CLM-P235

E·MINIMIS·MAXIMA

United Kingdom Atomic Energy Authority

RESEARCH GROUP

Preprint

MEASUREMENT OF SPECTRUM OF TURBULENCE WITHIN A COLLISIONLESS SHOCK BY COLLECTIVE SCATTERING OF LIGHT

C. C. DAUGHNEY
L. S. HOLMES
J. W. M. PAUL

Culham Laboratory
Abingdon Berkshire

1970

CLM-P 235

MEASUREMENT OF SPECTRUM OF TURBULENCE WITHIN A COLLISIONLESS SHOCK BY COLLECTIVE SCATTERING OF LIGHT

By

C.C. DAUGHNEY
L.S. HOLMES
J.W.M. PAUL

(To be submitted for publication in Physical Review Letters.)

ABSTRACT

We present experimental evidence for the presence of current driven ion wave turbulence within a certain class of collisionless shock waves. The frequency spectrum and wave number spectrum of this turbulence have been measured by scattering light from the shock front. These measurements are compared with the assumptions and predictions of non-linear theory.

U.K.A.E.A., Research Group,
Culham Laboratory,
Abingdon,
Berks.

March, 1970.

We are studying[1-4] a collisionless shock with low Alfven Mach Number ($M_A < 3$), which propagates perpendicular to a magnetic field. The shock is produced by the radial compression of an initial hydrogen plasma by a linear z-pinch. The initial plasma is 85% ionized with electron density $n_{e1} = 6.4 \times 10^{20}\ m^{-3}$, and temperatures $T_{e1} = T_{i1} = 1.2$ eV, and is in an axial magnetic field $B_{z1} = 0.12\ Wb\ m^{-2}$. The shock propagates radially inwards through the initial plasma and at 9 cm radius is in a steady state. The shock velocity $V_s = 240\ km\ s^{-1}$ ($M_A = 2.5$), the shock width $L_s = 1.4$ mm and the compression ratio is 2.5. The measured electron temperature behind the shock, $T_{e2} = 44$ eV, implies negligible ion heating and a mean resistivity within the shock which is 100 times the classical value. The corresponding effective electron-ion collision frequency $\nu^* \sim 3$ GHz.

The compression of B_{z1} in the shock front gives rise to an azimuthal current with an electron drift velocity $V_D \sim 4\ C_s$, where C_s is the local ion sound speed. For the conditions in the shock front, linear stability theory[5-7] predicts current driven ion wave instability. According to non-linear theory[8], this instability develops into turbulence with frequencies $\omega \lesssim \omega_{pi}$, the ion plasma frequency and wavelengths $\lambda \gtrsim \lambda_D$, the Debye length.

In a previous study[3] of the light scattered by this shock we measured a level of plasma fluctuations within the shock, which was over 100 times thermal. These supra-thermal fluctuations were identified as ion waves from their observed frequency. The measured level of fluctuation was shown, using certain assumptions, to be sufficient to yield stochastic heating of the electrons to the observed temperature.

When light is scattered in a given direction by plasma waves(9-11), the change of optical wave vector and frequency are related to the plasma wave vector $(\bar{k})$ and frequency (ω), both positive, as follows:

$$\bar{k}_s = \bar{k}_i \pm \bar{k} \quad \text{and correspondingly} \quad \omega_s = \omega_i \pm \omega$$

where the subscript s is for scattered and i for incident. The addition and subtraction signs correspond to scattering from plasma waves propagating in opposite directions. The scattering angle (θ) determines $|\bar{k}|$ of the plasma waves. The frequency spectrum of the plasma waves is given by the spectral profile of the scattered light and the relation

$$\omega = \pm (\omega_s - \omega_i) = \mp \left(\frac{2\pi c}{\lambda_i^2}\right) \Delta\lambda \text{ with } \Delta\lambda = \lambda_s - \lambda_i$$

Opposite wavelength shifts $(\pm \Delta\lambda)$ correspond to scattering from waves propagating in opposite directions. The differential scattering cross-section per electron, $\sigma_s(\omega, \bar{k})$, measures the level of electron density fluctuation in terms of its Fourier transform,

$$\sigma_s (\omega, \bar{k}) = \langle \delta n_e^2 (\omega, \bar{k}) \rangle / n_e = S(\omega, k)\sigma_e$$

where σ_e is the appropriate Thomson scattering cross-section for an electron. We define $S_k(\omega) = S(\omega, k)$ for a fixed $\bar{k}$ and $S(\bar{k}) = \int S(\omega, \bar{k})d\omega$.

