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by

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ABSTRACT

Using a general expression[4] for the power absorbed by transit time magnetic pumping the resonances in the heating due to the three low frequency electro-magnetic waves[5] which can propagate in a plasma in a magnetic field are calculated. The strengths and widths of the three resonances are compared under conditions when the electrons absorb most of the power. The ion acoustic resonance is the broadest and the magneto-acoustic (or compressional Alfvén) the strongest. The Alfvén (or shear Alfvén) wave is undamped when cyclotron damping and terms of order ω^2/ω_{ci}^2 are neglected. However, when terms of order ω^2/ω_{ci}^2 are included the Alfvén wave is shown to have a stronger resonance than the ion acoustic wave. Numerical results have also been obtained for the magneto-acoustic resonance under conditions where the ions are also heated and the width of the resonance is much broader than for pure electron heating.

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I. INTRODUCTION

The heating of a plasma to high temperatures is one of the main problems of controlled thermonuclear fusion research. A general method for doing this is by means of RF fields and one particular method, which was proposed by Spitzer[1] in 1953, is that of transit time magnetic pumping. This method becomes more effective the higher the plasma temperature. For a low temperature plasma any effect which increases the heating rate by this method would be very useful.

One such possibility is to operate with frequencies close to the ion acoustic frequency when the heating exhibits a resonance if $T_e \gg T_i$. This effect has been considered by Stepanov[2] and by Dawson and Uman[3]. However, under the conditions of TTMP (i.e. $\omega \ll \omega_{ci}$, $k_\perp \rho_i \ll 1$) there are three low frequency waves which can propagate in a plasma in a strong magnetic field. The other two branches are sometimes referred to as the compressional and shear Alfvén waves. Here we shall refer to them as the magneto-acoustic and Alfvén waves. The resonances in TTMP due to these waves has so far not been calculated (to the best of the authors' knowledge).

A simple model of RF heating has recently been analyzed by Lashmore-Davies[4] which allows any frequency range to be considered and in particular includes the effect of the three low frequency resonances. Here we shall use this model to obtain expressions for the power transferred to the plasma at these three resonances.

II. THE MODEL

The details of the model have already been described[4]. We consider an infinite uniform plasma in the presence of a static uniform magnetic field pointing along the z-axis. The plasma is

excited by an external current source flowing in the plasma along the x-axis. The current is taken to be of the form

$$\underline{J}_{ext} \sim \hat{\iota}_x \, J_{ext} \, e^{i(k_z z + k_y y - \omega t)} \quad .$$ (1)

This current source induces electromagnetic fields in the plasma which in turn produce a plasma current given by

$$\underline{J} = \underline{\underline{\sigma}} \, \underline{E}$$ (2)

where $\underline{\underline{\sigma}}$ is the conductivity tensor of a hot collisionless plasma. The power absorbed by the plasma is given by

$$P = \frac{1}{2} \, \text{Re} \, \underline{J}^* . \underline{E}$$ (3)

where $\underline{J}$ is given by equation (2).

The electric field which appears in equations (2) and (3) is the self consistent field produced by the sum of the external current and the plasma current. It is obtained from Maxwell's equations which can be written in the form

$$\underline{\underline{A}} \, \underline{E} = i \omega \mu_o \, \underline{J}_{ext.}$$ (4)

where

$$\underline{\underline{A}} \equiv -\underline{k} \, \underline{k} + \left(k^2 - \frac{\omega^2}{c^2}\right)\underline{\underline{I}} - i\omega\mu_o \, \underline{\underline{\sigma}} \quad .$$ (5)

Introducing the tensor $\underline{\underline{X}}$ where

$$\underline{\underline{X}} \equiv -\underline{k} \, \underline{k} + \left(k^2 - \frac{\omega^2}{c^2}\right)\underline{\underline{I}}$$

and taking the scalar product of equation (4) with $\underline{E}$ we obtain

$$\underline{J}^*_{ext.} . \underline{E} + E \, \underline{\sigma}^* \, \underline{E}^* = -\frac{1}{i\omega\mu_o} \, \underline{E} \, \underline{\underline{X}}^* \, \underline{E}^* .$$

Now, since $\underline{\underline{X}}$ is hermitian we have

$$\text{Re} \left(\underline{J}^*_{ext.} . \underline{E} + \underline{J}^* . \underline{E}\right) = 0$$ (6)

i.e. the power lost by the external circuit is absorbed by the plasma and power is conserved.

