width of the order of  $kv_i$ ,  $v_i$  being the ion thermal speed, and called the ion term. It may be remarked that in passing from the case of a single ionic species to that of several ionic species, not only is the numerator modified by the addition of extra terms, but the common denominator is modified by the inclusion of additional terms as well. This means that the fluctuation spectrum of the plasma containing several ionic species is more than simply the linear superposition of a set of spectra, one for each sort of ion.

Expanding the screening integrals and letting the distribution functions be Maxwellians, the following final form for equation (2) is obtained.

$$S(\alpha, x) = \frac{\left\{ \left[ \frac{1}{\alpha^2} - \sum b_j \frac{T_e}{T_j} R(a_j x) \right]^2 + \left[ \sum b_j \left( \frac{T_e}{T_j} \right) I(a_j x) \right]^2 \right\} a_e e^{-a_e^2 x^2}}{\left[ \frac{1}{\alpha^2} + R(a_e x) + \sum b_j \left( \frac{T_e}{T_j} \right) R(a_j x) \right]^2 + \left[ I(a_e x) - \sum b_j \left( \frac{T_e}{T_j} \right) I(a_j x) \right]^2} \\ + \frac{\left\{ \left[ R(a_e x) \right]^2 + \left[ I(a_e x) \right]^2 \right\} \sum b_j a_j e^{-a_j^2 x^2}}{\left[ \frac{1}{\alpha^2} + R(a_e x) + \sum b_j \left( \frac{T_e}{T_j} \right) R(a_j x) \right]^2 + \left[ I(a_e x) + \sum b_j \left( \frac{T_e}{T_j} \right) I(a_j x) \right]^2} \quad (3)$$

where  $\alpha \equiv (k\lambda_D)^{-1}$ , the ratio of scattering scale length to plasma Debye length, describes the degree to which the scattering depends upon plasma correlation effects, that is, the degree to which it is "co-operative".

$$a_e \equiv \sqrt{\frac{m_e}{M_1}} \sqrt{\frac{T_1}{T_e}} = \frac{\text{thermal speed of ions of the principal kind}}{\text{thermal speed of the electrons}} ;$$

$$a_j \equiv \sqrt{\frac{M_j}{M_1}} \sqrt{\frac{T_1}{T_j}} = \frac{\text{thermal speed of ions of the principal kind}}{\text{thermal speed of ions of the } j\text{th kind}} .$$

Frequency shift is expressed in units of  $kv_1$ , that is

$$x = \frac{\omega}{kv_1} = \frac{\omega}{k} \sqrt{\frac{M_1}{2KT_1}} \quad K \text{ being the Boltzmann constant.}$$

$$R(x) = 1 - 2xe^{-x^2} \int_0^x e^{t^2} dt = 1 + x \operatorname{Re} Z(x)$$

$$I(x) = \sqrt{\pi} xe^{-x^2} = x \operatorname{Im} Z(x)$$

where  $Z(x)$  is the plasma dispersion function of real variable discussed and tabulated by FRIED and CONTE (1961).

### 3. RESULTS OF THE CALCULATIONS

An analytic examination of equation (3) shows that the high frequency electron term is not greatly affected by the inclusion of impurity ions, so attention in this investigation was confined to the ion term and the machine calculation was accordingly terminated at  $x = 3.0$ . Throughout all the computations, the principal ionic component of the plasma was taken as hydrogen. In the first set of calculations, whose results are displayed in Fig.1, the change in the  $\alpha = 1$  spectrum brought about by introducing increasing amounts of fully stripped oxygen ions ( ${}_8O^{16}$ ) into plasmas whose electron/ion temperature ratio  $T_e/T_i$  varied between 5 and 0.2 is examined. Whereas the abscissa in each graph in this and subsequent figures runs from 0.0 to 3.0, the ordinate scale changes to accommodate the various peak heights of the spectra. Thus, in the first column, which illustrates the



well-known variation in shape and amplitude of the ion feature of a pure plasma resulting from alteration of the  $T_e/T_i$  ratio, the range of the ordinate scale changes from 0.0-0.05 to 0.0-0.20. The second, third, and fourth columns show the effect of adding 0.2%, 1.0% and 5% by number of  $O^{16}$  ions to the pure hydrogen plasma. A relatively narrow central feature whose height increases both with the increasing relative abundance of impurity and with decreasing  $T_e/T_i$  is superimposed upon the pure plasma spectra. When  $T_e/T_i = 0.2$  and the relative abundance of the impurity is 5% it will be seen that this central feature dominates the ion term. In fact, the peak height is about 7 times greater than the height attained by the ion term of a pure hydrogen plasma having the same temperature ratio.

The alteration in the shape of the spectra shown in Fig.1 is accompanied by an enhancement of the total amount of scattered light as measured by the integral under the curve:

$$2 \int_{0.0}^{3.0} S(\alpha, x) dx .$$

This integral is plotted as a function of the per cent abundance of the impurity ion, fully stripped oxygen ( $O^{16}$ ) in Fig.2, for a variety of values of  $T_e/T_i$ . The correlation parameter  $\alpha$  was unity in each case. The integrals over the ion features of impurity-containing plasmas are larger than one might expect from a simple linear superposition of the ion features of the two components computed separately. For example, the integral over the ion feature is given classically by

$$\int_{\text{ion}} S(\alpha, x) dx = \frac{Z\alpha^4}{(1 + \alpha^2) [1 + \alpha^2 (1 + Z T_e/T_i)]} .$$

For a hydrogen plasma having  $\alpha = 1$  and  $T_e/T_i = 1$ , this has the value 0.17. A fully-stripped oxygen plasma with the same values of  $\alpha$  and  $T_e/T_i$  has an integral over the ion feature of 0.4. If the weighted sum of the two integrals is formed, using as weighting factors the relative abundance of the two ions in the composite plasma, one might expect to get the integral under the ion term in that plasma if a linear superposition principle applied. For example, if the plasma were composed of 90% hydrogen and 10%  $_{8}O^{16}$ , this integral might be expected to be  $(0.9 \times 0.17) + (0.1 \times 0.4) = 0.19$ . In fact it will be seen from Fig.2 to lie in the neighbourhood of 0.32.

