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REALIZATION OF A NEW TYPE OF BIREFRINGENT FILTER

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ABSTRACT

A birefringent filter based upon a new principle described in a theoretical paper by one of the authors has been constructed. The theory developed there has been verified experimentally in all respects. Limitations on a filter of this type are discussed along with possible applications.

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INTRODUCTION

For many astronomical applications, especially in solar physics, it is desirable to be able to photograph an extended object in light restricted to a narrow spectral band. This task requires a filter of very narrow pass-band that is capable of transmitting image detail of high spatial resolution. The spectral pass-band should be centered at the same wave length for every point in the image.

It will be useful to examine briefly the limitations for this task of conventional optical filters such as the interference filter based on the Fabry-Perot etalon and the Lyot-Ohman birefringent filter since the new instrument described is a combination of the two. First consider the Fabry-Perot etalon: a well-known result relates the spectral resolvance R and the solid angle Ω of a cone of rays passing through the etalon i.e.,

$$R\Omega = 2\pi \ . \qquad (1)$$

Now if the Fabry-Perot be placed near the focal plane of an image whose chief rays are parallel to the optic axis, i.e., a telecentric image, the inclination of rays to the normal will be due entirely to the finite cone of rays forming each image point and we may relate the solid angle in Eq. (1) to the focal ratio F of the system, i.e.,

$$R = 8F^2 \ . \qquad (2)$$

The size of the telecentric image will be equal to the aperture of the F-P etalon assumed to be of diameter D. The size of the Airy disc of a diffraction-limited optical system is just 1.22 $F\lambda$. Defining R_S as the spatial resolvance of the image, i.e., the number of resolvable points in the image, we obtain from Eq. (2) the following expression

$$RR_S = 1.34 \left(\frac{D}{\lambda}\right)^2 \tag{3}$$

for the product of spatial and spectral resolvance. For a typical etalon diameter of 5 cm and a typical wavelength of the visible spectrum, we may write Eq. (3) in terms of the angular resolution δ in seconds of arc of a point on the solar disc the relation

$$R\delta^{-2} \sim 10^4 . \tag{4}$$

Thus a Fabry-Perot etalon designed to filter a solar image with an angular resolvance of one second of arc cannot have a spectral resolvance in excess of 10^4 or a pass-band narrower than 0.5 to 0.6 Å.

This simple analysis pertains to a perfect etalon. Ramsey [1] has made an extensive analysis of the effect of etalon defects on image quality as well as spectral resolvance the results of which require an optical tolerance on the flatness and parallelism of the etalon plates that is ~ N times that of a simple imaging surface where N is etalon finesse. These tolerances are presently at the limits of the optician's art so the result (Eq. 4) should be considered to be an upper limit to what can be achieved in practice.

For many purposes, particularly for the measurement of solar magnetic fields by the Zeeman effect, it would be desirable to have a filter with a pass-band an order of magnitude narrower than that given by Eq. (4), while still transmitting image detail with the angular resolution of a second of arc permitted by the best atmospheric seeing. A suitable birefringent filter permits the spectral resolvance to be increased with less stringent limitation on focal ratio, and hence image resolvance, i.e., such a filter is "field-widened". If we write the phase shift between the ordinary and extraordinary ray in a birefringent plate with its optic axes in the plane of the faces as

$$\gamma = \frac{A}{\lambda}\left(1 - \frac{B\theta^2}{2}\right) , \tag{5}$$

we can obtain an expression analogous to Eq. (1) by equating the variation of phase change (5), with wavelength at constant angle to the variation of this phase change with angle at constant wavelength. The result is

$$R\Omega = \frac{2\pi}{B} \, . \tag{6}$$

The factor B is equal to $\frac{|\epsilon - \omega|}{\omega^2 \epsilon}$ for a Lyot type I wide field element [2], where ϵ and ω are the extra-ordinary and ordinary indices of refraction of the crystal plate. The factor $1/B$ is equal to 47.5 for calcite and 830 for quartz. Thus a birefringent filter using elements of the Lyot type I of calcite would have a product of spectral and angular resolution 47.5 (that given by Eq. 4) and would not significantly limit the angular resolvance of the image for very large spectral resolvances.

The above considerations explain why the Lyot-Öhman Filter has been used almost exclusively for spectroheliography rather than the Fabry-Perot, although the latter is cheaper, less bulky, more versatile, and has a higher transmittance. The pass-band of a Lyot filter is however limited by the thickest plates of calcite of suitable optical quality that can be obtained. This is typically 30 mm which gives a pass-band of ~ 0.2Å.

To obtain a filter of still narrower pass-band with wide field properties, it would be desirable to combine the properties of the Fabry-Perot etalon and the Lyot filter. In a paper by one of the authors [3] a method of accomplishing this is developed theoretically. The present paper is an account of experimental work to test those theoretical ideas and to produce an extremely narrow band, wide-field filter.

THE GYMPI

The GYMPI (Gyromagnetic Polarizing Interferometer) consists of a single retarder (which may consist of two or more birefringent elements to form one of Lyot's wide field elements) placed between two Faraday rotators in an optical cavity. The entire cavity is placed between crossed polarizers. An analysis of this device was carried out[3] using the method of partial beams and the Jones' matrix calculus. The resulting instrumental function is given by

$$F(\gamma) = \frac{\tau_E}{2}\,\frac{1}{16}\left(\frac{\pi}{N_R}\right)\left(\frac{y^2}{1+y^2}\right)\frac{(1\pm\cos\gamma)^2}{1+\left(\frac{1}{1+y^2}\right)\left(\frac{N_R}{\pi}\right)^2\sin^2\gamma} \qquad (7)$$

where γ is the birefringent phase shift in the retarder i.e.,

$$\gamma = \frac{2\pi}{\lambda}\cdot(\omega-\epsilon)\;. \qquad (8)$$

N_R is the reflective finesse of the cavity as usually defined, τ is the transmissivity of the cavity and the quantity y defined as

$$y = \frac{2N_R}{\pi}\sin^2\alpha \qquad (9)$$

where α is the angle of Faraday rotation produced by each rotator for a single pass. The upper sign refers to the situation where the Faraday rotations are in the same sense while the lower sign refers to the situation where they have opposite sense. The instrumental function is essentially an Airy function of finesse

$$\frac{N_R}{(1+y^2)^{\frac{1}{2}}}\;.$$

The denominator of Eq. (7) becomes small when γ is an integer-multiple of π. Alternate maxima are cancelled by the factor in the numerator so that the maxima occur when γ is an odd multiple of π for the opposed Faraday rotations, and an even multiple of π for Faraday rotations in the same sense. According to Eq. (8) these maxima occur for those wavelengths for which the birefringent element is a half or whole wave plate respectively.

