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# RECOMBINATION BETWEEN ELECTRONS AND ATOMIC IONS

## Part I. OPTICALLY THIN PLASMAS


by

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## ABSTRACT

Consideration is given to the interacting collisional and radiative processes occurring in a plasma. A statistical theory describing the general loss mechanism, for which the name collisional-radiative recomination is proposed, is described. This theory enables the collisional-radiative recombination coefficient $\alpha$ to be determined knowing the relevant spontaneous transition probabilities and the rate coefficients for radiative recombination and collisional excitation and ionization.

Detailed calculations are carried out on hydrogen-ion plasmas which are optically thin. It is found that $\alpha$ is an increasing function of the number density of free electrons n(c) the increase being especially marked if the electron temperature T is low; for example if T is $250^{o}$K $\alpha$ becomes almost 20 times as great as the radiative recombination coefficient (which describes the loss in a very tenuous plasma) when n(c) is only about $10^{8}/cm^{3}$, whereas if T is $64,000^{o}$K $\alpha$ does not become as great as this until n(c) is about $10^{18}/cm^{3}$. From a similar investigation in which the ground level of the hydrogen atom is made inaccessible (in crude representation of an alkali atom) it is inferred that the value of $\alpha$ is probably not very sensitive to the species of singly-charged ion involved.

Recombination of electrons with bare nuclei of charge Ze to form hydrogenic ions is similarly tested for an optically thin plasma. It is shown that to a close approximation the reduced coefficient $\alpha/Z$ is a function of a reduced temperature $T/Z^2$ and a reduced density $n(c)/Z^7$ only. The values of the reduced coefficients are of comparable magnitude and have a similar dependence on the reduced temperature and density as the coefficients for hydrogen-ion plasmas.

The variation of the recombination coefficient with Z in the same plasma (i.e. same n(c) and T) is investigated. It may be expressed in the form $\alpha \propto Z^z$ where the index z depends on n(c) and T. Though z is generally positive as would be expected, it is negative if n(c) and T are very high. A physical explanation of this is presented.

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Fig. 1. Variation with log T of $\log\left[\underset{n(c)\rightarrow 0}{\mathrm{Lt}}\ \alpha\right]$ and of $\log\left[\underset{n(c)\rightarrow\infty}{\mathrm{Lt}}\ \frac{10^{10}\alpha}{n(c)}\right]$

where T is the temperature ($^{o}$K), n(c) is the number density of free electrons ($cm^{-3}$) and $\alpha$ is the collisional-radiative recombination coefficient ($cm^{3}\ sec^{-1}$) in an optically thin hydrogen ion plasma.

Fig. 2 Relative importance of the processes populating the ground level when the reduced temperature $T/Z^2$ is 4000$^{o}$K plotted against the logarithm of the reduced number density $n(c)/Z^7$ in $cm^3\ sec^{-1}$.
Curve a, radiative transitions from excited levels;
curve b, radiative recombination from the continuum;
curve c, collisional transitions from excited levels;
curve d, (not shown) three body recombination from the continuum (everywhere less than 0.5 percent).

## 1. Introduction

When an electron recombines with an atomic* ion in the absence of any third body the energy which is liberated is carried off as radiation. Extensive calculations have been done using the quantal formula giving the rate coefficient for the process (cf. Massey and Burhop 1952, Allen 1955, Bates and Dalgarno 1962). However, this rate coefficient only describes the actual recombination if the plasma is sufficiently tenuous. Account must in general be taken of the possibility that part of the energy liberated may be given to a neighbouring electron in a collision. Indeed if the plasma is sufficiently dense collisional processes control the recombination.

The term radiative recombination is commonly used for the loss mechanism in very tenuous plasmas. To parallel this it is natural to introduce the term collisional recombination for the loss mechanism in very dense plasmas. The two are limiting cases of a general loss mechanism which may appropriately be called collisional-radiative recombination. This general loss mechanism is, of course, not simply the sum of collisional and radiative recombination, it results from the combination of interacting collisional and radiative processes.

Notes by Bates and Kingston (1961) and McWhirter (1961) revealed that essentially similar work on collisional-radiative recombination was being carried out independently in Belfast and at the Culham Laboratory. To avoid unnecessary duplication it was judged best to collaborate and describe the work in the present joint paper.

Collisional-radiative recombination has also been investigated by D'Angelo (1961) in Princeton. His approach is different from the statistical approach adopted here in that he considers the complicated reaction paths followed by individual electrons.

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* Molecular ions, which may be destroyed by dissociative recombination, are excluded from the discussion.

## 2. General Theory

2.1 The discussion will be confin.. to the recombination of electrons e with bare nuclei $N^{Z+}$ of charge Ze to form hydrogen atoms or hydrogenic ions.

Write p, q, ..... for the principal quantum numbers of the discrete levels and c for the continuum; write n(p), n(q) for the number densities of atoms or ions in the levels indicated and n(c) and $n(N^{Z+})$ for the number densities of free electrons and bare nuclei. Let K(c,p) be the rate coefficient* for the three-body recombination process.

$$N^{Z+} + e + e \longrightarrow N^{(z-1)+}(p) + e \qquad .....(1)$$

K(p, c) be the rate coefficient for the inverse

$$N^{(z-1)+}(p) + e \longrightarrow N^{Z+} + e + e \qquad .....(2)$$

K(p,q) be the rate coefficient for the collisional excitation or de-excitation process

$$N^{(z-1)}(p) + e \longrightarrow N^{(z-1)}(q) + e \qquad .....(3)$$

and β(p) be the rate coefficient for radiative recombination

$$N^{Z+} + e \longrightarrow N^{(z-1)+}(p) + h\nu \qquad .....(4)$$

all these rate coefficients being at the electron temperature T of the plasma; and let A(p,q) be the spontaneous transition probability for

$$N^{(z-1)+}(p) \longrightarrow N^{(z-1)+}(q) + h\nu \qquad .....(5)$$

A uniform distribution amongst the degenerate states of a level is assumed. Elastic collisions must be sufficiently rapid to ensure such a distribution if inelastic and superelastic collisions populate and evacuate the level at a rate comparable with or greater than the rate at which radiative processes do so; and the distribution cannot influence the collisional-radiative recombination coefficient if inelastic and superelastic collisions are much less effective than radiative processes.

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* This rate coefficient is such that n(c) $n(N^{Z+})$ K(c,p) is the number of the collision concerned which occur per $cm^3$ per sec.

Neglecting electronic transitions due to atom-atom, atom-ion or ion-ion collisions (as is certainly justified), ignoring boundary effects and supposing for the present that the plasma is optically thin so that all the radiation emitted escapes without absorption it is seen that the rate of increase of $n(p)$ with time t is given by

$$\dot{n}(p) = - n(p)\ [n(c)\ K(p) + A(p)] + n(c) \sum_{q \neq p} n(q)\ K(q,p) + \sum_{q>p} n(q)\ A(q,p) + \frac{n(c)^2}{X}\ [K(c,p) + \beta(p)] \qquad \text{.....(6)}$$

where

$$K(p) = K(p,c) + \sum_{q \neq p} K(p,q), \quad A(p) = \sum_{q<p} A(p,q), \quad X = {}^{n(c)}/_{n(N^{z+})} \qquad \text{.....(7)}$$

It is convenient to introduce the ratios

$$\rho(p) = n(p)/n_E(p) \qquad \text{.....(8)}$$

where $n_E(p)$ is the number density of hydrogen atoms or hydrogenic ions in level p in Saha equilibrium at temperature T with free electrons and bare nuclei each of number density $n(c)$ and $n(N^{z+})$ so that

$$X\ n_E(p)/n(c)^2 = p^2\ \{h^2/2\pi\ mkT\}^{3/2}\ \exp\ (I_p/kT) \qquad \text{.....(9)*}$$

$I_p$ being the ionization potential (which may with sufficient accuracy be taken to be that of an isolated atom). Noting that

$$n_E(q)\ K(q,p) = n_E(p)\ K(p,q), \qquad (n(c)/_X)\ K(c,p) = n_E(p)\ K(p,c) \qquad \text{.....(10)}$$

* As is well known (Fowler 1936) formula (9) is not meaningful if the principal quantum number p is too large. The high levels concerned are fortunately unimportant in collisional-radiative recombination.

it is seen that (6) may be written

$$\dot{n}(p)/n_E(p) = -\rho(p)\,[n(c)\,K(p) + A(p)]$$

$$+ \sum_{q \neq p} \rho(q)\, n(c)\, K(p,q) + \sum_{q>p} \rho(q) \frac{n_E(q)}{n_E(p)} A(q,p) \qquad \ldots\ldots(11)$$

$$+ n(c)\, K(p,c) + \frac{n^2(c)}{n_E(p)} \frac{\beta(p)}{X}$$

The infinite set of coupled differential equations typified by (11) describes the course of the recombination.