We study the light forward-scattered from a 50 MW ruby laser beam of diameter 2 mm during the transit of the shock through the beam. The incident and forward scattered light paths intersect symmetrically in the mid-plane of the discharge tube at an angle $\theta = 5.1^o$(12). Both light paths are in a plane which is tangential to the shock front when the latter is at 9 cm radius.

The plasma waves responsible for this scattering have a mean wave vector $\bar{k}$ collinear with the azimuthal current within the shock front and mean $|\bar{k}| = k_m = 8.2 \times 10^5\ m^{-1}$. The nature of the plasma waves depends on the parameter $\alpha = 1/(k\lambda_D)$, in which λ_D varies through the shock. For $k = k_m$, the value of α varies from 3.8 to 1.0 between the front and rear of the shock with $\alpha = 1.1$ for the mean conditions in the shock. However there is no spatial resolution within the shock structure.

The scattered power is measured by an S20 photomultiplier and can be normalized to the incident laser power as measured by a photodiode. The observed pulse of scattered light has a width (half intensity) of 10 ns corresponding to the shock transit. This pulse is much shorter than the laser pulse (35 ns) and consequently the scattered light is easily distinguished from stray light. The timing of the shock relative to the laser pulse is obtained from an electric probe, which is displaced azimuthally out of the optical paths.

A viewing aperture of diameter 3.6 cm, accepts light scattered through a range of angles $\theta = 5.1 \pm 1.8^o$. These correspond to scattering from waves with a range of $|k|$ from 5.8 to $10.5 \times 10^5\ m^{-1}$. The angle φ between the scattering plane and the tangent plane to the shock varies over the range $-16^o < \varphi < +16^o$. Consequently the $\bar{k}$ of the scattering waves covers the same range of angles (φ) to $\bar{V}_D$, while remaining nearly perpendicular to $\bar{B}_z$. We now report measurements of the dependence of normalized scattered power, i.e. $S(\bar{k})$, on $|\bar{k}|$ and φ within these ranges.

The variation of $S(k)$ with $|\bar{k}|$ is obtained by placing various masks over the viewing aperture. Each mask has an arc shaped slot of constant $|\bar{k}|$. The measured $S(k)$, shown in Fig.1 decreases rapidly with increasing $|k|$ up to an effective cut-off at $|\bar{k}| \sim 8.0 \times 10^5\ m^{-1}$. As this cut-off occurs near the centre of the aperture the effective average $|\bar{k}|$ is not k_m.

Similarly, masks with fan shaped slots of constant φ are used to measure the dependence on angle φ. Within an experimental accuracy of $\pm$ 15%, there is no change of $S(\bar{k})$ over the range $\varphi = \pm 16^{\circ}$.

We also report(4) measurements of the spectral profile $S_k(\omega)$, of the scattered light accept by a viewing aperture of 2.0 cm diameter. From the measured $S(k)$ this aperture corresponds to an effective $|k| = k_e = 7.1 \times 10^5\ m^{-1}$. The profile is obtained by analysing the scattered light with a Fabry-Perot interferometer of 0.02 Å resolution. The measured spectral width of the incident laser line is 0.02 Å (after instrumental correction) as shown in Fig. 2(a). This narrow line is obtained by using a triple etalon resonant reflector in the laser cavity.

The spectral profiles are scanned, shot by shot, by varying the refractive index (pressure) of the gas in the interferometer. The transmitted light within the central fringe is detected by an S20 photomultiplier and is normalized to the incident scattered light.

The spectral profile of the light scattered from the shock is shown in Fig. 2(b). The width of the profile at half intensity $\delta\lambda = 0.055$ Å, which is three times broader than the incident laser line. The peak of the profile is shifted to the red by $\Delta\lambda_p = +\ 0.07$ Å from the laser line. The scattered powers at $\pm\ \Delta\lambda_p$ differ by a factor of more than 50.