Equation (7) may also be derived by superposing the intensities transmitted by an infinite series of Solc filters [4,5] of 1, 3, 5, 7 etc. plates weighted by the factors T^2, T^2R^4, etc., where T is the intensity transmissivity and R is the intensity reflectivity of the cavity mirrors. Although the original result was obtained by an amplitude superposition, averaged over all transit phase shifts, the final instrumental transmission results effectively from a superposition of intensities. Thus the optical tolerances on the cavity mirrors do not have to be as severe as for a Fabry-Perot etalon; it is only the birefringent defects that affect the instrumental function.

The image transmitted by the filter is the incoherent superposition of a succession of partial images, the spectral composition of each corresponding to the spectral transmission of a Solc filter of an odd number of plates.

We next consider the effect of defects in γ, the birefringence of the retarder. These defects arise both from variations in thickness or crystalline imperfections in the retarder or stray birefringence in the Faraday rotation elements. Following Chabbal's [6] treatment of the defects of the Fabry-Perot etalon, we can assume that the aperture of the filter is divided into infinitesimal elements, each of which is perfect with a finesse N_R, but the peak transmission of each element occurs at a slightly different wavelength. The resulting total transmission is a composite of the transmission of each element, but will be broader than that of an Airy function of finess N_R, it will have a certain effective finesse N_E which will be a convolution of the Airy function of finesse N_R with a defects function of finesse N_D. Assuming these functions to be Gaussian, Chabbal gives N_D as the number of half wavelengths in the r.m.s. deviation from flatness and parallelism of the etalon and the effective finesse is given by

$$\frac{1}{N_E^2} = \frac{1}{N_D^2} + \frac{1}{N_R^2} . \tag{10}$$

Since the area under the composite transmission curve is equal to that under the Airy function peak of the ideal etalon, because the elements of the imperfect etalon contribute to the same total aperture, the transmission will be reduced by a factor N_E/N_R. Thus the effect of defects is taken into account by substituting N_E for N_R throughout Eqs. (7) and (9) except for the leading factor of N_R^{-1} in (7) and to multiply the resulting expression by N_E/N_R. The resulting transmission thus depends on N_E/N_R^2 and it is even more important to use the lowest value of N_R consistent with the birefringence defects than for the corresponding situation for an ordinary Fabry-Perot etalon because transmission decreases with N_R^2 rather than N_R with effectively no improvement in resolution, as can be seen from Eq. (10).

EXPERIMENTAL RESULTS USING A CONFOCAL CAVITY

While the theory of the GYMPI was developed assuming a simple Fabry-Perot cavity formed by plane parallel mirrors, it is soon apparent that some other type of cavity must be employed for the practical realization of the instrument. Because of the large number of optical elements placed within the cavity, the mirrors must inevitably be much smaller in diameter than the distance between them. Not only does this fact raise problems of alignment, but the field widening property of the retarder would not be effective because rays at large angles of incidence would not remain within the cavity for more than a few passes.

What is required is a cavity in which each mirror is reimaged on itself by the other mirror. The simplest such cavity is the confocal Fabry-Perot first proposed by Connes [7]. It consists of two identical spherical mirrors separated by their common radius of curvature. Such a cavity is very insensitive to misalignment and once set up will maintain its adjustment indefinitely in the ordinary laboratory environment.

The theory of the GYMPI using a confocal cavity is slightly more complicated than for a simple Fabry-Perot etalon as there are four passes in the cavity between partial beams instead of two. The resulting instrumental function is still an Airy function with half the reflective finesse and half the transmission given by Eq. (7).

A confocal cavity was formed of two spherical mirrors 5 cm in diameter with a radius of curvature of 20 cm. The Faraday rotators were realized by cylinders of special glass 3 cm in diameter by 3 cm long placed in water-cooled solenoids which were able to produce a field of 1000 G. The glass used was Schott type SF-57 with a Verdet constant at the hydrogen Balmer alpha wavelength of 0.07 min/Gauss-cm. The particular glass was chosen not only for its high Verdet constant, but for its extremely low photoelastic constant. A sample of the glass placed between crossed polarizers gave almost complete extinction across the field. This latter requirement is very important to minimize the effect of stray birefringence.

The retarder was initially a simple plate of synthetic crystalline quartz of 1 cm nominal thickness flat and parallel to $\lambda/20$. Fig. 1 shows a photograph of the assembled filter.

Figure 2 shows a spectrophotometer trace of the filter transmission for white light. The upper trace is the transmission for opposed Faraday rotation while the central trace is for parallel rotation. The maxima are spaced by 48Å which is the period of the channel spectrum of the quartz plate. As predicted by theory the maxima occur for wavelengths for which the retarder is a half-wave plate for opposed Faraday rotation and for wavelengths for which the retarder is a whole-wave plate for Faraday rotations in the same sense. The lower trace shows the effect of switching off one of the Faraday rotators. Now both whole and half-wave maxima are present with an intensity about ⅓ that obtaining with both Faraday rotators active. This result is likewise in agreement with theory.

