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THE COLLISIONLESS HEATING OF A PLASMA BY MEANS OF OSCILLATING EXTERNAL CURRENTS

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THE COLLISIONLESS HEATING OF A PLASMA BY MEANS OF OSCILLATING EXTERNAL CURRENTS

by

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(Submitted for publication in Plasma Physics)

A B S T R A C T

The transfer of power between an external circuit and a collisionless plasma is described. A very simple model is used which enables heating for arbitrary values of the frequency to be analyzed. Transit time magnetic pumping and ion cyclotron heating are described in detail. The simple model used enables the mechanism of magnetic pumping to be described much more precisely than has been done previously. The mechanism is two-dimensional, i.e. if only one dimensional variations are included there is no power absorption. It is shown that for magnetic pumping the current parallel to the static magnetic field absorbs no power. It is also shown that the heating rate at the ion cyclotron frequency is very much greater than the magnetic pumping rate. Finally, the conditions are given for the electric field components (which give rise to the power absorption) to be vacuum fields.

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1. INTRODUCTION

Since the classical resistivity of a thermonuclear plasma will be too low for ohmic heating to be effective other methods of heating must be used. Two such methods are transit time magnetic pumping which operates well below the ion cyclotron frequency and ion cyclotron heating which works close to this frequency. Both of these methods involve the collisionless absorption of power from an external field, although both methods require a small number of collisions to maintain a near Maxwellian velocity distribution of particles in the region of the phase velocity of the imposed wave field. The cyclotron heating described below is just cyclotron damping and does not require a magnetic beach.

Transit time magnetic pumping was first suggested by SPITZER (1953) using the concept of the motion of the guiding centre and the force due to a magnetic field gradient. A number of papers (BERGER et al., 1958, DAWSON and UMAN, 1965, CANOBBIO, 1970), have appeared since, most of which make use of the magnetic field gradient force. In these formulations it is the oscillating magnetic field which appears to transfer power to the plasma. In fact, the force due to a magnetic field does no net work and it is the electric field which provides the transfer of energy from the external fields to the plasma. STEPANOV (1964) has calculated the power transfer from an external current source to a plasma by calculating the electric fields produced by such a source and then evaluating the work done by these fields on the plasma. The results of this calculation were essentially the same as other workers. Unfortunately, Stepanov's calculation which was more exact than the other treatments was obscured by the

cylindrical geometry he chose to use.

In this paper a similar model to Stepanov's will be analyzed but in plane geometry. The fields will be excited by an oscillating external current. In the treatment of other workers an oscillating magnetic field, which violated Maxwell's equations was assumed. Also, in these other treatments variation in only one spatial direction was allowed whereas any practical system will vary in at least two spatial coordinates.

Using the simple model to be described below the problem of transferring power to a plasma from an external circuit can be analyzed for arbitrary values of the frequency. Thus, after obtaining an expression for transit time magnetic pumping similar methods are applied to calculate the power absorbed in the vicinity of the ion cyclotron frequency.

2. FORCED OSCILLATIONS DUE TO AN EXTERNAL CIRCUIT

We choose the simplest model which still contains the essential ingredients of the problem. Thus, we consider an infinite uniform plasma with a static uniform magnetic field pointing in the z-direction. The external current is assumed to flow in the x-direction and to vary as

$$\tilde{J}_{\text{ext}} \sim \hat{i}_x J_{\text{ext}} e^{i(k_z z + k_y y - \omega t)}. \quad \dots (1)$$

Since the plasma is uniform we could obtain the solution for a more realistic current distribution by Fourier synthesis after we have obtained the solution for the current distribution given by equation (1).

The forced oscillation problem is much simpler than that for the natural or characteristic oscillations since ω and k are given and we can calculate the fields explicitly in terms of J_{ext} .

from Maxwell's equations. In order to calculate the fields and currents in the plasma we must know the conductivity tensor $\underline{\sigma}$. We now outline very briefly the calculation of $\underline{\sigma}$.

