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COLLISIONLESS SHOCKS

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ABSTRACT

This paper discusses naturally occurring collisionless shocks and the relevance of laboratory experiments. The results of such experiments are reviewed in relation to shock theory.

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1. INTRODUCTION

Collisionless shock waves appear with increasing frequency in the literature of space and astrophysics. The adjective collisionless is deceptive. The classical definition of a collision involves a series of small angled deflections by the THERMAL ELECTRIC FIELD FLUCTUATIONS, $\langle E^2 \rangle$, in the plasma. However many plasmas are far from thermal equilibrium (e.g. severe gradients in a shock) and the free energy can usually couple into the COLLECTIVE (i.e. wave) degrees of freedom of the plasma. This coupling is a plasma instability and leads to a SUPRA-THERMAL level of $\langle E^2 \rangle$.

If many such degrees of freedom are excited with random phases, the plasma is said to be TURBULENT. Non-linear wave-wave mode coupling can generate this 'wave chaos' in analogy with the particle-particle origin of 'molecular chaos'. There are interesting conceptual questions here which we must leave.

We restrict our discussion to electrostatic turbulence with
(i) scale $\ll L_s$ = shock width (i.e. microturbulence),
(ii) fluctuating potential $\phi \ll (\kappa T_e/e)$ so that large angle deflection, and trapping are not imported, (i.e. weak turbulence).
Under these conditions the turbulent $\langle E^2 \rangle$ results in more rapid deflection of the particles than for a thermal plasma. There is an effective or turbulent collision frequency i.e. $\nu^* > \nu_{thermal}$. The plasma can be described reasonably well by a FLUID MODEL with ENHANCED TRANSPORT coefficients derived from ν^*. The dependence of these transport coefficients is determined by the instability and the non-linear processes. The dissipation, i.e. entropy increase, arises through the randomness of the turbulence.

Experiments[1,2] have demonstrated that 'collisionless shocks' exist in which an enhanced collision rate arises from microturbulence and that the shock can be described by an MHD fluid model with enhanced transport coefficients.

2. COSMICAL COLLISIONLESS SHOCKS

We shall briefly mention some of the circumstances in which collisionless shock occur in the literature of space and astrophysics, recognizing that in the latter case it is often conjecture.

(i) Space: Satellites have observed both the earths bow shock[3,4] and interplanetary shocks in the solar wind.
(ii) Solar: Optical[5] and radio observations[6,7] demonstrate the emission of shocks from solar flares. Some models of flares invoke an internal shock as well.
(iii) Stellar: Flare stars and supernovae may involve shocks.
(iv) Galactic: Galactic 'jets' and 'explosions' may involve shocks as may the expanding 'plasma blobs' of double radio sources.

The appeal of a collisionless shock in situations (ii) to (iv) is that it can convert kinetic energy, through the mediation of plasma turbulence, into the observed NON-THERMAL ELECTROMAGNETIC EMISSION. Turbulence can stochastically accelerate particles to sufficient energy for synchrotron emission and can also directly emit at ω_{pe} and its harmonics. Both of these emission processes are observed.

It should be noted that the kinetic energy of the streaming plasma can generate the required turbulence even if no shock wave forms ahead of it. Both 'piston' and shock can be turbulent.

3. LABORATORY EXPERIMENTS

Just as laboratory spectroscopy and atomic physics have contributed to the understanding of classical (i.e. optical) astrophysics so we expect laboratory plasma physics to contribute to the understanding of modern (e.g. radio) astrophysics. Phenomena are more readily understood when they and the theories involved can be studied in the laboratory under the 'microscope', rather than the telescope.

Laboratory experiments on collisionless shocks can be divided into three classes depending on the nature of the flow and piston.

Flow experiments; A quasi-steady supersonic plasma flow can be produced by an arc or a nozzle. When the flow impinges on an obstacle, such as a magnetic dipole, a bow shock is produced.