The measured finesse from the trace is about 7. The defects finesse given by the tolerance of $\lambda/20$ on the retarder is 10. Silver mirrors of 85% reflectivity were used giving a usually defined reflective finesse of 20, but for a confocal cavity this becomes 10. From Eq. (10) we should thus expect an effective finesse of 7. The quantity y in Eq. (9) is 0.5 so the effect of the factor of $(1+y^2)^{-\frac{1}{2}}$ is to reduce the finesse by $\sim$10% which is within the errors of measurement.

To obtain an estimate of the defects finesse the mirrors were re-coated with multilayer dielectric coatings to a reflectivity of 95% giving a reflective finesse for the confocal cavity of 30. To reduce the effects of variable angles of incidence a Lyot type I wide-field retarder was used consisting of two matched quartz plates of 1 cm nominal thickness flat and parallel to $\lambda/60$. These were mounted with their optic axes perpendicular in a cell containing an index matching fluid. A mica half-wave plate was inserted between them with its axis at 45° to the axes of the quartz plate. If one assumes that the mica plate introduces no additional biregringent defects the resulting defects finesse should be 30. Thus one would expect an effective finesse of ~21. The factor y is now equal to 1.64 and the finesse of the measured profile would be reduced to ~11. The measured finesse is consistent with this value. A reduction in the angle of Faraday rotation was found to increase the finesse of the measured profile at the expense of the transmissivity of the filter as predicted by Eq. (7).

Figure 3 shows a spectrophotometer trace of the filter transmission for white light for the Lyot Type I field retarder. The free spectral range is now 24Å. Note that the immediate effect of wavelengths different from that for which the quarter-wave plate was made is an incomplete suppression of the maxima of opposite parity. The widths of the main transmission maxima are relatively unaffected.

It was found possible to replace the water-cooled solenoids by tubular permanent magnets. This removed the need for an external electric power supply and cooling water. Besides the inconvenience of supplying these external services, it was found that changes in cooling water temperature or magnet current induced thermal strains in the rotator glasses impairing the operation of a filter. Such problems were eliminated completely with the use of permanent magnets to supply the Faraday rotation field.

THE CONJUGATE CONCENTRIC-CAVITY; IMAGE-FORMING PROPERTIES OF THE GYMPI

The confocal cavity used in the preliminary work is the simplest type of self-imaging cavity but it suffers from the disadvantage of producing two exit images one of which is inverted with respect to the other. The image intensity is thus reduced by a factor of 2 and, more seriously, the filter must accomodate angles of incidence double that required by the angular extent of the image because of the need to separate the two images. This latter requirement becomes very serious when a thick retarder is used to obtain a very narrow bandwidth since the solid angle and resolution are fixed by Eq. (6).

The difficulty with the confocal cavity is the inversion of the alternate reflected images. What is needed is a cavity of magnification +1 instead of -1. Figure 4 shows two possible cavities with this property. In the first the mirrors are spaced by twice their radius of curvature i.e., their centres of curvature are coincident. In order that the cavity be self-imaging or stable a lens must be inserted at the common centre of curvature of the mirrors whose focal length equals that of the mirrors. This configuration is the conjugate concentric-cavity of Pole [8]. Another stable cavity of unity magnification can be formed using two lenses and two plane mirrors [9].

Another requirement to be met by the cavities is the minimization of aberrations. Any aberrations introduced by a double pass through the cavity will be multiplied in the higher order partial images with a subsequent degradation of image quality.

For our initial work we selected the first configuration and illuminated the cavity with an image near the plane of the lens i.e., at the common centre of curvature of the mirrors. Since the cavities are all symmetrical optical systems of unity magnification, coma and distortion of field should be absent. With the initial image at the common centre of curvature the aberrations of the lens have no effect as it is functioning essentially as a field lens. The spherical mirrors are imaging with unity magnification and hence contribute no spherical aberration. The only uncorrected aberration is the astigmatism of the mirrors. Figure 5 shows photographs of a standard resolution chart made through the instrument. With polarizers parallel the first few partial images contribute most to the final image and the resolution is quite good, exceeding 20 lines per mm. With polarizers crossed, however, the first few partial images are very faint because an insufficient number of Faraday rotations have accumulated to give appreciable transmission so partial images which result from several passes contribute principally to the final image, and the resolution has deteriorated to the order of 15 lines per mm. The effects of accumulating astigmatism can be seen by imaging a row of pinholes and slightly misaligning one of the cavity mirrors so as to separate the partial images along a direction perpendicular to the direction of the row. Subsequent partial images show astigmatism that increases with the order of the image, the rate of increase being greater for points further off the optic axis.

The effects of astigmatism and other off-axis aberrations can be eliminated by using the concentric cavity in a parallel beam with a stop at the center of curvature. In this case, however, the spherical aberration of the mirrors and lens must be corrected. This can be done for the mirrors by making them Mangin mirrors, or by inserting concentric meniscus corrector plates as in the Bowers-Maksutov objective. The spherical aberration of the lens must now be corrected by asphering, as in this configuration it is now an image-forming element rather than a field lens.

An alternative choice is the second configuration of Fig. 4 using plane mirrors. The two lenses must be aspherized to correct spherical aberation. To minimize the number of surfaces in the cavity the lenses could be realised by figuring the surfaces of the Faraday rotation elements.

To summarise, the optical design considerations for the cavity used in the GYMPI are essentially those of an iterative image relaying system such as a periscope. That such systems can be designed to produce satisfactory image quality is well-known. In this preliminary work we have been concerned only with the most elementary cavity designs employing simple spherical surfaces.

Figure 6 shows a spectrophotometer trace of white light transmitted through the conjugate concentric-cavity version of the GYMPI. The finesse is essentially the same as for the confocal cavity. The only remarkable additional feature is the complete suppression of the maxima of opposite parity, the transmission going to zero midway between the peaks. This effect is predicted by Eq. (7) which was derived for two passes through the cavity between partial images which obtains for the concentric cavity rather than four passes for the confocal cavity.