The direction of this shift $\Delta\lambda$ corresponds to scattering from plasma waves propagating in the same direction as the azimuthal electron current (i.e. $\bar{V}_D$) in the shock front. The vector directions are shown in Fig. 2. The direction of $\bar{V}_D$ in the shock front can be reversed simply by reversing B_{z1}. In a Z-pinch, with orthogonal drive and initial magnetic fields, such a reversal has no effect on the plasma behaviour other than to change the "hand". Reversal of the azimuthal current in this way, results in a reversal of the sign of the shift of wavelength of the scattered light as shown in Fig. 2(c). The smooth curves shown in Fig. 2(b) and (c) are the best fit to the whole set of observations irrespective of the sign of the wavelength shift.

The peak of the scattered spectrum occurs for a shift $\Delta\lambda_p$ corresponding to scattering from plasma waves with a frequency ω_o = 28 GHz. This is the range for ion waves in the shock, i.e. $\omega_o < \omega_{pi}$. However in the experiment $\omega_{pi} \approx \omega_{ce}$, the electron cyclotron frequency, and so electron Bernstein modes have to be considered[7]. We have studied a similar shock with almost the same Mach number (M_A = 2.3) but with ω_{pi} decreased by a factor 0.75 and ω_{ce} increased by a factor 1.3. The range of α within the shock is the same. In this scaled experiment, spectral profiles similar to Fig. 1 were obtained (i.e. shift, reversal and broadening) but with shift $\Delta\lambda_p' = 0.05$ Å and width $\delta\lambda' = 0.04$ Å. Both $\Delta\lambda$ and $\delta\lambda$ scale as ω_{pi} and not as ω_{ce}.

We now discuss the interpretation of these measurements of $S(\bar{k})$ and $S_k(\omega)$. The reversal of the shift $\Delta\lambda$ with reversal of $\bar{V}_D$, allows us to identify the electron drift current as the driving source for the observed supra-thermal fluctuation. We can identify the fluctuations as ion waves by considering the frequency ω of the peak of $S_k(\omega)$. This frequency scales as ω_{pi} and the mode (ω_o, k_e) fits the ion wave dispersion curve for the mean conditions in the shock, i.e. $\omega_o = 0.65\ \omega_{pim}$, $k_e\lambda_{Dm} = 0.78$. In the dispersion equation we assume $\gamma_e = 1$, adiabatic ion heating with $\gamma_i = 3$ or 5/3, and neglect magnetic effects[7]. Although ω amd k are related by dispersion, there is no confusion in the dependence of $S_k(\omega)$ on ω.

The profiles, $S_k(\omega)$, are similar in shift and reversal to those predicted by Rosenbluth and Rostoker[11] for a stable plasma with large $\bar{V}_D$ and $T_e > T_i$. However our observed profiles are more enhanced[3] and broader than predicted for a stable plasma. Asymmetric profiles, $S_k(\omega)$, have been observed in other experiments[10], but none of these have shown a well defined shift with an identified cause.

The width $(\delta\lambda)$ of $S_k(\omega)$ measures the lifetime (τ) of the mode (ω_o, k_e). The measured $\delta\lambda$ yields $\tau = (\lambda_i^2/c\delta\lambda) = 0.32$ ns. This time is roughly the coherence time between random changes of phase. As $\tau \sim (2\pi/\omega_o)$ there is appreciable randomness of phase; sufficient, we shall show later, for the fluctuations to constitute turbulence. This lifetime is much shorter than would be expected for a simple balance of linear growth and non-linear damping of the mode.

We can now compare our results with the assumptions and predictions of non-linear theory. Kadomtsev[8,4] has developed a theory of current driven ion wave turbulence in the absence of a magnetic field. He assumes a delta function for $S_k(\omega)$; this is not true for our turbulence. He then balances the linear growth rate against non-linear scattering of the ion waves by the ions and derives a spectrum of the form[13].

$$S(k) \propto k^{-3} \ln (k_c/k)$$

where the cut-off at k_c results from the absence of linear growth for $k > k_c \sim 1/\lambda_D$.

