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ON THE BOUNDARY CONDITIONS AT AN INSULATING WALL FOR HYDROMAGNETIC WAVES IN A CYLINDRICAL PLASMA

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ON THE BOUNDARY CONDITIONS AT AN INSULATING WALL FOR HYDROMAGNETIC WAVES IN A CYLINDRICAL PLASMA

by

L.C. WOODS*

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ABSTRACT

The order of the dispersion relation for the propagation of hydromagnetic waves along a magnetised cylindrical plasma falls by unity when the plasma resistivity, $1/\sigma$, tends to zero. A consequence of this is that the two boundary conditions necessary on an insulating wall are reduced to a single condition, a reduction brought about by the development of a current sheet. If the ratio, $\Omega \equiv \omega/\omega_{ci}$, of the wave frequency to the ion cyclotron frequency is also assumed to be vanishingly small, then the nature of the single boundary condition to be adopted in the limit $1/\sigma \rightarrow 0$ depends, for the slow hydromagnetic wave, on the limiting value of $\sigma^{\frac{1}{2}}\Omega^2$. Similarly, if $\Omega \gg 1$, and the fast hydromagnetic wave is being considered, then the relevant boundary condition is found to depend on the limiting value of $\Omega\sigma^{-\frac{1}{2}}$.

The 'resistive' waves that are found to accompany the fast and slow waves, in order to satisfy the boundary conditions for small but finite values of $1/\sigma$, are studied in some detail and their contribution to the wave damping is determined.

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CONTENTS

1. INTRODUCTION

1. In a previous paper published in this Journal (Woods, 1962), a dispersion relation was derived for the propagation of hydromagnetic waves along a magnetic field lying in the axial direction of a cylindrical, partially-ionized plasma. The neutral gas was assumed to possess viscosity and pressure, while for the ionized gas resistivity, $1/\sigma$, was added to these properties. This complexity yielded a rather involved relationship between the axial wave number k, the radial wave number k_c and the frequency ω, particularly as the ion-cyclotron frequency, ω_{ci}, was also taken into account. The main effect of the neutral gas - which will not concern us in this paper - was to modify the Alfven speed, $v_A = B_o(\mu\rho_o)^{-\frac{1}{2}}$, where B_o is the steady axial magnetic field and ρ_o is the density of the ionized gas, by replacing ρ_o by ρ_o/s where s is a complex number depending on the ion-neutral collision frequency and the ionization level (see (W59) - equation (59) of Woods 1962). For the special case in which pressure and viscosity can be neglected, the case usually realized in hydromagnetic wave experiments (Jephcott and Stocker 1962), the dispersion relation reduces to (cf. (W60)).

$$[k^2 - k_A^2(1 + i\delta k_t^2)][k_t^2 - k_A^2(1 + i\delta k_t^2)] = k^2 k_t^2 \Omega^2 , \qquad \ldots(1)$$

where $k_A^2 \equiv \omega^2/(v_A^2 s)$, $k_t^2 \equiv k_c^2 + k^2$ is the total wave number, $\delta \equiv 1/(\mu\omega\sigma)$ (mks units), and $\Omega \equiv \omega/\omega_{ci}$. For simplicity, in (1) we have not allowed for the anisotropic nature of the resistivity parameter $\delta^{\dagger}$, but it is easily verified from (W60) that $\delta_{\parallel}$ should replace δ appearing in the singular perturbation theory given in section 2 of this paper.

2. With the magnetic flux $\underline{B}$ expressed in cylindrical co-ordinates (r,θ,z) as

$$B(r,\theta,z,t) = B_o\underline{n} + \underline{B}_1(r)\exp\{-i(m\theta + kz - \omega t)\}, \qquad \ldots(2)$$

where $\underline{n}$ is unit vector along the Oz-axis, the theory gave for the components of $\underline{B}_1$:

