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# THE INCOHERENT SCATTERING OF RADIATION FROM A HIGH TEMPERATURE PLASMA

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1. INTRODUCTION

The calculations presented below are based upon the work of Pechacek and Trivelpiece[1] and Theimer and Sollid[2]. We are interested here only in incoherent scattering, therefore we may determine the scattered intensity for each electron independently. The scattered spectrum for the plasma as a whole is then found by adding the contributions from those electrons within the scattering volume which scatter into a given frequency interval. As is well known the incoherent spectrum is observed when $\alpha = \frac{1}{k\lambda_D} \ll 1$, where $k$ is the scattering wavenumber (equation 19) and $\lambda_D$ is the Debye length.

2. SCATTERING BY A SINGLE ELECTRON

The electric field radiated by an accelerating electron at a distance $R \gg \lambda_i$ from the electron, in the direction $\hat{s}$ and within the laboratory frame is given by[3]

$$\vec{E}_s(\vec{R},t) = -\frac{e}{c}\left[\frac{\hat{s} \times \{(\hat{s}-\vec{\beta}) \times \dot{\vec{\beta}}\}}{R^1(1-\hat{s}.\vec{\beta})^3}\right]_{ret} \qquad \ldots(1)$$

where $\vec{\beta} = \frac{\vec{\nu}}{c}$ and $\vec{\nu}(t^1)$ is the velocity of the electron. This quantity is evaluated at the retarded time, (see Figure 1.)

$$t^1 = t - \frac{|\vec{R} - \vec{r}(t^1)|}{c} \quad . \qquad \ldots(2)$$

We now need to determine the acceleration of the electron, at the retarded time, and in the laboratory frame, under the influence of the electromagnetic wave, for which,

$$\left.\begin{aligned} \vec{E}_i &= \vec{E}_i \ \mathrm{Cos}\ (\vec{k}_i . \vec{r} - w_i t^1) \\ \vec{H}_i &= \hat{i} \times \vec{E}_i \ . \end{aligned}\right\} \qquad \ldots(3)$$

The motion of the electron is determined by

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\frac{m_0\vec{\nu}}{\left(1-\frac{\nu^2}{c^2}\right)^{\frac{1}{2}}}\right) = -e\left(\vec{E}_i + \frac{\vec{\nu}}{c} \times \vec{H}_i\right) \qquad \ldots(4)$$

or $\gamma\, m_0 \dot{\vec{\nu}} + \gamma^3 m_0 \vec{\nu}\left(\frac{\vec{\nu}.\dot{\vec{\nu}}}{c^2}\right) = -e\left(\vec{E}_i + \frac{\vec{\nu}}{c} \times \vec{H}_i\right)$

where $\gamma = \left(1 - \frac{\nu^2}{c^2}\right)^{-\frac{1}{2}}$ .

We take the scalar product with $\vec{\nu}$ and

$$\dot{\vec{\nu}} = -\frac{e}{m_0}\left(1 - \frac{\nu^2}{c^2}\right)^{\frac{1}{2}}\{\vec{E}_i + \vec{\beta} \times \vec{H}_i - \vec{\beta}(\vec{\beta}.\vec{E}_i)\}. \qquad \ldots(5)$$

In total therefore, with $\beta$ included to all orders

$$E_s(R,t) = \frac{e^2}{m_0 c^2}\left(1 - \frac{\nu^2}{c^2}\right)^{\frac{1}{2}} \frac{\{\hat{s} \times [(\hat{s}-\vec{\beta}) \times (\vec{E}_i + \vec{\beta} \times (\hat{i} \times \vec{E}_i) - \vec{\beta}(\vec{\beta},\vec{E}_i))]\}}{R\,(1 - \hat{s}.\vec{\beta})^3}$$

$$.\ \mathrm{Cos}\,(\vec{k}_i.\vec{r}(t^1) - w_i t^1)\ , \qquad \ldots(6)$$

where we have substituted $R$ for $R^1$ in the denominator since we will be concerned only with observing distances (R) very large compared to the dimension (L) of the scattering volume.