Our measured spectrum is a good fit to the Kadomtsev form as is shown in Fig. 1. This comparison is obtained from the experimental data by plotting $k^3 S(k)$ against $\ln k$. The plot is linear and the logarithmic cut off dominates. The intercept on the $\ln k$ axis yields k_c and an effective $\lambda_D \sim 1.2 \times 10^{-6}$ m. This is in good agreement with the value for the mean shock conditions $\lambda_{Dm} = 1.1 \times 10^{-6}$ m. Thus the observations are consistent with the idea that k_c results from the absence of linear growth for $k > 1/\lambda_D$ and that the scattering originates from plasma with the mean shock parameters.

In Kadomtsev's model, for $V_D \gg C_s$ the turbulence is spread over the hemisphere of k-space surrounding the direction of $\bar{V}_D$. The observed anisotropy with respect to $\bar{V}_D$ and the lack of dependence of $S(\bar{k})$ on small angles φ, are consistent with this model.

Finally we reconsider some of the assumptions used previously[3,4] when constructing a self-consistent model of this collisionless shock. This model is based on the stochastic heating of electrons. Electrons with the mean thermal velocity (V_e), are assumed to interact with the effectively stationary $(C_s \ll V_e)$ electric field pattern of the ion wave turbulence. The stochastic behaviour arises from the finite coherence length of the ion waves, given by $d = C_s \tau$. The electron experiences a random phase change after a time $t = d/V_e = (C_s/V_e)\tau$. From the results presented here $t = 7.4 \times 10^{-12}$ s. This corresponds to 44 random changes of phase in one effective collision time, $\tau_{coll} = 1/\nu^* = 3.3 \times 10^{-10}$ s. Thus on the time scale of these collisions the Random Phase Approximation and consequently the stochastic approach is valid.

We have verified the important assumptions that there is a cut-off in $S(k)$ for $k > 1/\lambda_D$. Although less critical, we have also verified the Kadomtsev form of $S(k)$ that was assumed; except in that previously we used a sharp cut-off at $k\lambda_D = 1$.

Our previous simplifying assumption of isotropic turbulence is clearly wrong. Measurements of this anisotropy against angle are required before suitable corrections can be made.

We thank Dr R.J. Bickerton for his advice and Mr P.R.L. Hedley for his assistance.

REFERENCES

1. Paul, J.W.M. et al., 7th Intern. Conf. on Phen. in Ionized Gases, Belgrade, II, 819, 1965.
2. Paul, J.W.M. et al., Nature 208, 133, 1965 and 216, 363, 1967.
3. Paul, J.W.M., Daughney, C.C. and Holmes, L.S., Nature 223, 822, 1969 and 9th Intern. Conf. on Phen. in Ionized Gases, Bucharest p.650, 1969.
4. Paul, J.W.M. et al., Contrib. to Proceeding of Study Group on Collisionless Shocks, ESRO Report SP-51, p.97 and 207, 1969.
5. Stringer, T.E., J. Nucl. Energy, C, 6, 267, 1964.
6. Krall, N. and Book, D.L., Phys. Fluids 12, 347, 1969 and Physics Rev. Lett. 23, 574, 1969.
7. Gary, S.P. and Saunderson, J.J., Accepted by Journal of Plasma Physics.
8. Kadomtsev, B., Plasma Turbulence Academic Press, 1965.
9. Bekefi, G., Radiation Processes in Plasmas, Wiley, 1966.
10. Evans, D.E. and Katzenstein, J., Reports on Prog. in Physics 32, 207, 1969.
11. Rosenbluth, M.N. and Rostoker, N., Phys. Fluids 5, 776, 1962.
12. This is corrected from the previously quoted value of 4.5^{o}.
13. Kadomtsev quotes potential fluctuations but for ion waves these are proportional to the electron density fluctuations.