On the numerical substitution (9) becomes

$$X\, n_E(p)\, n(c) = 4.2 \times 10^{-16} \{n(c)\, p^2/T^{3/2}\} \exp(157890\, Z^2/p^2T) \qquad \ldots\ldots(12)$$

showing that for a wide range of plasmas the Saha equilibrium number density of excited systems is very much less than the number densities of free electrons and bare nuclei. It follows that for a still wider range

$$n(p) << p \neq 1 \qquad \ldots\ldots(13)$$

Table I which was prepared using (12) and the results of the computations on $\rho(p)$ to be described later, shows that this condition is only violated in the case of extremely dense plasmas.

Great simplification may be effected in the treatment of a plasma for which condition (13) is satisfied, and in which the mean thermal energy is much less than the first excitation energy so that in addition

$$n(p) << n(1), \quad p \neq 1 \qquad \ldots\ldots(14)$$

when the steady state is reached. In such a plasma a quasi-equilibrium number density of excited systems is established almost instantaneously without the number densities of free electrons and protons being appreciably altered; and therafter the rates per unit volume at which excited atoms are produced and destroyed by collision and radiative processes are much greater than the rates at which the number densities of these rare atoms change as the plasma decays. An instructive complementary description of the situation is that the relaxation times for the excited levels are very much shorter than the relaxation time for the ground level or for the free electrons.

In consequence the derivatives on the left of equations (11, $p \neq 1$) can be put equal to zero without causing significant error. The resulting set of simultaneous equations determines $\rho(p)$, $p \neq 1$ in terms of $n(c)$, $\rho(1)$, T, and various atomic parameters. Equation (11, $p = 1$) similarly determines $\dot{n}(1)$ which is the rate of disappearance of free charges so that

$$\dot{n}(1) = - \dot{n}(N^{z+}) = \frac{\gamma \, n^2(c)}{X} \qquad .....(15)$$

where $\gamma$ is an effective two-body rate coefficient (dependent on $n(c)$, $n(1)$, X and T) which will be called <u>collisional-radiative decay coefficient</u>. The decision to express the results in terms of a two-body rate coefficient does not imply that two-body processes predominate; it is quite arbitrary, based solely on considerations of convenience. It is felt that the description of the coefficient by the non-committal adjective <u>decay</u> is on the whole desirable since both recombination and ionization occur. When the steady state is reached these opposing processes naturally balance and $\gamma$ vanishes. In theoretical discussions the composite coefficient $\gamma$ may with advantage be replaced by two other rate coefficients (to be defined later).

The emphasis in this paper is on the decay of plasmas but the numerical results presented are also of direct relevance in connection with the growth of plasmas.

An infinite matrix may be avoided by taking advantage of the fact that when p is large enough collisional processes are much more important than radiative processes so that $n(p)$ satisfies the Saha equation. It is convenient in practice to group together the levels with p greater than some values s such that $\rho(s)$ is very close to unity. Denoting the group $\sigma$, taking

$$\rho(\sigma) = 1 \qquad .....(16)$$

and writing

$$K(p,\sigma) = \sum_{q>s} K(p,q) \qquad \text{.....(17)}$$

it is seen that (11) reduces to

$$\rho(p)\,[n(c)\,K(p) + A(p)] - \sum_{\substack{q\neq p\\ \leq s}} \rho(q)\,n(c)\,K(p,q) - \sum_{\substack{q>p\\ \leq s}} \rho(q)\,\frac{n_E(q)}{n_E(p)}\,A(q,p)$$

$$= n(c)\,[K(p,\sigma) + K(p,c)] + \frac{n(c)^2\beta(p)}{X\,n_E(p)} + \sum_{q>s} \frac{n_E(q)}{n_E(p)}\,A(q,p) \qquad \text{.....(18)*}$$

for

$$p \neq 1, \leq s \qquad \text{.....(19)}$$

Taking $\rho(1)$ as known, the solutions to this set of s-1 equations may be expressed

$$\rho(p) = r_o(p) + r_1(p)\,\rho(1) \qquad \text{.....(20)}$$

where $r_o(p)$ and $r_1(p)$, which are functions of $n(c)$, and T but not of X, are positive definite.

To check that the chosen s is large enough the computed $\rho(p)$ may be plotted against p and a curve drawn through the points. The line

$$\rho(p) = 1 \qquad \text{.....(21)}$$

should be tangential to this curve. If it is not, s should be increased.

Using (11, p = 1) and (15) it may be seen that the decay coefficient $\gamma$ is given by

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* It should be noted that the product $X\,n_E(p)$ and the ratio $n_E(q)/n_E(p)$ are independent of X.

$$\frac{\Upsilon}{X} = \frac{\dot{n}(1)}{n^2(c)}$$

$$= \frac{n_E(1)}{n(c)} \left[ \sum_{\substack{q>1 \\ <s}} \rho(q)\, K(1,q) + K(1,\sigma) + K(1,c) \right]$$

$$+ \left[ \sum_{\substack{q>p \\ \leqslant s}} \rho(q) \frac{n_E(q)}{n^2(c)} A(q,1) + \sum_{q>s} \frac{n_E(q)}{n^2(c)} A(q,1) + \frac{\beta(1)}{X} \right]$$

$$- \left[ \frac{n_E(1)}{n(c)} \rho(1)\, K(1) \right] \qquad \text{.....(22)}$$

the first term in square brackets representing the population of the ground level by collisional deactivation and by three-body recombination, the second representing the population of the ground level by radiative recombination and the third representing the evacuation of the ground level by collisional excitation and ionization. Because of the form of (20) this equation may be written

$$\Upsilon = \alpha - SX\, n(1)/n(c) \qquad \text{.....(23)}$$

where $\alpha$ and S are positive quantities dependent only on n(c), T and various atomic parameters. Clearly:

$$\frac{\alpha\, n(c)}{X} >> S\, n_E(1) \qquad \text{.....(24)}$$

in plasmas tenuous enough for radiative processes to be important and

$$\frac{\alpha\, n(c)}{X} \simeq S\, n_E(1) \qquad \text{.....(25)}$$

in plasmas dense enough for collision processes to be dominant. During at least the early part of the decay from a state in which n(1) is very low $\gamma$ may be put equal to $\alpha$ and the term in S may be beglected. It is convenient to refer to $\alpha$ as the collisional-radiative recombination coefficient and to refer to S as the collisional-radiative ionization coefficient. Contrary to what might at first be thought $\alpha$ and S do not correspond to coefficients giving the rate at which electrons enter and leave the ground level. They are smaller than such coefficients would be. Because of the dependence of $\rho(p)$, $p \neq 1$ on $\rho(1)$ the differential equation (15) with $\gamma/X$ as in (22) is of the form

$$\dot{n}(1) = n(c)\left[\frac{n(c)}{X}\alpha + n(1)\,L - n(c)\,n(1)\,(S + L)\right] \qquad .....(26)$$

$$S + L = K(1)$$

in which L is another rate coefficient. The two terms involving L may be regarded as describing electrons undergoing transitions between the excited levels and the ground level in a purely internal cycle that is without passage through the continuum.