We ignore the influence of the electromagnetic wave in determining the orbit of the electron, i.e. we limit the incident power so that $\frac{eE_i}{m_o w_i{}^2} << \left(\frac{kT_e}{m_e}\right)^{\frac{1}{2}}$ and solve

$\frac{d\vec{p}}{dt} = 0$, and find

$$\vec{r}(t^1) = \vec{r}(o) + \vec{\nu}\, t^1 \ . \qquad \ldots(7)$$

This we substitute into (2), and with $R>>L$ obtain

$$t^1 \simeq \frac{t - \frac{R}{c} + \frac{\hat{s}.\vec{r}(o)}{c}}{(1-\hat{s}.\vec{\beta})} \ . \qquad \ldots(8)$$

The argument of the cosine in (6) is now rewritten in terms of $t$, $\vec{\nu}$ and $\vec{r}(o)$ by the substitution of $\vec{r}(t^1)$ and $t^1$ from (7) and (8).

$$\vec{k}_i.\vec{r}(t^1) - w_i t^1 \Rightarrow k_s R - w_s t - \vec{k}.\vec{r}(O)$$

where $\vec{k}_s = \hat{s}k_s = \hat{s}\,\frac{w_s}{c}$ is the wave vector of the scattered radiation

$$\left.\begin{aligned} \vec{k} &= \vec{k}_s - \vec{k}_i \\ \text{and}\quad w_s &= \frac{(w_i - \vec{k}_i.\vec{\nu})}{(1-\hat{s}.\vec{\beta})} = w_i + \vec{k}.\vec{\nu} \end{aligned}\right\} \qquad \ldots(9)$$

In the expression for $w_s$ the numerator takes account of the Doppler shift in frequency as a result of the motion of the electron in the incident wave. The denominator gives the frequency shift owing to the motion of the electron in the direction of the observer. It is necessary to use the full expression for $w_s$ (equation 9) because finally, equations (25) and (28), it will lead to a correction term of order $\frac{\overline{\nu}}{c}$.

$$E_s(R,t) = \frac{e^2}{m_o c^2}\left(1-\frac{\nu^2}{c^2}\right)^{\frac{1}{2}} \frac{\{\hat{s}\times[(\hat{s}-\vec{\beta})\times(\vec{E}_i+\vec{\beta}\times(\hat{i}\times\vec{E}_i) - \vec{\beta}(\vec{\beta}.\vec{E}_i))]\}}{R(1-\hat{s}.\vec{\beta})^3} . \cos\left(k_s R - w_s t - \vec{k}.\vec{r}(0)\right) \quad ..(10)$$

This is not an approximation, there are no further relativistic corrections, provided we have $\lambda_i, \lambda_s >> 0.1$Å so that we may neglect the Compton effect.

## 3. RESTRICTION TO PLANE POLARISED RADIATION

Equation (10) may be conveniently rewritten[(4)] as

$$E_s(R,t) = -\frac{e^2}{m_o c^2} E_i \frac{(1-\beta^2)^{\frac{1}{2}}}{(1-\beta_s)^3}\Big[(1-\beta_i)(1-\beta_s)\hat{E}_i - \left\{(1-\beta_i)\cos\eta + (\cos\theta-\beta_s)\beta_E\right\}\hat{s} + \beta_E(1-\beta_s)\hat{i} + \left\{(1-\beta_i)\cos\eta - (1-\cos\theta)\beta_E\right\}\vec{\beta}\Big].\cos(k_s R - w_s t - \vec{k}.\vec{r}(0)) \quad ...(11)$$

(Note that it is a simple matter to show that $\hat{s}.\vec{E}_s = 0$ as is expected for a wave propagating in the direction $\hat{s}$)

$\beta_i$, $\beta_s$ and $\beta_E$ are the components of $\vec{\beta}$ parallel to the vectors $\hat{i}$, $\hat{s}$ and $\hat{E}_i$, and $\cos\eta = \hat{s}.\hat{E}_i$.