We assume that J_{ext} is sufficiently small such that the fields satisfy the small amplitude condition. We may then linearize the Vlasov equation to obtain

$$\frac{\partial f_{1j}}{\partial t} + \underline{v} \cdot \frac{\partial f_{1j}}{\partial \underline{r}} + \frac{q_j}{m_j} (\underline{v} \times \underline{B}_0) \cdot \frac{\partial f_{1j}}{\partial \underline{v}} = - \frac{q_j}{m_j} (\underline{E}_1 + \underline{v} \times \underline{B}_1) \cdot \frac{\partial f_{0j}}{\partial \underline{v}} \dots (2)$$

where j refers to electrons or ions, f_{0j} is the equilibrium solution which for simplicity is taken to be Maxwellian with $T_{\perp j} = T_{\parallel j}$. \underline{E}_1 , \underline{B}_1 are the electromagnetic fields resulting from J_{ext} and the plasma. All the other symbols have their usual meaning.

The solution of equation (2) is obtained in the standard way by integration along the equilibrium particle orbits

$$f_{1j} = - \frac{q_j}{m_j} \int_{-\infty}^0 \underline{E}_1(\underline{r}'(t'), t') \cdot \frac{\partial f_{0j}}{\partial \underline{v}} dt' \dots (3)$$

$$= - \frac{q_j}{m_j} \underline{E}_1 \cdot \frac{\partial f_{0j}}{\partial \underline{v}} \int_{-\infty}^0 e^{i(k_z z(t') + k_y y(t') - \omega t')} dt' \dots (4)$$

where the term $\underline{v} \times \underline{B}_1 \cdot \frac{\partial f_{0j}}{\partial \underline{v}} = 0$ since f_{0j} is a Maxwellian.

With the solution (4) we can calculate the plasma current produced by the electromagnetic fields

$$\underline{J} = \sum_j q_j \int f_{1j} \underline{v} d\underline{v} \dots (5)$$

and hence the conductivity tensor $\underline{\sigma}$. Performing these integrations we finally obtain for the elements of $\underline{\sigma}$

$$\sigma_{xx} = \sum_j \sum_{n=-\infty}^{\infty} \frac{in_0 q_j^2}{m_j} \left[\left(\frac{n^2}{k_y^2 \rho_j^2} + 2 \frac{k_y^2 \rho_j^2}{k_y^2 \rho_j^2} \right) I_n - k_y^2 \rho_j^2 I'_n \right] \frac{e^{-k_y^2 \rho_j^2}}{\sqrt{2} k_z v_{Tj}} Y(z_{nj}) \dots (6)$$

where v_{Tj} and ω_{cj} are the thermal velocity $(\kappa T_j/m_j)^{1/2}$ and cyclotron frequency $q_j B_0/m_j$ of the j^{th} species and ρ_j is the Larmor radius V_{Tj}/ω_{cj} . I_n is the n^{th} order modified Bessel function of the first kind and I'_n its derivative. The argument of the Bessel function will always be $k_y^2 \rho_j^2$. The function Y is $-Z$ where Z is the Plasma Dispersion function of FRIED and CONTE (1961). The argument of Y is $z_{nj} = (\omega + n\omega_{cj})/\sqrt{2} k_z v_{Tj}$. For the remainder of the elements of σ_{\sim} , \sum_{nj} will be used to denote $\sum_j \sum_{n=-\infty}^{\infty}$

For the other elements we have

$$\sigma_{xy} = \sum_{nj} \frac{n_0 q_j^2}{\kappa T_j} \frac{\omega_{cj}}{k_y^2} k_y^2 \rho_j^2 e^{-k_y^2 \rho_j^2} (I'_n - I_n) \frac{n\omega_{cj}}{\sqrt{2} k_z v_{Tj}} Y(z_{nj}) \quad \dots (7)$$

$$\sigma_{xz} = \sum_{nj} \frac{n_0 q_j^2}{\kappa T_j} \frac{\omega_{cj}}{k_y k_z} k_y^2 \rho_j^2 e^{-k_y^2 \rho_j^2} (I'_n - I_n) (1 - z_{nj} Y(z_{nj})) \quad \dots (8)$$

$$\sigma_{yx} = -\sigma_{xy} \quad \dots (9)$$

$$\sigma_{yy} = \sum_{nj} \frac{i n_0 q_j^2}{\kappa T_j} \frac{n\omega_{cj}}{k_y^2} e^{-k_y^2 \rho_j^2} I_n \left(-1 + \frac{n\omega_{cj}}{\sqrt{2} k_z v_{Tj}} Y(z_{nj}) \right) \quad \dots (10)$$

$$\sigma_{yz} = \sum_{nj} \frac{i n_0 q_j^2}{\kappa T_j} \frac{n\omega_{cj}}{k_z k_y} e^{-k_y^2 \rho_j^2} I_n \left(1 - z_{nj} Y(z_{nj}) \right) \quad \dots (11)$$

$$\sigma_{zx} = -\sigma_{xz} \quad \dots (12)$$

$$\sigma_{zy} = \sigma_{yz} \quad \dots (13)$$

$$\sigma_{zz} = \sum_{nj} \frac{i n_0 q_j^2}{\kappa T_j} \frac{(\omega + n\omega_{cj})}{k_z^2} e^{-k_y^2 \rho_j^2} I_n \left(-1 + z_{nj} Y(z_{nj}) \right) \quad \dots (14)$$