$$\left.\begin{aligned}
B_{1z} &= \sum_{i=1}^{2} k_{ci}\mathcal{A}_i\, J_m(k_{ci}r) , \\
B_{1\theta} &= -\sum_{i=1}^{2} \{\mathcal{C}_i\, J_m'(k_{ci}r) + (km/k_{ci}r)\mathcal{A}_i\, J_m(k_{ci}r)\} , \\
B_{1r} &= i\sum_{i=1}^{2} \{k\mathcal{A}_i J_m'(k_{ci}r) + (m/k_{ci}r)\,\mathcal{C}_i\, J_m(k_{ci}r)\} ,
\end{aligned}\right\} \qquad \ldots(3)$$

where

$$-\frac{\mathcal{C}_i}{\mathcal{A}_i} = \frac{k_{ti}^2 - k_A^2(1 + i\delta k_{ti}^2)}{k\Omega} = \frac{k\,k_{ti}^2\,\Omega}{k^2 - k_A^2(1 + i\delta k_{ti}^2)}\,, \quad i = 1,2\,, \qquad \ldots(4)$$

$k_{ti}^2 = k_{ci}^2 + k^2$, and k_{t1}^2, k_{t2}^2 are the roots of the quadratic in k_t^2 in (1).

3. Suppose the plasma has a radius r_o, then the boundary conditions at $r = r_o$ depend on the conductivity of the material that encloses the plasma: for infinity conducting walls

$$B_{1r} = 0\,, \quad \text{at} \quad r = r_o\,, \qquad \ldots(5)$$

while for perfectly insulating walls, it is shown by Woods (1962) that continuity of magnetic fields leads to

$$\left.\begin{aligned} \frac{m}{r_o}B_{1z} - kB_{1\theta} &= 0 \qquad ..(6a) \\ B_{1r} + iX_m B_{1z} &= 0 \qquad ..(6b) \end{aligned}\right\} \quad \text{at} \quad r = r_o$$

where

$$X_m \equiv K_m'(kr_o)/K_m(kr_o)\,. \qquad \ldots(7)$$

Equation (6a) is equivalent to requiring the radial current to vanish. The electric fields are assumed to be screened by an electric dipole layer on the walls (see discussion in earlier paper). The intermediate case of finite conductivity has also been considered.

4. Now for a given plasma, magnetic field strength B_o, and frequency ω, apart from the amplitude of the perturbations - which depends on the initial conditions - (3) contain five unknowns, viz. k, k_{c1}, k_{c2}, and the amplitude ratios,

$$a \equiv \mathcal{A}_2/\mathcal{A}_1\,, \quad c \equiv \mathcal{C}_2/\mathcal{C}_1\,. \qquad \ldots(8)$$

One restriction on a/c follows from (4), while (1) imposes two further relations between k, k_{c1}, and k_{c2}; consequently two boundary conditions can be satisfied. If only one radial mode is present, say that corresponding to k_{c1}, then a similar argument shows that only one boundary condition can be satisfied. Thus by (5), if the walls are infinitely conducting, each of the two radial modes summed in (3) can be propagated separately, whereas with insulating walls both modes must be present so that (6) can be satisfied.

5. Now suppose that the plasma resistivity is zero, i.e. $\delta = 0$, then (1) will yield only one root for k_t^2, and hence only one radial mode. Thus in

this limiting case the two boundary conditions in (6) must be replaced by a single condition. This suggest that when δ is very small, but not quite zero, a boundary layer will form providing a rapid transition in one or more of the magnetic field components, and so permiting both of (6) to be satisfied at the wall. Then in the limit $\delta = 0$, the boundary layer collapses into a current sheet.

6. This point was not pursued in the earlier paper, because at that time most of the Culham Laboratory experiments (Jephcott and Stocker, 1962), for which the theory was developed, involved waves with negligible B_{1r} and B_{1z} fields, so that it was necessary to satisfy only (6a). However recent experiments on fast waves in insulating tubes, involve relatively large B_{1r} and B_{1z} fields, and so require the analysis to be taken further for the special case of δ small but finite. Of course the three relations between a, c, k, k_{C1}, and k_{C2}, mentioned above, together with (6), provide a means of finding the solution for the general case, but this would necessarily be a a numerical rather than an analytic solution, and would throw no light on the limiting case described above.