To first order in $\beta$ this becomes

$$\vec{E}_s(R,t) = -\frac{e^2}{m_o c^2} E_i\Big[(1-\beta_i+2\beta_s)\hat{E}_i - \left\{(1-\beta_i+3\beta_s)\cos\eta + \beta_E \cos\theta\right\}\hat{s} + \beta_E i + \cos\eta.\vec{\beta}\Big].\cos(k_s R - w_s t - \vec{k}.\vec{r}(0)). \quad ...(12)$$

We will treat here only the common application of incoherent scattering that is the case where the incident light is plane polarised, the scattering plane is perpendicular to $E_i$ i.e. $\hat{s}.E_i = \cos\eta = 0$. In addition we will use a plane polariser $\hat{C}$ in the output oriented to accept scattered light polarised in the direction of $E_i$, and since $\hat{i}.\hat{E}_i = 0$ we have (see Figure 2).

$$E_{sc}(R,t) = -\frac{e^2 E_i}{m_o c^2 R}(1-\beta_i+2\beta_s)\cos(k_s R - w_s t - \vec{k}.\vec{r}(0)) \quad .(13)$$

However from (9) $\quad w_s = w_i \dfrac{(1-\beta_i)}{(1-\beta_s)}$ ...(14)

we substitute and

$$E_{sc} = -\frac{e^2 E_i}{m_o c^2 R}\,\frac{w_s^2}{w_i^2}(1+\beta_i)\,.\,\cos(k_s R - w_s t - \vec{k}.\vec{r}(0)) \quad ....(15)$$

## 4. SCATTERED POWER

The time averaged scattered intensity at R is given by

$$I_{sc}(R) = \frac{c}{4\pi}\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}|E_{sc}|^2\,dt\,. \quad ...(16)$$

We are only interested here in incoherent scattering, we will therefore eventually average over initial positions and we may drop the phase factor $(\vec{k}.\vec{r}(0))$.

The scattered power within the solid angle $d\Omega$ at the frequency $w_s$ is given to first order in $\beta$ by

$$P_{sc}(R)\,d\Omega = \frac{P_i}{A}\,r_o^2\,d\Omega\,\frac{w_s^4}{w_i^4}\,(1+2\beta_i) \quad ...(17)$$

where the incident power $P_i = \dfrac{cE_i^2}{8\pi}A$

and $r_o = \dfrac{e^2}{m_o c^2} = 2.82 \times 10^{-13}$ cm..

$d\Omega$ is the element of solid angle over which the scattered light is accepted, and A is the cross sectional area of the incident beam of radiation.

## 5. INCOHERENT SCATTERING FROM A PLASMA

Consider now a plasma with electron temperature $T_e$ and density $n_e = \frac{N}{V}$. We scatter from a volume V, which contains N electrons. The level of the scattered power at a given frequency $w_s$ is proportional to the number of electrons which have the component of velocity $\vec{v}_k$ along $\vec{k}$ such that $w_s = w_i + V_k k$. We have retained terms only to first order in $\beta$ and we may therefore use the non relativistic Maxwellian velocity distribution and integrate over velocity from $-\infty$ to $+\infty$.*

We restrict the calculation to an isotropic velocity distribution. In this situation it is then convenient to use a co-ordinate system with three mutually perpendicular directions; $\vec{k}$,; the perpendicular to $\vec{k}$ within the plane of $\hat{s}$ and $\hat{i}$; and the perpendicular to the plane of $\hat{s}$ and $\hat{i}$. See Figure 3. We denote these velocities by $V_k$, $V_{k\perp}$ and $V_{kT}$.

Thus

$$\beta_i = \frac{-V_k \text{Cos } \gamma}{c} + \frac{V_{k\perp}}{c} \text{Cos } (\frac{\pi}{2} - \gamma) \qquad \ldots(18)$$

$$k = \frac{(w_s^2 + w_i^2 - 2w_s w_i \text{ Cos } \theta)^{\frac{1}{2}}}{c} \qquad \ldots(19)$$

$$\left.\begin{aligned} \text{Cos } \gamma &= \frac{w_i - w_s \text{ Cos } \theta}{ck}, \quad V_k = \frac{(w_s - w_i)}{k} \\ \text{and} \quad dV_k &= \frac{(w_i^2 + w_i w_s)(1 - \text{Cos } \theta)}{k^2 c^2} \frac{dw_s}{k}. \end{aligned}\right\} \qquad \ldots(20)$$