Plasma pistons; A more dense plasma can be made to compress a less dense one and produce a shock. The more dense plasma can be produced within a plasma by (i) increasing the degree of ionization using pulsed UV light or by (ii) ionizing a solid target using a powerful pulse of laser light. Alternatively, plasmas of different density, separated by a negatively biassed grid, can be pulsed into contact. If the two plasmas interact sufficiently strongly a shock should be produced.

Magnetic piston: A rising magnetic field acts on a highly conducting plasma like a piston. Three cylindrical configurations are used; (i) Z-pinch with axial current, (ii) the theta-pinch with azimuthal current, both of which give radial compression, and (iii) the annular shock tube with radial current and axial compression.

The time and space scales must be right for piston and shock formation. In small apparatus, for example, the field may diffuse before any compression occurs or the driver plasma may not have time to interact with and hence compress the ambient plasma.

4. SHOCK DISCONTINUITIES

Assuming a shock forms, its nature depends on the piston and the compression wave which it can generate. In the absence of a magnetic field, there is only the one sound speed $c_o^2 = \gamma p/\rho$. However in a magnetized plasma there are three anisotropic sound speeds, slow (c_s) intermediate (c_i) and fast (c_f). All of these, except the intermediate wave, should theoretically steepen to form shocks. For MHD stability, the change of flow speed across the shock, in its frame, must jump only one sound speed (i.e. only one wave is trapped and steepens).

Regarding the shock as a discontinuous jump from state 1 to state 2, the conservation relations and Maxwell's equations determine the shock jumps (i.e. Rankine-Hugoniot or de Hoffman-Teller relations). These are uniquely defined by four parameters.

(i) Ratio of initial plasma to magnetic pressure $\beta = 2\mu_o p/B^2$

(ii) Ratio of shock to sound or Alfven speed (c_A), i.e. Mach Nos. $M = V_s/C_o$; $M_A = V_s/C_A = \sqrt{\mu_o\rho}\, V_s/B$ (S.I. units)

(iii) The angle between shock velocity vector $\bar{V}_s$ and $\bar{B}$

(iv) The ratio of specific heats γ.

For both fast and slow shock $\bar{V}_s$, $\bar{B}_1$, $\bar{B}_2$ are coplanar. For fast shocks $B_2 > B_1$ and the ratio $R = \kappa T_2/\frac{1}{2}MV_s^2$ tends to limit $1/\gamma$ as $M_A \to \infty$. For slow shocks $B_2 < B_1$ and R is not limited.

5. LIMITATION OF STEEPENING

The internal structure of the shock will be determined by the first process which can limit the steepening and satisfy the jump relations.

Dispersive limitations: Consider a collisionless plasma without instability. Through the dispersion relation, $D(\omega,k) = 0$, the phase velocity of a wave is related to its wave number, $k = 2\pi/\lambda$. When a dispersive effect decreases $d\omega/dk$ at a certain k_o, steepening results in a slower phase velocity and a trailing wave train; e.g. (i) non-magnetic ion sound waves at $k_o \sim 2\pi/\lambda_D$, (ii) fast waves with $\bar{V}_s \perp \bar{B}$, $\beta < 1$, at $k_o \sim 2\pi(\omega_{pe}/c)$. When $d\omega/dk$ increases, steepening will produce a forward wave train, but some damping is essential because it cannot extend indefinitely forward; e.g. fast wave propagating obliquely to B at $k_o \sim 2\pi(\omega_{pi}/c)$. Both these dispersive processes limit steepening and produce a large amplitude wave. Neither produce the required entropy increase for a shock without some additional process.

Dissipative Limitations: The gradients produced by simple steepening or the above large amplitude waves have free energy which can be dissipated through thermal collisions or instability driven turbulence. Both of these processes can be described by a transport coefficient and for a strong shock only viscosity (μ) and/or resistivity (η) need be considered. In a viscous shock dissipation through ∇v is dominant while in a resistive shock ∇B (i.e. current J) is dominant.