2. THE BOUNDARY CONDITION FOR SMALL PLASMA RESISTIVITY

7. On ignoring a term δ^2 (1) can be written

$$i\delta k_A^2 k_t^4 + \{k^2\Omega^2 + k_A^2 - k^2 - i\delta k_A^2(2k_A^2 - k^2)\}k_t^2 + k_A^2(k^2 - k_A^2) = 0 \quad \ldots(9)$$

If ε is small $i\varepsilon x^2 + bx + c = 0$ has approximate roots $-c/b$, ib/ε; so (9) has the approximate roots

$$k_{t1}^2 \approx \frac{(k_A^2 - k^2)k_A^2}{k^2\Omega^2 + k_A^2 - k^2 - i\delta k_A^2(2k_A^2 - k^2)} \approx \frac{k_A^2(k^2 - k_A^2)}{k^2 - k_A^2 - k^2\Omega^2}\,, \quad \ldots(10)$$

$$k_{t2}^2 \approx -i(k^2 - k_A^2 - k^2\Omega^2)/(\delta k_A^2)\,. \quad \ldots(11)$$

8. For simplicity we shall neglect here the effect of neutral gas and take k_A^2 to be real. Let η, ε, k_c and ζ denote real numbers defined by

$$k = \eta + i\varepsilon, \quad k_{c1} = k_c - i\zeta\,. \quad ..(11a)$$

For the wave to be propagated a significant distance, we require that $\varepsilon \ll \eta$; assuming this to be the case and neglecting second-order terms, we find from the first of (10) that

$$\eta^2 = k_A^2/h - \tfrac{1}{2}k_c^2 \pm G/2h , \qquad \ldots(12)$$

$$\varepsilon = (k_A^2\delta/2\eta h^2)\{(2-h)k_A^2 + \tfrac{1}{2}hk_c^2 \pm \tfrac{1}{2}G^{-1}[8k_A^4\Omega^2 + hk_c^2(hk_c^2 + 2\Omega^2k_A^2)]\}$$
$$+ (\zeta k_c/2\eta)\{1 \pm G^{-1}(2k_A^2 - hk_c^2)\} , \qquad \ldots(13)$$

where

$$h \equiv 1 - \Omega^2, \qquad G \equiv (k_c^4h^2 + 4k_A^4\Omega^2)^{\frac{1}{2}} . \qquad \ldots(14)$$

The positive sign in (12) gives a 'slow' wave and the negative sign a 'fast' wave.

9. To first order (11) becomes

$$k_{c2}^2 = -i\frac{\eta^2h - k_A^2}{\delta k_A^2} = \begin{matrix} -2iA_s^2; \text{ (slow wave)} \\ 2iA_f^2; \text{ (fast wave)} \end{matrix} \qquad \ldots(15)$$

where

$$A_f \equiv \tfrac{1}{2}\{(hk_c^2 + G)/\delta k_A^2\}^{\frac{1}{2}}, \quad A_s \equiv \tfrac{1}{2}\{(G - hk_c^2)/\delta k_A^2\}^{\frac{1}{2}} . \qquad \ldots(16)$$

Hence

$$k_{c2} = \begin{matrix} (1-i)A_s \quad \text{(slow wave)} \\ (1+i)A_f \quad \text{(fast wave)} \end{matrix} , \qquad \ldots(17)$$

where, as the plasma resistivity is assumed small, both A_s and A_f are large numbers. When Ω is small

$$A_f \approx (k_c/k_A)(2\delta)^{-\frac{1}{2}} \quad \text{and} \quad A_s \approx (k_A/k_c)\Omega(2\delta)^{-\frac{1}{2}} \qquad \ldots(18)$$

