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FEEDBACK EXPERIMENTS ON A HIGH ENERGY PLASMA IN THE PHOENIX MIRROR MACHINE

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FEEDBACK EXPERIMENTS ON A HIGH ENERGY PLASMA IN THE PHOENIX MIRROR MACHINE

by

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ABSTRACT

Experiments using a single feedback loop, by which electrostatic signals from a plasma were amplified and fed back as potentials to the plasma boundary, have been carried out in the PHOENIX II mirror machine working as a simple mirror. The results of experiments on normally stable plasma are compared with a theory which takes account of the frequency response of the feedback loop and reasonable agreement has been obtained. The effect of the feedback system on plasma losses caused by the flute instability has also been studied and correlation between plasma losses and the amplitude and frequency of the instability has been obtained. It is concluded that plasma losses can be controlled, to a certain extent, by a simple feedback system but that a more complex scheme would be necessary for complete stability.

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1. INTRODUCTION

Experiments in the OGRA II Mirror Machine using a feedback system[1-8] have been reported by Arsenin, Zhiltsov and Chuyanov[9] and it was shown that the flute instability which is normally present in that plasma was suppressed to the extent that a higher density and a lower fluctuation amplitude was achieved. Since then, feedback stabilisation of drift oscillations has been reported in plasmas with higher densities than those obtainable in OGRA II and a general treatment of feedback has been published by Taylor and Lashmore-Davies[10]. In the latter, two different types of instability are distinguished, those in which a single mode of oscillation is involved which may have positive or negative energy (the dissipative type) and those in which two waves are involved, one having positive and the other negative energy (the reactive type). The dissipative type is relatively easily stabilised by a small amount of feedback over a wide range of phase and has been shown to be so[11-16] in experiments on dissipative drift instabilities, whereas stabilisation of the reactive type demanded severe restrictions on the phase of the feedback. The flute instability present in OGRA II and PHOENIX II, when operated as simple mirror machines without energising the minimum B windings, is such a reactive type and causes serious plasma losses. If feedback methods are to become an alternative to minimum B systems for stabilising the flute instability in possible mirror reactors, it is necessary to determine which parameters are critical for stabilisation and what effect such a system has on plasma losses.

The theory of Arsenin and Chuyanov[2] showed two regions of stability with a feedback system, one when the potential feedback is such as to oppose the perturbed potential, here called negative feedback, and the other when the applied potential is in the same sense as that created by the plasma. The former region does not exist for plasmas with diffuse boundaries although some improvement in threshold density is possible.

Though an experiment was made using negative feedback and is described in Section 8, attention has been concentrated on the effects of positive feedback for which stability was predicted to be achievable at all densities. Section 4 describes experiments on stable plasmas and discusses the linear effects of a feedback system taking into account phase shifts introduced by the finite bandwidth of the feedback loop and the three succeeding sections describe experiments on unstable plasmas and the effects of feedback on plasma losses.

2. EXPERIMENTAL EQUIPMENT

In the PHOENIX II experiment[17,18] a monoenergetic plasma is built up by Lorentz ionization of a 15-20 keV neutral hydrogen atom beam as it is injected across the confining magnetic field which could be varied between 10 and 15 kG. The resulting plasma density was determined by the equilibrium between the trapped beam current and the ion loss processes which, in the case of stable plasmas, is charge exchange with background gas. However, at a density of 2-3 x 10^8 cm^{-3}, depending on the injection energy, an m = 1 flute instability appears and further attempts to increase the density beyond this value are frustrated by the losses due to this instability.

In these experiments a single feedback electrode was used and is schematically shown in Fig.1 which indicates the position of the electrodes relative to the plasma. The system consisted of an electrostatic probe in the form of a widely spaced grid on a circular frame inside an earthed electrostatic screen, an amplifier, a means of shifting phase and a control electrode symmetrically placed about the sensing probe. The entire system was placed in the chamber through a side window and arranged so that the probe was 8.5 cm from the chamber axis. No other diagnostic equipment was placed deeper in the chamber than this probe which therefore determined the radial dimension of the plasma; the plasma formed a disk with radius not exceeding 8.5 cm. The axial dimension of the plasma was not measured but according to measurements in similar geometry on PHOENIX 1A[19] the effective width of the axial density distribution was about 2 cm.

The measured input capacitance C_i was 110 pF, and stray feedback capacitance $C' \approx 0.03$ pF. The effective capacitance between the probe and the plasma was not precisely known; assuming that the distance between the effective surface of the plasma and the surface of the probe is one Larmor radius, then according to model measurements, the effective capacitance C^* was measured to be 0.1 pF, for a magnetic field B_o = 10 kG and an ion energy 20 keV. The input impedance of the amplifier was made large so that at flute frequencies the probe operated as a capacitative divider and the transfer coefficient (i.e. the ratio of the control electrode potential to the potential of the effective plasma surface) may be written

$$\delta = \frac{C^*}{C_i} \, \frac{K}{1 - C' K/C_i} \qquad \ldots (1)$$

where K is the amplifier gain.

Substituting the measured properties of the system into this formula gives a loop gain of unity when K is approximately 10^3; for $K > 3 \times 10^3$ the system must be self-excited and this was observed in the absence of the plasma. In the presence of plasma C' is altered and the system could therefore operate in practice even at higher amplification. The effective value of δ will evidently be affected by there being only one electrode forming a part of the boundary. From elementary arguments one would expect that the effect of the control electrodes would be diminished by a factor less than $\pi/\delta\theta$ where $\delta\theta$ ($\approx 80^o$) is the angular size of the electrode, i.e. by less than a factor of two. However, as there exists no theory for a one-electrode system and there may also be other influences this factor was ignored.

3. DIAGNOSTICS AND EXPERIMENTAL PROCEDURE

In these experiments the standard diagnostics for PHOENIX II[17,18] were used. A multi-grid end plate placed beyond the mirror measured the current of cold ions leaving the trap along the magnetic lines of force. A photomultiplier, collimated to receive charge exchanged ions from the centre of the plasma, recorded the flux density of fast atoms. The amplitude spectrum and spatial structure of the low frequency (less than 1 MHz) oscillations were recorded by electrostatic probes and the high frequency oscillations of the ion cyclotron harmonics by loop aerials. Relative changes of plasma density at the centre of the plasma were estimated from the photomultiplier signal and the line density ($\int n \, d\ell$ where ℓ is the effective length of the plasma) was quantitatively determined from the cold ion current and the decay time constant of the plasma.