Particle Effects: The above gradients have associated electric fields and these can reflect some of the incident ion distribution function. In the absence of a magnetic field these particles are lost, and no steady state is possible. The steepening can still be limited by the energy loss. In the presence of a magnetic field the reflected particles gyrate, gain energy and pass back through the shock. Theories exist which predict that the phase mixing of these gyrating particles behind the shock gives rise to the required entropy change for a shock to form. Trapping of particles within wave trains can also produce an entropy increase.

The structure of a shock will depend on which of these various processes limits the steepening.

6. SCALING LAWS

In relating large scale natural phenomenon ($L \sim 10^5 - 10^8$ m) to small scale laboratory experiments ($L \sim 10^{-3} - 10^0$ m) it is important to recognize a simple physical process and attempt to simulate it in the laboratory by scaling the relevant parameters.

For shock waves the important parameters are:

(i) Jump parameters: β, M_A, θ, γ
(ii) Plasma parameters: n_e, T_e, T_i, ion mass and charge

(iii) Flow parameters: V_s
(iv) Fields: B, E, ϕ (potential)
(v) Microscopic parameters: distribution $f^n s$, $f_e(v)$, $f_i(v)$,

lengths λ_D, λ_{coll}, r_{ce}, r_{ci} and $\alpha = \left(\frac{\omega_{ce}}{\omega_{pe}}\right)^2 = \left(\frac{r_{ce}}{\lambda_D}\right)^2$

Collisionless plasma effects should obey the Vlasov equation for which the exact scaling laws are (ref. 8)

$t \propto L$; $n \propto L^{-2}$; $B \propto L^{-1}$; $E \propto L^{-1}$ ($\lambda_D \propto L$, $r_c \propto L$)
α, β, γ, V_s, M_A, M, T, ϕ, f_e, f_i, independent.

Clearly this scaling does not include thermal fluctuation effects (e.g. λ_{coll}, N_D) or radiation effects.

For collisionless shocks we shall find that the shock width depends on (c/ω_{pe}) or (c/ω_{pi}) for magnetic shocks and λ_D for non-magnetic. All of these follow the Vlasov scaling $L \propto n^{-\frac{1}{2}}$ and fortunately the scaling curve also passes through the region of laboratory plasmas. For example

	L(m)	$n(m^{-3})$	B(T)
Bow shock	10^5	10^{+6}	10^{-9}
Laboratory	10^{-2}	10^{20}	10^{-2}

Qualitative scaling of thermal effects $L_s < \lambda_{coll}$ can also be obtained in the laboratory.

7. ELECTROSTATIC SHOCKS

In a collisionless unmagnetized plasma ion inertia produces dispersion at ω_{pi}. This gives rise to large amplitude trailing waves with $\lambda \sim \lambda_D$ and these have been observed in the laboratory[9,10].

At high Mach number $M > 1.6$, the E-field of the wave reflects particles forward and these are lost. The wave itself take up the reaction and becomes irregular, and non-steady.

At much higher M it is not even possible to produce a piston because the intersteaming velocity is too great for instability[10]. There is, as yet, no evidence of ion-ion streaming instability producing a viscous shock.

8. MAGNETIC SHOCKS : MACROSTRUCTURE

We shall restrict our discussion to fast shocks because these are observed in the laboratory. We shall classify by the jump parameters M_A, β and θ.

There are several critical Mach numbers in shock physics. The most important, M_A^*, occurs when the plasma is heated sufficiently for the flow behind a magnetic shock to become sub-sonic as well as sub-magnetosonic. Above M_A^* non-magnetic sound waves can steepen, on a scale length for which the magnetic field and plasma are resistively decoupled, and form a sub-shock. Below M_A^* we expect a resistive magnetic shock and above M_A^* an additional viscous electrostatic sub-shock.

