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OBSERVATIONS OF FLUTE WAVES IN SIMPLE MIRROR AND MINIMUM-B MAGNETIC FIELDS

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OBSERVATIONS OF FLUTE WAVES IN SIMPLE MIRROR AND MINIMUM-B MAGNETIC FIELDS

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ABSTRACT

A stable electrostatic flute wave, with $k_{\parallel} = 0$, has been observed in simple mirror geometry and in minimum-B geometry. In the simple mirror case the wave couples with the $\underline{\nabla B} \times \underline{B}$ drift motion of the ions to give the classic flute instability. In the minimum-B case (where the direction of the $\underline{\nabla B} \times \underline{B}$ drift is reversed) the wave has been shown to be stable up to the maximum density achieved. In both cases the variation of frequency with density for the m=1 mode is in reasonable quantitative agreement with the predictions of the available theoretical models.

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I. INTRODUCTION

Unstable low frequency flute instabilities have been observed in simple mirror plasmas by a number of workers. Until recently there had been no definitive observation of the associated stable flute waves.

Waves with $k_{\parallel} = 0$ arise in mirror geometry because of the different behaviour of ions and electrons in the magnetic field At low densities and $\omega \ll \omega_D$ (where ω_D is the ion $\underline{\nabla B} \times \underline{B}$ drift frequency) an electron wave exists in which the ions play little part because of their relatively fast drift motion through the wave. In simple mirror geometry as the density is raised so that $\omega \approx \frac{1}{2}\omega_D$ this wave couples with the ion drift motion to give the classic flute instability.

In minimum-B geometry, the ion drift motion is in the opposite direction to the electron wave and unstable coupling does not occur. In this case it should thus be possible to propagate the stable electron wave at densities where $\omega \gtrsim \omega_D$.

When $\omega_D \ll \omega \ll \omega_{ci}$ the wave becomes primarily dependent on the different behaviour of the ions and electrons due to their finite Larmor radius. When $\omega \gg \omega_{ci}$ a similar wave exists in which the ions are constrained by inertia[1].

In this paper we are concerned with the low and, to some extent, the intermediate frequency regime where the wave is dominated by the $\underline{\nabla B} \times \underline{B}$ drift frequency of the ions.

In 1967 some of our work was reported briefly[2] and since then Colchin, Dunlap and Postma[3] have described stable flute oscillations in their simple mirror plasma and compared their results with several theoretical models. In this paper we describe our own experiments on simple mirror plasmas where above some threshold density the stable electron flute wave is shown to couple with the ion wave and go over to the flute instability. We then go on to describe the observation of these stable waves in a minimum-B mirror geometry where they do not couple and remain stable up to high density.

II. EXPERIMENTAL METHOD

The PHOENIX II magnetic mirror trap has been described elsewhere[4]. The magnetic field is created by two coils mounted to form mirrors 30 cm apart giving a mirror field 1.8 times the central value. A quadrupole field can be superimposed on the simple mirror to produce a minimum-B system.

The plasma was created by injection of up to 50 mA equivalent of 20 keV neutral atoms across the centre of the trap where a small fraction of them was ionised and trapped by the Lorentz $\underline{v} \times \underline{B}$ field. In stable plasmas the main loss process was charge exchange on the residual gas and in order to alleviate this the background pressure was maintained at $\lesssim 10^{-9}$ torr by extensive gettering on the walls which were maintained at 77^{o}K. The equilibrium density was varied from 10^{7} cm^{-3} to 1.5×10^{9} cm^{-3} by introducing small quantities of helium gas. For the experimental conditions investigated the ion energy in the plasma was essentially the injected energy; the electron temperature was not an important parameter for these experiments but was typically in the range 10-30 eV.