The time sequence of the experiments was as follows. The central magnetic field strength (B_o) rose to between 10 and 15 kG depending on the chosen value and was maintained at that value for about 4 seconds. A beam of fast hydrogen atoms in the energy range 15-20 keV was injected twice, once for a period of 0.6 s and then for 1.5 s with an interval of about 0.5 s, when it was possible to observe the decay of the plasma and measure its decay time. At the end of the second injection pulse a gas valve was operated which injected hydrogen into the central chamber so that the plasma density fell to a low value (about 5-10 x 10^7 cm^{-3}). The injected gas pressure was so chosen that it was low enough for ionization of the neutral beam to be unimportant compared to the contribution by Lorentz ionization and yet high enough to reduce the plasma density to that at which it was completely stable and the only loss process was charge exchange. This enabled the resulting signals on the end plate and photomultiplier to be used as standards against which density changes, which were not due to the equilibrium between Lorentz trapping and charge exchange, could be measured.

The feedback system could be switched on and off as required during the experimental pulse and the kind of amplifier used in the feedback loop varied from experiment to experiment. The output amplifiers were capable of supplying 150 V peak-to-peak into the feedback probe. In the case of experiments on the dynamic range of the amplifier (Section 7), special arrangements were made to vary the maximum output voltage by using loads and transformers.

4. EXPERIMENTS ON NORMALLY STABLE PLASMA

Because application of feedback to a flute unstable plasma is known to change the plasma losses caused by the flute instability, it was preferable to study the linear effects of feedback on a normally stable plasma where the effect of density changes can be neglected and where higher radial and azimuthal modes were not expected to be present. Consequently the stability of a normally stable plasma was examined as a function of gain in the feedback loop, the criterion of instability being large amplitude potential fluctuations observable on electrostatic probes. Positive feedback ($\delta > 0$ in equation (2)) was used. Fig.2 shows central end plate current, frequency of electrostatic oscillations and injected beam and indicates the time sequence in those experiments. The feedback system was switched on for 0.4 s during the second period of beam injection and it can be seen that the plasma density as shown by the end-plate current did not change. However, at all gains but the lowest gains used, an instability was present as shown by the presence of detectable amplitude and frequency oscillations and Fig.3 shows the developing waveform from the moment the feedback system was switched on. In all cases except those with very low gain, the amplitude of the instability was limited to a value fixed by the overload characteristics of the amplifier system, but at low gain the limiting amplitude was smaller than this and presumably fixed by energy transfer rates within the plasma.

A plot of the frequency of the instability as a function of loop gain was made for a plasma whose density was just below the threshold of instability (by not more than 10%) and is shown in Fig.4 where the loop gain has been calculated from the known probe capacities and equations (1) and (4). The frequencies have been normalised to the magnetic drift frequency of the ions which in these experiments was 285 kHz (for 15 kV protons and B_o = 11 kG). The frequency of the unstable wave was also measured as a function of phase shift by introducing various lengths of delay cable and calculating the phase shift from the resultant unstable frequency. The results are shown in Fig.5 for a loop gain of 1.12.

These results are at variance with the predictions of the elementary theory[3,1]. In ref (3) the dispersion relation for flute oscillations ($\phi = \phi_o \; e^{i(m\theta-\omega t)}$) of a cylindrical plasma with radius a, a sharp boundary and a wall at radius b on which a potential is maintained at each azimuth equal to δ-times that on the plasma surface is given as:

$$\frac{\delta - \left(\frac{b}{a}\right)^{|m|}}{\left(\frac{b}{a}\right)^{|m|} - \left(\frac{a}{b}\right)^{|m|}} = \frac{\omega_{pi}^2}{2\,\omega_{ci}^2} + \frac{m\,\omega_{pi}^2}{2\,\omega_{ci}\,|m|}\left(\frac{1}{\omega} - \frac{1}{\omega + m\,\omega^*}\right) \qquad \ldots (2)$$

where ω^* is the ion precession frequency in the magnetic field gradient, ω_{pi} the ion plasma frequency and ω_{ci} is the ion cyclotron frequency.

If the transfer coefficient δ is independent of frequency eq.(2) is a quadratic in ω and its solution is

$$\omega = -\,m\,\omega^* \left\{ \tfrac{1}{2} \pm \sqrt{\tfrac{1}{4} - \frac{n^*}{(1-\delta_n)|m|}} \right\} \qquad \ldots (3)$$

where

$$\delta_n = \delta \Big/ \left\{ \left(\frac{b}{a}\right)^{|m|} + \frac{\omega_{pi}^2}{2\,\omega_{ci}^2}\left[\left(\frac{b}{a}\right)^{|m|} - \left(\frac{a}{b}\right)^{|m|} \right] \right\} \qquad \ldots (4)$$

is the normalised loop gain and

$$n^* = \frac{\left\{ \left(\frac{b}{a}\right)^{|m|} - \left(\frac{a}{b}\right)^{|m|} \right\} \dfrac{\omega_{pi}^2}{2\,\omega_{ci}\omega^*}}{\left\{ \left(\frac{b}{a}\right)^{|m|} + \dfrac{\omega_{pi}^2}{2\,\omega_{ci}^2}\left[\left(\frac{b}{a}\right)^{|m|} - \left(\frac{a}{b}\right)^{|m|} \right] \right\}} \qquad \ldots (5)$$

is the normalised density taking geometric factors and changes in dielectric constant into account. At the experimental densities $\omega_{pi} << \omega_{ci}$ and the factors are purely geometric. In this case for a plasma whose density is a little above the density threshold of instability for the m = 1 mode ($n^* \simeq \frac{1}{4}$), eq.(3) predicts two roots. For $0 < \delta_n < 1$ the two waves have the same frequency with the real part equal to $\omega^*/2$, one being stable and the other unstable, and this real part remains constant as δ_n increases while the imaginary part of the unstable wave increases rapidly as δ_n approaches 1. For $\delta_n > 1$ two marginally stable roots are predicted with waves travelling in opposite directions, one root tends to ω^* and the other to zero as δ_n approaches infinity.

In the experiments the real part of the frequency began to decrease at low values of gain and continued to decrease over the whole of the range explored with no sign of a discontinuity at a loop gain of unity.

Though the comparison is between a model incorporating an infinite number of infinitesimal feedback electrodes and an experiment with one electrode of finite size, the discrepancy between theory and experiment is not thought to be due to this difference. The finite size of the stabilising electrode will cause a coupling between azimuthal modes and generate higher harmonics but for m = 1 this effect is unlikely to be significant. Observations of unstable oscillations show that, at densities not exceeding the flute instability threshold, practically no generation of higher harmonics was seen except at very high gain.

Putting frequency dependent terms into the transfer coefficient would appear to be more significant, because it is known from the work of Taylor and Lashmore-Davies that the stability of the system is very sensitive to phase shift. If $d\delta/d\omega \neq 0$, eq.(2) is of a higher order (depending upon the function $\delta(\omega)$ used) and is more difficult to analyse, but there is no difficulty in solving this equation numerically. Consideration of the schematic diagram of the feedback amplifier shown in Fig.1 shows that a more exact comparison of theory and experiment could be made if the function $\delta(\omega)$ is expressed as

$$\delta(\omega) = \frac{\delta_o \, \omega \, \omega_h \, e^{i \Delta \theta}}{(\omega_h - i\omega)(\omega + i\omega_\ell)} \qquad \ldots (6)$$

where ω_h and ω_ℓ are the upper and lower limiting frequencies of the amplifier and are equal to $1/C_oR_o$ and $1/C_iR_i$ respectively; $\Delta\theta$ is a phase shift between the input and output signal which is assumed independent of frequency. Here δ is purely real only for a frequency ω_o such that $\omega_o^2 = \omega_h \, \omega_\ell$.