8.1 $\theta = 90^o$ (i.e. $V_{s\perp}B$), $\beta_1 \ll 1$, $M_A < M_A^* \sim 3$

Weak shocks with structures dominated by classical resistivity are observed[11]. These change to a broader structure with $L_s \sim 10(c/\omega_{pe})$ when the electron drift velocity (v_d) within the shock is sufficient to drive the ion wave instability (i.e. $v_d > c_o$).

Large amplitude trailing waves are observed at low densities[9,11]. These result from the dispersive effect of electron inertia and have $L_s \sim (c/\omega_{pe})$. For $\alpha \geq 1$, $v_d \to c$ and relativisitic limitation yields $L_s = \sqrt{\alpha}(c/\omega_{pe}) = v_A/\omega_{pi}$, which is observed[9]. At higher density there is time for the two-stream instability driven by $v_d \sim v_{eth}$ to grow within the shock, and then a non-oscillating structure with $L_s \sim 10(c/\omega_{pe})$ appears[9].

The characteristic non-classical (i.e. collisionless) shock for this θ, β, M_A, has $L_s \sim 10(c/\omega_{pe})$ and has been studied in many laboratories[1,2,9,11-14]. The 'collisionless' nature of these shocks is demonstrated by the inadequacy of classical transport coefficients to explain the observed electron heating which requires $\eta^* \sim 100\eta_{sp}$(classical)[13b]. Also in some cases $\tau_s (= L_s/V_s)$ is much shorter than the classical collision time (τ_{ei})[11]. As $r_{ci} \gg L_s \gg r_{ce}$, the ions are unmagnetized while the electrons experience drift motion due to ∇B, ∇n, ∇T and $\nabla\phi$ (i.e. E_L). This latter is dominant in most experiments. E_L arises from ∇p_e and the Hall effect and adiabatically slows down the ions. The electrons are irreversibly heated and satisfy the conservation relations.

The observed drift velocity exceeds the critical value for electrostatic instability. As the electrons are heated usually $T_e > T_i$, and so ion wave (I.W.) or I.W. coupled to electron cyclotron wave (E.C.W.) instability can occur[15]. There is experimental evidence for I.W. turbulence[1] and the η^* derived from the macrostructure scales[11] in agreement with the predictions of the Kadomtsev-Sagdeev theory for I.W. turbulence.

8.2 $\theta = 90^o$, $\beta_1 \ll 1$, $M_A > M_A^* \sim 3$

For $6 > M_A > M_A^* \sim 3$ the resistive shock with $L_s = L_R \sim 10(c/\omega_{pe})$ has a broad 'foot' in front with $L_F \simeq 2(c/\omega_{pi}) \sim 8\ L_R$[13]. The sharp rise L_R is the same as for $M_A < M_A^*$ and is dominated by I.W.

turbulence. The foot appears to be formed by the gyration of ions ($L_R \sim r_{ci}$) in front of the resistive structure. However the observed electric potential ϕ_R is not sufficient to reflect the required number of ions.

Theory predicts that above M_A^* there should be a viscous sub-shock at the rear of the resistive shock and that this sub-shock should heat ions. The observed electron heating is inadequate to satisfy the conservation relations and so ion heating is assumed to occur within a sub-shock. There are two suggested mechanisms for reflections of ions within the sub-shock. Firstly, the ion heating makes reflection more probable. Secondly the sub-shock can consist of damped ion inertia waves and the overshoot will reflect adequately.

The fraction of the jump ΔB across L_F increases until for $M_A \sim 6$ there is no L_R[13]. This structure has oscillations behind and is unsteady.

8.3 $\theta = 90^o$, $\beta_1 \gg 1$

There are only two experiments[2,11,12,14] in this regime. The most striking difference from $\beta \ll 1$ is that there is no structural change at M_A^* and that $L_s \sim (c/\omega_{pi})$ for $3 < M_A < 9$. There is a clearly observed change from electron heating below, to increasing ion heating above M_A^*, with the observed $T_e + T_i$ fitting the conservation relations. There is also direct evidence for microturbulence as for $\beta \ll 1$.

