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# NON-LINEAR TRANSMISSION OF AN OPTICAL SIGNAL

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# NON-LINEAR TRANSMISSION OF AN OPTICAL SIGNAL

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## A B S T R A C T

The dependence of phase and modulation depth on the frequency and intensity of an optical signal is examined for the case of transmission by a saturable absorber.

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The dynamic transmission equations for an optically thick saturable absorber were first given by Gires and Combaud[1] in their study of the response of phthalocyanine dye solutions to intense light pulses. Such absorbers are of interest for laser pulse shaping because of their intensity dependent transmission and finite relaxation time[2]. However, although the equations also possess periodic solutions[3], no systematic treatment of these exists. It is the purpose of this communication to discuss the perturbation solution for the transmission of a simple periodic light signal, paying particular attention to modulation amplitude and phase. Since a rate equation analysis is used, only non-resonant interactions are considered i.e. the signal frequency $\Omega$ satisfies the requirement $\Omega\tau_2 \ll 1$, where $\tau_2$ is the transverse relaxation time associated with the homogeneous width of the absorbing transition. No such approximation is necessary in respect of the longitudinal relaxation time $\tau_1$ of the excited state. Rather it is just that frequency range where $\Omega\tau_1 \sim 1$ which is of interest in examining signal transmission.

The transmission equation for a two-level model is[3]

$$\frac{dz}{d\tau} + z = 2\beta\, f(\tau) \left[ 1 - T_0 e^z \right] \qquad \ldots (1)$$

where $z = \ln T/T_0$, $T$ and $T_0$ are the instantaneous and initial transmittance respectively, $\tau$ is dimensionless time in units of the excited state lifetime $\tau_1$, $\beta$ is the ratio of the incident intensity to the saturation intensity, and $f(\tau)$ is the input signal. For

$$f(\tau) = 1 + \delta g(\tau) \qquad \ldots (2)$$

with $\delta \ll 1$ and the modulation function $|g(\tau)| \leqslant 1$, the solution of eq.(1) is

$$z(\tau,\delta) = z_o \left[ 1 + \delta G_1\ (\alpha,\tau) \ - \delta^2(\alpha - 1)G_2\ (\alpha,\tau) + \ \ldots \right]$$

where

$$z_o = 2\beta \left[ 1 - T_o e^{z_o} \right]$$

$$\alpha = 1 + 2\beta - z_o$$

$$G_1(\alpha,\tau) = e^{-\alpha\tau} \int g(\tau)\ e^{\alpha\tau}\ d\tau$$

$$G_2(\alpha,\tau) = e^{-\alpha\tau} \int \left[ g(\tau) + \tfrac{1}{2} z_o G_1\ (\alpha,\tau) \right] G_1\ (\alpha,\tau)\ e^{\alpha\tau}\ d\tau$$

... (3)

(omitting transients) and the transmitted signal takes the form

$$U(\tau) = T_1 \left[ 1 + \delta \left\{ g(\tau) + z_o G_1\ (\alpha,\tau) \right\} + O(\delta^2) \right]$$

with

$$T_1 = 1 - z_o/2\beta \ .$$ ... (4)

For single frequency modulation

$$g(\tau) = \sin\omega\tau$$

$$\omega = \Omega\tau_1 \ ,$$ ... (5)

it follows from (3) and (4) that

$$U(\tau) = T_1 \left[ 1 + \delta K \sin(\omega\tau - \psi) + O(\delta^2) \right]$$ ... (6)

where

$$K^2 = \frac{(\alpha + z_o)^2 + \omega^2}{\alpha^2 + \omega^2}$$

and

$$\tan\psi = \frac{\omega z_o}{\alpha(\alpha + z_o) + \omega^2} \ .$$

Thus the transmitted signal is undistorted to 1st order in $\delta$, the effect of the saturable absorption being to introduce a phase lag $\psi$ and to increase the modulation depth by a factor K (Fig.1). The value of $\psi$ is a maximum when $\omega = \omega_1$, where

$$\omega_1^2 = \alpha(\alpha + z_o) = \alpha(1 + 2\beta) \qquad \ldots (7)$$

whence

$$\tan \psi_m = z_o/2\omega_1$$

and

$$K(\psi_m) = (1 + z_o/\alpha)^{\frac{1}{2}}$$

Values of these parameters, and those of $z_o$, $\alpha$ and $T_1$, defined in eqs.(3) and (4), are given in Table 1 for a range of values of $\beta$, the incident intensity parameter. It can be seen that both the enhancement of modulation depth and the phase lag are maximised for $\beta \sim 1$.