The scattered power within the frequency range $w_s \rightarrow w_s + dw_s$ and solid angle $d\Omega$ is then

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*Note $$V_k = \frac{w_s - w_i}{k} = \frac{(w_s - w_i)}{\left((w_s - w_i)^2 + 2w_s w_i (1 - \text{Cos } \theta)\right)^{\frac{1}{2}}} \cdot C$$

It is clear that $w_s$ and $w_i$ must have the same sign otherwise we would have $|V_k| > C$, therefore strictly we should only integrate from -C to +C.

$$P_{sc}(R,w_s)\ dw_s d\Omega = N \iiint_{-\infty}^{+\infty} dv_k dv_{k\perp} dv_{kT} \frac{P_i}{A} r_o^2 d\Omega \frac{w_s^4}{w_i^4}(1+2\beta_i) \;.$$

$$\frac{\exp}{(\pi a^{\frac{3}{2}})} \left(\frac{-(V_k^2+V_{k\perp}^2+V_{kT}^2)}{a^2}\right) \delta\left(\nu_k - \left(\frac{w_s-w_i}{k}\right)\right), \qquad ...(21)$$

where $a^2 = \dfrac{2KTe}{m_e}$ .

We substitute from (18), (19) and (20) and find that

$$P_{sc}(R,w_s)dw_s d\Omega = P_i r_o^2 d\Omega\ n_e L\ \frac{w_s^4}{w_i^4}\left(1 - \frac{2(w_s-w_i)(w_i-w_s\ \text{Cos}\ \theta)}{k^2c^2}\right).$$

$$\left(\frac{w_i^2+w_i w_s))1-\text{Cos}\ \theta)}{k^2c^2}\right)\exp\left\{-\left(\frac{w_s-w_i}{k}\right)^2\right\}\cdot\frac{dw_s}{\pi^{\frac{1}{2}}ka} \qquad ...(22)$$

where $L$ is the length of the scattering volume ($V=AL$).

Now we want this result in terms of the wavelength shift $\Delta\lambda$ where

$$\lambda_s = \lambda_i + \Delta\lambda \;. \qquad ...(23)$$

To be consistent with our restriction to first order in $\frac{\nu}{c}$ we must only keep terms to first order in $\dfrac{\Delta\lambda}{\lambda_i}$ .

We note that $\dfrac{1}{k} \cong \dfrac{c}{2^{\frac{1}{2}}(1-\ \text{Cos}\ \theta)^{\frac{1}{2}} w_i}\left(1+\dfrac{\Delta\lambda}{2\lambda_i}\right)$

$$\frac{w_s^4}{w_i^4} \cong 1-4\frac{\Delta\lambda}{\lambda i}\ ,\quad dw_s \cong \frac{2\pi c}{\lambda_i^2}\left(1-2\frac{\Delta\lambda}{2\lambda_i}\right)d\lambda s\,;\ w_i^2+w_iw_s \cong 2w_i^2\left(1-\frac{\Delta\lambda}{2\lambda_i}\right) \qquad ..(24)$$

$$\text{and}\ \left(\frac{w_s-w_i}{ka}\right)^2 \cong \frac{c^2\Delta\lambda^2}{4\lambda_i^2\text{Sin}^2(\theta/2)a^2\left(1+\frac{\Delta\lambda}{\lambda_i}\right)}\ ;\quad \frac{2(w_s-w_i)(w_i-w_s\ \text{Cos}\theta)}{k^2c^2} \cong \frac{\Delta\lambda}{\lambda_i}\ .$$

Substituting,

$$P_{sc}d\lambda_s d\Omega = \frac{P_i r_o^2 d\Omega n_e L\ C}{2\pi^{\frac{1}{2}}a\text{Sin}(\theta/2)\lambda_i}\left\{1-4\frac{\Delta\lambda}{\lambda_i}+\frac{c^2\Delta\lambda^3}{4a^2\lambda_i^3\text{Sin}^2(\theta/2)}\right\}\ .$$

$$\exp\left(-\frac{c^2\Delta\lambda^2}{4a^2\lambda i^2\text{Sin}^2(\theta/2)}\right)\cdot d\lambda_s\ . \qquad ...(25)$$

We keep the term with $\frac{\Delta\lambda^3}{\lambda_i^3}$ because it is multiplied by $\frac{c^2}{a^2}$.