8.4 Oblique Shocks

For oblique propagation, Whistler dispersion produces a forward wave train which is observed in piston experiments[16]. Non-classical damping of the oscillations corresponds to an η^* similar to that observed for perpendicular propagation. For $M_A > M_A^*$, there is no 'foot' or other evidence of reflected ions, but the observed structures require a viscous sub-shock at the rear. Under certain conditions, which are not fully understood, the Whistler oscillations develop high frequency components as the shock propagates and these eventually destroy the regular structure.

In the steady flow experiments[17] there is no evidence of a forward wave train. Both in and behind the shock, $L_s \sim c/\omega_{pi}$, there is macroscopic electromagnetic turbulence. In these experiments there appears to be sufficient time for the high frequency instability, mentioned above, to convert the steady oscillations to turbulence.

These observations emphasise the problem of time scales in simulation experiments.

8.5 Parallel Propagation

For parallel propagations there are two classes of shock. The non-magnetic shock and the 'switch-on' shock. The latter generates a transverse component of B, and occurs in a limited region of parameter space defined by

$$\beta_1 \leq 2/\gamma \quad \text{and} \quad 1 \leq M_A \leq \hat{M}_A \; ; \quad \hat{M}_A = [\gamma(1 + \beta_1) + 1]^{\frac{1}{2}}/(\gamma - 1)^{\frac{1}{2}} .$$

Such shocks have been observed[1c] in the laboratory but not studied in detail.

9. MICROSTRUCTURE OF SHOCKS

The inadequacy of thermal transport leads directly to a search for collective effects and microturbulence. The micro-turbulent electric fields, $\langle E^2 \rangle$, will scatter and heat particles while the corresponding density fluctuations $\langle \delta n_e^2 \rangle$ can scatter photons and allow a direct measurement of the level and spectrum of the turbulence in terms of the Fourier transform $\langle \delta n_e^2(\omega,k) \rangle$. The nature of the fluctuations can be deduced and hence $\langle E^2 \rangle$ derived from the measured $\langle \delta n_e^2 \rangle$. This technique has been used on two perpendicular shock experiments.

The results from the first, TARANTULA[1], with $\beta \ll 1$, $M_A < M_A^*$, $T_e > T_i$ are summarized. The fluctuations are SUPRA-THERMAL by more than two orders of magnitude. For a given $\bar{k}$ there is a dominant mode with frequency ω such that (ω,k) fits and scales as for ION WAVES. The mode, however has a short coherence time $\tau \sim 2\pi/\omega$ demonstrating a high degree of RANDOMNESS OF PHASE. This turbulence is grossly ANISTROPIC being confined to within 50^o from the direction of the electron current within the shock. At present these measurements are restricted to the plane perpendicular to B. However, if the fluctuations are ion waves, as seems probable, this anisotropy should form a cone about the driving electron current. The wave number spectrum has the form $\langle \delta n_e^2(\omega,k) \rangle \propto (1/k^3)\ell n\,(1/k\lambda_D)$ in agreement with the predictions of non-linear theory[18,19].

The non-linear theory of ion wave turbulence is discussed in terms of a balance of linear growth at $k \sim 1/\lambda_D$ against non-linear diffusion to lower k. This diffusion results from the 3-wave process in the form of resonant wave decay[19] or non-resonant wave scattering on particles[18]. Both processes give the observed k-spectrum. The decay process is only possible because of the short coherence time but, when possible, as the resonant process it should dominate. However as yet there is no clear agreement between experiment and non-linear theory.

The measured level of turbulent energy ($\leq 2\%$ thermal), potential fluctuations ($e\phi \leq 1\% \,\kappa T_e$) and randomness of phase are used to justify a STOCHASTIC treatment of the electron heating by the turbulence. This yields a mean effective resistivity within the shock which is a half that required experimentally.