An attempt was made to use a Lyot Type I retarder of calcite in this instrument. This retarder consisted of two calcite plates each 2 cm thick with their axes crossed and a compound quartz half-wave plate inserted between them. Such a retarder had a free spectral range of 0.6Å, and, had a finesse of 10 been achieved as with the quartz retarder, the filter would have had a band width of 0.06Å. The results were negative, because the calcite plates were not of sufficiently good optical quality. The resulting compound retarder when placed between polarizers without any cavity or Faraday rotators gave a channel spectrum with a rather poor contrast so its performance in the GYMPI could not be expected to be very satisfactory.

CONCLUSIONS

The theory of the GYMPI as developed in Ref. 3 has been completely verified by experiment. The utility of the instrument as a practical tool for research in solar physics is another question. The chief drawback of the device is its inherently low transmission. Another disadvantage is the requirement for a relatively high resolution premonochromator filter to isolate an order as the practically achievable finesse is 10-12. Nonetheless if the existing instrument can be made to function with a thick calcite retarder it promises to extend the spectral resolvance of the birefringent filter by an order of magnitude. Hence many observations made heretofore with a spectroheliograph can now be accomplished with a birefringent filter with the considerable improvement of time resolution that comes from the simultaneous recording of all image points rather than the strip-by-strip recording of the former device. For certain problems such as the mapping of the solar magnetic field in active regions by Zeeman effect during the evolution of a solar flare this improved time resolution may be essential in revealing the phenomena involved. The chief outstanding problem with this instrument is whether it can be achieved with crystals of higher birefringence than quartz. Large pieces of calcite of the highest optical qualify are rare and expensive and, in view of the experience of Steel, Smartt, and Giovanelli [10], there is some question as to whether natural crystals of sufficient uniformity of birefringence can be obtained. Alternatively, synthetically grown crystals of high birefringence such as lithium niobate or rutile could be used. Such synthetic crystals can be expected to be free of lattice imperfections, so that birefringent defects arise only from the optical tolerances in working the surfaces.

ACKNOWLEDGMENT

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required tolerances by Mr. H. Yates of Optical Surfaces Ltd., of Kenley, Surrey.

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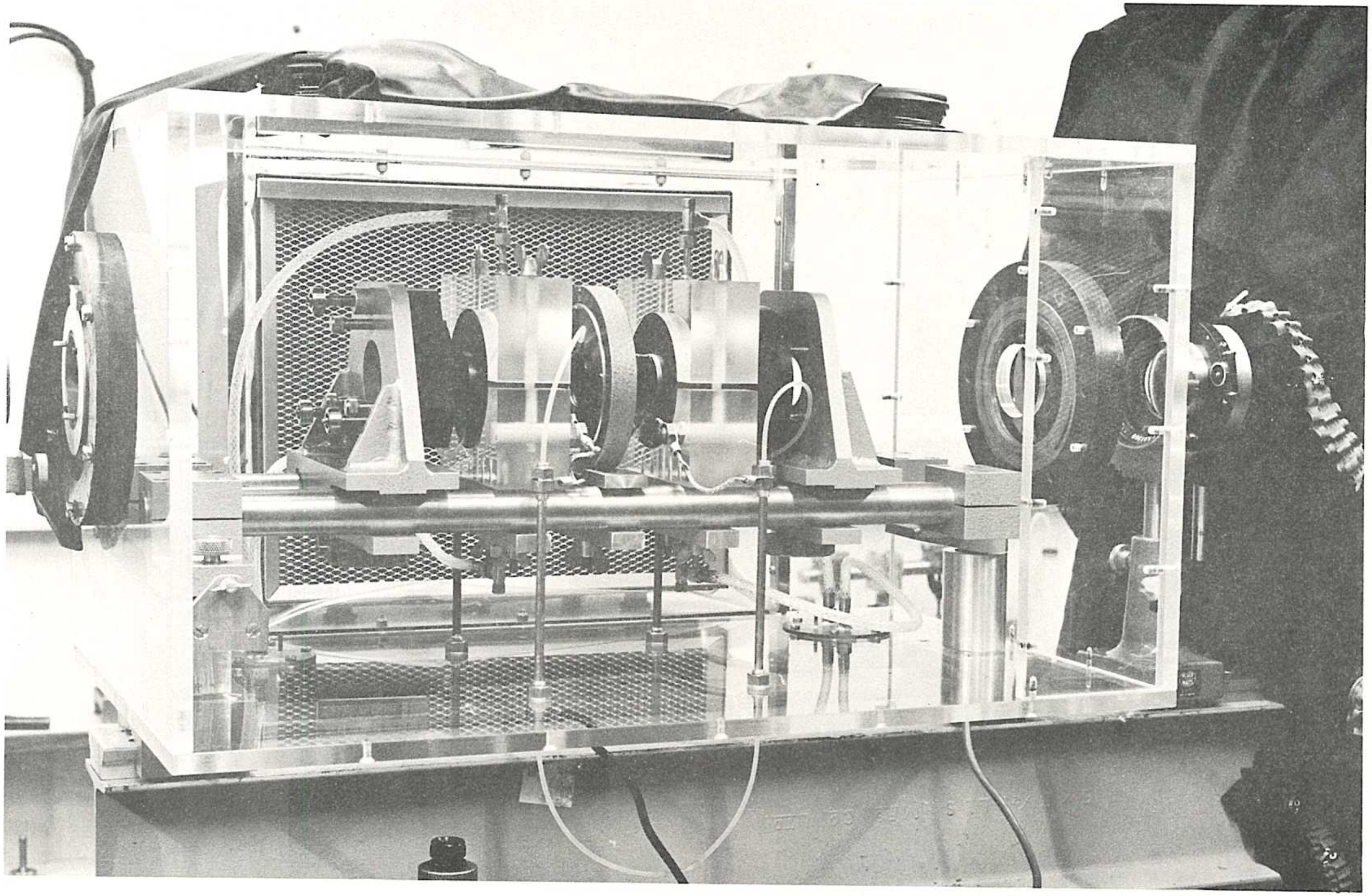