## 6. FINITE TRANSIT TIME EFFECTS

In evaluating the scattered intensity from one electron, (16), we assumed that this electron would remain within the scattering volume for the time $T$. In fact the incident beam has a finite diameter (D) and we will not be dealing over the whole time $T$ with the same group of $N$ electrons. This problem was first discussed by Pechacek and Trivelpiece(1). We now include this effect for the case of a long duration beam, i.e. $T \gg \frac{L}{\nu}$.

Let $v_\perp$ be the velocity of the electron at right angles to the beam. Then this electron will remain within the beam for the time $\Delta t^1 = \frac{D}{v_\perp}$. Now the density of electrons is constant, therefore on average for every electron which leaves the beam another one will enter. We see that rather than dealing with one electron we are dealing with the number $N_o(\nu_\perp) = \frac{v_\perp T}{D}$, which cross in the time $T$ and which each stay within the beam for the time interval $\Delta t^1$

$$\text{Now} \quad t = (1-\beta_s)\; t^1 + \frac{R}{c} - \hat{s}.\vec{r}_o. \qquad (8)^1$$

$$\text{Therefore} \quad \Delta t = (1-\beta_s)\; \Delta t^1 = (1-\beta_s)\; \frac{D}{v_\perp}$$

with this substitution (16) becomes

$$I_{sc}(R) = \frac{c}{4\pi} \lim_{T\to\infty} \frac{N_o}{T} \int_{-(1-\beta_s)\frac{D}{2v_\perp}}^{(1-\beta_s)\frac{D}{2v_\perp}} |E_{sc}|^2 dt \qquad \ldots(26)$$

and

$$P_{sc}(R)d\Omega = \frac{P_i}{A} r_o^2 d\Omega \frac{w_s^4}{w_i^4} (1+2\beta_i)(1-\beta_s)$$

$$= \frac{P_i}{A} r_o^2 d\Omega \frac{w_s^3}{w_i^3} (1+\beta_i) \; . \qquad \ldots(27)$$

The electrons which are moving away from the observer, (these gave a Doppler shift to the red) spend relatively longer in the beam, from the observers point of view, than the electrons of the opposite velocity which move towards the observer and give a Doppler shift to the blue. The factor $(1-\beta_s)$ which appears in equation (27) takes account of this effect.

An additional effect was proposed by Williamson and Clarke[4]. This was associated with the finite length of the laser beam. In fact the effect will only occur if the laser beam is shorter in length than the scattering volume, in which case electrons moving with the beam will scatter for longer than those moving away from it. This situation will not occur often in practice because for a typical scattering volume $L \sim 1$ cm, and we would require a laser pulse of duration less than 30 picoseconds to observe the effect. For this reason it is not included in this analysis.

It is assumed that $(1-\hat{s}.\vec{\beta})\,\frac{D}{2v_\perp} >> \frac{2\pi}{w_i}$.

Note that this scattered wave packet has a frequency width of order $\frac{v_\perp}{L}$, and this is a small broadening compared to that from the finite transit time effect, which is of order $\frac{v_\perp}{\lambda_i}\frac{\nu}{c}$.

i.e. we assume $L>>\lambda_i\,\frac{\nu}{c}$

with result (27) the scattered spectrum for the plasma becomes

$$P_{sc}(R,\lambda_s)\,d\lambda_s d\Omega = \frac{P_i r_o^2 d\Omega\, n_e L\, C}{2\pi^{\frac{1}{2}} a \operatorname{Sin}(\theta/2)\,\lambda_i}.\left\{1-\frac{7}{2}\frac{\Delta\lambda}{\lambda_i}+\frac{c^2\Delta\lambda^3}{4a^2\lambda_i{}^3\operatorname{Sin}^2(\theta/2)}\right\}.e^{-\frac{c^2\Delta\lambda^2}{4a^2\lambda_i{}^2\operatorname{Sin}^2(\theta/2)}}.d\lambda_s\,. \qquad ...(28)$$

This finite transit time effect has been observed in an electron beam, see Ward, Pechacek and Trivelpiece[5].