The second experiment[2] involving photon scattering has $\beta > 1$, $M_A \gtrsim M_A^*$ and $T_e < T_i$. The measurements are similar to the above although no scaling with ion plasma frequency is reported. This similarity is surprising because for $Te < Ti$ ion wave turbulence is not expected and the E.C.W. appears necessary[20].

10. COMPUTER SIMULATION OF MICROSTRUCTURE AND TURBULENCE

Particle simulation computations, usually one-dimensional, have followed the development of turbulence driven by a current across a magnetic field. However the conflicting results require more discussion than is possible here[21,22].

11. COLLISIONLESS SHOCKS OUTSIDE THE LABORATORY

Finally we return to the cosmical scene to consider two examples of collisionless shocks.

(i) The Earth's Bow Shock

Measurements from spacecraft show clearly the existence of a collisionless bow shock but equally clearly it is NOT STEADY in position or structure. It is, in general, a fast oblique shock with $M_A \sim 8 > M_A^*$, $\beta \gtrsim 1$, $T_{e1} \gtrsim T_{i1}$, $T_{e2} < T_{i2}$. Unfortunately the shock moves in position with a velocity comparable to or greater than that of the space craft. This results in multiple crossings and ambiguous length scales. The length scales for the shock transition vary considerably for different parameters and from crossing to crossing. Detailed results from OGO V[3] and VELA 4[4] show magnetic field changes in distance $L_s \sim 10\ c/\omega_{pe}$ and less frequently c/ω_{pi}, while temperature changes are over $L \sim 10\ c/\omega_{pe}$ for ions and $30\ \lambda_D$ for electrons (rare observation). In one crossing a reversible wave train with $\lambda \sim c/\omega_{pe}$ (i.e. dispersive wave train) was observed. Some of these results are surprising for $\beta \geq 1$ and $M_A > M_A^*$.

High frequency FLUCTUATING ELECTRIC FIELDS are observed in regions of high magnetic field gradient. However the ambiguity of velocity makes the k and ω scales uncertain. The frequency spectrum appears as discrete modes which tend to broaden and merge towards the rear. There is some similarity here with the current driven turbulence observed in laboratory shocks[1,2].

(ii) Solar Flare Shocks

A shock like disturbance has been observed, by both optical[5] and radio[6] emission, to emanate from the sudden release of energy in a solar flare on the disc of the sun. The radio emission at ω_{pe}

and 2 ω_{pe} (Type II) must arise from some COLLECTIVE EFFECT within the shock front.

A limb flare has been observed[7] to give rise to type IV radio emission propagating outwards from a solar flare like a shock wave. Analysis suggests that this synchrotron emission commences when the disturbance forms a collisionless shock through two-stream instability and that it disappears when the shock broadens at $M_A = M_A^*$. The emission is thought to be compatible with synchrotron emission from a TURBULENT PLASMA with a few electrons STOCHASTICALLY accelerated to a few MeV. This calculation assumed current driven ion wave turbulence of the Kadomtsev form[18].

12. CONCLUSIONS

There is now clear evidence for the turbulent nature of collisionless resistive ($M_A < M_A^*$) shocks in the laboratory. A reasonable degree of self-consistency and agreement with theory has been obtained. Some of these features are also observed in space and solar shocks.

Future laboratory effort should move on to particle effects, including acceleration, in both electrostatic and high M_A magnetic shocks and, if possible, also onto electromagnetic emission from shocks.

The task of understanding natural phenomena is intrinsically difficult because of irreproducibility and uniqueness. Also physical processes can not be isolated as in the laboratory. Fortunately there is no need to understand cosmic plasmas in great detail. If the basic processes involved can be understood as a result of theory and experiment, then an adequate model can be constructed. While not accurate in detail it should then have a high degree of plausibility.

REVIEWS

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