10. From the asymptotic form of the Bessel functions we have for the fast wave

$$J'_m(k_{c2}r) = J'_m\{(1 + i)A_fr\} \sim -F_f(r), \qquad \ldots(19)$$

where

$$F_f = \tfrac{1}{2}\left(\frac{\sqrt{2}}{A_f\pi}\right)^{\frac{1}{2}} e^{i\pi(m+5/4)/2}\, e^{A_f(1-i)r}\, r^{-\frac{1}{2}} , \qquad \ldots(20)$$

provided, of course, that $r \neq 0$. Also $J_m(k_{c2}r) \sim iF_f(r)$. It now follows from (5), (6) and (8), that when δ is sufficiently small to make A_fr_o large, the boundary conditions for insulating walls can be expressed:

$$\begin{aligned} J'_m + \lambda_1\{(k^2 + k_{c1}^2)/k_{c1}\}J_m &= F_f(r_o)\{c - \lambda_1 aA_f(1-i)\} \\ kJ'_m + \{k_{c1}X_m + \lambda_2/k_{c1}\}J_m &= F_f(r_o)\{c\lambda_2/[A_f(1-i)] - X_m aA_f(1-i)\} \end{aligned} \qquad \ldots(21)$$

where $\lambda_1 \equiv (m/kr_o)(\mathcal{A}_1/\mathcal{C}_1)$, $\lambda_2 \equiv (m/r_o)(\mathcal{C}_1/\mathcal{A}_1)$, and the argument $k_{c1}r_o$ of the Bessel functions has been omitted for brevity. The corresponding boundary conditions for the slow wave are obtained by replacing $A_f(1+i)$ by

$A_s(1-i)$ in (20) and (21), but notice from (18) that at low frequencies, for the asymptotic theory to be valid in this case, δ must be small enough to satisfy $(2\delta)^{\frac{1}{2}} \ll (k_A r_o \Omega / k_c)$.

11. From (4),(8) and (11)

$$a/c = (\mathscr{C}_1/\mathscr{A}_1)(k^2 - k_A^2 - i\delta k_{c2}^2 k_A^2)/(k\, k_{c2}^2 \Omega) = (\mathscr{C}_1/\mathscr{A}_1)k\Omega[A_f(1-i)]^{-2} \approx 0$$

and hence on eliminating F_f from (21) one finds

$$\{kJ'_m + (k_{c1}X_m + \frac{m}{r_o k_{c1}}\frac{\mathscr{C}_1}{\mathscr{A}_1})J_m\} \equiv k(1+i)\ell_f\{J'_m + \frac{m}{r_o k}\frac{k^2+k_{c1}^2}{k_{c1}}\frac{\mathscr{A}_1}{\mathscr{C}_1}J_m\} \quad \ldots(22)$$

where

$$\ell_f \equiv (\mathscr{C}_1/\mathscr{A}_1)(m/r_o k - X_m\Omega)/2A_f \,. \quad \ldots(23)$$

As A_f has been assumed large, $\ell_f \ll 1$ unless $(\mathscr{C}_1/\mathscr{A}_1)$ is large. From (4) and (12) we find that for the fast wave there are no critical frequencies or wave numbers at which $(\mathscr{C}_1/\mathscr{A}_1)$ is unduly large, and so ℓ_f is small, except perhaps at high frequencies. Thus in evaluating it we can neglect the small imaginary parts of k and k_{c1}. To first order then, by (4) and (11a) $\mathscr{C}_1/\mathscr{A}_1 = \Omega\eta(k_c^2+\eta^2)/(k_A^2-\eta^2)$, and using (12) to eliminate η^2 from this expression (take the negative sign in (12)) and then substituting the result in (23) we find

$$\ell_f = 2\eta\Omega k_A^3\,\delta^{\frac{1}{2}}(G + hk_c^2)^{-\frac{3}{2}}(\frac{m}{r_o\eta} - X_m\Omega) \,, \quad \ldots(24)$$

where A_f has been eliminated by (16).

12. The corresponding boundary condition for the slow wave is like (22) except that $(1+i)\ell_f$ is replaced by $(1-i)\ell_s$, where

$$\ell_s = \tfrac{1}{4}(\eta/\Omega^2)k_A^{-3}\,\delta^{\frac{1}{2}}(G + hk_c^2)^{\frac{3}{2}}\,(\frac{m}{r_o\eta} - X_m\Omega) \,. \quad \ldots(25)$$

Notice that as $(G + hk_c^2) \rightarrow 2k_c^2$ when $\Omega \rightarrow o$, and $k_A \equiv \omega/v_A$, ℓ_f tends to zero like ω^4 at small frequencies, whereas ℓ_s tends to infinity like ω^{-4}.