Figure 1. Photograph of the assembled filter. CLM-P256

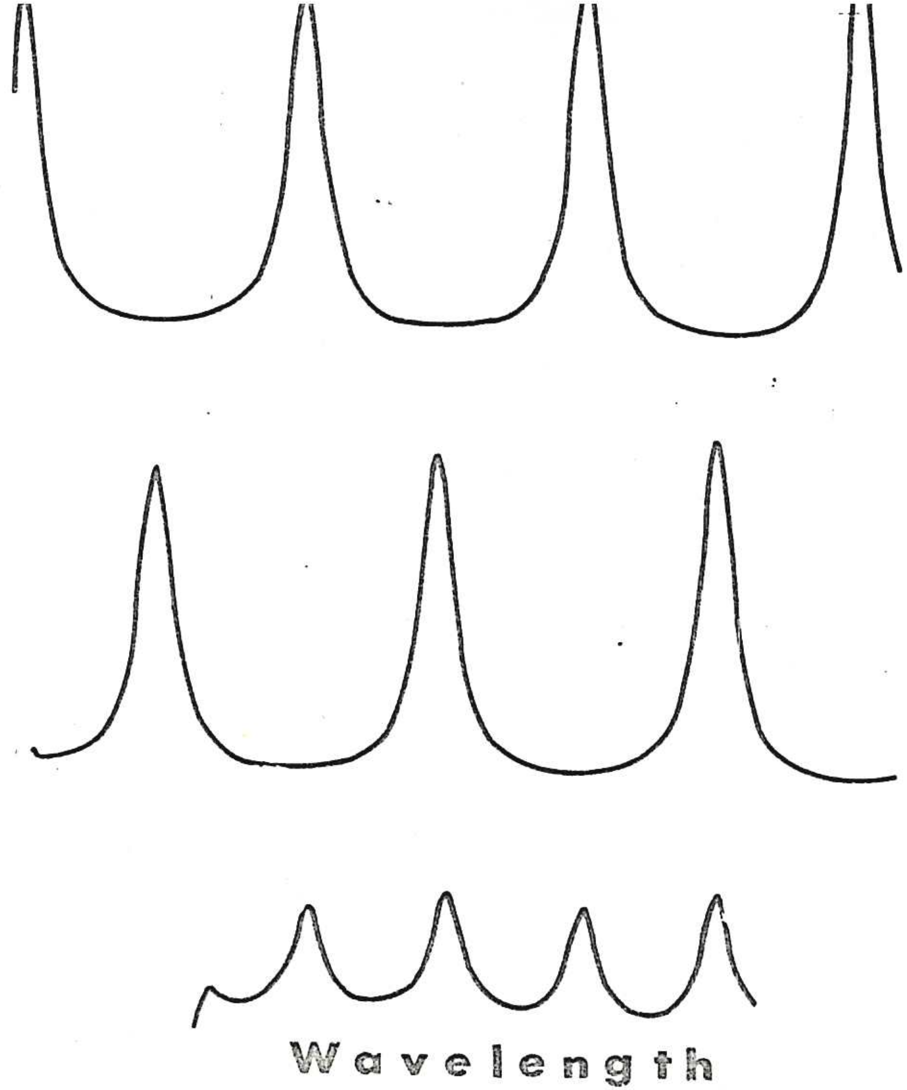


Figure 2. Spectrophotometer traces of the filter transmission for white light. A. Opposed Faraday Rotations, B. Parallel Faraday Rotations, C. One Faraday rotator switched off. The spectral interval between maxima in A and B is 48Å.