The plasma density was obtained by measuring the cold ion current emerging through the mirrors. This current is produced by ionisation of the gas by the fast ions and to some extent by the electrons and was measured using a small area Faraday cup with secondary electron suppression. The fast ion density (n_+) was calculated using the relationship

$$I_+ = n_o \left(\overline{\sigma v_e} + \overline{\sigma v_i}\right) A \int n_+ \, d\ell$$

where I_+ is the current emerging in area A (suitably corrected for the mirror ratio), n_o is the gas density, the rate coefficients are appropriate for electrons and ions, and the density is integrated along a line of force. The effective length of the plasma appropriate for comparison with theory will be discussed later.

n_o was determined by observing the time constant (τ) for the decay of the trapped ion plasma after beam switch off (using fast atom detectors) and using the relationship

$$\tau = \frac{1}{n_o \, \sigma_x \, v}$$

where σ_x is the charge exchange cross section of the trapped ions of velocity v.

III. SIMPLE MIRROR EXPERIMENTS

The radial and axial density distribution in simple mirror geometry has been described previously[5]. Essentially the plasma is a short cylinder of total length 5 cm and radius decided by a plasma limiter. The magnetic field as a function of radius (r) is given to good approximation by the expression

$$B = B_o (1 - \alpha_1 r^2)$$

where B_o is the central field and $\alpha_1 = 3.7 \times 10^{-3}$ cm^{-2}. The ion energy was 20 keV.

Electrostatic flute oscillations were excited using a plate located near the surface of the plasma. For the simple mirror experiments this plate was longer than the extent of the plasma in the direction of the magnetic field, was mounted at a radius of 8.5 cm and subtended a 75^o angle: it served also as the plasma limiter. The radius of the vacuum chamber is 20 cm.

The oscillations were detected using a small (~ 1 cm^2) electrostatic probe screened from the field of the exciting electrode. After amplification they could be displayed on an oscilloscope or their frequency measured using a counting technique which produced an output on a LINC-8 computer data handling system(6).

Two methods were used to excite the oscillations: a single frequency and white noise.

For the single frequency experiments an oscillator was employed which could be swept slowly over the range 0.2 - 500 kHz while the plasma density was kept constant. Alternatively the frequency of the oscillator could be kept fixed and the density allowed to decay. Figure 1 shows the response of the plasma in these two cases. In all cases a strong principal resonance was observed. This was identified as the first azimuthal mode (m=1) by using two electrostatic probes separated in azimuth. Other modes were observed which corresponded to higher azimuthal, and possibly radial, mode numbers.

The finite width of the resonance is probably associated with non-linear effects. Certainly the shape of the swept frequency resonance curves illustrated by that in Figure 1 is reminiscent of the non-linear response observed in other experiments[7]. As the density was raised the resonances became narrower and the number of observed modes increased. At these higher densities ($\gtrsim 2 \times 10^8$ cm^{-3}) there also appeared a new phenomenon: excitation at the fundamental frequency became increasingly difficult and, eventually, for the strongest excitation the oscillator had to be tuned to $\frac{1}{2}$ the excited frequency.

For densities very close to the threshold for instability, short bursts of oscillation at the unstable frequency ($\sim \frac{1}{2}\,\omega_D$) were observed followed by slowly damped oscillations whose frequency depended on density in exactly the same way as that found for externally stimulated oscillations. The damping time of these oscillations was typically 10 ms and was of the order of the time required for electron replacement by ionisation of the background gas.

Figure 2 shows the frequency of the principal resonance plotted against line density ($\int n\, d\ell$). As an alternative experimental approach we have obtained a similar plot directly by excitation using white noise. Figure 3 shows such a plot obtained as the plasma builds up after beam switch on and decays after beam switch off. This data is also in good agreement with that obtained using a single frequency source of excitation.

Some of the scatter of data such as that shown in Figure 2 is ascribed to the Doppler shift of the frequency due to plasma rotation in the radial electric field associated with the positive plasma potential. Residual microinstabilities cause fluctuations of potential[8] sufficient to account for the observed frequency shifts.

Attempts were made to excite the ion branch of the wave with frequency between ω_D and $\frac{1}{2}\,\omega_D$. This proved difficult to excite - a phenomenon which we believe to be due to the heavy damping caused by the variation of ion drift velocity with radius.

IV. MINIMUM-B EXPERIMENTS

A minimum-B magnetic well was created by passing current through the quadrupole windings. The general dimensions and conditions were as described previously[9]. The magnetic field variation with radius (r) is given to a good approximation over the range of interest by

$$B = B_o\ (1 + \alpha_2\ r^2)$$

where B_o is the central field (12.5 kGauss for these experiments) and $\alpha_2 = 4.7 \times 10^{-3}\ cm^{-2}$ for the value of quadrupole current used in these experiments. The ion energy was again 20 keV.