## 7. THE EXTRA TERMS LEAD TO A SHIFT OF THE CENTRE OF THE SCATTERED SPECTRUM

We differentiate (28) with respect to $\Delta\lambda$ and set the result equal to zero, with the shift of the centre of the spectrum denoted by $\Delta\lambda_m$. Now $\frac{\Delta\lambda_m}{\lambda_i} \ll 1$, and with this approximation we find

$$\Delta\lambda_m = - \frac{(7a^2 \sin^2(\theta/2)}{c^2} \lambda_i \; . \qquad \ldots(29)$$

That is a shift to the blue side of the spectrum.

Now $\frac{a^2}{c^2} \cong 4 \times 10^{-6}\, T_e(eV)$ therefore

$$\Delta\lambda_m \cong - 2.8 \times 10^{-5} T_e(ev) \sin^2(\theta/2)\lambda_i \qquad \ldots(30)$$

For $\theta = 90^\circ$, $T_e = 100ev$ and $\lambda_i = 6943$Å

$$\Delta\lambda_m \cong 10\text{Å} \; .$$

This compares favourably with the measured value of 12 ± 3Å, see Gondhalekhar, Kronast and Benesch[6].

## 8. TEMPERATURE MEASUREMENT

The temperature is customarily obtained from a plot of ln $(P_{sc})$ against $\Delta\lambda^2$ on the blue side of the spectrum, i.e. $\Delta\lambda < 0$.

If the correction terms $\left(-\frac{7}{2}\frac{\Delta\lambda}{\lambda_i} + \frac{c^2\Delta\lambda^3}{4a^2\lambda_i{}^2 \sin^2(\theta/2)}\right)$ are ignored and we take the gradient between the points $\Delta\lambda_1$ and $\Delta\lambda_2$, then we find

$$\left(\frac{2KT_e}{me}\right)_{approx.} = \frac{c^2}{4\sin^2(\theta/2)}\left[\left(\frac{\Delta\lambda_2}{\lambda_i}\right)^2 - \left(\frac{\Delta\lambda_1}{\lambda_i}\right)^2\right] \Big/ G_o \qquad \ldots(31)$$

where $G_o = \ln[P_{sc}(\Delta\lambda_1)] - \ln[P_{sc}(\Delta\lambda_2)]$ .

If we include the correction terms

$$\left(\frac{2KT_e}{me}\right)_{correct} = \frac{c^2}{4\sin^2(\theta/2)}\left[\left(\frac{\Delta\lambda_2}{\lambda_i}\right)^2 - \left(\frac{\Delta\lambda_1}{\lambda_i}\right)^2\right] \Bigg/ \left\{ G_o + \ln\left[\frac{1 - \frac{7}{2}\frac{\Delta\lambda_2}{\lambda_i} + \frac{c^2\Delta\lambda_2^3}{4a^2\lambda_i{}^3\sin^2\theta/2}}{1 - \frac{7}{2}\frac{\Delta\lambda_1}{\lambda_i} + \frac{c^2\Delta\lambda_1^3}{4a^2\lambda_i{}^3\sin^2\theta/2}}\right]\right\} . \qquad \ldots(32)$$

We now for brevity set

$$\frac{c^2 \Delta\lambda_1^2}{4a^2\lambda_i^2 \sin^2(\theta/2)} = X_1 \qquad \frac{c^2 \Delta\lambda_2^2}{4a^2\lambda_i^2 \sin^2\theta/2} = X_2 \text{ and then } G_o = X_2 - X_1 \quad ..(33)$$

We use the result $\ln(1+\epsilon) \cong \epsilon$ for $\epsilon<1$ and find on the blue side of the spectrum that

$$T_{e_{app}} = T_{e_{corr}} \left\{ \frac{X_2 - X_1 + [X_2^{\frac{1}{2}}(1 - \frac{X_2}{3.5}) - X_1^{\frac{1}{2}}(1 - \frac{X_1}{3.5})]7\frac{a}{c}\sin\theta/2}{(X_2 - X_1)} \right\} \quad ..(34)$$

where $\frac{a}{c} = 2 \times 10^{-3} \quad [T_{e_{corr}}(eV)]^{\frac{1}{2}}$ .