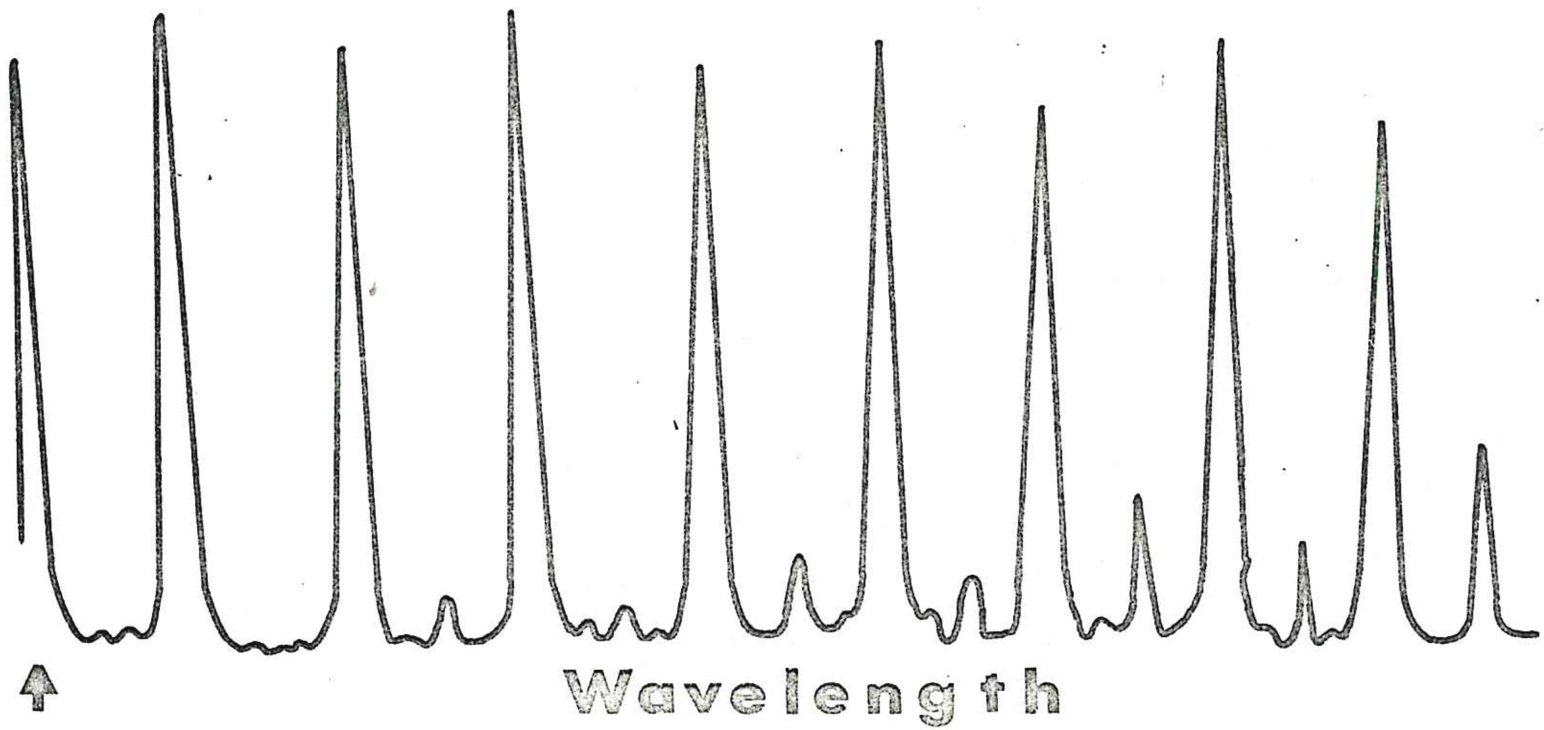


Figure 3. Spectrophotometer trace of the filter transmission for white light using a Lyot Type I wide field retarder. The spectral interval between peaks