CLM - P 296

UKAEA RESEARCH GROUP

Preprint

ON THE NATURE OF THE INSTABILITY CAUSED BY ELECTRONS TRAPPED BY AN ELECTRON PLASMA WAVE

R N FRANKLIN
S M HAMBERGER
H IKEZI
G LAMPIS
G J SMITH

CULHAM LABORATORY
Abingdon Berkshire

1972

CLM - P 296

ON THE NATURE OF THE INSTABILITY CAUSED BY ELECTRONS TRAPPED BY AN ELECTRON PLASMA WAVE

by

R.N. FRANKLIN*, S.M. HAMBERGER, H. IKEZI†,
G. LAMPIS and G.J. SMITH

(Submitted for publication in Phys. Rev. Letts.)

ABSTRACT

The side-bands which accompany a large amplitude electron wave on a plasma column are shown to arise from selectively amplified noise; the amplification mechanism is parametric coupling between the side-band frequencies and the oscillations of the electrons trapped in the large wave.

*Also at Department of Engineering Science, Oxford University, Oxford, England

†On leave from Institute of Plasma Physics, Nagoya University, Nagoya, Japan

UKAEA Research Group,
Culham Laboratory,
Abingdon,
Berks

January, 1972

ON THE NATURE OF THE INSTABILITY CAUSED BY ELECTRONS TRAPPED BY AN ELECTRON PLASMA WAVE

It is now well established[1-4] that a finite amplitude, monochromatic, electron wave, frequency ω_o, wave number k_o, propagating in a one-dimensional collisionless plasma may be accompanied by other frequencies, ω, which appear as one or more 'side-bands' to ω_o. These fluctuations increase in intensity along the direction of propagation of the wave, causing it to be damped, and have frequency displacements from that of the original wave $\Delta\omega \equiv |\omega_o - \omega|$ which increase with the initial wave amplitude φ_o, and which, at least in the earliest observation[1], obeyed the approximate relation $\Delta\omega \propto \varphi_o^{\frac{1}{2}}$.

The observation stimulated considerable theoretical work on the stability of plasma supporting a large-amplitude monochromatic wave, all of which invokes the existence of resonant electrons which, by virtue of their having velocity components parallel to the direction of propagation closely equal to the phase velocity of the wave $v_\varphi = \omega_o/k_o$ can become trapped in the electrostatic potential well of the wave. These oscillate in the well with a characteristic frequency $\omega_B = k_o(e\varphi_o/m)^{\frac{1}{2}}$, where e, m are the electronic charge and mass respectively and can therefore couple to other oscillations in the system provided certain resonance conditions are satisfied.

Because the original experiment[1] showed $\Delta\omega \simeq \omega_B$ this has tended to be an important criterion (as it turns out, misleading) for testing the various theories. In addition, the relative importance of upper and lower side-bands, or, indeed, the existence of the former, is in some dispute.

Detailed results of various theoretical approaches depend largely on the assumptions made. Manheimer[5] ascribed the unstable side-bands to propagating plasma waves (ω,k) resonant with the (Doppler shifted) oscillation frequency of the trapped electrons, leading to

$$\omega - k\,v_\varphi = \pm\,\omega_B\,, \qquad \ldots (1)$$

where (ω,k) satisfies the dispersion relation $\varepsilon(\omega,k) = 0$. This relation is implicit in the earlier work of Kruer et al[6]. Mima and Nishikawa[7] assumed the trapped electrons oscillate in an infinite parabolic potential well as in reference 8 and found also higher order resonances, analogous to the energy levels of a harmonic oscillator:

$$\omega - k v_\varphi = \pm (2N+1)^{\frac{1}{2}} \omega_B \qquad \ldots (2)$$

where $N = 0, 1, 2 \ldots$. Bud'ko, Karpman and Shklyar[9] have recently extended the earlier treatment of O'Neil[10] assuming a sinusoidal potential well. The lowest order side-bands then become

$$\omega - k v_\varphi \approx \pm 0.9 \omega_B . \qquad \ldots (3)$$

One can deduce that higher order resonances occur between integer multiples of ω_B. These last results[9] are valid if

$$e\varphi_o / k_B T < (v_e / v_\varphi)^2 \qquad \ldots (4)$$

where $v_e^2 = 2 k_B T/m$, k_B is Boltzmann's constant.