It may be mentioned here in parenthesis that in addition to being necessary for the vital simplification leading to (15) condition (13) happens to ensure that the life of an electron in any level of importance in recombination exceeds the time the electron takes to describe its orbit; that is that

$$\tau_{lf}(p) > \tau_{obt}(p), \quad p < s \qquad .....(27)$$

where

$$\tau_{lf}(p) = {}^{1}/\{A(p) + n(c)\,K(p)\} \qquad .....(28)$$

and

$$\tau_{obt}(p) = \{2\pi\,\hbar^3\,p^3/Z^3\,m\,e^4\}$$

$$= 1.5 \times 10^{-16}(p^3/Z^3)\ \text{sec} \qquad .....(29)$$

If $n(c)$ were high enough to make (27) untrue a collision could not be regarded as a distinct event. The saturation effect which sets in prevents $\alpha$ from continuing to increase as the first power of $n(c)$ indefinitely.

## 3. Atomic Parameters

No difficulty arose in connection with the radiative processes as they had already been treated with high precision. The values of the spontaneous transition probabilities $A(p,q)$ were obtained from the tables of Baker and Menzel (1938) and of Green, Rush and Chandler (1957) and those of the radiative recombination coefficients $\beta(p)$ were obtained from the tables of Seaton (1959).

The position regarding the collision processes is much less satisfactory precise quantal results not being available. If it were not for the possible errors in the rate coefficients of these processes the calculations on collisional-radiative decay would be effectively exact within the specified limitations.

Neutral atoms Gryzinski (1959) has recently adduced evidence that fair accuracy may be achieved by classical calculations.

According to his formulation the rate coefficient for collisional ionization of atomic hydrogen in level p is given by

$$K(p,c) = \frac{5.45}{T^{3/2}} \int_1^\infty I \exp\left(\frac{-157890\,x}{p^2\,T}\right) dx \ \text{cm}^3/\text{sec} \qquad \ldots\ldots(30)$$

where

$$I = \frac{4\sqrt{2}}{3} \frac{(x-1)^{3/2}}{(x+1)^{3/2}} \qquad x < 2$$

$$= \frac{1}{3} \frac{(5x-6)}{(x+1)^{3/2}} \, x^{\frac{1}{2}} \qquad x > 2 \qquad \ldots\ldots(31)$$

and where T is as usual the temperature in degrees absolute.

The concept of excitation to a discrete level is in a sense alien to the classical theory which gives directly only the cross section, $Q(p,\varepsilon)d\varepsilon$ for a collision in which energy between $\varepsilon$ and $\varepsilon + d\varepsilon$ is transferred to the target. However, if $\varepsilon$ is in atomic units it may be assumed that the cross section for the excitation of the $p \longrightarrow q$ transition is

$$Q(p,q) = \frac{1}{q^3} Q(p,\varepsilon_{qp}), \quad q > p \qquad \text{.....(32)}$$

where $\varepsilon_{qp}$ is the excitation energy (cf. McCarroll 1957). Using Gryzinski's expression $Q(p,\varepsilon_{qp})$ it is then found that in the case of atomic hydrogen the rate coefficient for collisional excitation is

$$K(p,q) = \frac{10.9}{T^{3/2}} \frac{p^2}{q(q^2-p^2)} \int_1^\infty E \exp\left\{\frac{-157890\,(q^2-p^2)y}{p^2q^2\,T}\right\} dy \ \text{cm}^3/\text{sec} \qquad \text{.....(33)}$$

where

$$E = \begin{cases} \dfrac{\{2-a+y\,(1+4a)\}\,(y-1)^{\frac{1}{2}}\,(a+1)^{\frac{1}{2}}}{3a^{\frac{1}{2}}\,(a+y)^{3/2}}, & y < 1+a \\[2ex] \dfrac{\{-3a+y\,(3+4a)\}\,y^{\frac{1}{2}}}{3(a+y)^{3/2}}, & y > 1+a \end{cases} \qquad \text{.....(34)}$$

with

$$a = q^2/(q^2 - p^2) . \qquad \text{.....(35)}$$

Classical theory cannot, of course, describe the collisions properly; and it gives the wrong threshold law; nevertheless comparison with the experimental data of Fite and his associates (1958, 1959, 1960) suggests that the derived rate for collisions between electrons and hydrogen atoms are unlikely to be in error by more than a factor of three.

Ions  On the basis of an approximate quantal solution of the problem of collisional excitation of positive ions Seaton (1962) has proposed an expression for the cross section for energies of the incident electron close to the threshold. The corresponding rate coefficient was compared with the rate coefficient derived from the Coulomb-Born approximation of Burgess (1961) in the case of $Z = 2$, $p = 1$, $q = 2$. Agreement to within a factor of two, over a range of $kT/\varepsilon_{qp}$ from 0 to 10 is obtained if the effective Gaunt factor $\bar{g}$ is taken to be 0.4 (instead of 0.2 as suggested by Seaton). With this value of $\bar{g}$ the expression for the rate coefficient is

$$K(p,q) = 4.75 \times 10^{-5} \frac{p^2 q^2}{q^2 - p^2} \frac{f(p,q)}{Z^2 T^{\frac{1}{2}}} \exp\left( \frac{-157890\, Z^2 (q^2 - p^2)}{p^2 q^2 T} \right) \text{cm}^3/\text{sec} \qquad \ldots\ldots(36)$$

where $f(p,q)$ is the absorption oscillator strength indicated.

Seaton's approximate expression for the ionization coefficient near the threshold does not lend itself to the same simple treatment. However, it was judged sufficient to adopt a form similar to (36): explicitly

$$K(p,c) = \frac{1.4 \times 10^5 p^2}{Z^2 T^{\frac{1}{2}}} \exp\left( \frac{-157890\, Z^2}{p^2\, T} \right) \text{cm}^3/\text{sec}\ . \qquad \ldots\ldots(37)$$

In the case $Z = 2$, $p = 1$ this reproduces the ionization coefficient computed from the Coulomb-Born approximation of Burgess to within a factor of two over a $kT/I_p$ range from $\frac{1}{4}$ to 4.

## 4. Results

### 4.1 Single charged positive ions

The values of the collisional-radiative recombination and ionization coefficients $\alpha$ and $S$ are displayed in Tables IIA and IIB respectively. Through (23) these parameters determine $\gamma$ the collisional-radiative decay coefficient. The values of $S + L$ of (26) are for interest also displayed in Table IIB (last row). They depend only on the temperature.

Table III gives

$$n_s(1) = \alpha\ n(c)/XS \qquad .....(38)*$$

the number density of normal atoms in the steady state and, for comparison, $n_E(1)$ the corresponding number density in Saha equilibrium. Inspection of it shows that Table IIB may often be disregarded and the decay coefficient identified with the entries in Table IIA.

The tables have been carried to values of n(c) above the limit set by condition (13) and indicated by Table I so that the capacity of the electron-reservoir provided by the excited levels is large. If the plasma is not much denser than the limit the decay coefficient given underestimates the rate of disappearance of free charges during the initial stage of recovery from a state of complete ionization because the electron reservoir formed by the excited levels is then being filled; and it overestimates the rate during the later stages because this reservoir is then being emptied. As already mentioned saturation effects occur if condition (27) is violated. In consequence the last row of each of the tables is without physical significance.

Again at the highest value of T considered, 64,000°K condition (14) cannot be said to be satisfied. There is insufficient time for the quasi-equilibrium values of ρ(p), p ≠ 1 adopted in computing α to be reached. This does not affect the derived value of α if n(c) is low enough for radiative recombination to be dominant; but it makes the derived value less than it should be if n(c) is high enough for collisional processes to be important.

If n(c) is sufficiently low α is independent of n(c) the recombination being then purely radiative; and if n(c) is sufficiently high α is proportional to n(c) the recombination being then purely collisional. As is seen from table IIA the increase occurs earlier and is more pronounced in cool than in hot plasmas.† Another manifestation of the same effect is that the

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* If the plasma is electrically neutral and if $H^+$ is the only species of ion then X is unity.