We may now calculate the power absorbed by the plasma from

$$P = - \frac{1}{2} \text{ Re } \underline{J}^*_{ext.} \cdot \underline{E} \; . \tag{7}$$

Using equations (1) and (4) we obtain the final expression for P

$$P = \frac{\omega \mu_0}{2} \left| J_{ext.} \right|^2 \text{ Im } (A_{xx}^{-1}) \; . \tag{8}$$

The resonant character of equation (8) becomes evident since

$$A_{xx}^{-1} \; \alpha \; 1/(\det \underline{\underline{A}})$$

where

$$\det \underline{\underline{A}} = 0 \tag{9}$$

is the dispersion relation of electromagnetic waves in a plasma.

III. THE RESONANCES

We shall always assume $\omega^2 \ll \omega_{ci}^2$ and $k_y^2 \rho_i^2 \ll 1$. Neglecting terms of this order the conductivity tensor can be written

$$\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_{xx} & 0 & \sigma_{xz} \\ 0 & \sigma_{yy} & 0 \\ \sigma_{zx} & 0 & \sigma_{zz} \end{pmatrix} \; . \tag{10}$$

For the case when

$$k_z^2 v_{T_i}^2 \ll \omega^2 \ll k_z^2 v_{T_e}^2$$

it is possible to obtain analytic solutions of the dispersion equation (9). Neglecting terms of order ω^2/ω_{ci}^2, equation (9) can be factorized into two equations (c.f. Shafranov[5])

$$N^2 = \frac{i \sigma_{xx}}{\omega \varepsilon_0} - \frac{i \sigma_{xz} \sigma_{zx}}{\omega \varepsilon_0 \sigma_{zz}} \tag{11}$$

$$N^2 = \frac{i \sigma_{yy}}{\omega \varepsilon_0 \cos^2 \theta} \tag{12}$$

where N^2 is the refractive index $c^2 k^2/\omega^2$. In order to solve equations (11) and (12) for the three low frequency branches of the dispersion diagram we use the values for the elements of the conduct-

ivity tensor given in reference 4. Equation (11) gives two branches. These can be written[5]

$$\omega = k_z c_s \left(1 - \frac{i\sqrt{\pi}}{2} \left[\left(\frac{m_e}{2m_i} \right)^{\frac{1}{2}} + \frac{1}{\sqrt{2}} \left(\frac{T_e}{T_i} \right)^{3/2} e^{-T_e/2T_i} \right] \right) \qquad (13)$$

where $c_s = \left(\kappa T_e / m_i \right)^{\frac{1}{2}}$ is the sound speed and we shall refer to equation (13) as the ion acoustic wave. The second solution of equation (11) is[5]

$$\omega = k c_A \left(1 + \frac{1}{2} \frac{k_y^2}{k^2} \beta_i \left[1 + \frac{T_e}{2 T_i} \right] \right)$$
$$- i\sqrt{\pi} \, k c_A \frac{k_y^2}{k^2} \beta_i \left(z_{oi} \, e^{-z^2_{oi}} \left[1 + \frac{T_e}{T_i} + \frac{T_e^2}{2 T_i^2} \right] + z_{oe} \frac{T_e}{2 T_i} \right) \qquad (14)$$

which we shall refer to as the magneto-acoustic wave. $c_A = B_o / \left(n_o m_i \mu_o \right)^{\frac{1}{2}}$ is the Alfvén speed, $z_{oj} = \omega / \sqrt{2} k_z v_{Tj}$ where v_{Tj} is the thermal speed of the j-species and $\beta_j = 2\mu_o n_o \kappa T_j / B_o^2$. The other symbols have their usual meaning. The ion-acoustic wave and the magneto-acoustic wave are both weakly damped in this limit.