The above expressions are valid for arbitrary frequencies and an equilibrium distribution which is Maxwellian and where $T_{\perp j} = T_{\parallel j}$.

3. TRANSIT TIME MAGNETIC PUMPING

The frequency range for this form of heating is such that $\omega \ll \omega_{ci}$. In the original conception of this heating mechanism the particles traversed a finite region over which the magnetic field was changed periodically in time but was effectively constant outside this region. The magnetic field inside and outside this region differed typically by a factor ~ 2 . In more recent forms (DAWSON and UMAN, 1965, CANOBBIO, 1970) of this heating mechanism the magnetic field oscillates about its equilibrium value over the whole of the plasma but the ratio of the amplitude of the field oscillations to the background field is only a few per cent. In our model the oscillating fields are created by the external current in the x-direction.

For the frequency range already mentioned ($\omega \ll \omega_{ci}$) and for $k_y^2 \rho_j^2 \ll 1$ the conductivity tensor simplifies to the form

$$\underline{\sigma} \approx \begin{pmatrix} \sigma_{xx} & 0 & \sigma_{xz} \\ 0 & \sigma_{yy} & 0 \\ \sigma_{zx} & 0 & \sigma_{zz} \end{pmatrix}$$

and where terms $k_z^2 \rho_j^2$ have also been neglected. We now calculate the electromagnetic fields produced by the external current with the aid of Maxwell's equations. From the two curl equations we have

$$\left(-\underline{k} \underline{k} + \left(k^2 - \frac{\omega^2}{c^2} \right) \underline{I} - i\omega \underline{\mu}_0 \underline{\sigma} \right) \underline{E} = i\omega \underline{\mu}_0 \underline{J}_{ext.} \hat{t}_x \quad \dots (15)$$

where $k^2 = k_y^2 + k_z^2$ and \underline{I} is the unit tensor. Solving equation (15)

for the electric field we obtain

$$\underline{E}_x = \frac{i\omega \underline{\mu}_0}{A} \underline{J}_{ext.} \left\{ \frac{\omega^4}{c^4} - \frac{\omega^2}{c^2} k^2 - i\omega \underline{\mu}_0 \sigma_{zz} \left(k_z^2 - \frac{\omega^2}{c^2} \right) - i\omega \underline{\mu}_0 \sigma_{yy} \left(k_y^2 - \frac{\omega^2}{c^2} \right) - \omega^2 \underline{\mu}_0^2 \sigma_{yy} \sigma_{zz} \right\} \quad \dots (16)$$

$$\underline{E}_y = -(\omega \underline{\mu}_0)^2 k_y k_z \underline{J}_{ext.} \sigma_{zx} / A \quad \dots (17)$$

$$\underline{E}_z = -(\omega \underline{\mu}_0)^2 \underline{J}_{ext.} \left(k_z^2 - \frac{\omega^2}{c^2} - i\omega \underline{\mu}_0 \sigma_{yy} \right) \sigma_{zx} / A \quad \dots (18)$$

where

$$A = \det. \left(-\underline{k} \underline{k} + \left(k^2 - \frac{\omega^2}{c^2} \right) \underline{I} - i\omega \underline{\mu}_0 \underline{\sigma} \right). \quad \dots (19)$$

Having calculated the electric fields excited in the plasma by the external current we may now obtain an expression for the power absorbed by the plasma. This is given by the usual expression

$$P = \frac{1}{2} \operatorname{Re} \underline{J} \cdot \underline{E}^* \quad \dots (20)$$

where \underline{J} in equation (20) is the plasma current

$$\underline{J} = \underline{\sigma} \underline{E}.$$

Owing to the form of the conductivity tensor $\underline{\sigma}$

$$\operatorname{Re} (J_y E_y^*) = 0$$

and therefore

$$P = \frac{1}{2} \operatorname{Re} (J_x E_x^* + J_z E_z^*). \quad \dots (21)$$

However, from equations (16) and (18) and the condition

$$\begin{aligned} \sigma_{zx} &= -\sigma_{xz} \\ \operatorname{Re} (J_z E_z^*) &= 0. \end{aligned} \quad \dots (22)$$