† It may be mentioned that there are peculiar slight changes in the slopes of the α *versus* n(c) curves when T is above about 8000 K. These changes are associated with the approach of ρ(2) to its limiting value, unity.

rate coefficients in the limiting cases of pure radiative recombination and pure collisional recombination differ greatly in their dependence on T the temperature (figure 1), thus as T is raised $\alpha$ falls off slightly faster than as $T^{-\frac{1}{2}}$ in the former case, but as $T^{-x}$ in the latter case with x increasing from about 5 near 250°K to about 8 near 6000°K and then decreasing to about 2 near 64,000°K.

In his calculations (which are based on a representation of the processes involved which is less complete than that developed here) D'Angelo (1961) took T to be 1000°K, 3000°K and 10,000°K and took n(c) to be between rather less than $10^{12}/cm^3$ and rather more than $10^{13}/cm^3$. His values of $\alpha$ for the lowest temperature are about 10 times smaller than the corresponding values given in table II; but the two sets of values for the highest temperature are in fair accord. It is significant that whereas collisional processes are dominant at 1000°K radiative recombination is of major importance at 10,000°K.

The conflict between the two theories is much more profound that even the factor of 10 which arises in the collisional region would suggest. Thus, though D'Angelo obtained smaller values of $\alpha$ he used larger values of K(p,c) and K(c,p) than those of the present investigation. The explanation of the apparent anomaly is that he neglected collision induced transitions between discrete levels. These transitions are, in fact, of great importance.

Alkali ions To obtain some indication of the recombination coefficient to other charged positive ions it was thought worth carrying out calculations similar to those discussed in the early part of this section, but with the ground level of the atom rendered inaccessible. This modified hydrogen atom provides a crude model of an alkali atom Ak with excitation potential 1.9 eV and ionization potential 3.4 eV

The derived values of the parameters $\alpha$ and S which appear in formula (23) for the decay coefficient, are given in tables IVA and B. Condition (13) imposes about the same restriction as before (table I) on n(c), but condition (14) restricts T much more severely. It should be noted that the S term of IVB cannot be neglected compared with the $\alpha$ term of IVA unless n(c) and T are quite low.

Study of the detailed computations on which tables II and IV are based shows that for the denser hotter plasmas the pseudo-alkali ion decay coefficient is greater than the corresponding hydrogen ion decay coefficient when

$$n(Ak\ (2s,2p)) < n(H\ (2s,\ 2p)) \qquad \ldots\ldots(39)$$

the n's being as usual the number densities indicated; but that for all plasmas it is smaller when

$$n(Ak\ (2s,2p)) > n(H\ (2s,2p)) \qquad \ldots\ldots(40)$$

The differences arise partly because the collisional ionization of Ak(2s,2p) and H(2s,2p) are important in some plasmas and partly because certain radiative processes which contribute to hydrogen ion recombination do not contribute to pseudo-alkali ion recombination. They are not large. This suggests that the value of the collisional-radiative decay coefficient is not very sensitive to the species of ion.

4.2 Multiple charged ions

The computational labour is reduced and the presentation of the results is made more compact by taking advantage of certain scaling laws. These laws may be established by choosing the basic independent variables to be, not the electron temperature T and number density $\eta(c)$, but reduced values of these quantities defined

$$\Theta = T/Z^2, \quad \eta(c) = n(c)/Z^7. \qquad \ldots\ldots(41)$$

Writing the Saha equilibrium formula and the expressions for the spontaneous transition probabilities and for the rate coefficients for radiative recombination and for collisional excitation and ionization in terms of $\Theta$ and $\eta(c)$ it is found from (18) that $Xn(p)/Z^{11}$ is a function of $\Theta$ and $\eta(c)$ only and hence that $\alpha/Z$ and $Z^3S$ are also functions of $\Theta$ and $\eta(c)$ only. Though the scaling laws are correct to within the approximation of the calculations, they are not exact because the effective Gaunt factors $\bar{g}$ concealed in formulae (36) and (37) are not quite independent of Z and $\Theta$ as assumed.

The computed values of $\alpha/Z$, $Z^3S$ and $Z^3(S+L)$ for an optically thin plasma are presented in terms of $\Theta$ and $\eta(c)$ in tables VA and B. As would be expected the entries are of the same order as the corresponding entries in tables IIA and B and vary in a similar manner. At the higher densities the value of $\alpha/Z$ in table VA exceeds that of $\alpha$ in table IIA and at all densities the value of $Z^3S$ in table VB exceeds that of S in table IIB. This is a manifestation of the fact that the reduced collisional rate coefficients $Z^3$ K(p,q or c) for hydrogenic ions are larger than the corresponding collisional rate coefficients K(p, q or c) for hydrogen atoms.

Table VII gives $X\,\eta_s(1)/Z^{11}$ and $X\,\eta_E(1)/Z^{11}$ the reduced number densities of normal hydrogenic ions, in the steady state and in Saha equilibrium.

Interest is attached to how $\alpha$ depends on Z with T and n(c) held constant. The variation of $\alpha/Z$ over a small area of the $[\Theta, \eta(c)]$ plane may be represented by

$$\alpha/Z = \propto \Theta^{-x}\,\eta(c)^y \qquad .....(42)$$

x and y being assigned some suitable (positive) values. Using (41) it may be seen that over this area

$$\alpha \propto T^{-x}\,n(c)^y\,Z^z \qquad .....(43)$$

with

$$z = 1 + 2x - 7y\ . \qquad .....(44)$$

Inspection of table VA reveals that in a very tenuous plasma z is slowly increasing function of $\Theta$ its value remaining close to 2.4; and it reveals also that in a very dense plasma z is a rapidly decreasing function of $\Theta$ - for example if $\eta(c)$ is $10^{18}$ $cm^{-3}$ z falls from about 4 when $\Theta$ is 250°K to about - 1.6 when $\Theta$ is 64,000°K.

The occurrence of negative values of the index z is at first rather surprising. It may readily be understood. If $\eta(c)$ is sufficiently high the Saha equation (9) describes the distribution among the excited levels, and if $\Theta$ is sufficiently high the exponentials in this equation may be replaced by unity. In these circumstances the number densities $\eta(p)$ are independent of Z. Now figure 2 shows that when $\eta(c)$ is high the ground level is populated mainly by collisional transitions from excited levels. Moreover, according to (36) combined with (10) the rate coefficient for de-excitation is proportional to $Z^{-2}$ when $\Theta$ is high. Hence the recombination coefficient is also proportional to $Z^{-2}$ in the case of a very dense, very hot plasma.

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TABLE I

| Temperature parameter $T/Z^2$ (with T in $^oK$) | Greatest value of $n(c)/Z^3$ X for which condition (13) is valid in the case of an optically thin plasma ($cm^{-3}$) |
|---|---|
| 250 | $10^{12}$ |
| 1000 | $10^{14}$ |
| 4000 | $10^{16}$ |
| 16000 | $10^{18}$ |
| 64000 | $10^{20}$ |

## TABLE II

### Optically thin hydrogen ion plasma

Quantities α and S appearing in formula (23) for the collisional-radiative decay coefficient