Experimental results are presented here which show that the approach used in most of the above work (i.e. of the stability of a test wave (ω, k) such that $\varepsilon(\omega, k) = 0$) does offer a good description of the effect, and that higher order resonances, leading to several possible growing side-bands, do occur, in broad agreement with equations (2) and (3), and that the side-bands are simply selectively amplified plasma noise.

Our experiments were conducted in a single-ended, thermally ionized Na plasma column, diameter 2.5 cm, temperature 2500 °K, density $\sim 3 \times 10^7$ cm^{-3} ($\omega_{pe}/2\pi \sim 30$-60 MHz) and uniform over its 80 cm length to within ~1%, radially confined by a strong (~2 kG) uniform axial magnetic field. The waves were excited using either fine wire probes or the cold end-plate, and their amplitude φ measured using a high input impedance probe[11]. Allowing for uncertainties regarding the probe-plasma impedance and its frequency variation the absolute accuracy of φ was estimated to be ~ ± 20%, corresponding to an uncertainty in ω_B ~± 10%.

In general it was found that for waves which in the linear regime are lightly damped ($\omega \lesssim \omega_{pe}$) two lower side-bands and one upper appear, but for $\omega \gtrsim \omega_{pe}$ only the first lower side-band, with an appreciable frequency spread, is strongly excited. By directly recording the signals received from two probes with small but variable separation the phase velocity of individual frequency components within one side-band was measured confirming that they are electron waves on the same dispersion curve as (ω_o, k_o).

The absolute level of all the side-bands could be increased by injecting bands of low-level noise into the plasma at frequencies close to one of them. Fig.1 shows typical spectra observed for side-bands which were (a) spontaneously occurring and (b) enhanced by the signal (c). Notice that two lower side-bands and one upper can clearly be seen. By accurately measuring the real part of the dispersion of small amplitude waves[12] in the presence of

(ω_o, k_o) (which causes small but significant changes from the linear dispersion particularly when $\omega \gtrsim \omega_{pe}$) the measured side-band frequencies were compared with the theoretical predictions. For example, equation (2) suggests that the quantities $\omega' = \omega - k\, v_\varphi$ for the $N = 1, 0$ lower side-bands and the $N = 0$ upper should be related in the ratio $\sqrt{3} : 1 : 1$ respectively. For $e\varphi_o \simeq 0.25\, k_B T$, and a range of frequencies between 42 MHz and 46 MHz ($\omega_{pe}/2\pi = 43.5$ MHz), the values of ω' were found to be in the ratio $(1.7 \pm 0.1):1:(1.0 \pm 0.1)$ thus tending to support (2), which is to be expected since inequality (4) was not satisfied.

More quantitative data have been obtained for the first lower side-band ($N = 0$) by (a) varying φ_o at constant ω_o (i.e. constant v_φ), (b) varying ω_o (i.e. v_φ) at constant φ_o. The experimental results are summarized in Fig. 2, where the frequency difference $\Delta\omega$ is plotted against the experimentally varied parameter. The solid lines represent the prediction of equation (2) for $N = 0$; the shaded area corresponds to the experimental uncertainties in both φ_o and the plasma dispersion. The dashed line in each case is for $\Delta\omega = \omega_B$, which clearly offers a far poorer description of the data than equation (2).

Notice that the actual form of the variation $\Delta\omega(\omega_B)$ depends on the shape of the dispersion curve. It is simple to show that for $\Delta\omega \ll \omega_o$, $N = 0$ equation (2) becomes:

$$\Delta\omega \simeq \omega_B / \left(v_\varphi \frac{dk}{d\omega} - 1 \right) \qquad \ldots (5)$$

If, as in the experimental condition of reference 1, $(d\omega/dk) \approx 0.5\, v_\varphi$, then $\Delta\omega \approx \omega_B$.