Thus in this case the ion magnetic drift was in the opposite direction to the electron wave and had rotation frequency to first order independent of radius and equal to -240 kHz.

The plasma radius was limited to 5 cm because, at larger radii, the magnetic drift surfaces were no longer closed. A somewhat different method of excitation was used for these experiments; the stable wave was impulse excited by a negative pulse, of one micro-second duration, applied to a 1 cm diameter plate placed at a radius of 6 cm on the median plane. A slow repetition rate (30/sec) allowed the plasma to recover between pulses. As for the simple mirror case the oscillations were picked up on a similar probe placed at a radius of 8 cm directly opposite the excitation probe.

The slowly decaying oscillation was amplified, filtered, displayed on an oscilloscope and recorded using a film camera. A high pass filter with a cut off in the range 3-10 kHz was necessary to reject slow potential excursions, mains pickup and machine generated interference. The response of the system was otherwise flat up to 10 MHz.

Over most of the density range investigated a clearly defined dominant mode was present. It was again identified as an m=1 wave travelling in the direction of the electron wave. The decay time of the oscillation again agreed with estimates of the electron replacement time. The frequency of this dominant mode is plotted against line density in Figure 4. Data from five different plasma decays are shown superimposed.

At the highest densities obtainable the mode structure became very complicated. The composite frequencies present were analysed and an attempt made to associate them with specific modes. However, the scatter was too great to allow positive identification of individual modes.

V. COMPARISON WITH THEORY

A number of low density theories of the flute instability have been developed. Early workers, for example Mikhailovskii(10), discussed these waves in plain geometry with consequent difficulties of translation into cylindrical geometry terms. More recently, Kuo et al.(11), Guest and Beasley(12), Varma(13) and Arsenin and Chuyanov(14) have obtained dispersion relations for cylindrical geometry with varying assumptions about boundary conditions.

In cylindrical geometry at low densities $\left(\omega_{pi}^{\ 2} \ll \omega_{ci}^{\ 2}\right)$ and when finite Larmor radius effects are neglected, the dispersion relation is of the form

$$\frac{\omega_{ci}}{\omega_{pi}^{\ 2}} + \frac{\omega_D A_1}{\omega(\omega + m\omega_D)} = 0 \tag{1}$$

where ω_{ci}, ω_{pi}, ω_D are the ion cyclotron, ion plasma and ion magnetic drift frequencies respectively, m is the azimuthal mode number and A_1 is a factor which depends on the particular model. ω_D is taken as positive in simple mirror systems and negative in minimum-B systems. Solving for ω

$$\omega = -\frac{m\omega_D}{2}\left(1 \pm \left(1 - \frac{4\,\omega_{pi}^2 A_1}{m^2\omega_D\omega_{ci}}\right)^{\frac{1}{2}}\right) . \tag{2}$$

The physical significance of this expression becomes more apparent if the low density approximation is taken

$$\frac{4\,\omega_{pi}^2 A_1}{m^2\,\omega_D\,\omega_{ci}} \ll 1 , \tag{3}$$

in which case the two roots become

$$\omega_1 \simeq -m\,\omega_D + \frac{\omega_{pi}^2 A_1}{m\,\omega_{ci}} \tag{4}$$

and

$$\omega_2 \simeq -\frac{\omega_{pi}^2 A_1}{m\,\omega_{ci}} . \tag{5}$$

The first root (ω_1) corresponds to oscillations near the ion magnetic drift frequency. These occur as phase bunching of particles and are therefore easily damped by dispersion in the drift velocity (which may account for the difficulty in exciting them in the present experiments). The second root (ω_2) corresponds to oscillations of the electrons about the ion cloud, the ions remaining essentially unperturbed because of their rapid ∇B drift through the wave. Note that this root is independent of ω_D and ω_2 is proportional to n/B.