Although Fig.2 shows that the total scattered light increases with increasing relative abundance of impurity ion, it also shows that the amount of the increase is governed by the electron /ion temperature ratio  $T_e/T_i$ , and somewhat unexpectedly it shows that if  $T_e/T_i$  is great enough ( $= 10$  in this Figure) increasing the impurity abundance leaves the total scattering unaltered.

It was seen in Fig.1 that the ion spectra of composite plasmas all of whose ionic species have the same temperature display narrow central features. The relative widths of the part that can be attributed to the pure hydrogen plasma and the central feature which is associated with the presence of the impurity are approximately in the inverse ratio of the square roots of the masses of the two ion species present. For example, the spectrum for 5% contamination by  $_{8}O^{16}$  at  $T_e/T_i = 5$  exhibits two maxima located near  $x = 0.4$  and  $x = 1.45$ . The ratio of these is 3.6 and this is in the neighbourhood of  $\sqrt{M_O/M_H} = 4$ . To investigate further the dependence of the spectrum on the mass of the impurity ion, a set of curves for the hypothetical impurity  $_{8}Fe^{56}$  (iron stripped of 8 electrons) was computed, and these



are displayed in Fig.3. Once again, the spectra display two distinguishable parts whose relative widths are in the ratio of the square roots of the masses. The central impurity features are greater in amplitude than those characterizing the less massive impurity  $^{16}\text{O}$ , but it was found that the integrals are virtually identical to those found in the  $^{16}\text{O}$  cases. We may conclude that the integrals under the ion feature of a composite plasma are independent of the mass of the impurity.

The way in which the charge  $Z$  of the impurity ion affects the spectrum was studied by assuming a plasma contaminated by 1% of  $^{56}\text{Fe}$  and allowing  $Z$  to vary between 0 and 15. As in the previous calculations, the value of  $\alpha$  was kept at unity and the temperature ratio  $T_e/T_i$  was permitted to vary so that the influence of hot ions or hot electrons could be observed. The curves obtained are shown in Fig.4. Here, proceeding from left to right corresponds to increasing  $Z$ . It will be observed that increasing the effective charge of the impurity ion produces effects in the scattered light spectra broadly similar to those brought about by increasing the abundance of the impurity. The corresponding total scattering cross sections as measured by the integral under the spectrum from 0.0 to 3.0 are shown as functions of  $Z$  in Fig.5. The same integrals calculated for  $\alpha = \infty$  are shown in this Figure as well.

Because situations can occur in which the impurity ions and the principal plasma ions may possess different temperatures, calculations in which the ratio of temperature of the impurity  $^{16}\text{O}$  to that of the hydrogen,  $T_o/T_H$ , was allowed to vary were performed. Here, the electron temperature was maintained equal to the principal ion temperature and the ratio  $T_o/T_H$  took the values 0.2, 0.5, 1.0, 2.0, 5.0

and 16.0. The last value was chosen to make the velocities of the principal ions and the impurity ions equal. There are circumstances, for example during the implosion phase of a theta-pinch plasma, in which this state of affairs is thought to exist. Fig.6, which displays the result of this calculation, shows that changing the temperature of the impurity changes the width of the feature associated with it on the ion term. Also there is found to be weak dependence of the total scattering intensity on  $T_O/T_H$ . When the impurity ions are cold relative to the hydrogen the impurity feature is reduced in width relative to its width for equal temperatures ( $T_O/T_H = 1$ ) and the integral over the ion term is marginally smaller than in the  $T_O/T_H = 1$  case. On the other hand, when the impurity ions are relatively hot, the impurity feature is wider and at the same time the total amount of scattered light is increased, though only by a few per cent. For example, the ratio of total cross section for equal velocities to total cross section for equal temperatures in the present computation with 1% fully stripped oxygen was 1.17.

All the spectra described so far were calculated for the case of  $\alpha = 1$ . The calculations were then repeated in the long wavelength limit, that is, with  $\alpha \rightarrow \infty$ . Figs. 7 and 8 illustrate the results. Comparison of Fig.7, which shows  $_{80}^{16}$  impurity spectra for different relative abundance and various values of  $T_e/T_i$ , with Fig.1 demonstrates that the presence of impurities leads to the same general spectral features in both the  $\alpha = 1$  and the  $\alpha = \infty$  cases, but that the influence of impurities in those plasmas in which electron temperature exceeds ion temperature is less marked for strongly correlated scattering ( $\alpha = \infty$ ) than for moderate correlation ( $\alpha = 1$ ). It has already been pointed out that the scattering enhancement, measured