Solving equation (2) for ω with such a frequency dependence gives four roots, one of which, with a frequency near ω_ℓ, is stable for all values of δ_o and is ignored. The real parts of the remaining three roots, normalised to ω^*, are shown schematically as a function of loop gain in Fig.6 for a plasma whose density is less than the threshold density for flute oscillations in the absence of feedback. It can be seen that, of these roots, two originate from waves in the plasma, being derived from the negative energy ion branch and the positive energy electron branch of the dispersion equation with $\delta = 0$. This theory predicts that one of these roots will be unstable for all but the lowest densities. The unstable root is determined by the sign of the imaginary part of $\delta(\omega)$ i.e. on whether the feedback system is feeding energy into or taking it out of the plasma. In either case the growth rate for large amplification is of the order of ω_h being the maximum rate of transfer of energy from the feedback system. The remaining wave, propagating in the opposite direction to the others, can be considered as an amplifier root and is predicted to be stable for moderate gains but can be unstable for high gains with growth rates much lower than that of the unstable plasma root.

In the experiments $\omega_\ell/\omega^* = 0.008$ and $\omega_h/\omega^* = 1.35$; the measured phase angle at the centre frequency ω_o was zero to within $\pm 5^\circ$. The unstable root for the experimental conditions is plotted as a function of gain for zero phase shift in Fig.4 and as a function of phase shift for a gain of 1.12 in Fig.5. The agreement between theory and experiment shown in Fig.4 is good except at high values of loop gain. Comparing the results considered as a function of phase shift the agreement can be said to be no more than reasonable. At higher gains the discrepancy became much greater and at a loop gain of 1.6 the frequency became almost independent of phase shift, whereas theoretically near to zero phase it should be very sensitive to phase shift. The discrepancies at high values of loop gain are attributed to stray capacities not accounted for in the theoretical model; these are such as to cause the feedback system to oscillate in the absence of plasma at a loop gain of about three.

Measurements of growth rates of the instability from photographs such as shown in Fig.3 give values which are an order of magnitude lower than those predicted by the theory. This may be a consequence of the fact that the precessional drift frequency of the ions is not constant throughout the plasma, thus giving rise to some damping of the wave.

It seems, therefore, that the frequency response of the feedback loop is very important in determining the properties of the feedback-plasma system and that the predictions of a simple theoretical model are reasonably well borne out by experiment. If, as seems likely, the theoretical model is adequate then, in contrast to the case of dissipative instabilities, a simple feedback system will not be sufficient to stabilise the flute instability and a more complicated phase-gain relationship would be necessary for such a purpose. However, even with this simple system, it is possible to change the frequency of the instability and, because the growth rate is of the order of ω_h, to have an arbitrarily low growth rate. This effect combined with the damping that already appears to be present in the plasma could lead to stability at least for plasmas of moderate densities.

5. EXPERIMENTS WITH FLUTE UNSTABLE PLASMAS AND PLASMA LOSSES

The interest in feedback experiments with unstable plasmas is in its effects on plasma losses, normally caused by the flute instability. In a simple mirror field without feedback the PHOENIX II plasma shows intense flute oscillations of the first azimuthal mode at frequencies near to $\omega^*/2$[19]. These oscillations develop when the critical density is reached and restrict the build-up of the plasma at densities of at most 30-40% above the threshold of the instabilities. Measurement of the threshold of flute instability, made in relatively poor vacuum conditions shows that at B_o = 12 kG the threshold value of the line density was 6×10^8 cm^{-3}, i.e. for a length of 2 cm the threshold density was 3×10^8 cm^{-3}. This compares favourably with the predicted value of 6.5×10^8 cm^{-3} for the line density calculated from the results of electron drift wave experiments reported elsewhere[20].

In the experiments on unstable plasma the feedback system was switched on from the beginning of the magnetic field pulse until some time during the second beam injection as shown in Fig.7 where it can be seen that when the feedback system is switched off the end plate current falls to a low value and the flute amplitude and frequency rise, indicating a loss rate caused by the instability which was not present while the feedback system was switched on.

To study the effect of feedback on the gross plasma parameters two series of experiments were carried out using a narrow band feedback system ($\omega_h/2\pi \simeq 300$ kHz) and a wideband amplifier ($\omega_h/2\pi \sim 10$ MHz) and the effect of feedback loop gain on the line density, amplitude and frequency of the low frequency signals is shown in Fig.8. The results for the narrow band amplifier are for a delay of 1.46 μs which was found to have the optimum effect upon the density. As far as the frequency of the instability is concerned for $\delta_n \lesssim 1$ the results of both series are in qualitative agreement with the theory developed here but the quantitative agreement is not a good as in the case of stable plasmas due, at least in part, to the fact that a constant plasma density could not be maintained. When the loop gain was increased beyond $\delta_n \approx 1$ the plasma suddenly changed to a new state in the wideband case; its density increased and the oscillations changed in frequency and mode. The oscillations became highly irregular and were a mixture of m = 1, 2 and 3. This sudden change occurred at gain values for which the calculated value of δ_n was close to 1, using b = 8.5 cm and a = 8.5 cm - a_i, where a_i is the Larmor radius of the ions. This agreement however, appears to be somewhat fortuituous because experiments with various amplifiers have shown that the position of the discontinuity varied slightly and depended on the high frequency cut-off of the amplifiers used. As this cut-off was lowered the position of the discontinuity moved to higher values of gain and the density became larger before the jump; consequently, the size of the discontinuity became smaller and so, for the narrow band case there was a continuous transition. For a fixed set of amplifiers the gain value at which the discontinuity occurred was reproducible to within less than 3dB. The discontinuity seems to be associated with changes in the frequency and growth rate as functions of the gain and with a non-linear limitation of the oscillation amplitude. The nature of this phenomenon is not yet clear. At still higher gains near the threshold of self-excitation of the amplifier the plasma density fell and there appeared to be a loss of control by the feedback amplifier. Under almost all conditions sporadic bursts of oscillations at the flute frequency occurred; these were attributed to loss of control by the amplifier system which could be paralysed by chance increases of noise.