The third low frequency wave solution is obtained from equation (12) and is[5]

$$\omega = k_z c_A \qquad (15)$$

which we will refer to as the Alfvén wave. This solution is interesting since it is an undamped solution. However, this is due to the neglect of the small number of particles in the "tail" of the Maxwellian distribution that can produce cyclotron damping but more importantly to the neglect of terms $\sim \omega^2 / \omega_{ci}^2$. When we come to calculate the power absorbed at the Alfvén resonance it will be necessary to retain terms $\sim \omega^2 / \omega_{ci}^2$ (although the cyclotron damping contribution will still be negligible). We will now calculate expressions for the power absorbed at the three resonances.

Equation (8) for the power P can be written

$$P = \frac{\omega\mu_o}{2} \left|J_{ext.}\right|^2 \, \mathrm{Im}\left\{ \frac{\left(k_z^2 - \frac{\omega^2}{c^2} - i\omega\mu_o\sigma_{yy}\right)\left(k_y^2 - \frac{\omega^2}{c^2} - i\omega\mu_o\sigma_{zz}\right) - k_y^2 k_z^2}{\det A} \right\} .$$

$$(16)$$

(i) The Ion-Acoustic Resonance.

This resonance has already been considered in detail by Stepanov[2]. When $T_e \gg T_i$ most of the power is absorbed by the electrons and equation (16) takes a particularly simple form in this limit. We give this result for the purpose of comparison.

$$P = \frac{\omega\mu_o}{2} \left|J_{ext.}\right|^2 \frac{k_y^2}{k^4}\left(\frac{2}{\pi}\right)^{\frac{1}{2}} \beta_e \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} \qquad (17)$$

where $\beta_e \ll (m_e/m_i)^{\frac{1}{2}}$. This result is equivalent to Stepanov's[2] and represents an enhancement in the power away from the resonance by a factor of the order $(T_e/T_i)(m_i/m_e)^{\frac{1}{2}}$. The width of the resonance under these conditions is $\Delta\omega/\omega_{res.} \sim T_i/T_e$.

(ii) The Magneto-Acoustic Resonance.

Whereas the ion-acoustic resonance can occur for very low density plasmas this resonance requires $\omega_{pi}^2 \gg c^2 k^2$. The power absorbed by both ions and electrons can be enhanced by this resonance. However, in order to compare this resonance with the ion acoustic one we first consider the limit when the electrons absorb nearly all the power i.e.

$$\omega \gg k_z v_{Ti} .$$

Taking $T_e \sim T_i$ and $k_z \sim k_y$ then at the magneto-acoustic resonance equation (16) gives

$$P = \frac{\omega\mu_o}{2} \left|J_{ext}\right|^2 \frac{k^2}{k_z^2 k_y^2} \, 2\left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{1}{\beta_e}\left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} \qquad (18)$$

where

$$\frac{m_e}{m_i} \ll \beta_e \ll 1 .$$

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This is a very similar expression to the ion acoustic resonance except that now $P \propto \beta_e^{-1}$ and so this is a much stronger resonance. Notice however that the resonance is narrow: its width is roughly

$$\frac{\Delta\omega}{\omega_{res}} \simeq \left(\frac{m_e}{m_i} \frac{T_i}{T_e} \right)^{\frac{1}{2}} \beta_e^{\frac{1}{2}} \tag{19}$$

which is narrower than the ion acoustic resonance.

For a more detailed account of this resonance in the regime where a significant fraction of the power input is directly to the ions, we write (16) in explicit form as

$$P = \frac{\omega\mu_o}{2} \left| J_{ext} \right|^2 \frac{1}{k_y^2} \frac{\beta_i \chi}{\left[1 + K + \beta_i \left(\xi - Kz_{oi}^2 \right) \right]^2 + \beta_i^2 \chi^2} . \tag{20}$$

Here $K \equiv k_z^2/k_y^2$, and the real functions χ and ξ are defined by (see Ref.(4))

$$\xi - i\chi = - z_{oi} Z(z_{oi}) - \frac{T_e}{T_i} z_{oe} Z(z_{oe}) +$$

$$+ \frac{1}{2} \frac{\left(z_{oi} Z(z_{oi}) - z_{oe} Z(z_{oe}) \right)^2}{\left(z_{oi} Z(z_{oi}) + \frac{T_i}{T_e} z_{oe} Z(z_{oe}) + 1 + \frac{T_i}{T_e} \right)} \tag{21}$$

with Z being the Fried and Conte[6] plasma dispersion function. These functions ξ and χ may readily be tabulated.