**Collisional-radiative recombination coefficient**

**IIA α in cm$^3$ sec$^{-1}$**

| n(c) (cm$^{-3}$) \ T(°K) | 250 | 500 | 1000 | 2000 | 4000 | 8000 | 16000 | 32000 | 64000 |
|---|---|---|---|---|---|---|---|---|---|
| Lt n(c) → 0 | $4.8^{-12}$ | $3.1^{-12}$ | $2.0^{-12}$ | $1.3^{-12}$ | $7.9^{-13}$ | $4.8^{-13}$ | $2.9^{-13}$ | $1.7^{-13}$ | $1.0^{-13}$ |
| $10^8$ | $8.8^{-11}$ | $1.4^{-11}$ | $4.1^{-12}$ | $1.8^{-12}$ | $9.2^{-13}$ | $5.1^{-13}$ | $3.0^{-13}$ | $1.8^{-13}$ | $1.0^{-13}$ |
| $10^9$ | $4.0^{-10}$ | $3.8^{-11}$ | $7.5^{-12}$ | $2.5^{-12}$ | $1.0^{-12}$ | $5.3^{-13}$ | $3.0^{-13}$ | $1.8^{-13}$ | $1.0^{-13}$ |
| $10^{10}$ | $2.8^{-9}$ | $1.6^{-10}$ | $1.9^{-11}$ | $4.1^{-12}$ | $1.4^{-12}$ | $6.1^{-13}$ | $3.2^{-13}$ | $1.8^{-13}$ | $1.0^{-13}$ |
| $10^{11}$ | $2.7^{-8}$ | $1.0^{-9}$ | $6.9^{-11}$ | $9.1^{-12}$ | $2.2^{-12}$ | $8.1^{-13}$ | $3.4^{-13}$ | $1.8^{-13}$ | $1.0^{-13}$ |
| $10^{12}$ | $2.6^{-7}$ | $9.0^{-9}$ | $3.9^{-10}$ | $2.9^{-11}$ | $4.4^{-12}$ | $1.2^{-12}$ | $4.3^{-13}$ | $2.1^{-13}$ | $1.0^{-13}$ |
| $10^{13}$ | $2.6^{-6}$ | $8.9^{-8}$ | $3.1^{-9}$ | $1.4^{-10}$ | $1.2^{-11}$ | $2.1^{-12}$ | $6.2^{-13}$ | $2.4^{-13}$ | $1.1^{-13}$ |
| $10^{14}$ | $2.6^{-5}$ | $8.8^{-7}$ | $2.9^{-8}$ | $9.8^{-10}$ | $5.1^{-11}$ | $5.1^{-12}$ | $1.0^{-12}$ | $3.1^{-13}$ | $1.2^{-13}$ |
| $10^{15}$ | | $8.8^{-6}$ | $2.9^{-7}$ | $8.7^{-9}$ | $2.7^{-10}$ | $1.7^{-11}$ | $2.3^{-12}$ | $4.9^{-13}$ | $1.6^{-13}$ |
| $10^{16}$ | | | $2.9^{-6}$ | $8.5^{-8}$ | $2.3^{-9}$ | $8.4^{-11}$ | $5.0^{-12}$ | $7.3^{-13}$ | $1.9^{-13}$ |
| $10^{17}$ | | | | $8.4^{-7}$ | $2.1^{-8}$ | $3.4^{-10}$ | $1.4^{-11}$ | $1.8^{-12}$ | $4.4^{-13}$ |
| $10^{18}$ | | | | | $2.0^{-7}$ | $2.5^{-9}$ | $9.6^{-11}$ | $1.2^{-11}$ | $2.8^{-12}$ |
| Lt n(c) → ∞ | $2.6^{-19}$n(c) | $8.8^{-21}$n(c) | $2.9^{-22}$n(c) | $8.4^{-24}$n(c) | $1.9^{-25}$n(c) | $2.4^{-27}$n(c) | $9.1^{-29}$n(c) | $1.1^{-29}$n(c) | $2.7^{-30}$n(c) |

**Collisional-radiative ionization coefficient**

**IIB S in cm$^3$ sec$^{-1}$**

| n(c) (cm$^{-3}$) \ T(°K) | 250 | 500 | 1000 | 2000 | 4000 | 8000 | 16000 | 32000 | 64000 |
|---|---|---|---|---|---|---|---|---|---|
| Lt n(c) → 0 | | | | | $1.4^{-26}$ | $9.7^{-18}$ | $3.4^{-13}$ | $8.2^{-11}$ | $1.5^{-9}$ |
| $10^8$ | | | | | $1.6^{-26}$ | $1.1^{-17}$ | $3.6^{-13}$ | $8.4^{-11}$ | $1.6^{-9}$ |
| $10^9$ | | | | | $1.8^{-26}$ | $1.2^{-17}$ | $3.8^{-13}$ | $8.8^{\ 11}$ | $1.6^{-9}$ |
| $10^{10}$ | | | | | $2.7^{-26}$ | $1.4^{-17}$ | $4.2^{-13}$ | $9.2^{-11}$ | $1.6^{-9}$ |
| $10^{11}$ | | | | | $4.5^{-26}$ | $1.9^{-17}$ | $4.9^{-13}$ | $1.0^{-10}$ | $1.7^{-9}$ |
| $10^{12}$ | | | | | $1.0^{-25}$ | $3.0^{-17}$ | $6.5^{-13}$ | $1.2^{-10}$ | $1.9^{-9}$ |
| $10^{13}$ | | | | | $3.6^{-25}$ | $6.8^{-17}$ | $1.1^{-12}$ | $1.7^{-10}$ | $2.4^{-9}$ |
| $10^{14}$ | | | | | $2.2^{-24}$ | $2.3^{-16}$ | $2.5^{-12}$ | $2.9^{-10}$ | $3.6^{-9}$ |
| $10^{15}$ | | | | | $1.8^{-23}$ | $1.3^{-15}$ | $8.6^{-12}$ | $6.9^{-10}$ | $6.6^{-9}$ |
| $10^{16}$ | | | | | $1.5^{-22}$ | $5.9^{-15}$ | $1.9^{-11}$ | $1.0^{-9}$ | $8.5^{-9}$ |
| $10^{17}$ | | | | | | $1.0^{-14}$ | $2.3^{-11}$ | $1.1^{-9}$ | $8.8^{-9}$ |
| $10^{18}$ | | | | | | $1.1^{-14}$ | $2.3^{-11}$ | $1.1^{-9}$ | $8.8^{-9}$ |
| Lt n(c) → ∞ | | | | | $8.3^{-22}$ | $1.1^{-14}$ | $2.3^{-11}$ | $1.1^{-9}$ | $8.8^{-9}$ |
| S + L | | | | | $4.9^{-21}$ | $1.3^{-14}$ | $2.4^{-11}$ | $1.1^{-9}$ | $8.8^{-9}$ |

The indices give the power of 10 by which the entries in the α and S columns and the S + L row must be multiplied

## TABLE III

### Optically thin hydrogen ion plasma

Number density of normal hydrogen atoms in steady state X $n_s(1)$ and in Saha eqilibrium X $n_s(1)$ in $cm^{-3}$

| $n(c)$ ($cm^{-3}$) \ $T(^\circ K)$ | 4000 | 8000 | 16000 | 32000 | 64000 |
|---|---|---|---|---|---|
| | | number densities $n_s(1)$ | | | |
| Lt $n(c) \longrightarrow 0$ | $5.7^{13}\ n(c)$ | $5.0^{4}\ n(c)$ | $8.6^{-1} n(c)$ | $2.1^{-3} n(c)$ | $6.5^{-5} n(c)$ |
| $10^8$ | $5.6^{21}$ | $4.8^{12}$ | $8.4^{7}$ | $2.1^{5}$ | $6.4^{3}$ |
| $10^9$ | $5.5^{22}$ | $4.6^{13}$ | $8.1^{8}$ | $2.0^{6}$ | $6.3^{4}$ |
| $10^{10}$ | $5.3^{23}$ | $4.5^{14}$ | $7.6^{9}$ | $2.0^{7}$ | $6.2^{5}$ |
| $10^{11}$ | $4.9^{24}$ | $4.3^{15}$ | $7.1^{10}$ | $1.9^{8}$ | $6.0^{6}$ |
| $10^{12}$ | $4.3^{25}$ | $3.9^{16}$ | $6.5^{11}$ | $1.8^{9}$ | $5.6^{7}$ |
| $10^{13}$ | $3.4^{26}$ | $3.2^{17}$ | $5.7^{12}$ | $1.5^{10}$ | $4.8^{8}$ |
| $10^{14}$ | $2.3^{27}$ | $2.2^{18}$ | $4.1^{13}$ | $1.1^{11}$ | $3.6^{9}$ |
| $10^{15}$ | $1.5^{28}$ | $1.4^{19}$ | $2.6^{14}$ | $7.0^{11}$ | $2.4^{10}$ |
| $10^{16}$ | $1.5^{29}$ | $1.4^{20}$ | $2.6^{15}$ | $6.9^{12}$ | $2.3^{11}$ |
| $10^{17}$ | | $3.4^{21}$ | $6.1^{16}$ | $1.6^{14}$ | $5.0^{12}$ |
| $10^{18}$ | | $2.3^{23}$ | $4.2^{18}$ | $1.1^{16}$ | $3.2^{14}$ |
| $n_s(1)$ in Lt $n(c) \rightarrow \infty$ ) $n_E(1)$ for all $n(c)$ ) | $2.3^{-4} n(c)^2$ | $2.2^{-13} n(c)^2$ | $4.0^{-18} n(c)^2$ | $1.0^{-20} n(c)^2$ | $3.0^{-22} n(c)^2$ |

The indices give the power of 10 by which entries in the columns for the number densities of normal atoms must be multiplied.