This result is in agreement with DOLGOPOLOV and STEPANOV (1963). The expression for the power now becomes

$$P = \frac{1}{2} \operatorname{Re} \left\{ \sigma_{xx} |E_x|^2 + \sigma_{xz} E_z E_x^* \right\}. \quad \dots (23)$$

The power gained by the plasma is of course lost by the external circuit. One can easily check this by making use of equations (16) - (18) to obtain

$$\operatorname{Re} (J_{\text{ext}} E_x^* + \underline{J} \cdot \underline{E}^*) = 0. \quad \dots (24)$$

Assuming $\omega^2 \ll c^2 k_z^2 (k_z \sim k_y)$ the power per unit volume absorbed by the plasma is

$$\begin{aligned} P &= \frac{(\omega \mu_0)^4}{2} \frac{|J_{\text{ext.}}|^2}{|A|^2} k_z^4 |\sigma_{zz}|^2 \left| \left(1 + \frac{k_y^2}{k_z^2} \frac{\sigma_{yy}}{\sigma_{zz}} + \frac{i\omega \mu_0}{k_z^2} \sigma_{yy} \right) \right|^2 \operatorname{Re}(\sigma_{xx}) \\ &- \frac{(\omega \mu_0)^4}{2} \frac{|J_{\text{ext.}}|^2}{|A|^2} (k_z^2 - i\omega \mu_0 \sigma_{yy}) k_z^2 |\sigma_{zz}|^2 \times \\ &\times \operatorname{Re} \left\{ \frac{\sigma_{xz} \sigma_{zx}}{\sigma_{zz}} \left(1 + \frac{k_y^2}{k_z^2} \frac{\sigma_{yy}^*}{\sigma_{zz}^*} + \frac{i\omega \mu_0}{k_z^2} \sigma_{yy}^* \right) \right\}. \end{aligned} \quad \dots (25)$$

The factor $|A|^2$ in the demoninator of the expression for P gives rise to resonances in the heating when ω is one of the normal modes of the plasma. The ion acoustic resonance has already been discussed by STEPANOV (1964). However, there are two other low frequency waves that can give rise to resonant heating. The magnetic acoustic or compressional alfvén wave and the shear alfvén wave. These resonances will be considered in a later publication.

Now under the conditions assumed the elements of the conductivity tensor are given by the following expressions

$$\sigma_{xx} = \sum_j \left\{ \frac{i 2 n_o \kappa T_j}{B_o^2} \frac{k_y^2}{\sqrt{2} k_z v_{Tj}} Y(z_{oj}) - \frac{i n_o \omega m_j}{B_o^2} \right\} \dots (26)$$

$$\sigma_{yy} = \sum_j - \frac{i n_o \omega m_j}{B_o^2} \dots (27)$$

$$\sigma_{xz} = \sum_j \frac{n_o q_j}{B_o} \frac{k_y}{k_z} (z_{oj} Y(z_{oj}) - 1) \dots (28)$$

$$\sigma_{zz} = \sum_j \frac{i n_o q_j^2}{\kappa T_j} \frac{\omega}{k_z^2} (z_{oj} Y(z_{oj}) - 1) \dots (29)$$

Substituting equations (26) - (29) into equation (25) gives the power absorbed by the plasma in transit time magnetic pumping as a function of T_e/T_i and the phase velocity of the forced wave. We will not write down the expression for the general case but only for the special case when $T_e = T_i$ and $\omega/k_z \approx v_{Ti}$. From these two conditions and equations (25) - (29) we obtain

$$P = - \frac{1}{4} \frac{|J_{ext.}|^2}{\epsilon_o} \omega \beta \frac{k_y^2}{c^2 (k_y^2 + k_z^2)^2} \operatorname{Im} \left\{ z_{oi} Y(z_{oi}) \left(2 + \frac{z_{oi} Y(z_{oi})}{2 - z_{oi} Y(z_{oi})} \right) \right\} \dots (30)$$