As the density is raised so that that condition (3) is no longer satisfied then the difference between simple mirror and minimum-B geometry becomes apparent. In the simple mirror case the waves are in the same direction and they couple to give the classic flute instability at a threshold density (n_c). At threshold the real part of the frequency is given by

$$\omega_c \simeq \tfrac{1}{2}\, m\, \omega_D \;.$$

Below threshold the variation of frequency with density given by equation (2) is close to parabolic (since, for $\omega_{pi}^2 \ll \omega_{ci}^2$, A_1 is independent of density).

In comparing with the various theoretical assumptions we take the parameters shown in Table I. The average density is evaluated taking a length equal to the total plasma length(15) given in the above table. Taking these values and allowing A_1 as the only undetermined parameter we fit a parabolic curve of the form given by equation (2).

We find

$$\left.\begin{array}{ll}\text{simple mirror} & A_1 = 0.23 \pm 0.05 \\ \text{minimum-B} & A_1 = 0.18 \pm 0.04\end{array}\right\} \text{(experimental)}$$

The main errors come from difficulty in deciding the correct plasma length to assume in calculating the average density. The errors in determining the frequency and line density are less than 10%.

Shown in Figures 2 and 4 are theoretical curves obtained from equation (2) using the calculated value of ω_D and the empirically determined value of A_1 given above. We now compare these experimental values of A_1 with the available theoretical models.

Kuo et al[11] consider an infinite cylinder with an essentially parabolic radial distribution. The walls are taken to be at infinity. They match the radial boundary condition exactly and find

$$A_1 = \frac{2\,m^2}{Z_m}$$

where Z_m is given by the argument of the Bessel function when $J_{m-1}(Z_m^{\frac{1}{2}}) = 0$. For $m=1$, $Z_m = 5.76$ then for both simple mirror and minimum-B cases

$$A_1 = 0.35 \text{ (Kuo et al.)}.$$

In a numerical calculation for a radial density distribution closer to the experimental one they find a value of A_1 14% higher than the above.

Guest and Beasley[12] are primarily concerned with the stabilising effect of end plates and the line tying effects of cold plasma; the radial boundary conditions are treated only approximately. They consider a plasma of length L coupled to the end walls by a cold plasma of length L_v and they match the field and potential at these boundaries. Taking their dispersion relation in the limit of zero cold plasma density (appropriate for the present experimental conditions) we find

$$A_1 = \frac{m\,L \tanh\,(mL_v/Rp)}{m\,L \tanh\,(mL_v/Rp) + 2Rp} \quad ,$$

where $1/Rp = (dn/dr)/n$. For our case Rp is approximately equal to 3 cm in the simple mirror case and 2 cm in the minimum-B case. Using these values we find

$$\left.\begin{array}{ll} \text{simple mirror} & A_1 = 0.45 \\ \text{minimum-B} & A_1 = 0.40 \end{array}\right\} \text{(Guest and Beasley)}.$$

Varma[13] considers the effect of the conservation of magnetic moment which changes the dispersion relation to a cubic with a third root near ω_D. However, the change to the other roots is very small for the experimental conditions, and for the present purposes we may separate out the quadratic form given in equation (1). Varma also considers the effect of finite plasma length and treats both square and parabolic radial density profiles with walls either at infinity or at the plasma surface. For the infinite length case, with a parabolic radial density profile, agreement is obtained with the calculation of Kuo et al. quoted above. He calculates numerically the change of the instability threshold as the plasma length is decreased but does not carry the computation to values of L smaller than 3 Larmor diameters at which, for the case of the parabolic density distribution the threshold for instability is given by

$\omega_{pi}^2/\omega_{ci}\ \omega_D = 0.9$. From this we may set an upper limit for both the simple mirror and minimum-B cases

$$A_1 < 0.28 \text{ (Varma)}.$$

Arsenin and Chuyanov[14] consider an infinitely long plasma with a square radial distribution and investigate the effect of changing the radial boundary conditions using feedback techniques. If we take their basic dispersion equations with no feedback present but conducting walls at radius b (plasma radius = a), and assume $\omega_{pi}^2 \ll \omega_{ci}^2$ then we find

$$A_1 = \frac{|m|}{2}\left(1 - \left(\frac{a}{b}\right)^{2|m|}\right).$$

We take $b - a \approx a_i$, where a_i is the ion Larmor radius. For the simple mirror case $b = 3.5$ cm, and for the minimum-B case $b = 6$ cm. Thus for $m=1$

$$\left.\begin{array}{ll} \text{simple mirror} & A_1 = 0.21 \\ \text{minimum-B} & A_1 = 0.28 \end{array}\right\} \text{(Arsenin and Chuyanov)}.$$

The comparison of the results from the various theoretical models and the experimental values is summarised in Table II.