The effect of the correction terms is indicated by the following examples where we compare the approximate and correct temperatures when the gradient is taken respectively between the points $e^{-x} = 0.9 \rightarrow 0.3$ and $e^{-x} = 0.9 \rightarrow 0.5$ .

Case (1) $X_1 = 0.1$, $X_2 = 1.2$,

$$T_{e_{app}} = T_{e_{corr}} [1 + 5.8 \times 10^{-3} \sin(\theta/2)(T_e(eV))^{\frac{1}{2}}] \quad ..(35)$$

Case (2) $X_1 = 0.1$ $X_2 = 0.7$

$$T_{e_{app}} = T_{e_{corr}} [1 + 8.4 \times 10^{-3} \sin(\theta/2)(T_e(ev))^{\frac{1}{2}}] \quad ..(36)$$

A comparison of some approximate and correct temperatures (for 90° scattering on the blue side of the spectrum) is made in Table 1 below.

| $T_{e_{corr}}$ eV | $T_{e_{app}}$ 0.90→0.30 eV | $T_{e_{app}}$ 0.90→ 0.50 eV |
|---|---|---|
| 100 | 104 | 106 |
| 400 | 433 | 447 |
| 900 | 1011 | 1058 |
| 1600 | 1865 | 1980 |
| 2500 | 3000 | 3240 |

For measurements on the red side of the line the correction term takes the opposite sign and we would underestimate the temperature with the approximate formula (26).

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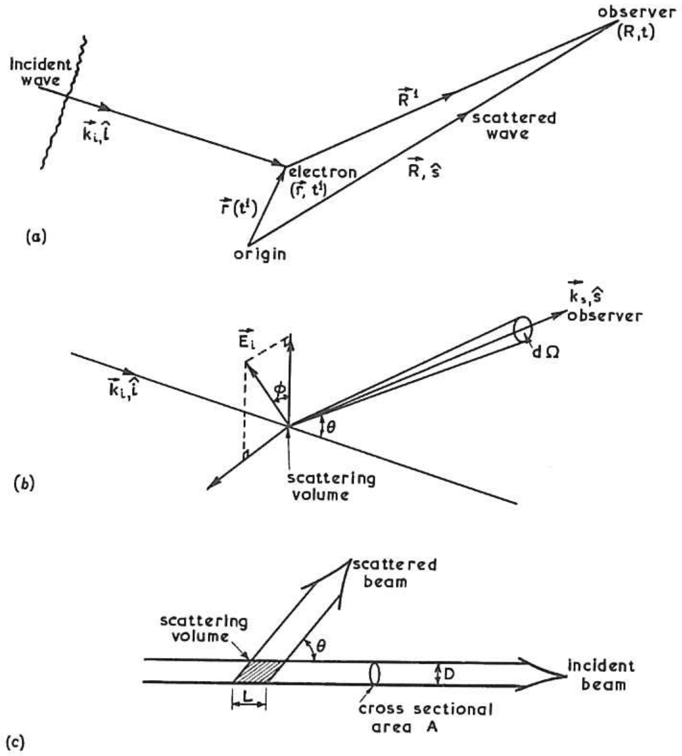


Fig. 1

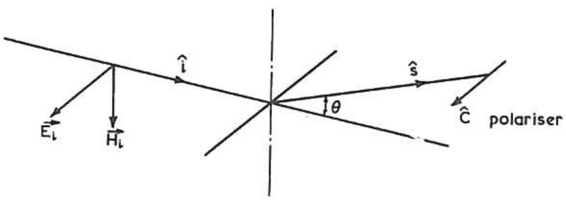


Fig. 2

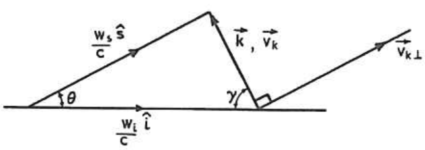


Fig. 3