where β , the ratio of plasma pressure to magnetic pressure, has been assumed to be much less than unity. The expression in the curly brackets in equation (30) is exactly Dawson and Uman's function H . Also, like Dawson and Uman P is proportional to $\omega\beta$. The only significant difference between the above expression and that of Dawson and Uman is that equation (30) is proportional to k_y^2/k^2 and hence $P = 0$ if $k_y = 0$. The reason for this difference is that Dawson and Uman simply assume the existence of an oscillating magnetic field in the z -direction which does not satisfy Maxwell's equations. When the source of the fields is included the z -component of the oscillating magnetic field is found to depend on k_y and we obtain the result that $P = 0$ if $k_y = 0$. Subject to the restriction that $k_y \rho_i \ll 1$, k_y should be as large as possible.

The above analysis shows that the power absorption in transit time magnetic pumping is due to the plasma current transverse to the static magnetic field. However, the necessary phase difference between this current and the electric field is due to the motion of the resonant particles along the static magnetic field. Thus magnetic pumping is due to a rather subtle interplay of transverse and longitudinal motion.

Previous estimates of the power absorbed in magnetic pumping have shown it to be proportional to the square of the amplitude of the magnetic field perturbation. However, this was due to the approximation of replacing particles by their guiding centre motion. In fact, as shown by equation (23) it is the electric field components E_x and E_z which give the power absorption.

From the point of view of experiment it is worth considering the origin and magnitude of these electric field components.

Comparing these components with the aid of equations (16) and (18)

$$\frac{E_x}{E_z} \sim \frac{\sigma_{zz}}{\sigma_{zx}} \sim \frac{1}{k_y \rho_i} \gg 1$$

where we have again assumed $\omega^2 \ll c^2 k_y^2$ or $c^2 k_z^2$. In other words the electric field excited is predominantly transverse. However, due to the difference in magnitude of the elements of $\underline{\sigma}$ the term in the power proportional to E_z is comparable to the part proportional to $|E_x|^2$. Since $\sigma_{xx}/\sigma_{zx} \sim k_y \rho_i$ we have

$$\frac{\sigma_{xx} |E_x|^2}{\sigma_{zx} E_z E_x^*} \sim 1.$$

Finally, for a low- β plasma

$$A \approx -i\omega\mu_0 \sigma_{zz} k_z^2 (k_y^2 + k_z^2) \quad \dots (31)$$

and from equations (16) and (18) we have

$$E_x \approx \frac{i\omega\mu_0 J_{\text{ext.}}}{(k_y^2 + k_z^2)} \quad \dots (32)$$

$$E_z \approx -\frac{\sigma_{zx}}{\sigma_{zz}} E_x \quad \dots (33)$$

i.e. the E_x component is a vacuum field for low β whereas E_z is due to the presence of plasma. In any experiment designed to demonstrate the effect of transit time magnetic pumping it should be E_x or its equivalent which should be maximized.

4. POWER ABSORPTION NEAR THE ION CYCLOTRON FREQUENCY

We will now use the same model of forced electro-magnetic waves to obtain an expression for the power absorbed by the plasma when the particles 'see' the electromagnetic field oscillating close to the ion cyclotron frequency. In other words we choose ω and k_z

such that

$$\omega \sim \omega_{ci} + \sqrt{2} k_z v_{Ti} = \omega_{ci} (1 + \sqrt{2} k_z \rho_i) \quad \dots (34)$$

This corresponds to what STIX (1962) calls cyclotron damping and not ion cyclotron resonance heating which requires a 'magnetic beach' (cf STIX, 1962). Nevertheless, this is a resonance method in the sense that the condition given by equation (34) must be satisfied within certain limits which will be considered after we have obtained an expression for the power absorbed. By contrast transit time magnetic pumping works for any frequency provided it satisfies