13. In (22) we now have one combined boundary condition from which $k_{c1} = k_c - i\zeta$ can be determined. When ℓ_f is small, the dominant condition is the vanishing of the first term in curly brackets, which corresponds to (6b), whereas if ℓ_f is large, as is possible at high frequencies, the vanishing of its bracketed coefficient is the dominant condition, corresponding to (6a). For the slow wave the dominant condition is (6a) at low frequencies and (6b) at high frequencies. The imaginary part of k_{c1} i.e. ζ

is likely to be largest when ℓ_f and ℓ_s are near unity in value, and by (13) the wave attenuation is likely to be largest in this case.

3. AXI-SYMMETRIC WAVES (m = 0)

14. If m = 0 the theory simplifies a little, and as this is an important case in experimental work, we shall consider it in some detail in this section. The analysis to be given below could easily be extended to the more general case of $m \neq 0$.

When m = 0 (11a) and (22) give

$$(\eta + i\varepsilon)(1 - \ell_f - i\ell_f)J_1([k_c - i\zeta]r_o) - (k_c - i\zeta)X_oJ_o([k_c - i\zeta]r_o) = 0 \quad \ldots(26)$$

where $X_o = -K_1([\eta + i\varepsilon]r_o)/K_o([\eta + i\varepsilon]r_o)$ and ℓ_f is a function of k_c and η. Equation (26) provides two relations from which the values of k_c and ζ can be determined. If ε, ζ and ℓ_f are assumed small it can be written

$$(\eta + i\varepsilon)(1 - \ell_f - i\ell_f)(J_1 - i\zeta r_oJ_1') + (k_c - i\zeta)(X_o + i\varepsilon r_oX_o')(J_o + i\zeta r_oJ_1) = 0 \quad \ldots(27)$$

where the argument of the J's is k_cr_o and of X_o is ηr_o. The real and imaginary parts of (27) yield

$$\frac{1}{x}\frac{J_1(x)}{J_o(x)} + \frac{f(y)}{1 - \ell_f(y)} = 0 . \quad \ldots(28)$$

$$\zeta r_o\{1 + x/y + 1/f(y) + x^2f(y)\} - x\{\ell_f(y) + (\varepsilon/\eta)yf'(y)/f(y)\} = 0 , \quad \ldots(29)$$

where

$$f(y) \equiv K_1(y)/\{yK_o(y)\}, \quad x \equiv k_cr_o, \quad y \equiv \eta r_o . \quad \ldots(30)$$

From (12) and (24),

$$\ell_f(y) = 2y^2\Omega^2b^2f(y)(k_A^2\delta)^{\frac{1}{2}}\{2[b^2 - hy^2]\}^{-\frac{3}{2}} , \quad \ldots(31)$$

where $b \equiv k_Ar_o$. Equation (12) can be written

$$y^2 = b^2/h - x^2/2 - (h^2x^4 + 4b^4\Omega^2)^{\frac{1}{2}}/2h \quad \ldots(32)$$

and the problem is now reduced to that of solving (28) and (32) simultaneously for x and y, a solution which will depend on the three non-dimensional numbers b, Ω, and $(k_A^2\delta)^{\frac{1}{2}}$.

15. When x and y are found, (13) and (29) yield for the damping ratio

$$\frac{\varepsilon}{\eta} = \frac{k_A^2\delta}{2h^2y^2Q}\{(2 - h)b^2 + \tfrac{1}{2}hx^2 - \tfrac{1}{2}[8b^4\Omega^2 + hx^2(hx^2 + 2b^2\Omega^2)][h^2x^4 + 4b^4\Omega^2]^{-\frac{1}{2}}\}$$

$$+ (\Omega^2 - 1)x^2\ell_f\{Q(2b^2 - 2hy^2 - hx^2)\}^{-1} , \quad \ldots(33)$$

where

$$Q \equiv 1 + f'hx^2y(2b^2 - 2hy^2 - hx^2)^{-1}\{1 + x^2f^2 + (1 + x/y)f\}^{-1}. \qquad \ldots(34)$$

Notice that the additional damping due to the imaginary part of the radial wave number, k_{c1}, is proportional to $(k_A^2\delta)^{\frac{1}{2}}$ so that at small resistivity and large Alfvén wave velocity, v_A, this component would be the dominant term in (33) except near $\Omega = 1$.