The amplification process was demonstrated by introducing from a second launching antenna a small amplitude, coherent test wave, propagating in the same direction as (ω_o, k_o) and whose frequency was varied. This wave grew exponentially along the direction of propagation of the large wave (ω_o, k_o) with a maximum growth rate when at the same frequency as a side-band maximum. Measured values of the amplification coefficient (for values of $\varphi_o \lesssim 60\,\text{mV}$) correspond typically to $k_i/k_r \sim (1\text{-}2) \times 10^{-2}$, and vary with φ_o like $k_i/k_r \propto \omega_B \propto \varphi_o^{1/2}$. A similar proportionality has been found in computer simulation[3] for the <u>temporal</u> growth rate γ/ω_{pe}. Further, the process is clearly distinguishable from non-linear growth arising from either decay instabilities[13,14] or non-linear Landau damping[15], for which the growth is proportional to φ_o^2. A test wave propagating in the opposite direction to (ω_o, k_o) did not grow.

Typical data for spatial amplitude variations are seen in Fig.3 : it shows (i) the normal Landau damping of a small amplitude wave; (ii) the increased attenuation of the same wave when its amplitude was increased sufficiently to exhibit the first lower side-band; (iii) the exponential growth of the peak of the side-band, and (iv) a test wave at the same frequency as that of the peak of the side-band and propagating in the same direction as (ω_o, k_o) which clearly has the same exponential growth as (iii).

Further evidence that the side-bands originate from a linear amplification of initial noise was provided by deliberately increasing the plasma noise level (by connecting the test wave launcher to a noise source) until it was just observable (in the absence of the large wave) on the spectrum analyzer. For sufficiently low noise levels the side-band amplitude increased proportionately to the r.m.s. noise level.

To demonstrate that the above effects are associated with trapped electrons, the amplitude of the side-band was measured in the presence of a second perturbing wave (ω_1, k_1) whose frequency was well removed from the unstable regions. The side-band amplitude (and hence the amplification) decreased with increasing amplitude of the second large wave. When both waves had comparable amplitudes, thus effectively displacing the time invariant potential well to a phase velocity[16] $(\omega_1-\omega_0)/(k_1-k_0)$ and thus destroying the necessary resonance, the side-bands completely disappeared. The side-bands also disappeared when the electron collision frequency was increased by adding neutral gas.

The effects described here, which will be described in greater detail elsewhere, can and usually do occur concurrently with other non-linear effects which lead to enhanced decay, e.g. induced decay into ion waves[13] and into other electron modes[14].

Finally, notice that these effects suggest that stable electron wave equilibria of the BGK[17] type cannot occur if the system can support other (electrostatic) waves which satisfy resonance conditions such as (2). Instability of BGK modes by decay into ion waves was noted by Sagdeev and Galeev[18].

Results closely related to these have recently been obtained in Nagoya[19] for ion waves propagating in a collisionless plasma with $T_e \sim 20\ T_i$.

We are happy to acknowledge the technical assistance of W.J. McKay and W.H.M. Clark.

REFERENCES

[1] C.B. Wharton, J.H. Malmberg, and T.M. O'Neil, Phys. Fluids 11, 1761 (1968).

[2] R.N. Franklin, G.J. Smith, S.M. Hamberger, and G. Lampis, in Proceedings of the Fourth European Conference on Fusion and Plasma Physics, Rome, Italy, 1970 (unpublished), p.158.

[3] W.L. Kruer and J.M. Dawson, Phys. Fluids 13, 2747 (1970).

[4] J. Denavit and W.L. Kruer, Phys. Fluids 14, 1782 (1971).

[5] W.M. Manheimer, Phys. Rev. A. 3, 1402 (1971).

[6] W.L. Kruer, J.M. Dawson, and R.N. Sudan, Phys. Rev. Lett. 23, 838 (1969).

[7] K. Mima and K. Nishikawa, J. Phys. Soc. Japan 30, 1722 (1971).