TABLE IV

Optically thin pseudo-alkali ion plasma

Quantities α and S appearing in formula (23) for the collisional-radiative decay coefficient

**Collisional-radiative recombination coefficient**

IVA α in $cm^3\ sec^{-1}$

| n(c) ($cm^{-3}$) \ T(°K) | 250 | 500 | 1000 | 2000 | 4000 | 8000 |
|---|---|---|---|---|---|---|
| Lt n(c) → 0 | $3.8^{-12}$ | $2.4^{-12}$ | $1.5^{-12}$ | $9.0^{-13}$ | $5.4^{-13}$ | $3.1^{-13}$ |
| $10^8$ | $7.8^{-11}$ | $1.2^{-11}$ | $3.2^{-12}$ | $1.3^{-12}$ | $6.4^{-13}$ | $3.3^{-13}$ |
| $10^9$ | $3.8^{-10}$ | $3.3^{-11}$ | $6.0^{-12}$ | $1.8^{-12}$ | $7.5^{-13}$ | $3.6^{-13}$ |
| $10^{10}$ | $2.8^{-9}$ | $1.5^{-10}$ | $1.6^{-11}$ | $3.1^{-12}$ | $9.8^{-13}$ | $4.1^{-13}$ |
| $10^{11}$ | $2.7^{-8}$ | $1.0^{-9}$ | $6.1^{-11}$ | $7.1^{-12}$ | $1.6^{-12}$ | $5.1^{-13}$ |
| $10^{12}$ | $2.6^{-7}$ | $9.0^{-9}$ | $3.6^{-10}$ | $2.4^{-11}$ | $3.2^{-12}$ | $7.4^{-13}$ |
| $10^{13}$ | $2.6^{-6}$ | $8.8^{-8}$ | $3.0^{-9}$ | $1.2^{-10}$ | $9.3^{-12}$ | $1.3^{-12}$ |
| $10^{14}$ | $2.6^{-5}$ | $8.8^{-7}$ | $2.9^{-8}$ | $9.4^{-10}$ | $4.0^{-11}$ | $3.2^{-12}$ |
| $10^{15}$ | | $8.8^{-6}$ | $2.9^{-7}$ | $8.5^{-9}$ | $2.5^{-10}$ | $1.6^{-11}$ |
| $10^{16}$ | | | $2.9^{-6}$ | $8.4^{-8}$ | $2.3^{-9}$ | $1.4^{-10}$ |
| $10^{17}$ | | | | $8.4^{-7}$ | $2.3^{-8}$ | $1.4^{-9}$ |
| $10^{18}$ | | | | | $2.3^{-7}$ | $1.4^{-8}$ |
| Lt n(c) → ∞ | $2.6^{-19}n(c)$ | $8.8^{-21}n(c)$ | $2.9^{-22}n(c)$ | $8.4^{-24}n(c)$ | $2.3^{-25}n(c)$ | $1.4^{-26}n(c)$ |

**Collisional-radiative ionization coefficient**

IVB S in $cm^3\ sec^{-1}$

| n(c) ($cm^{-3}$) \ T(°K) | 250 | 500 | 1000 | 2000 | 4000 | 8000 |
|---|---|---|---|---|---|---|
| Lt n(c) | | | $1.1^{-25}$ | $7.7^{-17}$ | $2.7^{-12}$ | $6.6^{-10}$ |
| $10^8$ | | | $2.6^{-25}$ | $1.3^{-16}$ | $3.6^{-12}$ | $7.8^{-10}$ |
| $10^9$ | | | $5.2^{-25}$ | $1.9^{-16}$ | $4.4^{-12}$ | $9.0^{-10}$ |
| $10^{10}$ | | | $1.6^{-24}$ | $3.6^{-16}$ | $6.6^{-12}$ | $1.1^{-9}$ |
| $10^{11}$ | | | $8.4^{-24}$ | $1.1^{-15}$ | $1.3^{-11}$ | $1.8^{-9}$ |
| $10^{12}$ | | | $8.8^{-23}$ | $6.3^{-15}$ | $4.6^{-11}$ | $4.2^{-9}$ |
| $10^{13}$ | | | $1.5^{-21}$ | $6.9^{-14}$ | $2.8^{-10}$ | $1.6^{-8}$ |
| $10^{14}$ | | | $1.4^{-20}$ | $4.9^{-13}$ | $1.1^{-9}$ | $3.6^{-8}$ |
| $10^{15}$ | | | $3.4^{-20}$ | $1.0^{-12}$ | $1.7^{-9}$ | $4.3^{-8}$ |
| $10^{16}$ | | | | $1.2^{-12}$ | $1.8^{-9}$ | $4.3^{-8}$ |
| Lt n(c) → ∞ | | | $3.9^{-20}$ | $1.2^{-12}$ | $1.8^{-9}$ | $4.3^{-8}$ |
| S + L | | | $1.8^{-16}$ | $1.1^{-11}$ | $2.8^{-9}$ | $5.0^{-8}$ |

The indices give the power of 10 by which the entries in the α and S columns and the S + L row must be multiplied.

## TABLE V

### Optically thin pseudo-alkali ion plasma

Number density of normal alkali atoms in steady state X $n_s(1)$ and in Saha eqilibrium X $n_E(1)$ in $cm^{-3}$

| $n(c)$ ($cm^{-3}$) \ $T(^{\circ}K)$ | 1000 | 2000 | 4000 | 8000 |
|---|---|---|---|---|
| | | number densities $n_s(1)$ | | |
| Lt $n(c) \to 0$ | $1.4^{13}\,n(c)$ | $1.2^{4}\,n(c)$ | $2.0^{-1}\,n(c)$ | $4.7^{-4}\,n(c)$ |
| $10^{8}$ | $1.2^{21}$ | $1.0^{12}$ | $1.8^{7}$ | $4.3^{4}$ |
| $10^{9}$ | $1.1^{22}$ | $9.9^{12}$ | $1.7^{8}$ | $4.0^{5}$ |
| $10^{10}$ | $9.7^{22}$ | $8.6^{13}$ | $1.5^{9}$ | $3.6^{6}$ |
| $10^{11}$ | $7.2^{23}$ | $6.6^{14}$ | $1.2^{10}$ | $2.9^{7}$ |
| $10^{12}$ | $4.1^{24}$ | $3.9^{15}$ | $6.9^{10}$ | $1.8^{8}$ |
| $10^{13}$ | $2.0^{25}$ | $1.8^{16}$ | $3.3^{11}$ | $8.5^{8}$ |
| $10^{14}$ | $2.0^{26}$ | $1.9^{17}$ | $3.5^{12}$ | $8.9^{9}$ |
| $10^{15}$ | $8.5^{27}$ | $8.1^{18}$ | $1.5^{14}$ | $3.8^{11}$ |
| $10^{16}$ | | $7.1^{20}$ | $1.3^{16}$ | $3.3^{13}$ |
| $n_s(1)$ in Lt $n(c) \to \infty$; $n_E(1)$ for all $n(c)$ | $7.3^{-3}n(c)^2$ | $7.0^{-12}n(c)^2$ | $1.3^{-16}n(c)^2$ | $3.2^{-19}n(c)^2$ |