16. Now consider the slow wave under conditions such that the parameter ℓ_s is large. By (25) and (30).

$$\ell_s = \tfrac{1}{4}(\Omega b^4)^{-1}y^2f(y)(k_A^2\delta)^{\frac{1}{2}}\{r_o^2G + hx^2\}^{\frac{3}{2}} . \qquad \ldots(35)$$

By the same type of analysis as that used for the fast wave, corresponding to (28),(29) and (32) we find

$$\frac{1}{x}\frac{J_1(x)}{J_o(x)} - \frac{f(y)}{2\ell_s(y)} = 0, \qquad \ldots(36)$$

$$\zeta r_o = xf(y)/2\ell_s \qquad \ldots(37)$$

and

$$y^2 = b^2/h - x^2/2 + (h^2x^4 + 4b^4\Omega^2)^{\frac{1}{2}}/2h . \qquad \ldots(38)$$

From (13) and (37), the additional damping, ε^* say, due to ζ is

$$\frac{\varepsilon^*}{\eta} = \frac{x^2}{2y^2\ell_s} f(y)\{1 + (2k_A^2 - hk_c^2)/G\} \qquad \ldots(39)$$

and at small values of Ω (13) and (39) yield

$$\varepsilon/\eta = k_A^2\delta(k_A^2 + k_c^2)/2\eta + 2\Omega k_A^6/\{r_o\eta^4k_c^3k_A\delta^{\frac{1}{2}}\} . \qquad \ldots(40)$$

For the case, $\ell_s \ll 1$, which corresponds to Ω remaining finite and δ tending to zero, we need only replace ℓ_f in (28) and (29) by ℓ_s and $-\ell_s$ respectively.

4. THE EFFECTS OF ELECTRON INERTIA

17. The singular perturbation problem studied in this paper arises from the fact that putting resistivity equal to zero lowers the order of the differential equation. That this is a consequence of being able to neglect electron inertia and hence to admit the possibility of current sheets, in the theory of hydromagnetic waves can be shown as follows. Let $\underline{j}$ be the current and n the number density, then inclusion of electron inertia adds a term $(m_e/e^2n)\partial\underline{j}/\partial t$ to $\underline{j}/\sigma$ in Ohm's law. Thus for oscillatory perturbations the coefficient of $\underline{j}$ is changed from $1/\sigma$ to $1/\sigma - i(\omega m_e/e^2n)$, and in place of δ there is now

$$\delta = 1/(\sigma\mu\omega) - i(c/\omega_{pe})^2 , \qquad \ldots(41)$$

where c is the speed of e.m. waves and ω_{pe} is the electron plasma frequency.

18. In the limit $\sigma \to \infty$, $\delta \to -i(c/\omega_{pe})^2$, and as a consequence $k_{c2} \to R_1(\omega_{pe}/c)$, and $(1+i)\ell_f \to R_2(c/\omega_{pe})$, where R_1 and R_2 are real numbers. Thus the limit $\delta = 0$ is not physically attainable.

5. EXPRESSIONS FOR THE MAGNETIC FIELD STRENGTH

19. From (3),(8),(20) and (21) we find that for $m = 0$, the magnetic field perturbations are given by

$$\begin{aligned} B_{1z} &= \mathcal{A}_1\{k_{c1} J_0(k_{c1}r) - \mathcal{L}T(r)\}, \\ B_{1\theta} &= -\mathcal{C}_1\{J_0'(k_{c1}r) - J_0'(k_{c1}r_0)T(r)\}, \\ B_{1r} &= ik\mathcal{A}_1\{J_0'(k_{c1}r) - \tfrac{a}{c} J_0'(k_{c1}r_0)T(r)\} \\ &\approx ik\mathcal{A}_1 J_0'(k_{c1}r), \end{aligned} \qquad \ldots(42)$$