[8] L.M. Al'tshul and V.I. Karpman, Zh. Eksp. Teor. Fiz. 49, 515 (1965) [Sov. Phys. JETP 22, 361 (1966)].

[9] N.I. Bud'ko, V.I. Karpman, and D.R. Shklyar, Preprints N4, N5, Institute of Terrestrial Magnetism, Ionosphere and Radiowave Propagation, Moscow, (1971).

[10] T.M. O'Neil, Phys. Fluids 8, 2255 (1965).

[11] H. Ikezi, P.J. Barrett, R. White, and A.Y. Wong, Phys. Fluids 14, 1997 (1971).

[12] R.N. Franklin, S.M. Hamberger, G. Lampis, and G.J. Smith, in Proceedings of the Third International Conference on Quiescent Plasmas, Elsinore, Denmark, 1971, p.144.

[13] R.N. Franklin, S.M. Hamberger, G. Lampis, and G.J. Smith, Phys. Rev. Lett. 27, 1119 (1971).

[14] R.N. Franklin, S.M. Hamberger, G. Lampis, and G.J. Smith, Phys. Lett. 36 A, 473 (1971).

[15] K. Gentle and A. Malein, Phys. Rev. Lett. 26, 625 (1971).

[16] E. Ott and C.T. Dum, Phys. Fluids 14, 959 (1971).

[17] I.B. Bernstein, J. Greene, and M. Kruskal, Phys. Rev. 108, 546 (1957).

[18] R.Z. Sagdeev and A.A. Galeev, Nonlinear Plasma Theory (W.A. Benjamin, New York, 1969), p.43.

[19] H. Ikezi, Y. Kiwamoto, K. Mima, and K. Nishikawa, in Proceedings of the Third International Conference on Quiescent Plasmas, Elsinore, Denmark, 1971, p.334.

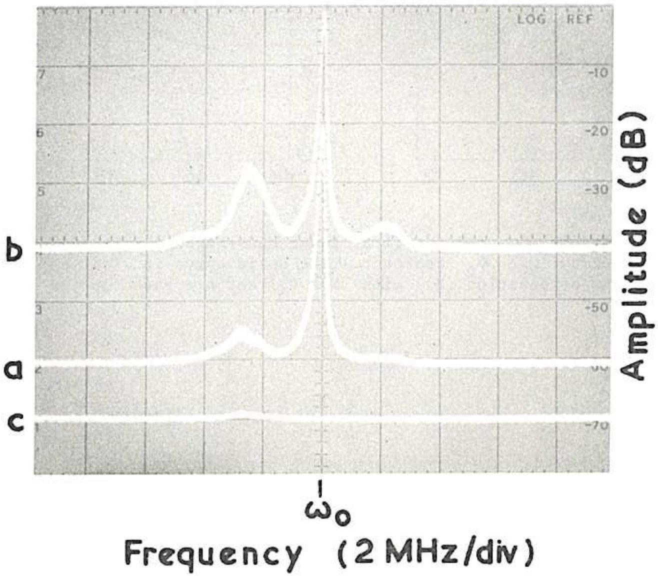

Fig.1 Spectra showing (a) spontaneous side-bands and (b) side-bands enhanced by noise (c). $\omega_o/2\pi$ = 44 MHz.