by the integral of  $S(\alpha, x)$ , depends upon the temperature ratio  $T_e/T_i$  and is greatest when the ions are hotter than the electrons. Fig.8, which displays these integrals as functions of impurity abundance in the long wavelength limit, shows two noteworthy features. The first is that even where impurities are absent, the integral is not monotonically decreasing with increasing  $T_e/T_i$ , but goes through a minimum. The second, shown by the curve labelled  $T_e/T_i = 10$  exhibiting an actual decrease in the total intensity of scattered light with increased abundance of impurity, is the displacement of that minimum towards higher values of  $T_e/T_i$  as the amount of impurity increases. This behaviour stems from the fact that for large values of the electron/ion temperature ratio, the contribution to the ion spectrum of the first, or electron term in the form factor is not negligible, and even dominates  $S(\alpha, x)$  at last. It is connected with the thermal excitation of the ion-acoustic resonance which is only lightly damped at large  $T_e/T_i$ , and becomes the principal contribution to the electron density fluctuations at long wavelengths. A more complete discussion of this point is to be found in EVANS and KATZENTSTEIN (1969). The rate of increase in the amount of scattered light with increasing abundance of impurity in the hot ion cases is not very dependent upon the value of  $\alpha$ : thus when  $T_e/T_i = 0.1$  and  $\alpha = 1$ , increasing the relative abundance of  $sO^{16}$  from 0.2% to 10% increases the total scattering from 0.29 to 0.81, that is, by a factor of 2.8 times. The same increase in the abundance of this impurity for this same temperature ratio when  $\alpha = \infty$  increases the total scattering from 0.86 to 2.6, that is, by a factor of 3.

#### 4. DISCUSSION

A physical explanation of these results can probably be found in the increased clumping of electrons about the relatively highly charged impurity ions. In cooperative scattering, Doppler effects stemming from the motion of bunches of electrons and not from individual electrons determine the frequency shifts which will be observed, and such bunches, adhering to impurity ions are clearly the origin of the relatively narrow impurity feature. But in addition, any departure of the electrons from a random spatial distribution may be expected to lead to enhancement in the total scattering. This is because any change from randomness is towards some sort of order, the limiting case being a grating-like distribution. Any change of this sort can lead to an increase in light scattering through the effect of constructive interference. It is not surprising then, that the increase in clumping which the impurity ions are expected to effect can lead to an increase in the total amount of light scattered.

#### 5. CONCLUSIONS

The addition to a plasma of a small amount, as little as 0.2% of an impurity in the form of ions of relatively high charge and mass compared to the charge and mass of the ions of which the plasma is chiefly composed (fully stripped oxygen in a hydrogen plasma was adopted as an example) can bring about a striking change in the amplitude and frequency distribution of the cooperative electron density fluctuations, confined principally to the ion term. The effect of the impurity is to alter the spectrum of fluctuations of a pure plasma by the addition of a central feature, symmetric about the zero (laser) frequency, whose width compared to that of the pure plasma ion term is in the ratio of the thermal velocities of the two sorts of ions



and whose height depends upon the charge and abundance of the impurity ions. But the composite spectrum is not a simple linear superposition of the spectra of the principal ion and the impurity ion calculated separately. The influence of the impurity becomes more marked as the ion temperature is increased over the electron temperature so that for example, in a hydrogen plasma contaminated by 5% of fully stripped oxygen whose electron/ion temperature ratio  $T_e/T_i = 0.2$ , the height of the spectrum at the centre was found to be 10 times greater than the spectrum of pure hydrogen plasma, and the total scattering intensity was increased by a factor of 3.

#### ACKNOWLEDGEMENTS

I am indebted to Dr A.A. Newton for helpful discussion and to Mr E. Detyna and Mr I. Pasco who carried out the computing.

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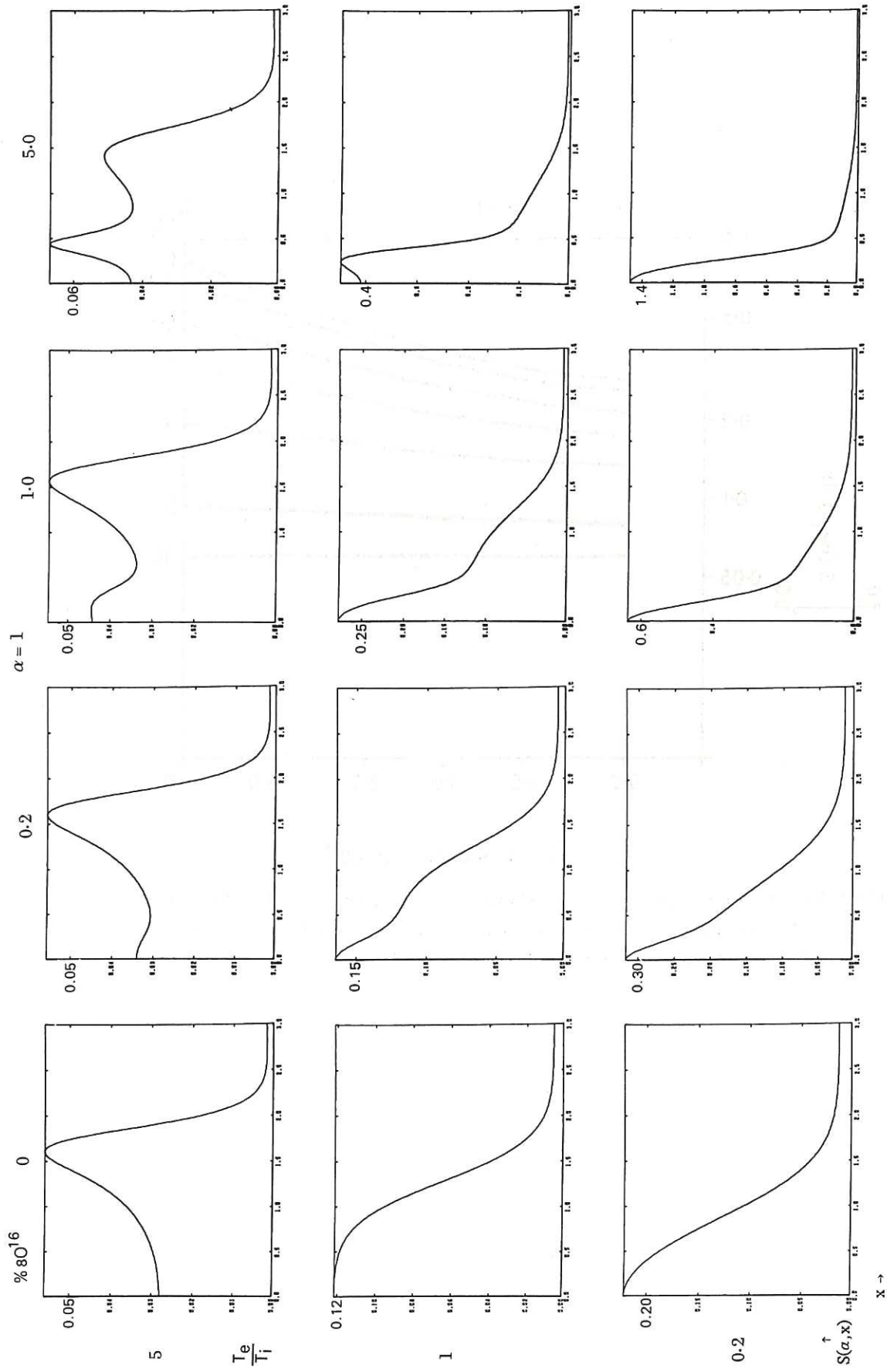