Thus by incorporating saturable absorbers in an opto-electronic network it is possible to increase the intensity modulation, a result known from experimental studies of mode-locking in laser oscillators[4], and one which is relevant to the generation and detection of signals in optical communication systems.

TABLE 1

Dependence of various parameters on $\beta$

$T_o = 0.01$

| $\beta$ | $z_o$ | $\alpha$ | $T_1$ | $K(\psi_m)$ | $\omega_1$ | $\psi_m$(deg.) |
|---|---|---|---|---|---|---|
| $\infty$ | 4.606 | $\infty$ | 1.00 | 1.00 | $\infty$ | 0 |
| 10 | 4.37 | 16.63 | 0.782 | 1.12 | 18.7 | 6.7 |
| 5 | 4.07 | 6.93 | 0.593 | 1.26 | 8.73 | 13.1 |
| 2 | 3.11 | 1.89 | 0.223 | 1.63 | 3.08 | 26.8 |
| 1 | 1.87 | 1.13 | 0.065 | 1.63 | 1.84 | 26.9 |
| 0.5 | 0.97 | 1.03 | 0.03 | 1,39 | 1.43 | 18.7 |
| 0.25 | 0.49 | 1.01 | 0.02 | 1.22 | 1.25 | 11.1 |
| 0.1 | 0.2 | 1.00 | 0.01 | 1.1 | 1.1 | 5.2 |
| 0 | 0 | 1.00 | 0.01 | 1.00 | 1.00 | 0 |

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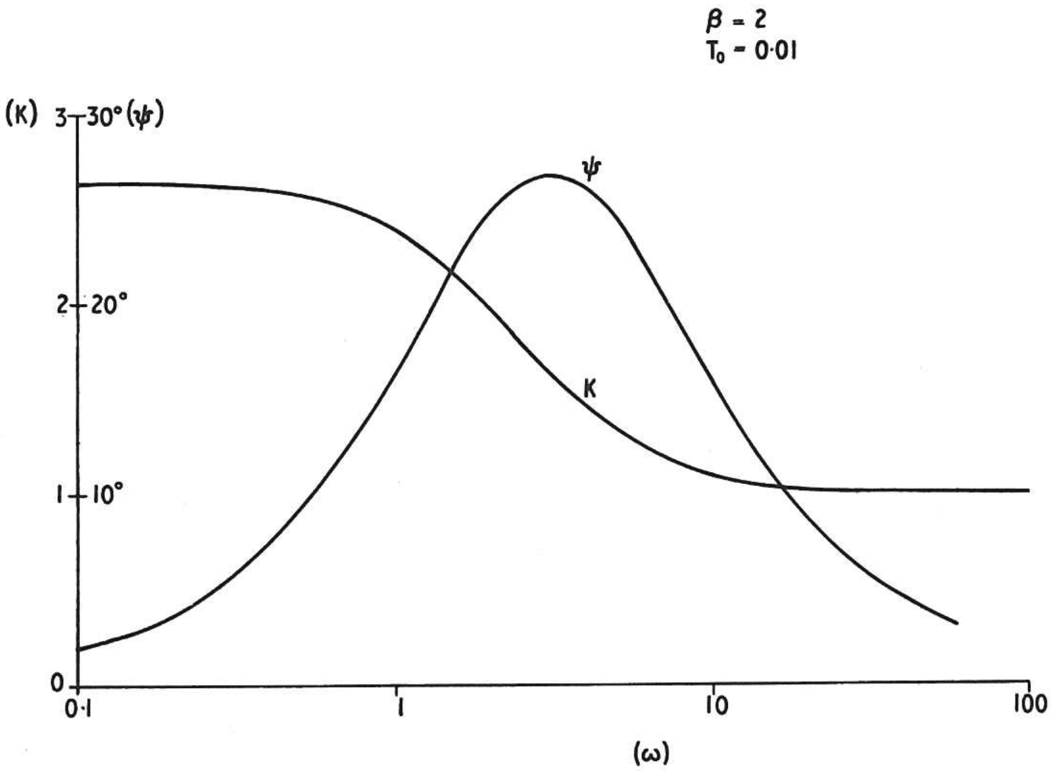


Fig. 1. Frequency variation of K and ψ.

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