To determine the extent of losses and to ascertain their nature, plots of plasma parameters as a function of the density expected from the equilibrium between Lorentz ionization and charge exchange loss rate were made by varying the injected beam. These are shown in Fig.9 for cases with and without feedback. The absolute density was calculated from the cold ion current along the magnetic field lines. The cold ion current, I, from a unit volume of plasma is given by the expression[21]

$$I = n(1 + \sigma_i/\sigma_{cx})/\tau_{cx} \quad (7)$$

where n is the hot ion density, σ_i the cross section for the production of electrons, σ_{cx} the charge exchange cross section of the hot ions with the background gas and τ_{cx} the plasma decay time due to charge exchange. It was not possible to measure τ_{cx} in the presence of flute losses and therefore the value of τ_{cx} obtained in a minimum B field under similar beam injection and vacuum conditions was used. There was no evidence that the dynamic background pressure changed appreciably with injected current. Taking all the uncertainty into account it is estimated that the absolute line density is known to within 40%. Errors in the absolute value of the line density, however, do not affect the shape of these curves.

The dependence of the central line density and the central photomultiplier signal on the injected current for the case with feedback shows that there were no losses until the density reached a certain critical value about 2.2 times the threshold for the flute in the absence of feedback instability. At higher densities losses occurred and these put a practical limit on the build-up though not so sharply as when the feedback system was absent. When injection stopped the plasma density fell rapidly to a value at which the plasma appeared to decay stably. The level at which this stable decay of the central photomultiplier signal began is marked as the threshold of stable decay.

Although the threshold behaviour of the losses is seen very clearly from the dependence of the density on the injected current, no such threshold was visible in the corresponding dependence of the low frequency oscillation amplitude. However, as the injected current was increased there was a noticable change in the frequency spectrum and spatial structure of the low frequency oscillations. With the feedback system in operation very low frequency (20-30 kHz) oscillations of the first azimuthal mode were observed at densities below the flute instability threshold. With increasing injected current, oscillations of the second azimuthal mode ($m = 2$) with approximately twice the frequency appeared and became stronger. Fig.10 shows the signals from two electrostatic probes 90° apart in azimuth; there are evidently two modes of oscillation to be seen. Under these experimental conditions the amplitude of the second mode was slightly greater than that of the first mode. At the highest currents, oscillations were observed in modes up to $m = 5$. Although these experimental results do not allow a precise determination of the threshold for occurrence of the $m = 2$ mode or a correlation of it with the loss threshold, such a connection appears very probable since a single electrode system cannot suppress higher modes of flute oscillations. The second mode would be expected to occur at a density of about twice the threshold for the first mode and this is approximately the density at the threshold of stable decay and also that at which losses were first observed.

To elucidate the radial structure of the low frequency oscillations, measurements were made with a movable electrostatic probe at the side of the plasma and 5 cm from the median plane. The signals from these electrostatic probes at different radii showed that the oscillations of the first and second azimuthal modes were in phase, although the ratio of amplitudes was different at different radii. The $m = 2$ mode was more predominant near the centre of the plasma and these measurements also showed that both the $m = 1$ and $m = 2$ modes were the simplest radial modes. The increase of the $m = 2$ amplitude with density caused an overloading of the stabilisation system and bursts of ordinary flute oscillations with corresponding sudden changes of plasma density.

As in earlier experiments on PHOENIX IA[18,22] and OGRA II[23] oscillations at the ion cyclotron frequency and its harmonics develop in a flute unstable plasma. These oscillations did not appear to have a serious effect on the operation of the feedback system except in that intermittent overloading of the system and consequent loss of control may have been caused by bursts of these ion cyclotron oscillations. They also caused a broadening of the ion energy spectrum which may have resulted in a slightly modified magnetic drift frequency and would certainly have the effect of lowering the growth rate of the flute instability. The feedback system, on the other hand, suppressed the ion cyclotron oscillations characterised by axial currents but did not affect the azimuthal mode. This effect became less marked at magnetic fields above 14 kG. The explanation for this phenomenon should be sought in the effects of electron damping in a finite length plasma.

6. RELATION BETWEEN THE AMPLITUDE AND FREQUENCY OF THE LOW FREQUENCY OSCILLATIONS

Whereas the dependence of $\mathrm{Re}\,\omega$ on the gain permits a simple comparison with experiment and shows good agreement, the same is not true of the growth rate of the oscillations. The theory which takes into account the function $\delta(\omega)$ predicts that the growth rate of the instability should increase with δ. The experiments indicate that the oscillation amplitude and the losses decrease, and therefore comparisons should be made with non-linear theory and not with the solution of eq.(2).

Dupre[24] put forward a non-linear theory of low frequency oscillations. According to this theory the wave amplitude would be limited by particle-wave interaction. Since the frequencies of the oscillations are small compared to the ion cyclotron frequency the particles execute a drift motion under the action of the wave field. If the phase velocity of the wave differs from the drift velocity of the particles the latter will undergo oscillations in the wave field whose amplitude increases as the phase velocity approaches the particle drift velocity. Finally when the velocities are exactly equal, the amplitude of the particle displacement in the wave field becomes infinite. Thus the condition for a strong interaction between the wave and the particles may be written

$$v_{dr} = \frac{c\,E_r}{B} = v_{ph} = r\,\omega/m \qquad \ldots (8)$$

where v_{dr} is the particle drift velocity. This formula gives excessively high values for the flute oscillation amplitude.

In a bounded plasma of small dimensions another non-linear effect may occur; the radial oscillations of the particles may lead to losses at the walls and consequently a limitation of the wave amplitude. In the conditions of the present experiment when the distance from the effective plasma boundary to the wall was of the order of the ion Larmor radius, one expects an effective limitation of the wave amplitude if the radial displacement of the particles in the wave field is of the order of the ion Larmor radius.

$$\Delta r = v_{dr}/\omega \approx a_i \qquad \ldots (9)$$

whence

$$E_\theta \approx B\,a_i\,\omega/c \qquad \ldots (10)$$

where E_θ is the amplitude of the azimuthal field component. When m = 1 $E_\theta \simeq E_r$ and the wave amplitude is again proportional to the oscillation frequency. This equation gives an electric field of 70 V/cm at the flute frequency as the limiting electric field and this is close to the experimentally measured values.

The above estimates are valid only for electrons. To apply them to ions one must take into account the ion drift in the inhomogeneous magnetic field. This leads to a Doppler shift of frequency and for ions formulae (9) and (10) become

$$\Delta_{ri} = v_{dr}/|\omega - \omega^*| \approx a_i \qquad \ldots (9a)$$

and

$$E_\theta \simeq B a_i |\omega - \omega^*|/c \qquad \ldots (10a)$$

We see that the ion and electron oscillation amplitudes in a given field are equal only if $\omega = \omega^*/2$. When $\omega < \omega^*/2$ the wave interacts mainly with the electrons and its amplitude is limited by this interaction; when

$\omega > \omega^*/2$ the main interaction is with the ions. Thus one expects the wave amplitude to be described by the formula

$$E = A\,\omega \qquad \text{for} \quad \omega < \omega^*/2$$

$$= A(\omega^* - \omega) \quad \text{for} \quad \omega > \omega^*/2 \qquad \ldots (11)$$

that is, the oscillation amplitude is greatest for $\omega = \omega^*/2$ and decreases at both lower and higher frequencies. For oscillations with $\omega < \omega^*/2$ small losses of ions are expected, but for $\omega > \omega^*/2$ one expects greater losses of ions despite the smaller amplitude of the oscillations.