Fig.1 Ion term frequency spectra computed for plasmas composed of hydrogen and various amounts of fully stripped oxygen ions ( $80^{16}$ ), for various ratios of electron-to-ion temperature ( $T_e/T_i$ ). Scattering is moderately cooperative ( $\alpha = 1$ ) in all cases. The variable plotted as abscissa is  $x = \frac{\omega}{k} \sqrt{\frac{2kT_H}{M_H}}$  where H refers to hydrogen. Note that while the abscissa scale runs from  $x = 0.0$  to  $x = 3.0$  in all cases, the ordinate is varied to accommodate spectra of different peak amplitudes.



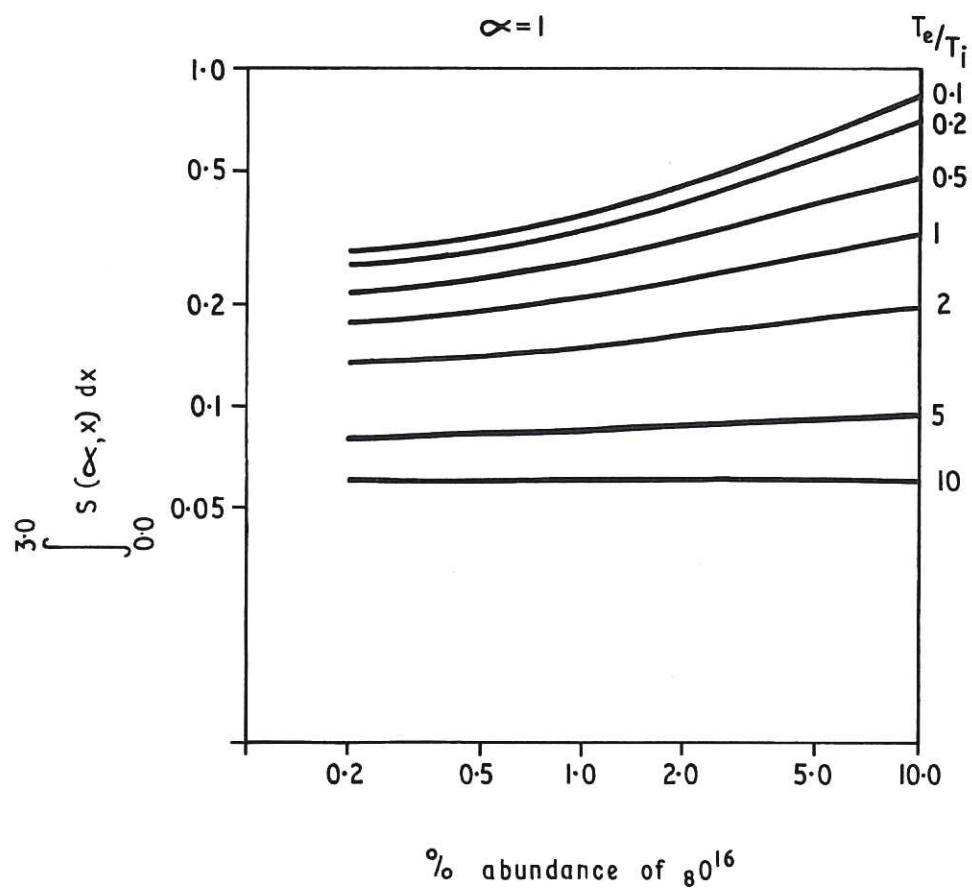


Fig.2 The integrals under the curves shown in Fig.1 are displayed as functions of the per cent abundance of the impurity  $g_0^{16}$ . The family of curves is generated by varying the parameter  $T_e/T_i$ .