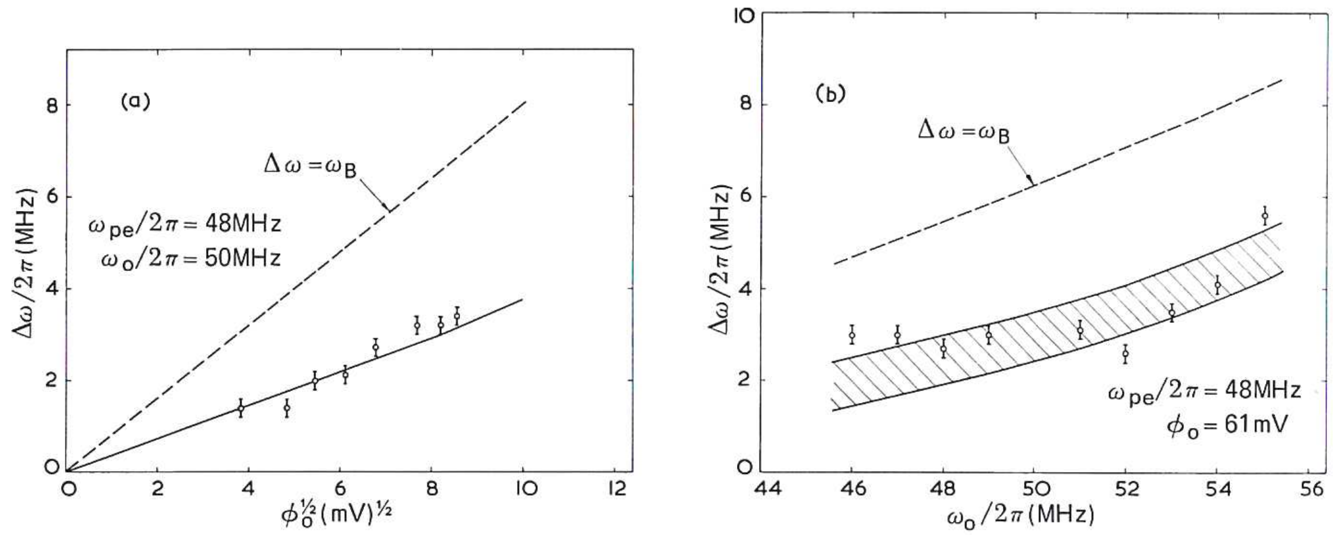

Fig.2 Observed frequency difference $\Delta\omega$ for the first lower side-band versus (a) wave amplitude ϕ_o, ω_o constant; (b) wave frequency ω_o, ϕ_o constant. The solid curve in (a) and the shaded area in (b) are theoretical predictions of equation (2) with N = 0, and are based on the measured dispersion.

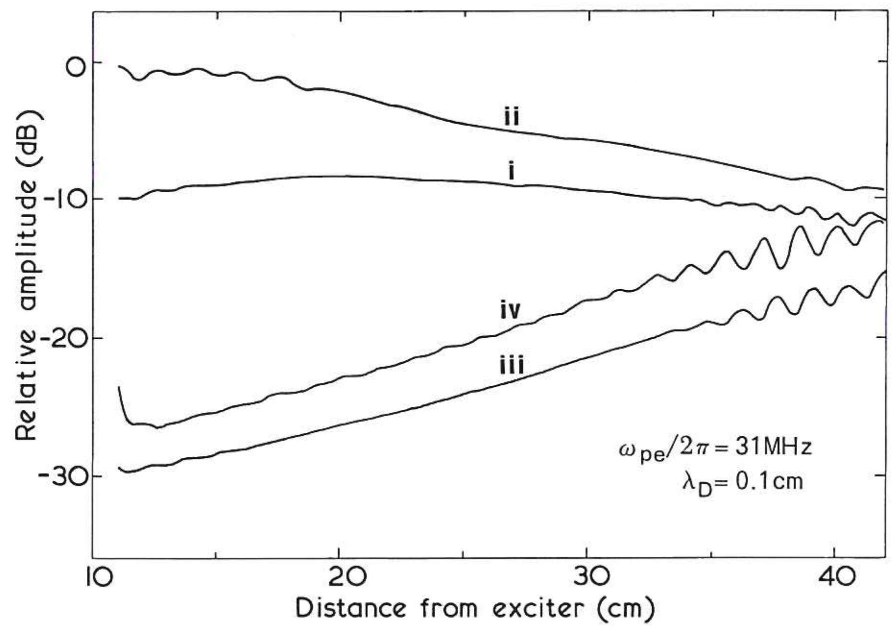

Fig.3 Spatial variation of (i) small amplitude wave ω_o (ii) large amplitude wave ω_o exhibiting side-bands (iii) first lower side-band frequency ω, and (iv) test wave frequency ω. $\omega_o/2\pi$ = 34 MHz; $\omega/2\pi$ = 30 MHz.