Although only of potential relevance to a restricted class of experiments, the regime where β_i is not very much smaller than unity is interesting, in that the region of the resonance can be $z_{oi} \sim 1$, and there is substantial heating of the ions in a direct way. Some typical numerical results are displayed in figure 1 for $\beta_i = 0.5$ (where ions and electrons are heated equally), and for $\beta_i = 0.1$ (where we are essentially back at the limiting result (18) above):

these examples assume $K = 1$ and $T_e = T_i$.

(iii) The Alfven Resonance.

This resonance requires $\omega_{pi}^2 \gg c^2 k_z^2$. As already mentioned, in order to calculate the power absorbed at this resonance we must retain terms $\sim \omega^2/\omega_{ci}^2$. To do this we use the following ordering scheme

$$\frac{\omega^2}{\omega_{ci}^2} \sim k_y^2 \rho_i^2 \sim \beta_e \sim \frac{k_z^2}{k_y^2} \sim \frac{\omega_{ci}^2}{\omega_{pi}^2}$$

where $T_e \sim T_i$ and where

$$\frac{m_e}{m_i} \ll \beta_e \ll 1.$$

Under these assumptions the correction to the Alfven solution is

$$\omega \approx c_A k_z \left(1 + \frac{k_y^2 c_s^2}{\omega_{ci}^2} \left[1 - \frac{\omega_{ci}^2}{\omega_{pi}^2} \frac{\omega_{ci}^2}{k_y^2 c_s^2} - \frac{T_i}{T_e} - \frac{k_z^2}{k_y^2} \frac{2}{\beta_e} \right] \right)$$

$$- i c_A k_z \sqrt{\pi} \frac{k_y^2 c_s^2}{\omega_{ci}^2} \left(\frac{m_e}{\beta_e m_i} \right)^{\frac{1}{2}}. \qquad (22)$$

The Alfvén wave is now very weakly damped and the damping is due to the electrons. However, this is because we have assumed $k_z^2 v_{Ti}^2 \ll \omega^2 \ll k_z^2 v_{Te}^2$. If this condition were relaxed then the ions would evidently contribute to the damping and hence the power absorption.

Evaluating equation (16) for the power absorbed at the Alfvén resonance under conditions when most of the power goes to the electrons we obtain

$$P = \frac{\omega \mu_o}{2} \left| J_{ext} \right|^2 \frac{1}{k^2} \left(1 + \frac{k_z^2}{k_y^2} \frac{2}{\beta_i} \right) \frac{1}{(\pi)^{\frac{1}{2}}} \frac{T_i}{T_e} \beta_e^{\frac{1}{2}} \left(\frac{m_i}{m_e} \right)^{\frac{1}{2}}. \qquad (23)$$

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The width of this resonance is $\Delta\omega/\omega_{res} \sim (m_e/m_i)^{\frac{1}{2}} \times k_y^2 \rho_i^2 \times \beta_e^{-\frac{1}{2}}$ which is comparable to the width of the magneto-acoustic resonance but narrower than the ion acoustic one. Comparing the strengths of the three resonances it can be seen that this resonance is stronger than the ion acoustic case but weaker than that due to the magneto acoustic wave.

IV. CONCLUSIONS

We have calculated the power absorbed by a plasma due to transit time magnetic pumping at the three low frequency resonances. The ion acoustic resonance has been previously considered[2] but the other two resonances are evaluated here for the first time. When the power is absorbed mainly by the electrons the ion-acoustic resonance turns out to be the weakest of the three and the magneto-acoustic resonance very much the strongest. However, the width of the ion acoustic resonance is greater than the other two whose widths are of the same order. The ultimate strength of all three resonances will be determined by non-linear effects which have not been considered here.

The strength and width of each resonance is not the only consideration. For a very low β plasma only the ion-acoustic resonance will be accessible. This resonance requires the plasma to be initially non-isothermal i.e. $T_e > T_i$. For the ion-acoustic resonance the resonant condition will be lost when the electrons are heated significantly since $c_s \propto T_e^{\frac{1}{2}}$.

When the value of β is high enough $(\beta \gg m_e/m_i)$ the other two resonances become accessible and neither of these require $T_e \gg T_i$.