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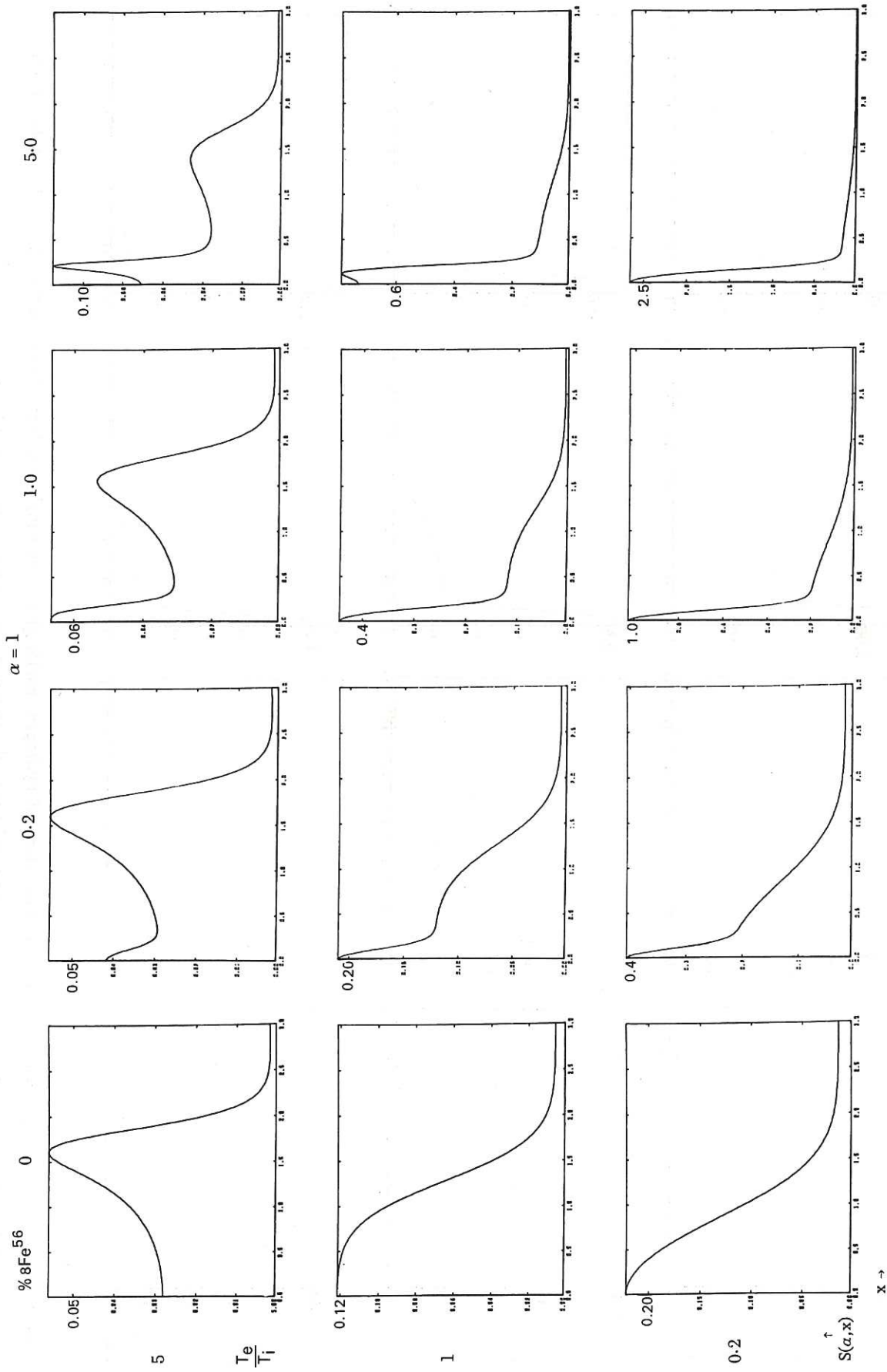


Fig.3 Ion term frequency spectra for composite plasmas consisting of hydrogen and iron ions have effective charge  $8(g\text{Fe}^{56})$ ,  $\alpha = 1$   
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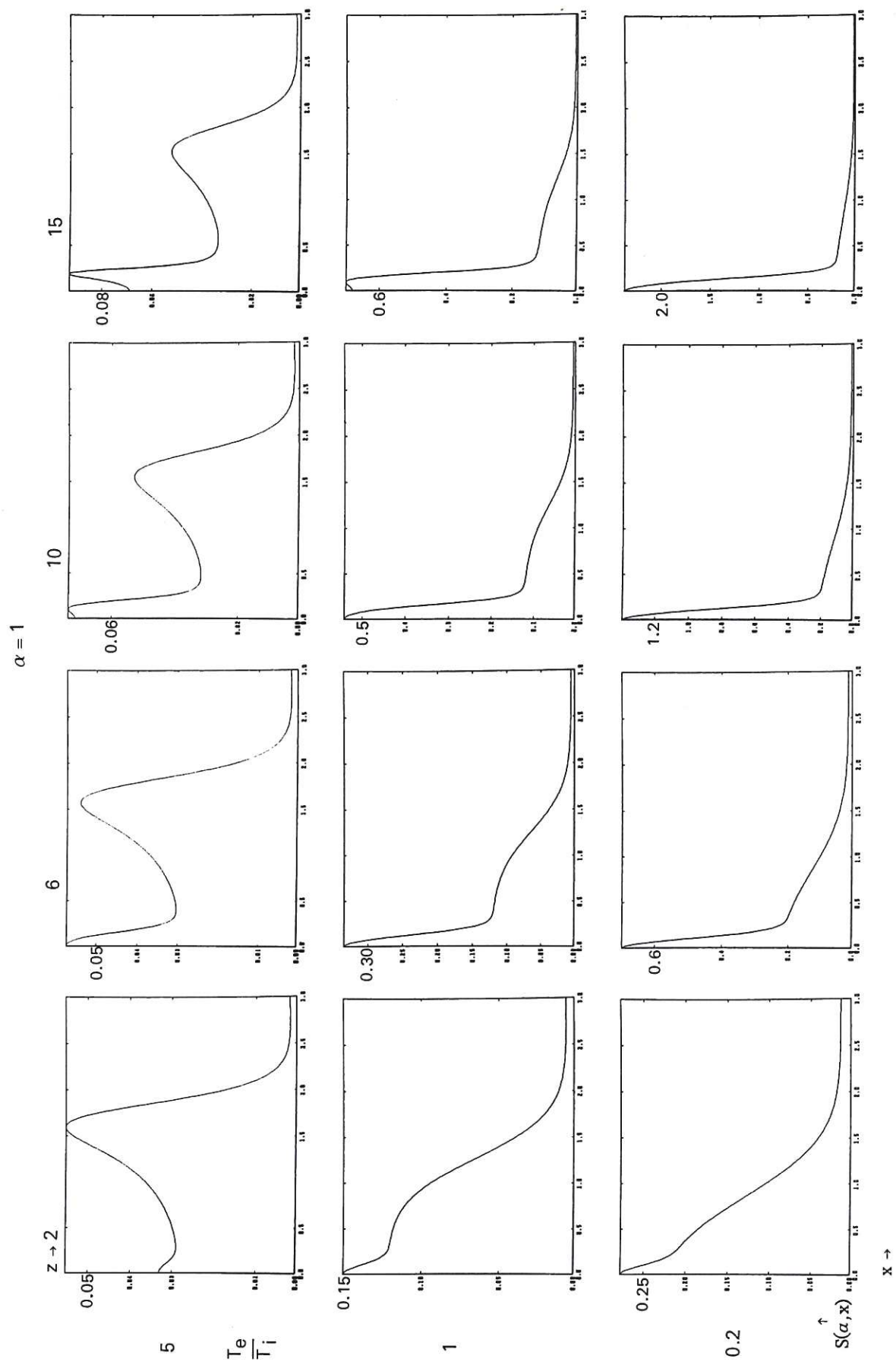