is 24Å. The arrow denoted the wavelength of 6700Å for which the mica wave plate is an exact half wave plate.

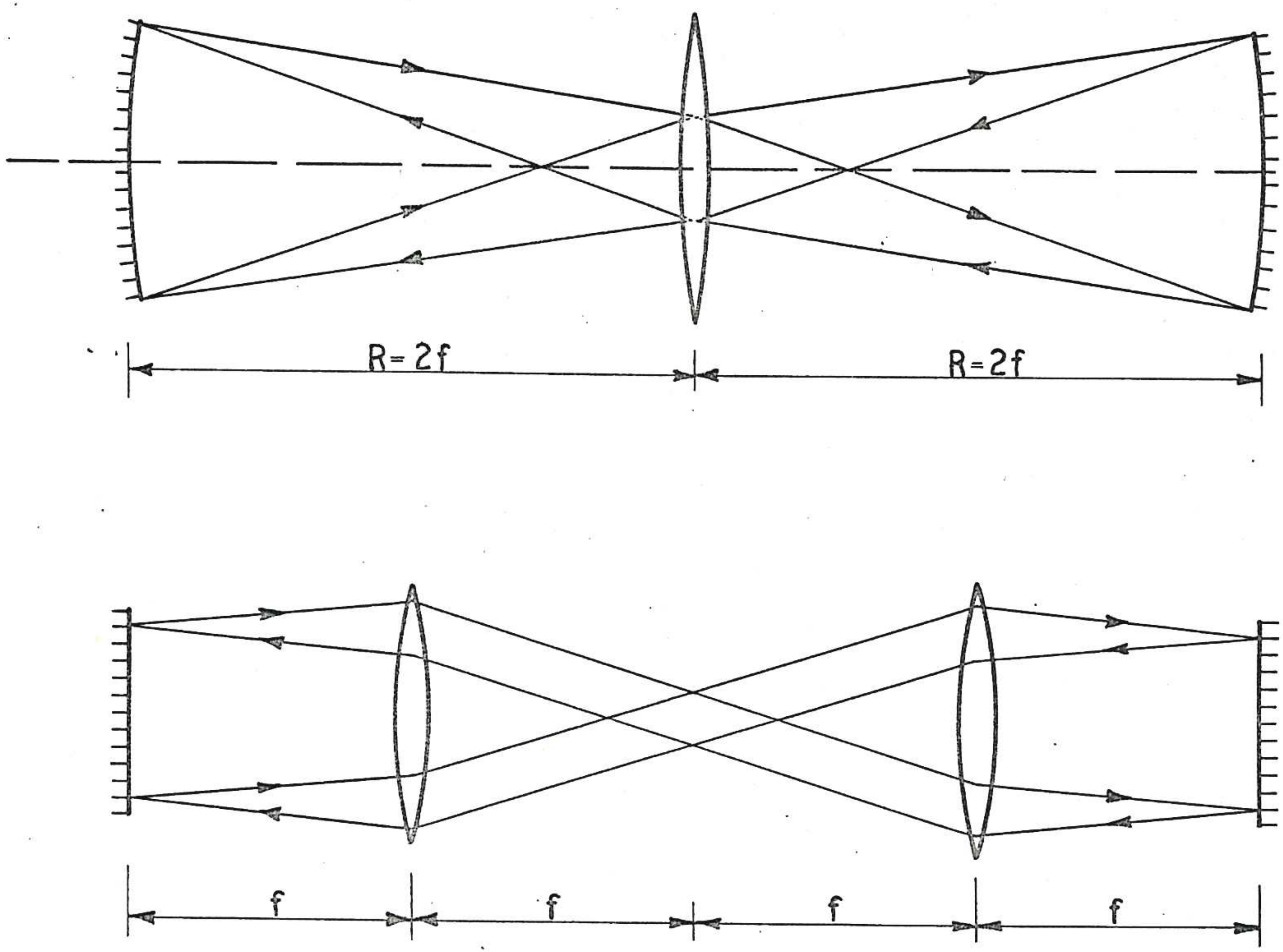
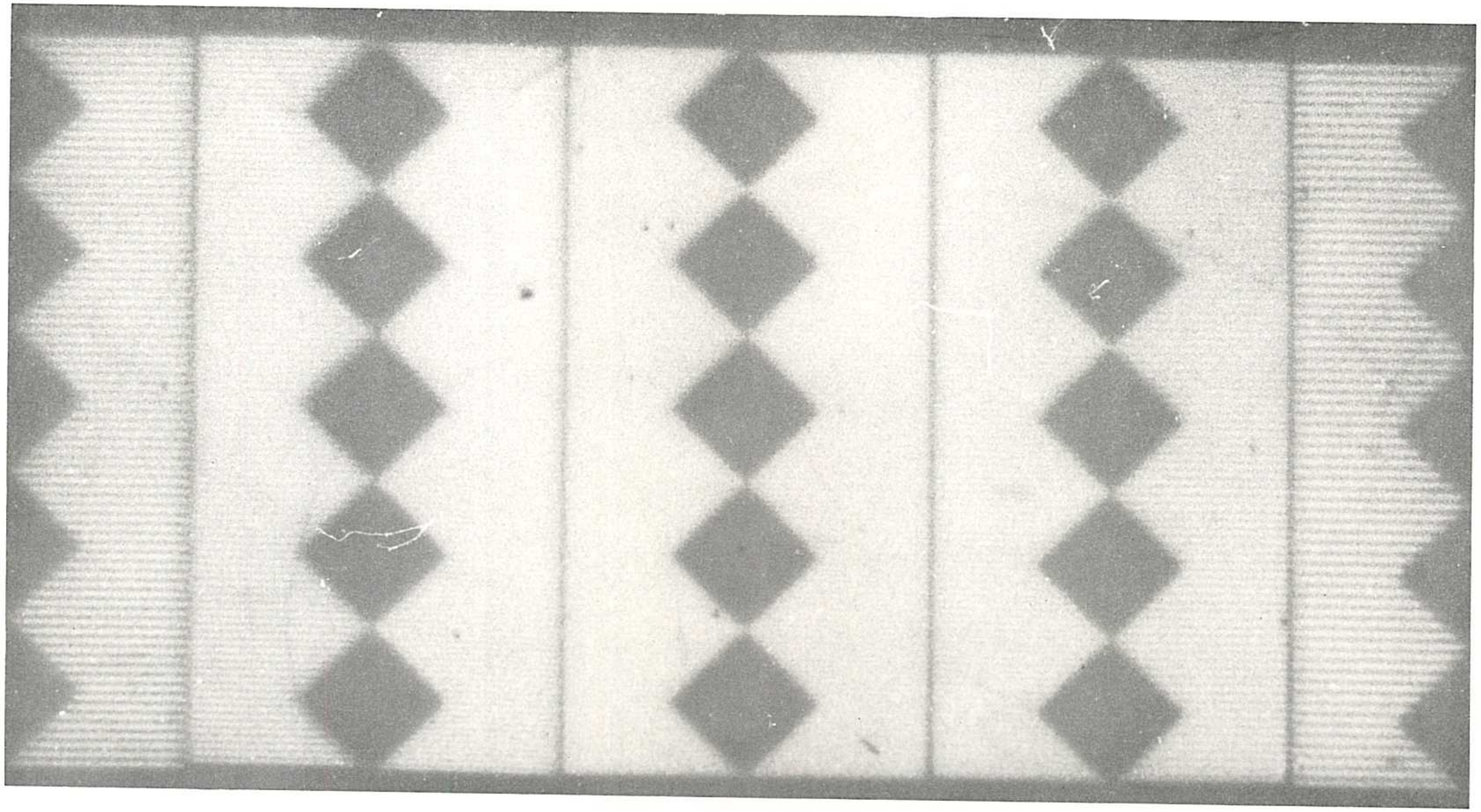