$\omega \ll \omega_{ci}$. The conductivity tensor does not simplify so much as in the previous case, all the elements being non-zero even though $k_y^2 \rho_j^2 \ll 1$ is again assumed. Solving for the electric field components from Maxwell's equations, as before, we can again calculate the power absorbed by the plasma

$$\begin{aligned} \frac{1}{2} \operatorname{Re} \underline{J} \cdot \underline{E}^* &= \frac{1}{2} \operatorname{Re} (\sigma_{xx}) |\underline{E}_x + i \underline{E}_y|^2 + \frac{1}{2} \operatorname{Re} (\sigma_{zz}) |\underline{E}_z|^2 \\ &\quad - \operatorname{Im} (\sigma_{xz}) \operatorname{Im} (\underline{E}_z \underline{E}_x^*) + \operatorname{Re} (\sigma_{yz}) \operatorname{Re} (\underline{E}_z \underline{E}_y^*) \end{aligned} \quad \dots (35)$$

where we have used equation (34), the symmetry properties of $\underline{\sigma}$ and the approximations $\sigma_{xx} \approx \sigma_{yy}$, $\sigma_{xy} \approx i \sigma_{xx}$ for $k_y^2 \rho_i^2 \ll 1$. If we further assume

$$\frac{\omega_{pi}^2}{c^2 k_z^2} \frac{1}{k_z \rho_i} \ll 1, \quad k_y \rho_i \ll 1 \quad \dots (36)$$

conditions which are satisfied by the proposed heating experiment on Proto-Cleo the electric field components are given by

$$\underline{E}_x = (\omega_{pi}^2)^2 k_z^2 J_{\text{ext.}} \sigma_{zz} / A \quad \dots (37)$$

$$E_y = -(\omega\mu_0)^2 J_{\text{ext.}} \sigma_{yx} (k_y^2 - i\omega\mu_0 \sigma_{zz})/A \quad \dots (38)$$

$$E_z = -(\omega\mu_0)^2 J_{\text{ext.}} k_y k_z \sigma_{yx}/A \quad \dots (39)$$

where A is of course numerically different from the previous case but is still given by equation (19).

For this case the elements of $\underline{\sigma}$ are given by

$$\sigma_{xx} \approx \frac{i n_0 q_i^2}{2 m_i} \frac{1}{\sqrt{2} k_z v_{Ti}} Y(z_{-1i}) \quad \dots (40)$$

$$\sigma_{xz} \approx \frac{n_0 q_i}{2 B_0} \frac{k_y}{k_z} \left(3 - z_{-1i} Y(z_{-1i}) - \frac{z_{oe}}{2} Y(z_{oe}) \right) \quad \dots (41)$$

$$\sigma_{yz} \approx \frac{i n_0 q_i}{2 B_0} \frac{k_y}{k_z} \left(-1 + z_{-1i} Y(z_{-1i}) \right) \quad \dots (42)$$

$$\sigma_{zz} \approx \frac{i n_0 q_e^2}{\kappa T_e} \frac{\omega}{k_z^2} \left(-1 + z_{oe} Y(z_{oe}) \right). \quad \dots (43)$$

The remaining elements are related to these through symmetry conditions or the $k_y^2 \rho_j^2 \ll 1$ approximation.

Using equations (37) - (43) and again considering $T_e \sim T_i$ we can simplify the power expression given in equation (35). First E_x is again the dominant electric field component since

$$E_y \sim E_z$$

and $E_z/E_x \sim \sigma_{yx}/\sigma_{zz} \sim k_z \rho_i$. Secondly, the ratio of the second, third and fourth terms to the first in equation (35) are $(m_e/m_i)^{\frac{1}{2}}$, $k_y^2 \rho_i^2$ and $k_y^3 \rho_i^3$ respectively. Thus, we may write

$$P \approx \frac{1}{2} \text{Re} (\sigma_{xx}) |E_x|^2. \quad \dots (44)$$

It follows from the approximations already made that

$$A \approx - (k_y^2 + k_z^2) k_z^2 i\omega\mu_0 \sigma_{zz}. \quad \dots (45)$$

This means that the dominant electric field excited by the oscillating external current

$$E_x \approx \frac{i\omega\mu_0 J_{\text{ext.}}}{(k_y^2 + k_z^2)} \quad \dots (46)$$

is again a vacuum field. Using equations (40) and (46) the final expression for the power per unit volume absorbed by the plasma in the vicinity of the ion cyclotron frequency is

$$P = \frac{1}{4} \left(\frac{\pi}{2} \right)^{\frac{1}{2}} \frac{|J_{\text{ext.}}|^2}{\epsilon_0} \exp(-z_{-1i}^2) \frac{\omega_{pi}^2 \omega_{ci}}{k_z \rho_i \times c^4 (k_y^2 + k_z^2)^2} \quad \dots (47)$$

The variation of P with ω has a resonant behaviour due to the exponential factor. The maximum for P occurs when $\omega = \omega_{ci}$ and the half width is approximately $2\sqrt{2} k_z v_{Ti}$.