The indices give the power of 10 by which the entries in the columns for the number densities of normal atoms must be multiplied

**TABLE VI**

**Optically thin hydrogenic ion plasma**

**Reduced collisional-radiative recombination coefficient** $\alpha/Z$ in $cms^3\ sec^{-1}$

| $\eta(c)$ ($cm^{-3}$) \ $\theta$ ($^oK$) | 250 | 500 | 1000 | 2000 | 4000 | 8000 | 16000 | 32000 | 64000 |
|---|---|---|---|---|---|---|---|---|---|
| Lt $\eta(c) \rightarrow 0$ | $4.8^{-12}$ | $3.1^{-12}$ | $2.0^{-12}$ | $1.3^{-12}$ | $7.9^{-13}$ | $4.8^{-13}$ | $2.9^{-13}$ | $1.7^{-13}$ | $1.0^{-13}$ |
| $10^8$ | $1.5^{-10}$ | $1.7^{-11}$ | $4.3^{-12}$ | $1.8^{-12}$ | $9.2^{-13}$ | $5.1^{-13}$ | $3.0^{-13}$ | $1.8^{-13}$ | $1.0^{-13}$ |
| $10^9$ | $9.4^{-10}$ | $5.6^{-11}$ | $8.1^{-12}$ | $2.5^{-12}$ | $1.0^{-12}$ | $5.3^{-13}$ | $3.0^{-13}$ | $1.8^{-13}$ | $1.0^{-13}$ |
| $10^{10}$ | $8.4^{-9}$ | $3.1^{-10}$ | $2.4^{-11}$ | $4.3^{-12}$ | $1.4^{-12}$ | $6.1^{-13}$ | $3.2^{-13}$ | $1.8^{-13}$ | $1.0^{-13}$ |
| $10^{11}$ | $8.9^{-8}$ | $2.7^{-9}$ | $1.2^{-10}$ | $1.1^{-11}$ | $2.2^{-12}$ | $7.8^{-13}$ | $3.4^{-13}$ | $1.8^{-13}$ | $1.0^{-13}$ |
| $10^{12}$ | $8.2^{-7}$ | $2.6^{-8}$ | $9.1^{-10}$ | $4.8^{-11}$ | $5.2^{-12}$ | $1.2^{-12}$ | $4.3^{-13}$ | $2.0^{-13}$ | $1.0^{-13}$ |
| $10^{13}$ | $8.2^{-6}$ | $2.6^{-7}$ | $8.6^{-9}$ | $3.3^{-10}$ | $2.0^{-11}$ | $2.5^{-12}$ | $6.3^{-13}$ | $2.4^{-13}$ | $1.1^{-13}$ |
| $10^{14}$ | $8.2^{-5}$ | $2.6^{-6}$ | $8.5^{-8}$ | $2.9^{-9}$ | $1.1^{-10}$ | $7.6^{-12}$ | $1.2^{-12}$ | $3.3^{-13}$ | $1.2^{-13}$ |
| $10^{15}$ | | $2.6^{-5}$ | $8.5^{-7}$ | $2.8^{-8}$ | $8.6^{-10}$ | $3.7^{-11}$ | $3.0^{-12}$ | $5.0^{-13}$ | $1.6^{-13}$ |
| $10^{16}$ | | | $8.5^{-6}$ | $2.7^{-7}$ | $8.0^{-9}$ | $1.7^{-10}$ | $7.9^{-12}$ | $9.2^{-13}$ | $2.1^{-13}$ |
| $10^{17}$ | | | | $2.7^{-6}$ | $7.6^{-8}$ | $1.1^{-9}$ | $3.5^{-11}$ | $3.5^{-12}$ | $6.9^{-13}$ |
| $10^{18}$ | | | | | $7.6^{-7}$ | $1.0^{-8}$ | $3.1^{-10}$ | $2.9^{-11}$ | $5.6^{-12}$ |
| Lt $\eta(c) \rightarrow \infty$ | $8.2^{-19}\eta(c)$ | $2.6^{-20}\eta(c)$ | $8.5^{-22}\eta(c)$ | $2.7^{-23}\eta(c)$ | $7.6^{-25}\eta(c)$ | $1.0^{-26}\eta(c)$ | $3.1^{-28}\eta(c)$ | $2.9^{-29}\eta(c)$ | $5.6^{-30}\eta(c)$ |

**Reduced collisional-radiative ionization coefficient** $Z^3$ S in $cms^3\ sec^{-1}$

| $\eta(c)$ ($cm^{-3}$) \ $\theta$ ($^oK$) | 250 | 500 | 1000 | 2000 | 4000 | 8000 | 16000 | 32000 | 64000 |
|---|---|---|---|---|---|---|---|---|---|
| Lt $\eta(c) \rightarrow 0$ | | | | | $9.1^{-26}$ | $4.9^{-17}$ | $1.3^{-12}$ | $2.4^{-10}$ | $3.4^{-9}$ |
| $10^8$ | | | | | $1.1^{-25}$ | $5.3^{-17}$ | $1.4^{-12}$ | $2.4^{-10}$ | $3.4^{-9}$ |
| $10^9$ | | | | | $1.3^{-25}$ | $5.8^{-17}$ | $1.4^{-12}$ | $2.4^{-10}$ | $3.4^{-9}$ |
| $10^{10}$ | | | | | $1.9^{-25}$ | $6.8^{-17}$ | $1.5^{-12}$ | $2.5^{-10}$ | $3.5^{-9}$ |
| $10^{11}$ | | | | | $4.0^{-25}$ | $9.2^{-17}$ | $1.8^{-12}$ | $2.7^{-10}$ | $3.7^{-9}$ |
| $10^{12}$ | | | | | $1.0^{-24}$ | $1.7^{-16}$ | $2.4^{-12}$ | $3.2^{-10}$ | $4.0^{-9}$ |
| $10^{13}$ | | | | | $6.1^{-24}$ | $4.1^{-16}$ | $4.2^{-12}$ | $4.4^{-10}$ | $4.8^{-9}$ |
| $10^{14}$ | | | | | $4.6^{-23}$ | $2.2^{-15}$ | $1.2^{-11}$ | $8.5^{-10}$ | $7.3^{-9}$ |
| $10^{15}$ | | | | | $3.7^{-22}$ | $1.6^{-14}$ | $4.3^{-11}$ | $2.0^{-9}$ | $1.4^{-8}$ |
| $10^{16}$ | | | | | $2.0^{-21}$ | $3.8^{-14}$ | $7.2^{-11}$ | $2.8^{-9}$ | $1.8^{-8}$ |
| $10^{17}$ | | | | | $2.8^{-21}$ | $4.4^{-14}$ | $7.7^{-11}$ | $2.9^{-9}$ | $1.8^{-8}$ |
| $10^{18}$ | | | | | $3.5^{-21}$ | $4.5^{-14}$ | $7.8^{-11}$ | $2.9^{-9}$ | $1.8^{-8}$ |
| Lt $\eta(c) \rightarrow \infty$ | | | | | $3.8^{-21}$ | $4.5^{-14}$ | $7.8^{-11}$ | $2.9^{-9}$ | $1.8^{-8}$ |
| $Z^3(S+L)$ | | | | | $3.2^{-20}$ | $6.3^{-14}$ | $8.3^{-16}$ | $3.0^{-9}$ | $1.8^{-8}$ |