where

$$\begin{aligned} \mathcal{L} &\equiv \{k/X_0(kr_0)\}J_0'(k_{c1}r_0) + k_{c1}J_0(k_{c1}r_0) \\ &= \{k/X_0(kr_0)\}(1+i)\ell_f J_0'(k_{c1}r_0), \end{aligned} \qquad \ldots(43)$$

by (26), and $T(r) \equiv \exp\{-(1-i)A_f r_0(1 - r/r_0)\}\,(r_0/r)^{\frac{1}{2}}$. For the slow wave replace $A_f(1+i)$ and $\ell_f(1+i)$ by $A_s(1-i)$ and $\ell_s(1-i)$. The additional wave appearing in B_{1z} and $B_{1\theta}$ can be appropriately termed the 'resistive' wave; its effect on B_{1r} is negligible, while its effect on B_{1z} and $B_{1\theta}$ is only important near the boundary.

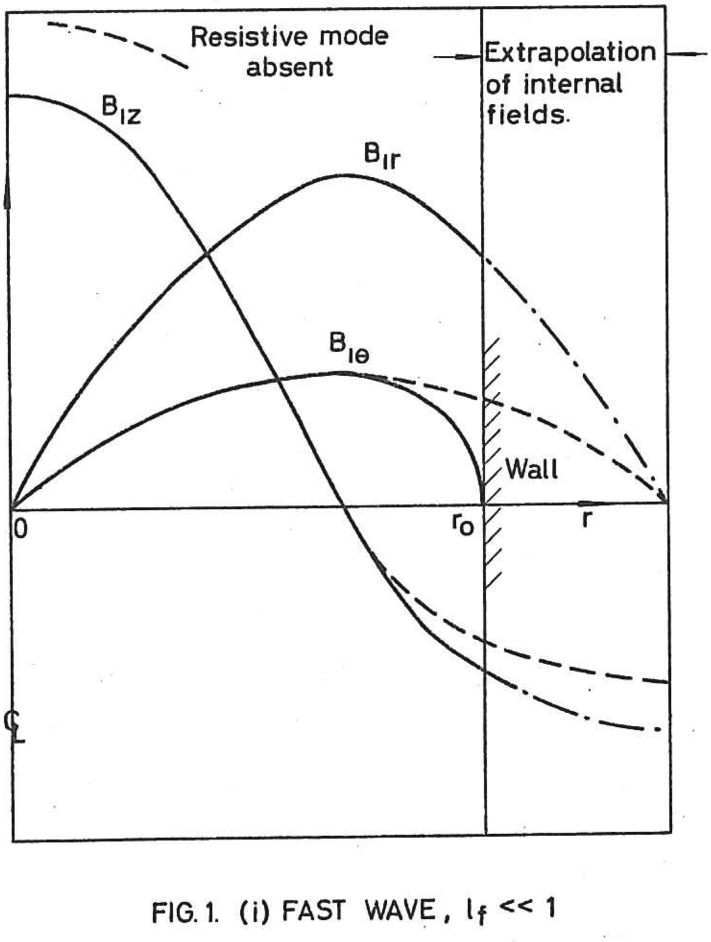

FIG. 1. (i) FAST WAVE, l_f << 1

(i) Fast Wave: $\ell_f \ll 1$

For this case $\mathcal{L}$ is small and so is the contribution of the resistive mode to B_{1z}. As $T(r_0) = 1$, $B_{1\theta} = 0$ at $r = r_0$, no matter how small the resistivity is, and the effect of the resistive mode on $B_{1\theta}$ is quite large as shown in Fig.1.

The axial current is (see (W38)) $\mu j_{1z} = \mathcal{C}_1\{k\,J_0(k_{c1}r) + (1-i)A_f J_0'(k_{c1}r)T(r)\}$, which becomes very large at $r = r_0$, matching the rapid fall in $B_{1\theta}$. Thus the insulating wall appears to behave like an infinitely conducting wall, by producing large axial currents in the plasma adjacent to it. However from (3) and (5) it follows that for an infinitely conducting wall B_{1r}, and as a consequence $B_{1\theta}$ (recall that $m = 0$ in the present discussion) tend to zero smoothly at $r = r_o$, while B_{1z} has a discontinuity, corresponding to an azimuthal current sheet, at $r = r_o$. The cases are therefore quite different.