We may now compare the power absorbed by the plasma due to transit time magnetic pumping with that due to cyclotron damping. For the same exciting current and $T_i \sim T_e$,

$$\frac{P_{\text{cycl.}}}{P_{\text{TMP}}} \sim \frac{1}{k_y^2 \rho_i^2 \times k_z \rho_i} \frac{\omega_{ci}}{\omega} \quad \dots (48)$$

where we have taken the exponential factor in equation (47) and the function of z_{oi} in equation (30) both to be of order unity. In equation (48), ω is the frequency for magnetic pumping so that $\omega_{ci}/\omega \gg 1$. Thus, power is evidently absorbed much more efficiently by the plasma close to the ion cyclotron frequency than by transit time magnetic pumping.

5. CONCLUSIONS

Although the cyclotron heating rate is much greater than that due to magnetic pumping there is one respect in which the latter method is superior. It does not depend critically on the value of the magnetic field. On the other hand, in the ion cyclotron case, if ω departs far from ω_{ci} the heating rate becomes exponentially small.

Heating methods of the type described in this paper may be applied to plasmas in magnetic traps where very complex magnetic fields are used. To carry out an analysis similar to the above would be extremely difficult in such fields. All we shall attempt here is a very simple estimate of the fractional magnetic field and temperature variation which could be tolerated.

In so far as variations of the confining magnetic field effect those particles resonant with the forced wave, both methods of heating will be equally sensitive to such variations. However, these effects are not covered by this analysis. We are only concerned with the way in which the heating rates calculated in this paper depend on magnetic field and temperature. Magnetic pumping should not be very sensitive to small changes of the magnetic field and significant ion heating will take place provided

$$0 < \omega < 3 k_z v_{Ti} \text{ (approx.)}$$

Since ω and k_z are fixed by the circuit and v_{Ti} increases during the heating this condition would be satisfied throughout.

For $\omega \ll k_z v_{Ti}$ the heating becomes linearly small.

As already mentioned, heating near the cyclotron frequency is very sensitive to variations in the magnetic field and temperature. The condition for a significant fraction of the ions to 'feel' the electromagnetic fields oscillating at their cyclotron frequency is

$$\omega \sim \omega_{ci} + \sqrt{2} k_z v_{Ti}.$$

If we denote the mean value of ω_{ci} by ω_{ci}^0 , its variation by $\Delta\omega_{ci}$ and let v_{Ti}^0 and Δv_{Ti} by the initial ion thermal velocity and its increase then we can get an estimate of the amount of variation that can be tolerated by taking

$$-\sqrt{2} \leq z_{-1i} \leq \sqrt{2}.$$

This then gives for the band of allowed ω_{ci} values

$$-(2-\sqrt{2}) k_z \rho_i^0 - 2 k_z \Delta \rho_i \leq \Delta\omega_{ci}/\omega_{ci}^0 \leq (2 + \sqrt{2}) k_z \rho_i^0 + 2 k_z \Delta \rho_i.$$

All the analysis in this paper has been linear. Without a discussion of the non-linear development of the heating no definitive answer to the feasibility of these methods can be given. The linear expressions do provide an upper limit to the heating rate however. Since both methods depend on resonant particles they both require a sufficient collision rate to re-Maxwellianize the distribution of these particles. In the magnetic pumping method this results in there being a critical density below which the effect will not occur. It seems reasonable to expect that the corresponding density for the cyclotron case may not be very different but that due to the larger rate of heating a smaller external current amplitude could be used.

In summary, then, we have analysed a sufficiently simple model to enable the linearized problem of transit time magnetic pumping to be solved exactly. The exact treatment shows much more clearly the

mechanism of the power absorption which depends on the transverse current (transverse to the uniform background magnetic field) and the E_x and E_z electric field components. The longitudinal current gives no power absorption but it is the longitudinal motion of the particles which produces the phase shift between \underline{J} and \underline{E} resulting in the power absorption. For a low- β plasma the E_x field is a vacuum field whereas E_z is due to the presence of plasma. The same model has also been used to obtain the power absorbed near the ion cyclotron frequency. The power absorption was much greater than in the magnetic pumping case. Finally, in the cyclotron case the power absorption is dominated by E_x which is again a vacuum field for $\omega_{pi}^2/c^2 k_z^2 \ll 1$.

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