VI. DISCUSSION

The above theoretical models reflect various aspects of the experimental situation. However, none of them offer simultaneously a complete description of both the radial and axial boundary conditions. The models with reasonable radial boundary conditions give values of A_1 which differ by a factor of less than two from the experimental values. The work of Varma[13] has shown that axial boundary conditions can make differences of about this value. In view of this the agreement between the predictions of the present theories and the experiments is considered reasonable.

In the simple mirror case the theoretical curve plotted using the value of A_1 determined by fitting a parabolic relationship is only in reasonable agreement with the experimental results (Fig. 2). The experimental data appears to lie below the parabolic curve at low density. A similar discrepancy was observed by Colchin et al.[3] However, once the plasma has become unstable, the flute frequency appears to be higher experimentally than the $\frac{1}{2}\omega_D$ predicted by the theoretical models. The most likely explanation of this is in terms of the plasma potential, averaged over the plasma volume, which rises from ~ 100 volts in the stable case to > 1000 volts in an unstable one.

Plasma potential effects are incorporated in the modified theoretical treatment of Damm et al[16] and Dnestrovskii and Pavlova[17] who assume a difference in ion and electron charge density that is proportional to the ion density. The net effect is that the $\underline{E} \times \underline{B}$ rotation resulting from the consequent radial electric field Doppler shifts the observed wave frequency. To illustrate the order of magnitude of this effect a parabolic distribution of potential, $\varphi = \varphi_o(1 - r^2/a^2)$, gives a frequency of rotation that is independent of radius,

$$\omega = 2 \times 10^8 \varphi_o/a^2 B.$$

If $\varphi_o = 10^3$ volts, $a = 8$ cm, $B = 10^4$ Gauss, then

$$\omega /2\pi = 50 \text{ kHz},$$

which would easily account for the excess of the observed unstable flute frequency (180 kHz) over the calculated flute frequency (153 kHz). During the excitation of the stable oscillations the plasma potential is less than 100 volts and this gives only a small Doppler shift. The finite intercepts of the minimum-B curve with the frequency axis at 4 kHz is probably accounted for by this effect as similar frequency shifts have been introduced by artificially changing the plasma potential with respect to the walls.

In minimum-B geometry, at the highest densities obtainable, the simple parabolic expression given by equation (2) appears to predict too low a frequency. The neglect of terms in $\omega_{pi}^2/\omega_{ci}^2$ does not account for this. The explanation may most probably be sought in the neglect of finite Larmor radius effects. Recent theoretical work by Cordey[15] shows that, in the regime $\omega_D << \omega << \omega_{ci}$, the wave is primarily dependent on the relative displacement of ions and electrons due to the finite ion Larmor radius. In the regime of interest here finite Larmor radius effects raise the frequency at the highest densities and make the frequency more nearly proportional to density.

VII. CONCLUSIONS

The electron flute wave has been observed in both simple mirror geometry and in minimum-B geometry. In the simple mirror case the wave couples with the ion $\underline{\nabla B} \times \underline{B}$ drift to give the classic flute instability and in the minimum-B case the wave may be propagated stably up to the maximum density achieved. In both cases the variation of frequency with density for the $m=1$ mode are in substantial qualitative agreement with theoretical expectations. Quantitatively the absolute values are predicted quite accurately if due allowance is made for the radial and axial boundary conditions, though none of the theoretical models cover quite the cases reported here. There is some evidence from the experimental data for Doppler shift caused by rotation due to the finite plasma potential and in the minimum-B case for finite Larmor radius effects at high density.