By varying the gain and the phase shift in the feedback circuit the real part of the oscillation frequency can be altered and the amplitude frequency relationship examined. When the feedback parameters are varied, not only the frequency but also the growth rate of the oscillations change; but when these parameters are correctly chosen the change in the growth rate may be small. For instance, for the amplifier and 1.46 μs delay used in this work at a density near the flute instability threshold, the calculated value of the growth rate is constant when δ_n varies from 0.2 to 0.9. In this case therefore, one may expect to find a relation of the type (11). Fig.11 shows the results of some 2000 simultaneous measurements of the amplitude and frequency of low frequency oscillations under various experimental conditions and with various feedback characteristics, including the case of no feedback. In all cases however, the oscillations frequency was not greater than $\omega^*/2$ so that this diagram should correspond to the low frequency branch of equation (11) as is in fact, observed. There are no such extensive data for oscillations in the high frequency branch. If the feedback parameters were made such that the real part of the frequency exceeded $\omega^*/2$ then the plasma losses became greater. Since the basic idea of the experiment was to make the plasma density as high as possible such conditions were, in general, avoided. However, the few results obtained at oscillation frequencies close to ω^* show that the amplitudes in these cases were less than those for $\omega = \omega^{*}/2$ but despite the small amplitudes the oscillations caused even greater losses than those caused by the natural flute instability. A similar result regarding the amplitudes and losses was also observed in OGRA II[9] with a particular excitation of the instability near $\omega = \omega^{*}$. Thus one may suppose that equation (11) is in qualitative agreement with experiment.

The dependence of the low frequency oscillation amplitude and plasma losses on the oscillation frequency seems to explain why, despite the theoretically predicted higher growth rates, the application of feedback actually reduces the oscillation amplitude and ion loss rate.

7. THE DYNAMIC RANGE OF THE FEEDBACK SYSTEM

The threshold behaviour of the system at a certain gain value in the feedback circuit made it possible to study the effect of the dynamic range of the feedback amplifier on the stabilisation. The dynamic range of the output amplifier was gradually reduced from 150 V peak-to-peak to 40 V peak-to-peak whilst ensuring that the small signal amplification of the system was maintained constant. The results showed that when the dynamic range was reduced below 100 V peak-to-peak the plasma density decreased, the oscillation frequency increased, and the mode of oscillation changed exactly as if the gain of the small signal amplification has been decreased. Fig.12 shows a comparison of the dependence of the plasma density on the gain and also on the dynamic range. Here the results of both measurements are shown on the same diagram, the scales having been so chosen that the gain corresponding to $\delta_n = 1$ coincides with the value of the dynamic range beyond which the latter begins to affect the behaviour of the plasma. It is easy to see that with this normalisation the results of the two series of experiments lie on the same curve. The interpretation is that for a dynamic range exceeding 100 V peak-to-peak the quantity δ_n is determined by the small signal gain, whereas for a dynamic range of less than 100

$$\delta_n = \frac{\text{Dynamic Range}}{2(\text{Wave Amplitude})} \quad . \qquad \ldots (12)$$

Hence the amplitude of the "noise" potential on the plasma surface is of the order of 50 V. This value is clearly too large for actual noise, especially as direct measurements of noise in the absence of feedback give a much smaller value. Thus, this noise must be due to the interaction of the plasma with the feedback system. No new effects were found with a dynamic range of 600 V peak-to-peak except that there were no longer any bursts of large amplitude high frequency oscillations for δ_n somewhat less than 1. The relation between these bursts and the dynamic range indicates that they are due to non-linear instability; if the oscillation amplitude is close to the dynamic limit of the amplifier a chance increase of the amplitude above this limit due, for instance, to a burst of ion cyclotron activity, may decrease the effective gain because the input signal increases while the output signal is held constant. The decreased gain in turn causes an increased oscillation frequency which causes a further increase of amplitude because of the latter's dependence on frequency. Thus, the oscillation amplitude and frequency will increase until the frequency reaches a value at which the condition

$$\omega = f(\delta_n) = f(A_{max}/\beta\omega) \qquad \ldots (13)$$

is satisfied where $f(\delta_n)$ is the dependence of the frequency on the transfer coefficient obtained by solving the dispersion relation (2) $\beta = A/\omega$ is the ratio of the amplitude of the non-linear wave to the oscillation frequency and A_{max} is the maximum amplitude transmitted by the amplifier.

8. STABILISATION SYSTEM WITH NEGATIVE FEEDBACK

The excitation of oscillations in the presence of a feedback system is seen to be a consequence of the use of positive feedback. It would, therefore, be desirable to achieve stability with negative feedback. Equation (3) shows that there is such a region of stability for large negative values of δ_n. This region has not been found experimentally[1] and one can easily see[25,26] that its existence depends directly on the assumption of a sharp boundary to the plasma. If one calculates a more realistic distribution, i.e. parabolic, there is no longer any stability region for $\delta < 0$, and even for $|\delta_n| \to \infty$ the instability threshold rises only very slightly. The situation is improved if one can sense fluctuations inside the plasma and since the PHOENIX II plasma is a flat disk and $k_{\parallel} = 0$ for the flute instability, fluctuations within the plasma can be measured without actually putting a probe inside the plasma. The theory suggests that for best results the gain should be high and the probe should measure the perturbed potential in the region of the maximum density gradient. These conclusions accord with the experimental results shown in Fig.13, which show the dependence of the flute instability threshold on the gain for a fixed probe position and also on the probe position for a fixed gain. For comparison the radial distribution of plasma density taken from ref.21 is shown. The threshold is seen to rise with increasing gain and the optimum position of the probe coincides with the maximum gradient although the rise of threshold is fairly small, not exceeding 50%. It was also observed that at high injection currents when the densities reach a new threshold value, the m = 1 flute instability was again excited and the density decreased to the instability threshold found in the absence of feedback. This is presumed to be due to overloading of the amplifiers. The unstable oscillations were then damped, the feedback system resumed control, and the density rose again to the new threshold. Thus the plasma density oscillated between the old and the new instability thresholds. When the injection current was still higher the plasma built up more rapidly and the relaxation frequency was higher.

9. CONCLUSIONS

It has been shown that the frequency response of a feedback scheme is a very important factor to be taken into account in designing a stabilising system for a reactive instability and that, for the case of the flute instability, if the frequency response is correctly taken into account, then the linear behaviour of the system can be reasonably well described by existing theory. It has also been shown that the plasma loss rate can be controlled, even when the system is unstable, by altering the frequency of the instability so that the unstable wave is decoupled from the motion of the ions in the magnetic field gradient.

There are good prospects of being able to increase density further and suppressing higher azimuthal modes with a higher order feedback system. With more advanced techniques and careful design of feedback loops there seem to be possibilities, at least in principle, of suppressing all the modes of the flute instability.