Figure 4. Two examples of self-conjugate optical cavities that have only a single output beam.



13 16·5 19·5 23 21 18 14·5 10·5

Fig.5A Resolution Chart photographed through the filter; with the polarizers parallel. Numbers refer to line pairs per mm. CLM-P 256

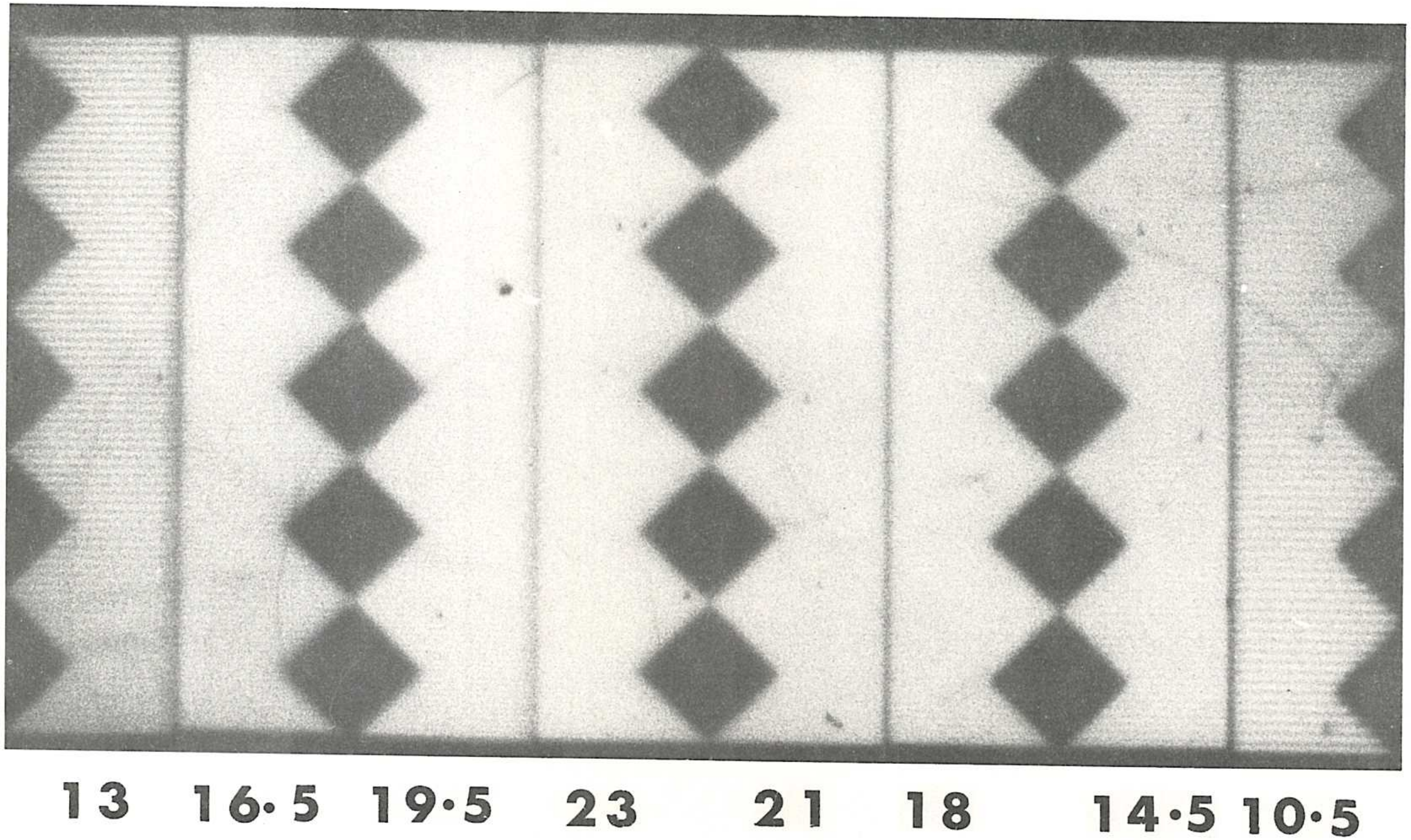


Fig.5B Resolution Chart photographed through the filter; with the polarizers crossed.
Numbers refer to line pairs per mm. CLM-P 256

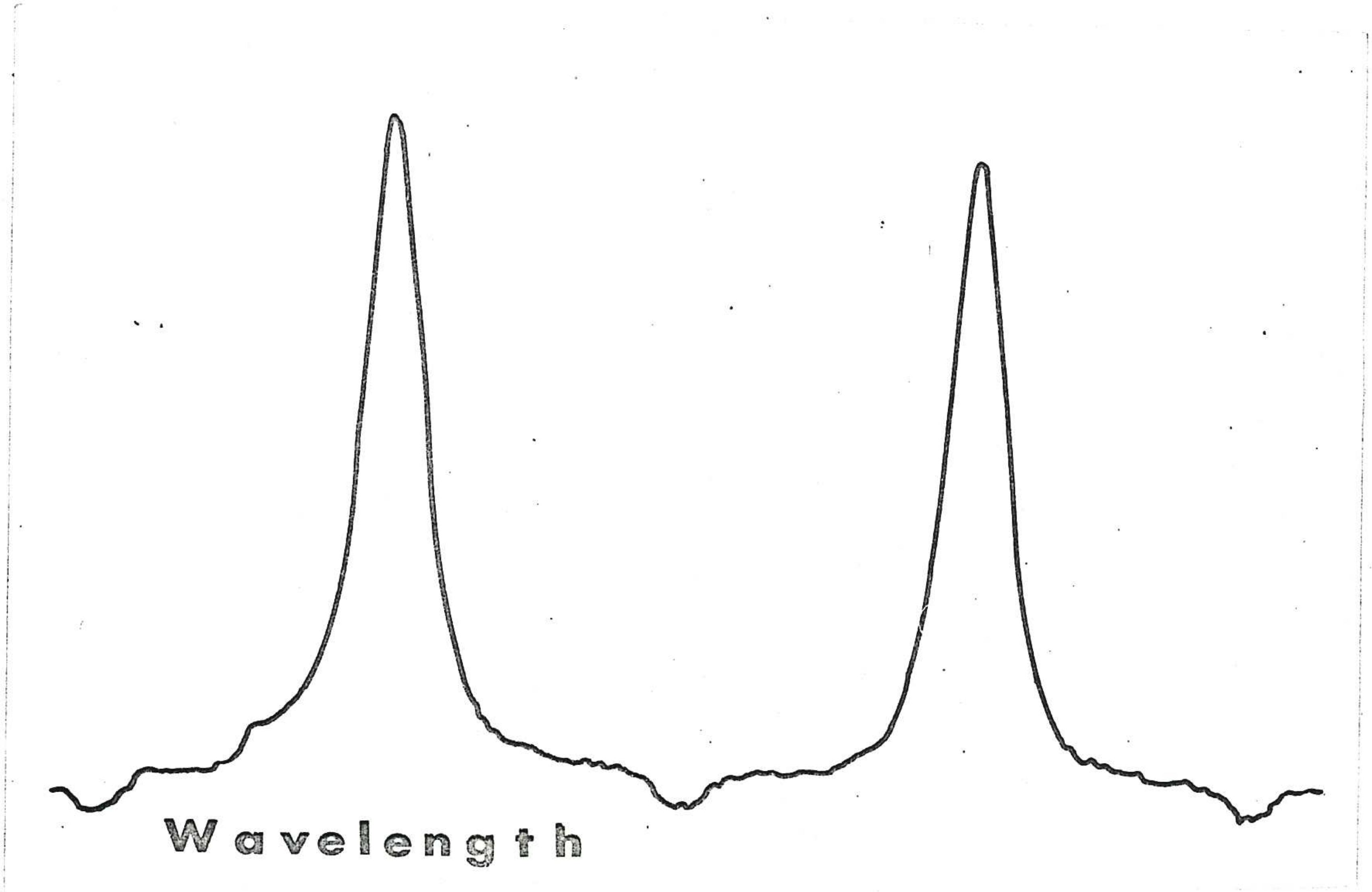


Figure 6. Spectrophotometer trace of the filter transmission using the conjugate-concentric cavity.