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<u>TABLE I</u>

	Simple Mirror	Minimum-B
B_o (Gauss)	10.7×10^3	12.5×10^3
α (cm^{-2})	3.7×10^{-3}	4.7×10^{-3}
Ion Energy (keV)	20	20
ω_D (rad/sec)	1.92×10^6 (306 kHz)	-1.5×10^6 (-240 kHz)
Plasma Length, L(cm)	5	10
Length, plasma to end wall, L_v(cm)	5	2

<u>CAPTION</u>: Summary of parameters used in the comparison of theory with the experimental results. B_o is the magnetic field at the centre of the apparatus and α is defined by the relationship $B = B_o (1 \pm \alpha r^2)$.

TABLE II

	Simple Mirror	Minimum-B
Experimental	0.23 ± 0.05	0.18 ± 0.04
Kuo et al	0.35	0.35
Guest and Beasley	0.45	0.40
Varma	<0.28	<0.28
Arsenin and Chuyanov	0.21	0.28

CAPTION: Summary of values of A_1 obtained from the experimental data and the various theoretical models for the simple mirror and minimum-B cases. A_1 is a constant defined by the dispersion relation $A_1 = - \omega \omega_{ci} (\omega + m \omega_D) / \omega_D \omega_{pi}^2$.

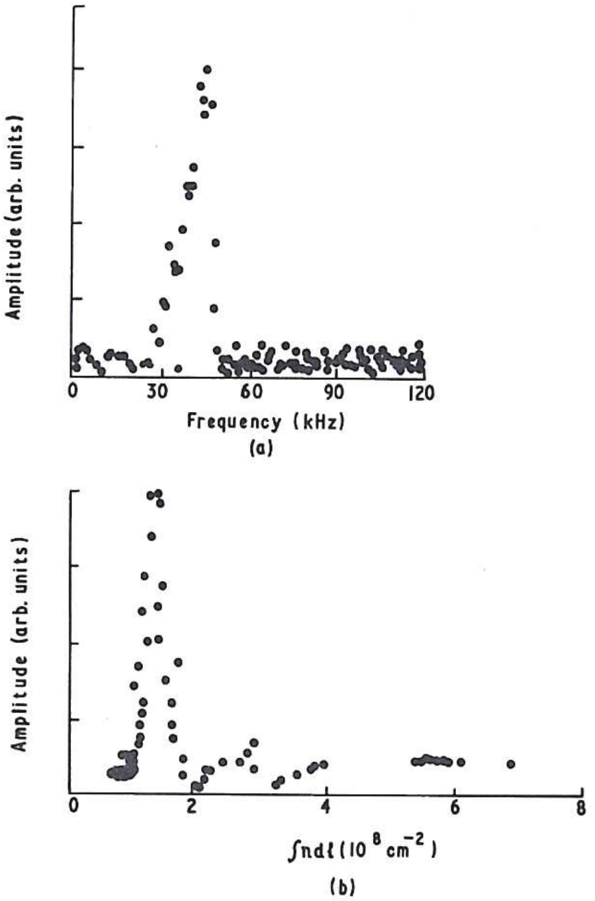

Fig.1 Amplitude of flute wave (a) as a function of frequency at constant density and (b) density at constant frequency for a simple mirror.