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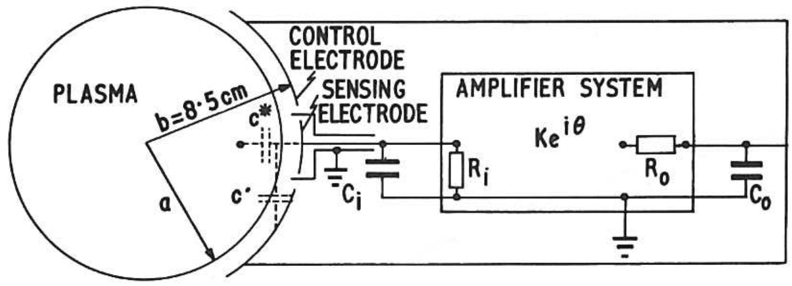

Fig. 1 Schematic diagram of feedback system.

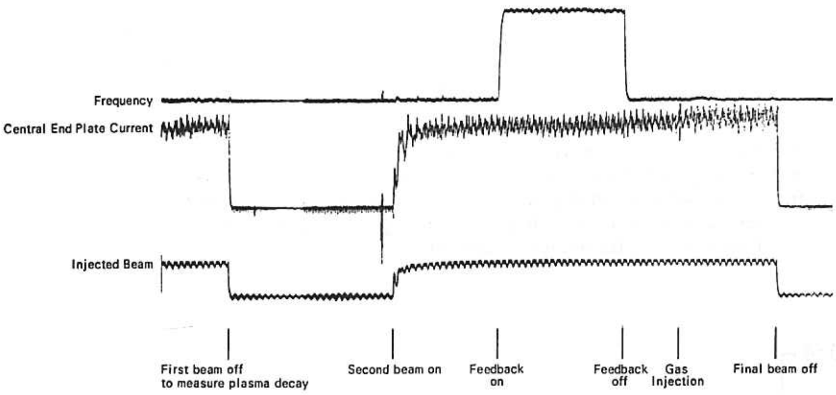

Fig. 2 Central end plate current, frequency of electrostatic oscillations and injected beam as a function of time for an experiment with a normally stable plasma. The feedback was switched on for 0.4 s during the second period of beam injection.

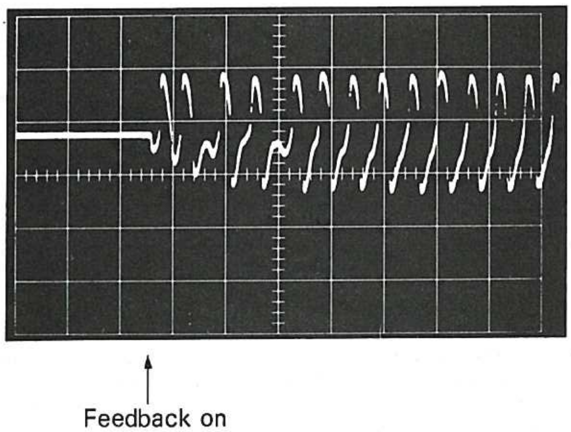

Fig. 3 Amplitude of electrostatic oscillations from the time that feedback was applied. Sweep speed 50 μs/division. CLM-P 272

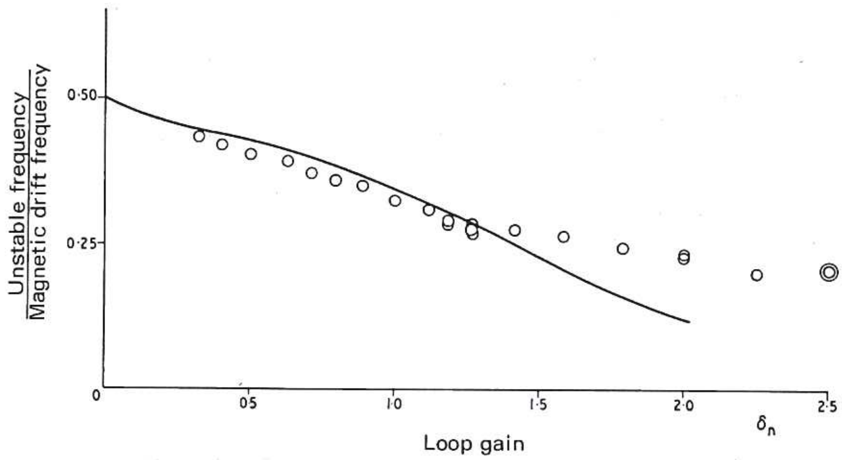

Fig. 4 Frequency of instability as a function of a feedback loop gain. The frequency is normalised to the magnetic drift frequency and the loop gain is normalised as in equation (4). The solid line is the solution of equation (2) with the frequency dependence of equation (6) for phase shift $\Delta\theta = 0$, $\omega_\ell/\omega^* = 0.008$, $\omega_h/\omega^* = 1.35$, $m = 1$ and $n^* = \frac{1}{4}$ (threshold density)

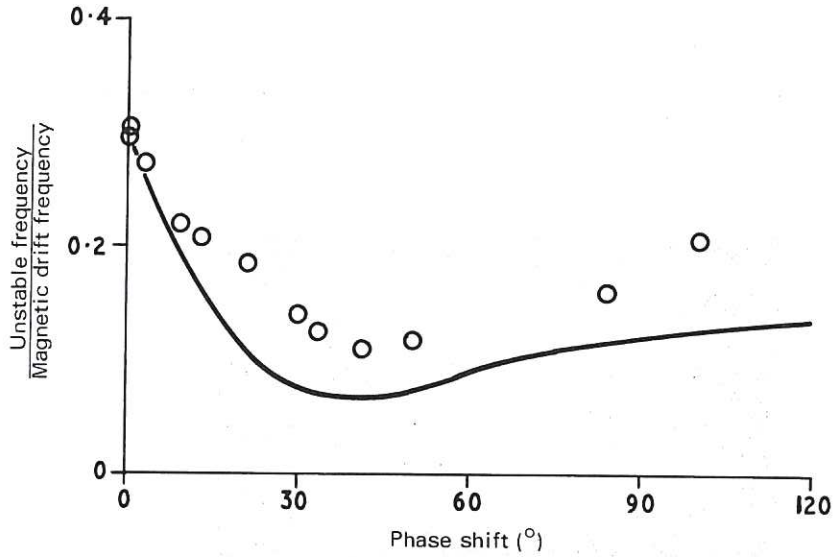

Fig. 5 Frequency of the instability as a function of phase shift in the freeback loop. The frequecny is normalised to the magnetic drift frequency and the phase shift is the lag of the applied potential behind the detected potential. The solid line is calculated for a loop gain of 1.12 and all other conditions are as for Fig. 4