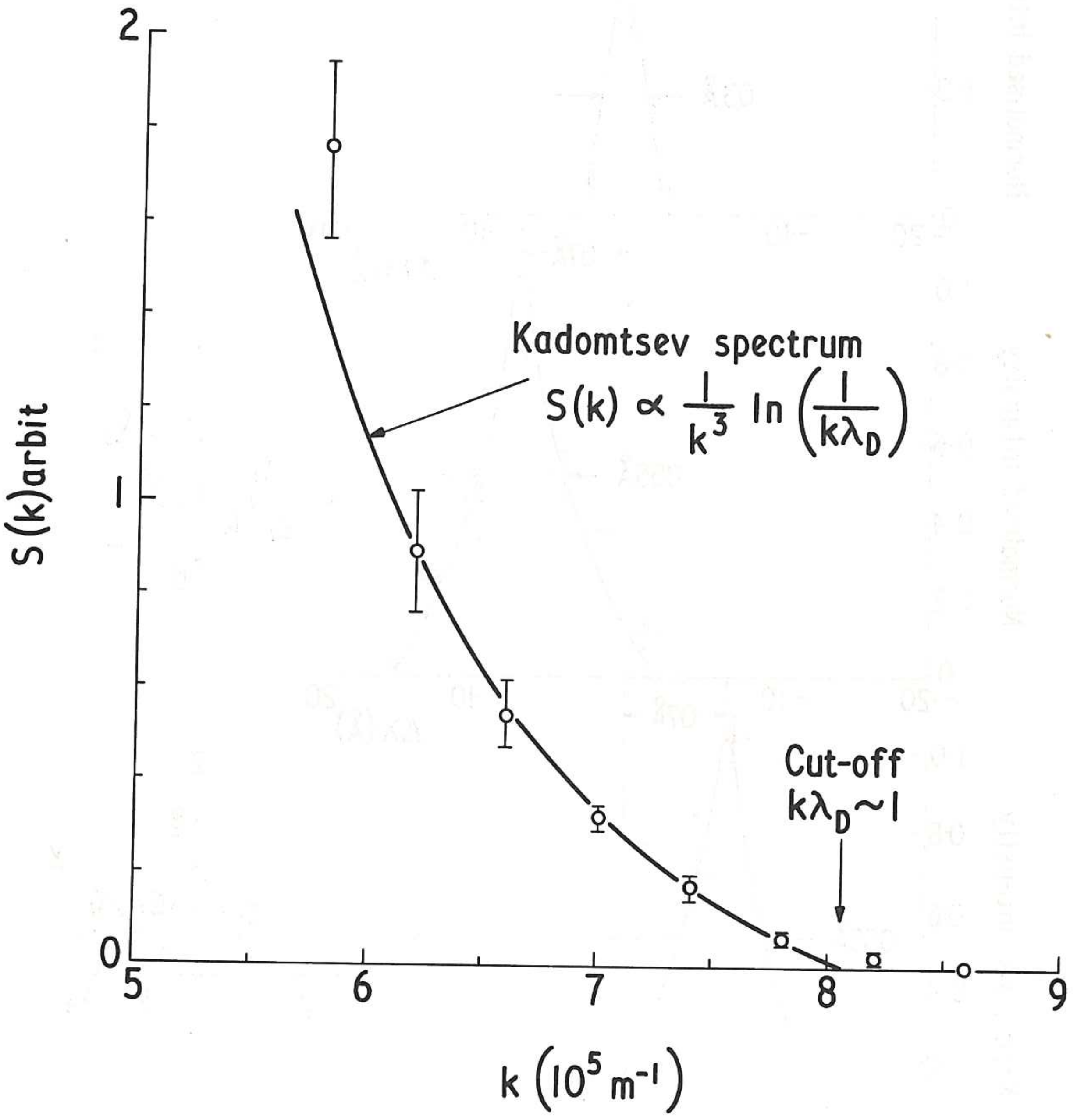

Fig.1 Wave number spectrum S(k):
Experimental points are mean of 5 measurements and error bars are standard deviation of the mean. The curve is a Kadomtsev spectrum.