(ii) Slow Wave: $\ell_s \gg 1$

In this case, as $\mathcal{L}$ remains finite it follows from (43) that $J_o'(k_{c1} r_o) \approx 0$, and so $\mathcal{L} \approx k_{c1} J_o(k_{c1} r_o)$, and $B_{1z} \approx \mathcal{A}_1 k_{c1} \{ J_o(k_{c1} r) - J_o(k_{c1} r_o) T(r) \}$ $B_{1\theta} \approx \mathcal{C}_1 J_o'(k_{c1} r)$. The situation is now that the resistive mode has a big effect on B_{1z} on the wall, but little effect on $B_{1\theta}$ and B_{1r} (see Fig.2). It is this case that corresponds closely to the conducting wall case - paradoxically, the case that arises when the resistivity is not zero, but is large enough to satisfy the inequalities

$$\tfrac{1}{2} f(k_A r_o)(r_o k_c)^2 \gg r_o (k_A/k_c) \Omega (2\delta)^{-\frac{1}{2}} \gg 1, \qquad \ldots(44)$$

which follow from (18),(35) and $A_s r_o \gg 1$, $\ell_s \gg 1$, $\Omega \ll 1$.

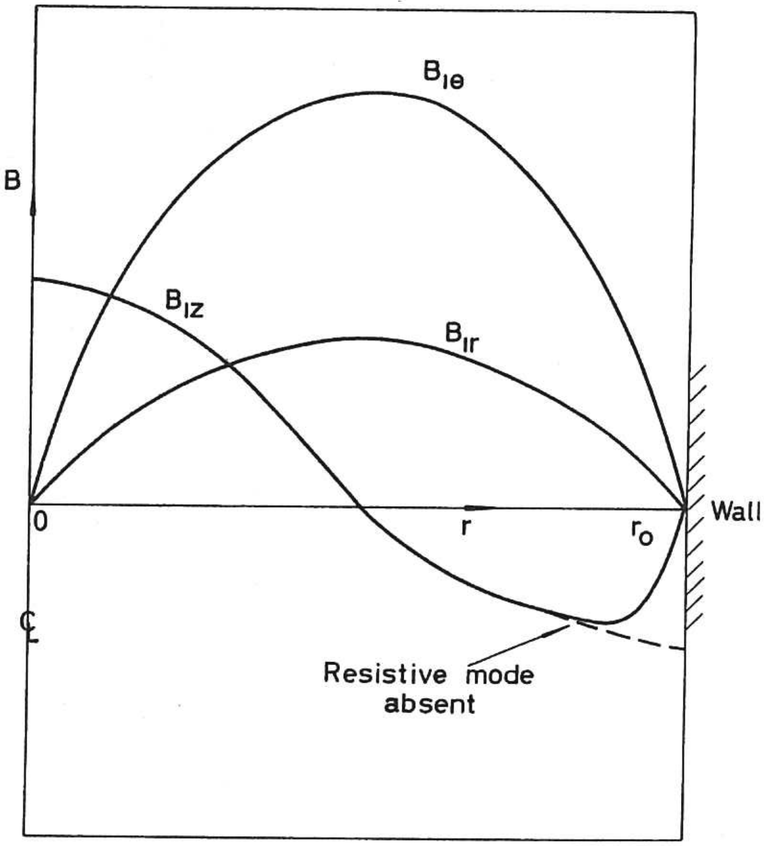

FIG. 2. (ii) SLOW WAVE, ls >> 1

20. The magnetic field distributions for the two other cases, viz. (iii) fast wave, $\ell_f \gg 1$ and (iv) slow wave, $\ell_s \ll 1$ will resemble (ii) and (i) respectively, except that the relative amplitudes will not be as shown in the figures. In (iii) B_{1z}, $B_{1r} \gg B_{1\theta}$, while in (iv) $B_{1\theta} \gg B_{1z}$, B_{1r}, except near the critical frequency $\Omega = 1$.

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7. REFERENCES

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