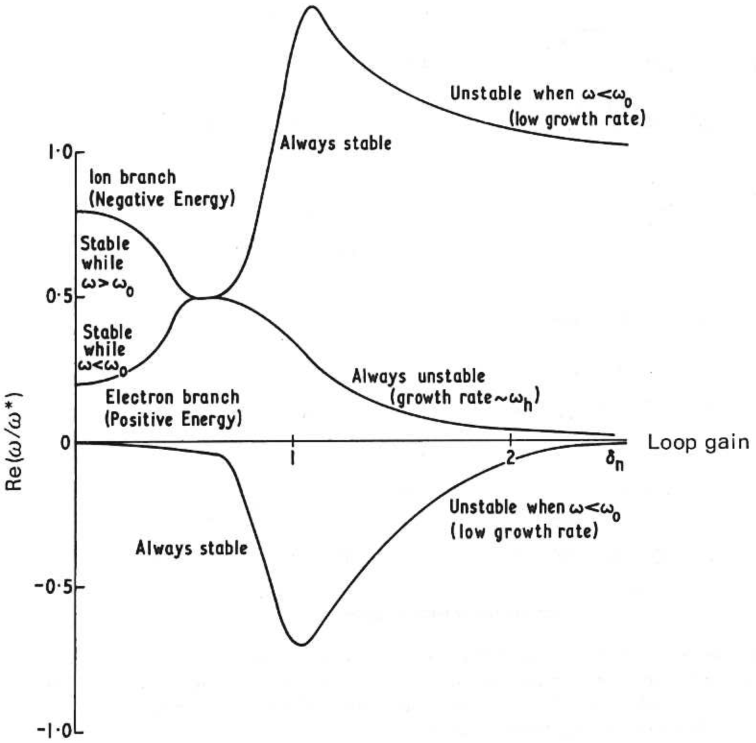

Fig. 6 Typical solutions for the real part of the frequency as a function of gain calculated from equation (2) using the frequency dependence of equation (6). The fourth stable root near ω_ℓ is always stable and is not shown. The solutions shown here are for a density somewhat less than the threshold density. At higher densities the ion and electron branches converge to 0. 5 and above that density are degenerate at that frequency and unstable. $\omega_o^2 = \omega_\ell \omega_h$

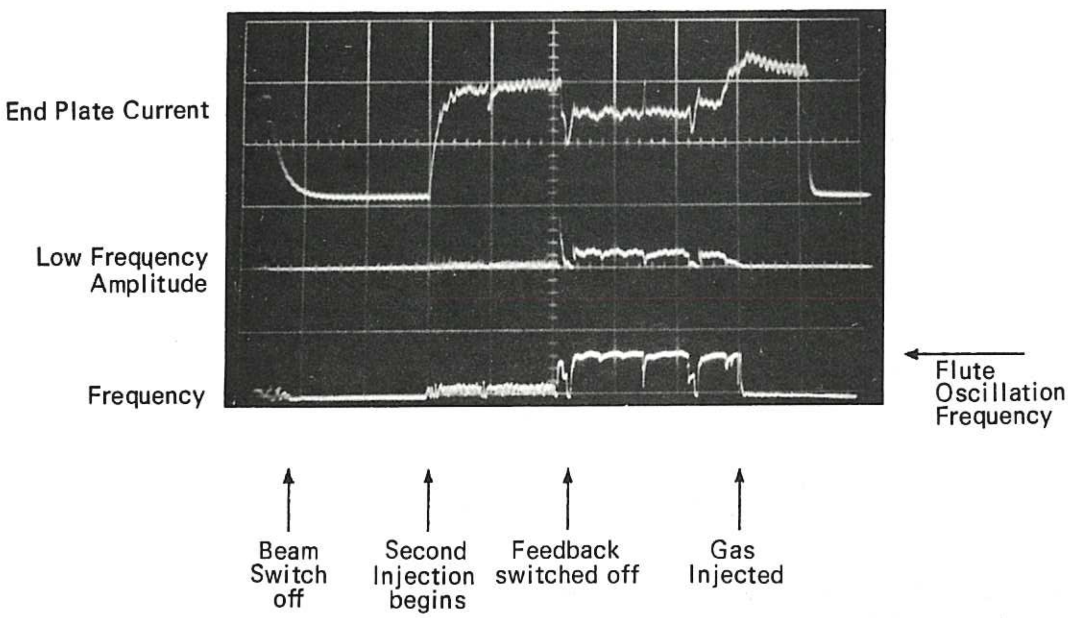

Fig. 7 End plate current, amplitude and frequency of electrostatic oscillations as a function of time for an experiment with an unstable plasma. Sweep speed 200 ms/div.

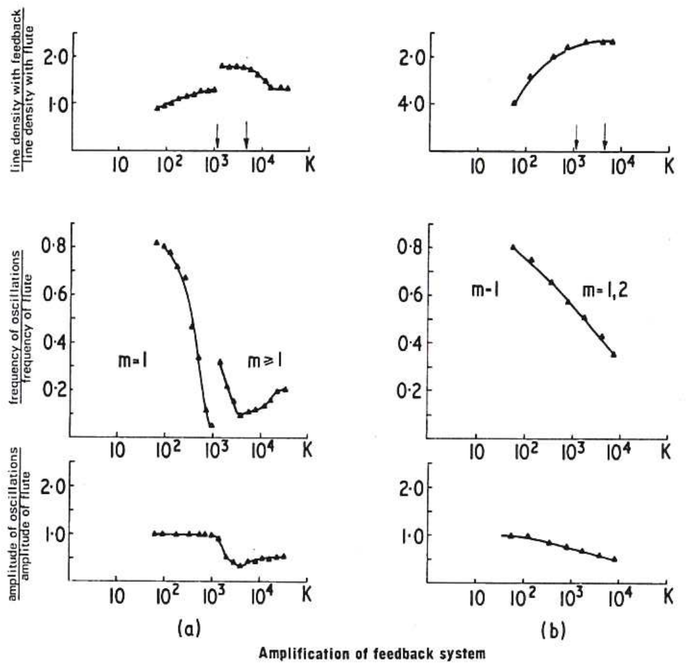

Fig. 8 End plate current, amplitude and frequency of electrostatic oscillations as a function of amplification for two amplifier systems: (a) a wide band amplifier with $\omega_h/2\pi \sim 10$ MHz and (b) a narrow band amplifier ($\omega_h/2\pi \sim 0.3$ MHz) with a 1.46 μs delay line for optimum effect.