Fig.4 Ion term spectra for composite plasma consisting of hydrogen and 1%  $Z\text{Fe}^{56}$  showing the effect of varying the charge,  $Z$ , of the impurity.  
 $\alpha = 1$  throughout. Hot electrons, cold electrons and equal electron and ion temperatures are examined.



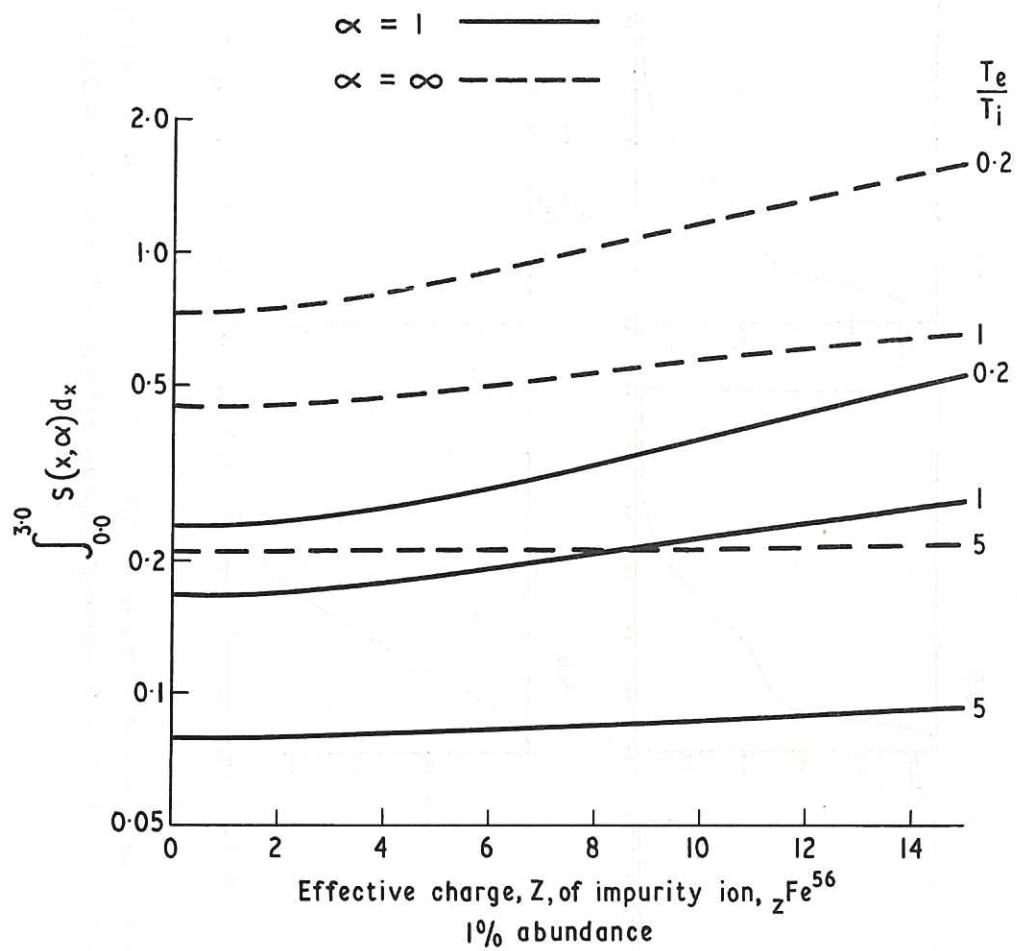


Fig.5 The integrals under the spectra shown in Fig.4 demonstrate the way in which varying the charge of the impurity ion affects the total scattering intensity. Integrals for  $\alpha = 1$  and for  $\alpha = \infty$  are displayed.