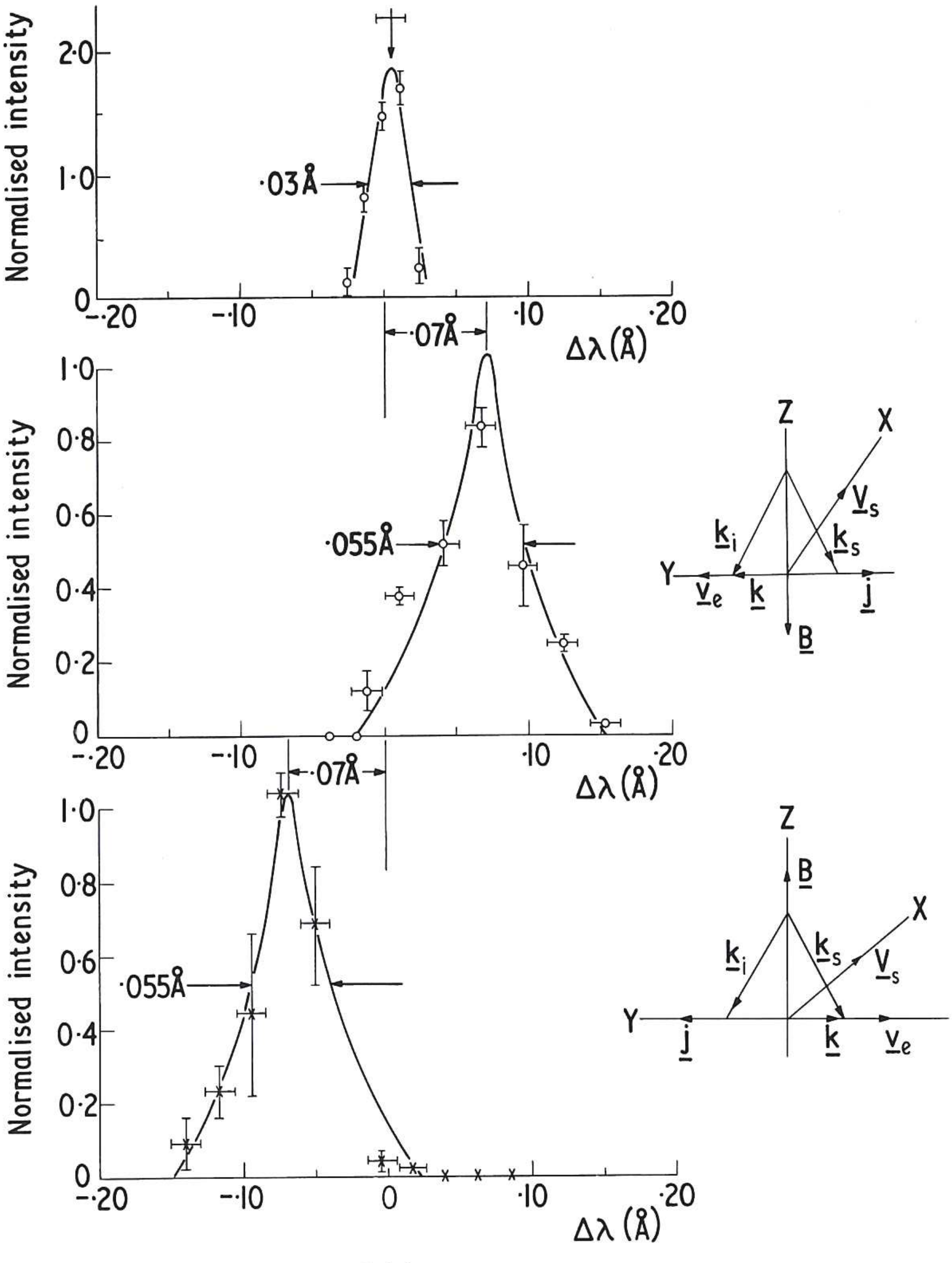

Fig.2 Frequency spectrum $S_k(\omega)$:
Spectral profiles of (a) incident laser light, (b) and (c) scattered light, i.e. $S_k(\omega)$, for opposite directions of $\overline{B}_{z1}$ and $\overline{V}_D$ as shown. Experimental points are mean of 5 measurements and error bars are standard deviation of the mean.

CLM-P 235