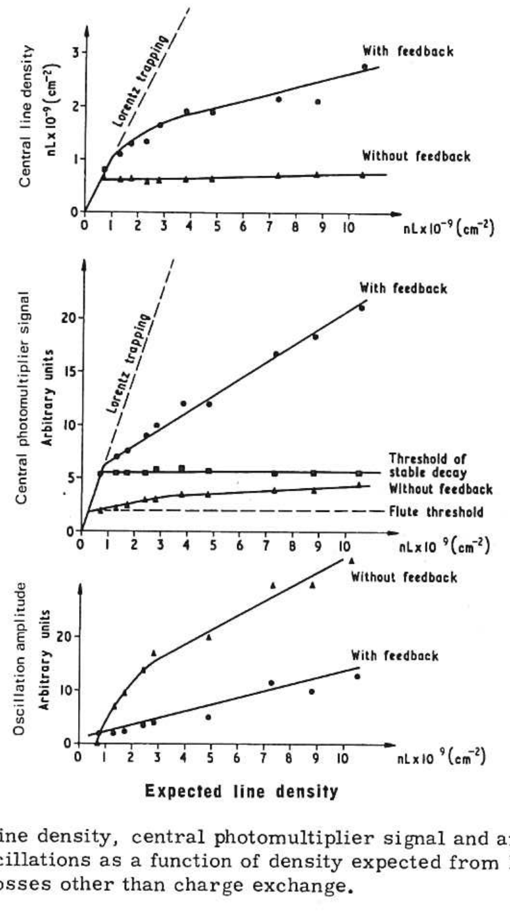

Fig. 9 Central line density, central photomultiplier signal and amplitude of electrostatic oscillations as a function of density expected from Lorentz trapping with no losses other than charge exchange.

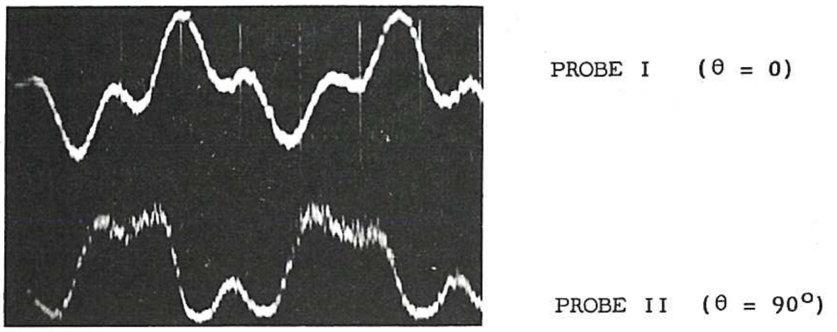

Fig. 10 Signals from two electrostatic probes offset azimuthally by 90°. Sweep speed = 5 μs/div.

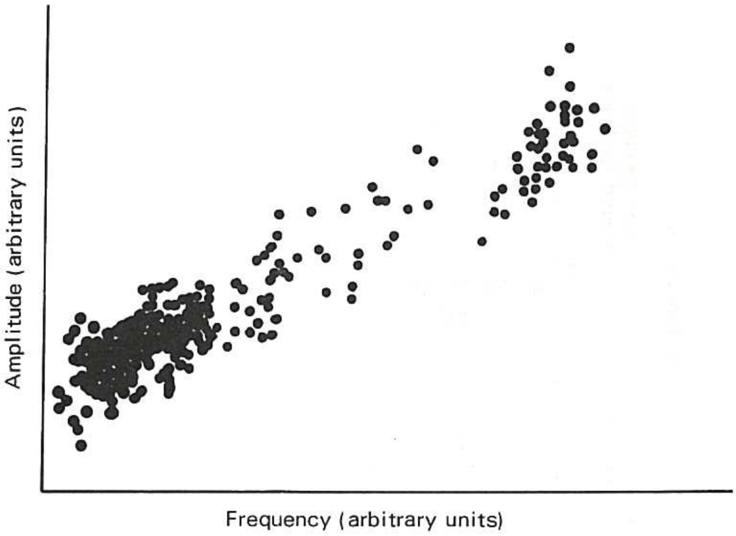

Fig. 11 Dependence of the amplitude of electrostatic oscillations on frequency showing the results of 2000 simultaneous measurements of amplitude and frequency in a variety of different experimental conditions obtained by varying the parameters of the feedback system and including, at the highest frequencies, the case of pure flute oscillations.

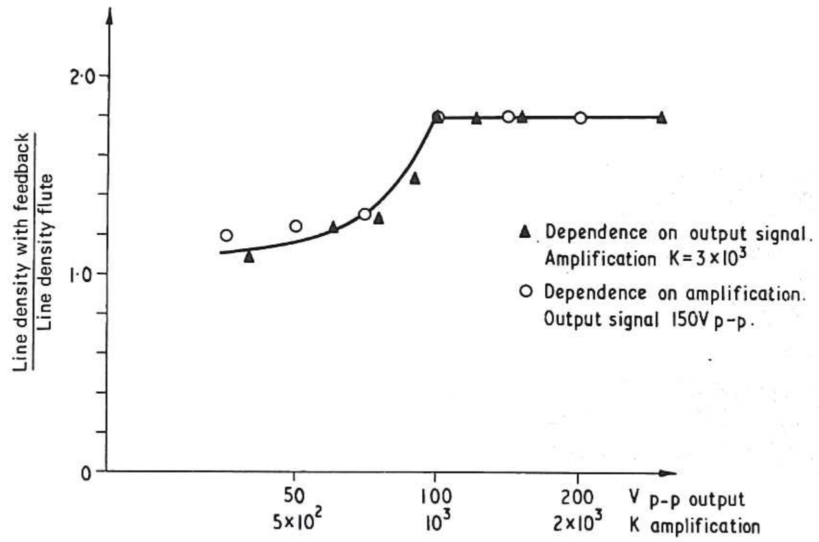

Fig. 12 Dependence of line density on amplification for a fixed dynamic range and on dynamic range for fixed amplification. The abscissae have been normalised to the point on the knee of the curve.

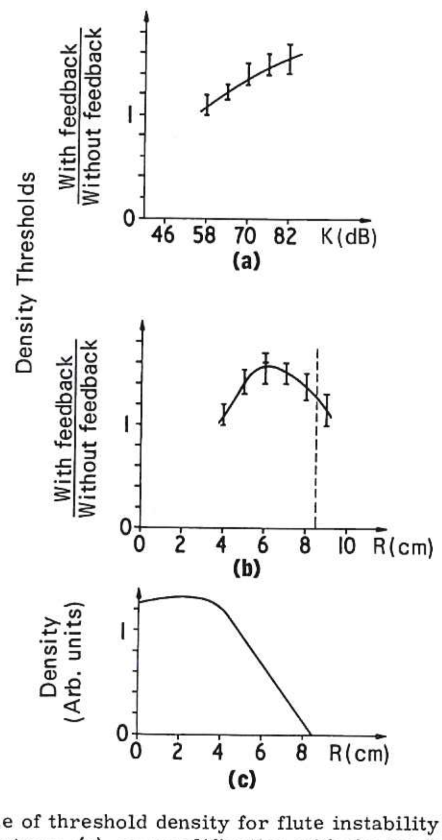

Fig. 13 Dependence of threshold density for flute instability for a negative feeback system: (a) on amplification with the feedback probe at a radius of 6 cm (b) on probe position with the feedback electrode at 8.5 cm indicated by the dashed line (c) shows the expected radial density distribution.