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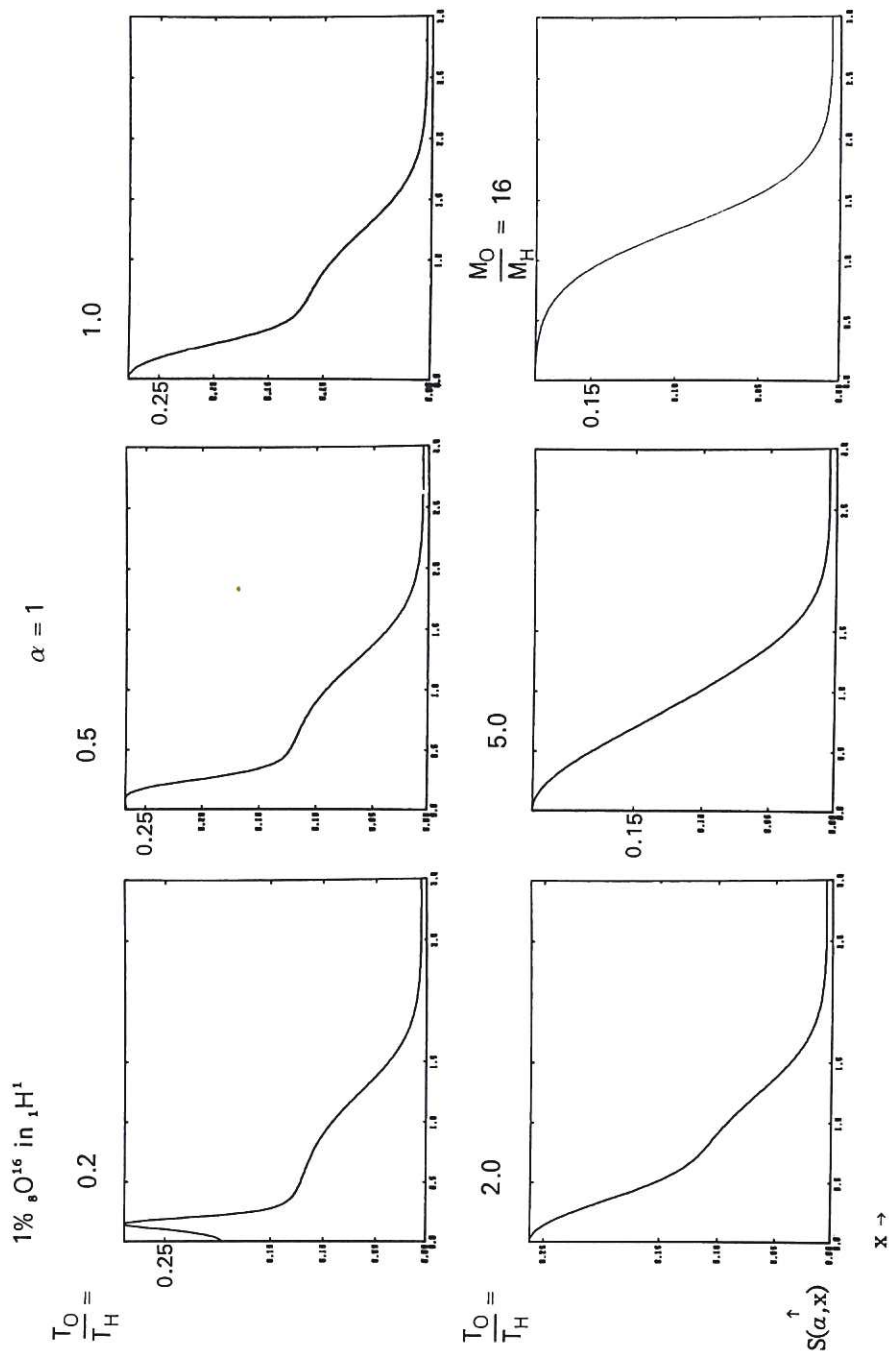


Fig.6 The result of having impurity ions at a temperature different from the temperature of the principal ion species is shown.  $\alpha = 1$ .  $T_O/T_H = 16$  represents impurity and principal ion species at the same velocity.