In both of these cases the resonant frequency depends on the magnetic field and the density and so the resonance condition will not be affected as the plasma is heated.

Since the Alfvén resonance is $\omega \sim k_z c_A$ whereas the magneto-acoustic one is $\omega \sim k\,c_A$ the Alfvén resonance will be accessible more readily since there is more control over k_z than k_y and k_z can be chosen to be much less than k.

Although the ultimate aim of any plasma heating method is to raise the ion temperature, electron heating is valuable for two reasons. Firstly, transit time magnetic pumping has yet to be demonstrated experimentally. Therefore for this purpose it is immaterial whether electron heating or ion heating is produced. Secondly, since magnetic pumping is more effective the higher the temperature any method which increases the temperature of either species would help this heating mechanism in its initial stages.

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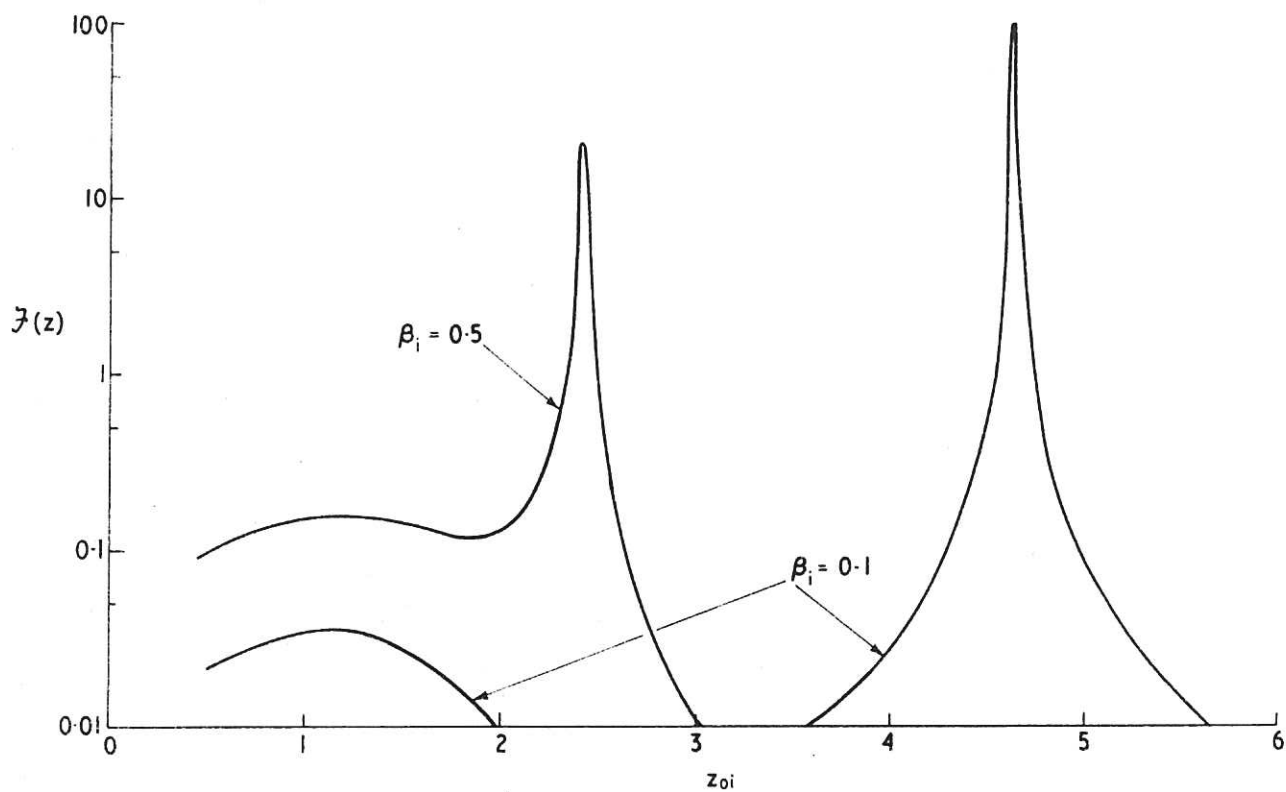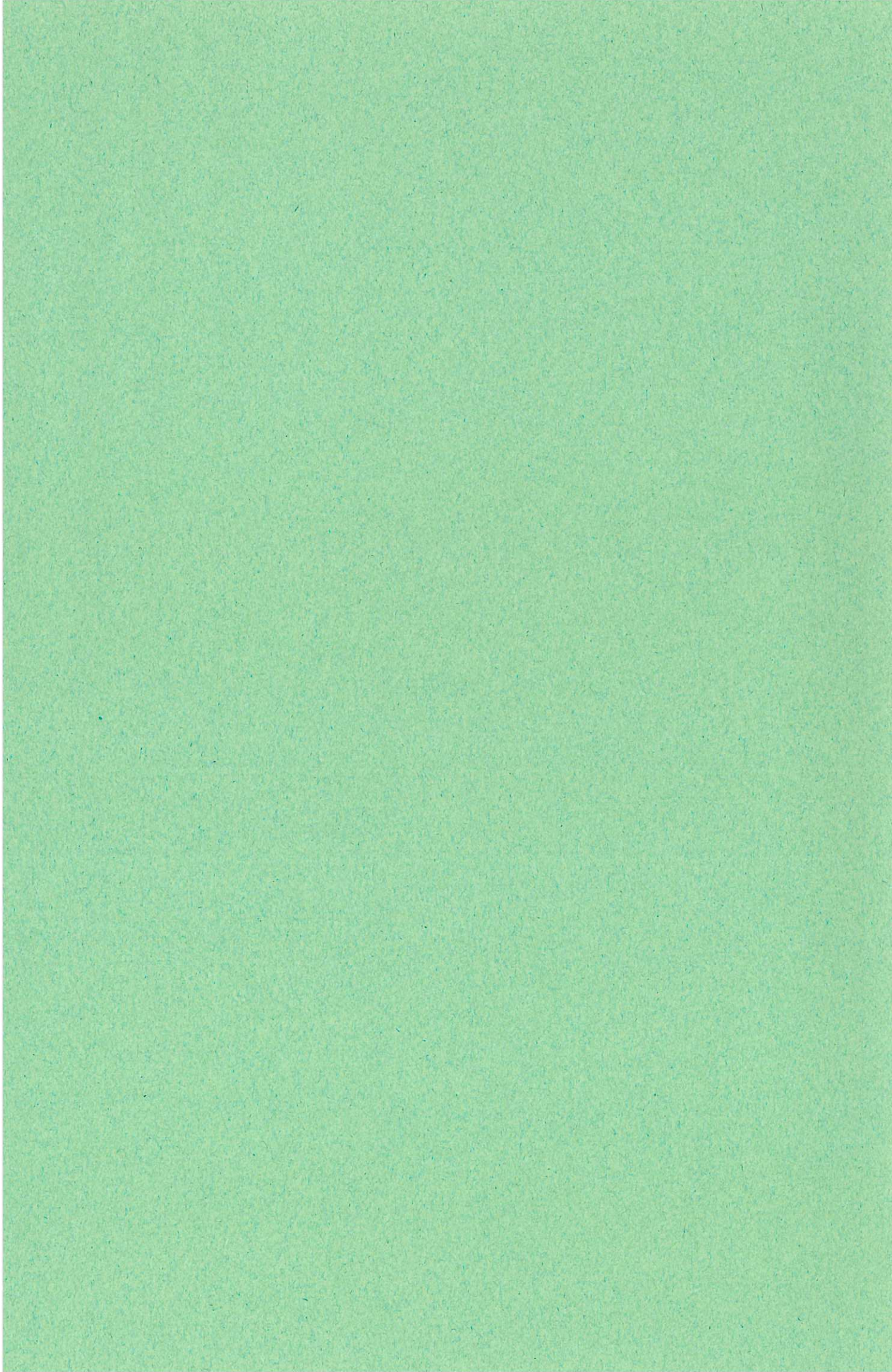6. B.D. Fried and S.D. Conte, The Plasma Dispersion Function (Academic Press, New York, 1961)

Fig.1.   The factor F in $P = \frac{1}{2} \omega\mu_o k_y^{-2} \left| J_{ext} \right|^2$ F, plotted as a

function of z_{oi} when $T_e = T_i$ and $k_y = k_z$ for (a)

$\beta_i = 0.5$, and (b) $\beta_i = 0.1$.