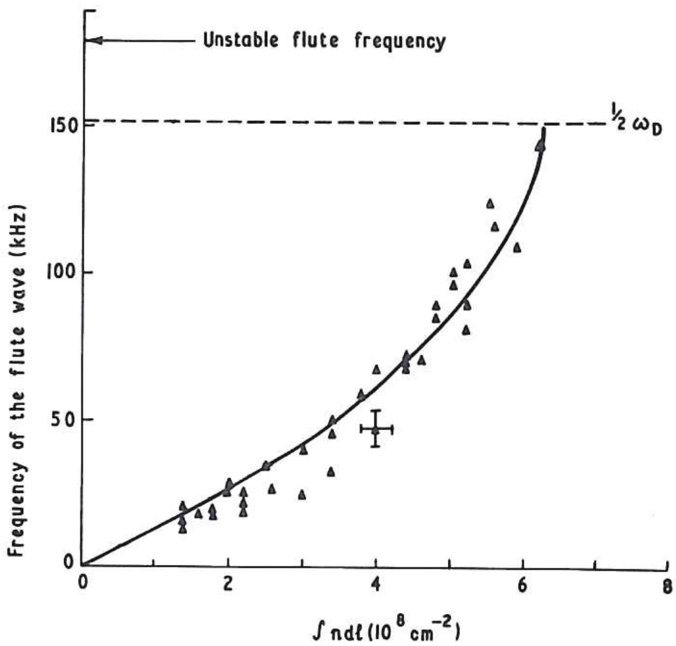

Fig.2 Frequency of flute wave as a function of line density in a simple mirror. The wave was excited by pure frequencies. Also shown is $\frac{1}{2}\omega_D$ and the unstable flute frequency.

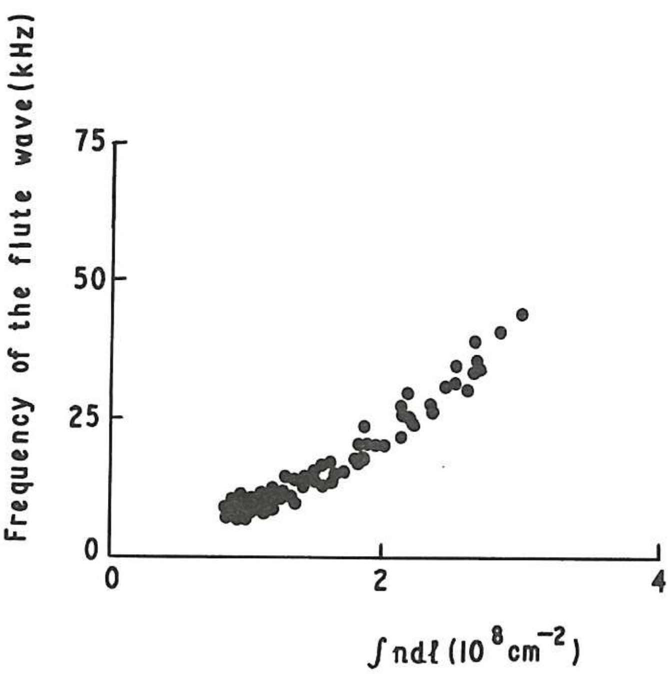

Fig.3 Frequency of flute wave in a simple mirror excited by white noise. The frequency and density are sampled by a LINC-8 computer and data from several shots superimposed.

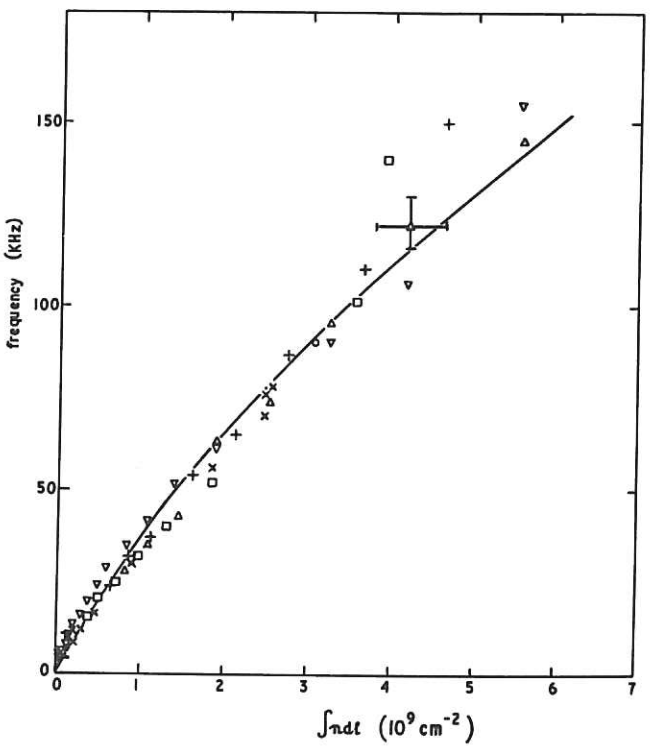

Fig.4 Frequency of flute wave in minimum-B geometry as a function of line density.