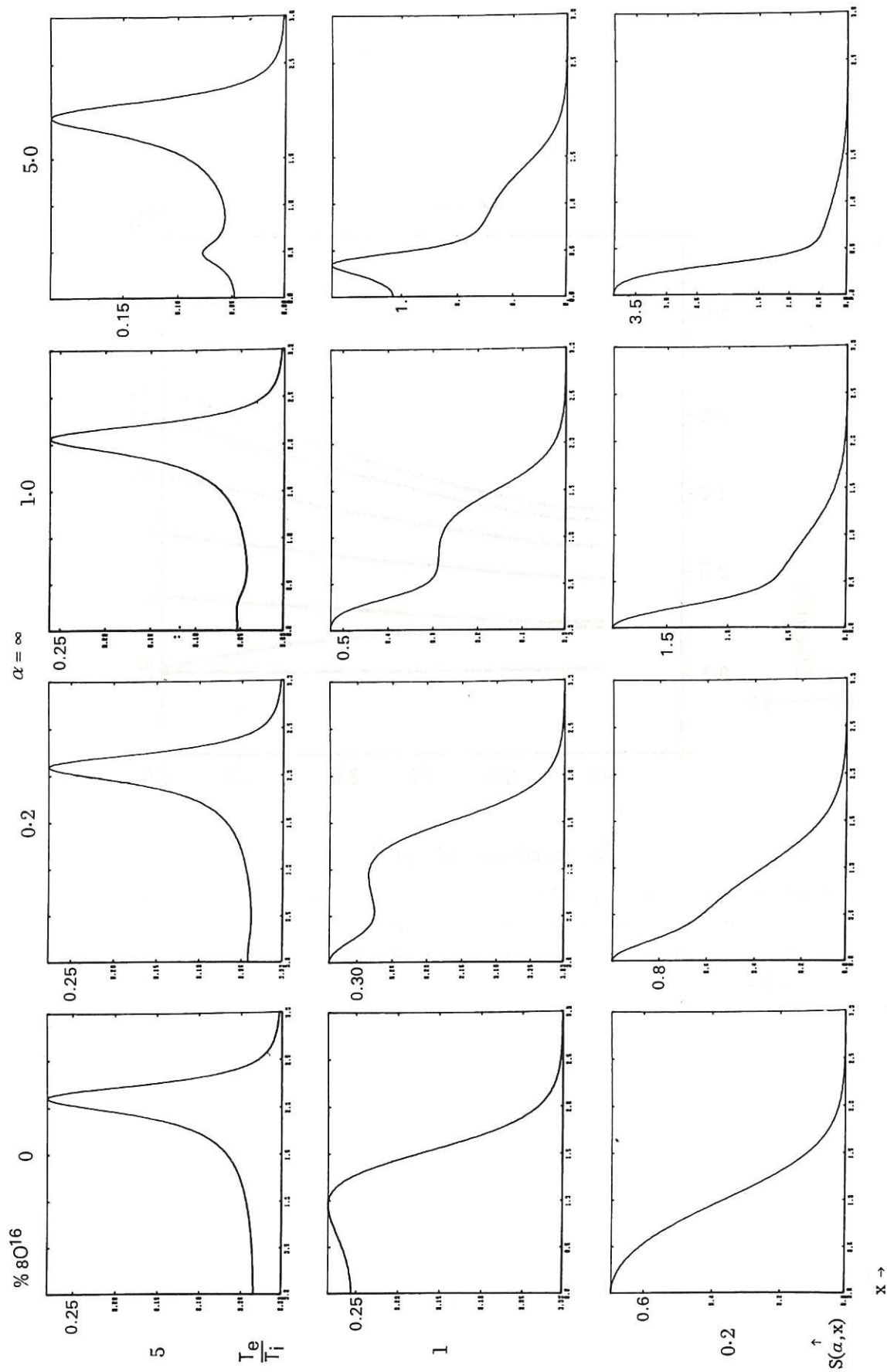


Fig.7 Ion term spectra for composite plasmas consisting of hydrogen and fully-stripped oxygen ions, where scattering is highly cooperative.  
 $\alpha = \infty$ .  
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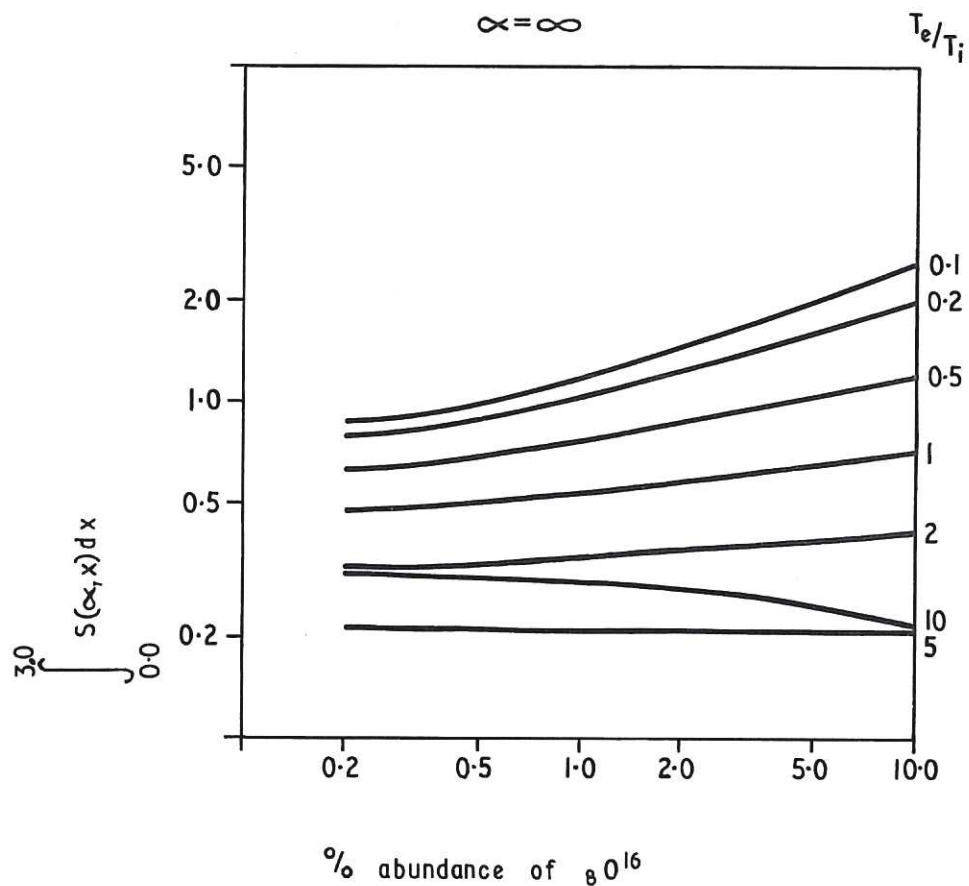


Fig.8 The integrals under the curves of Fig.7, giving the total scattering cross section of the ion term as functions of the abundance of the impurity ion, when  $\alpha = \infty$ . The family of curves is generated by varying the parameter  $T_e/T_i$ .