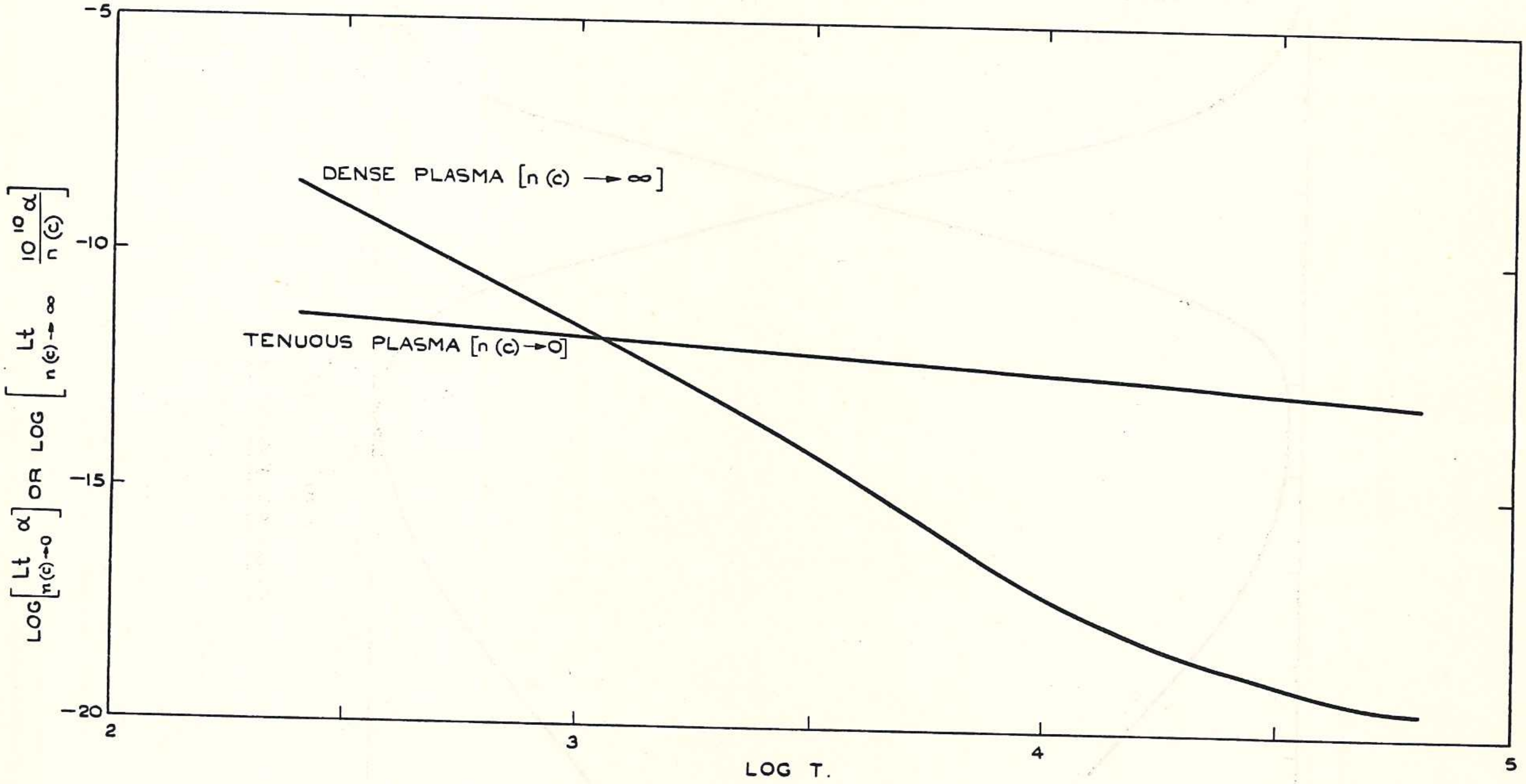
The indices give the power of 10 by which the entries in the $\alpha/Z$ and $Z^3S$ columns and the $Z^3(S+L)$ row must be multiplied

## TABLE VII

Optically thin hydrogenic ion plasma

Reduced number density of normal hydrogenic ions in steady state $\frac{X\, n_s(1)}{Z^{11}}$ and in Saha equilibrium $\frac{X\, n_s(1)}{Z^{11}}$ in $cm^{-3}$

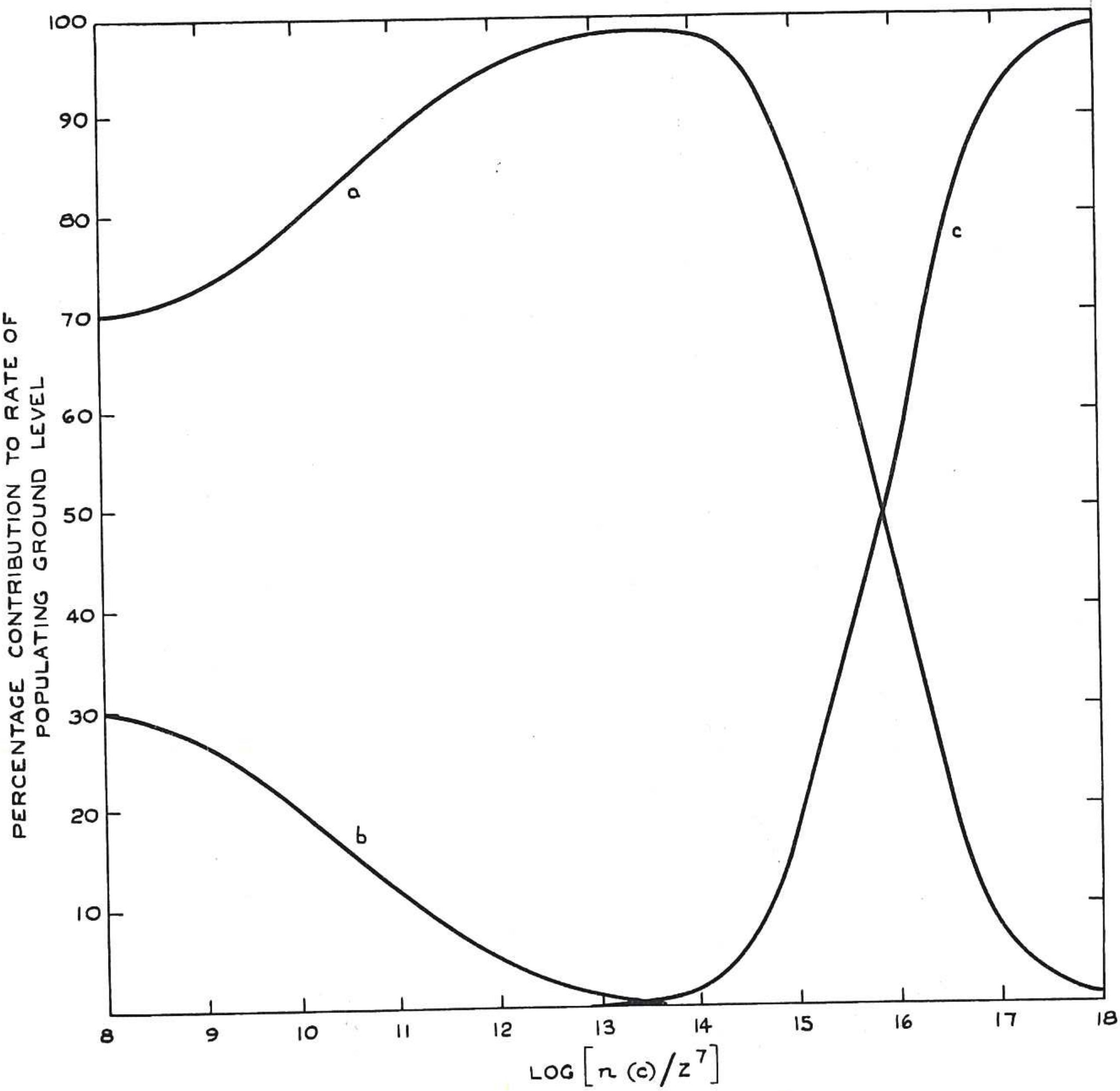
| $\eta(c)$ ($cm^{-3}$) \ $\Theta(^oK)$ | 4000 | 8000 | 16000 | 32000 | 64000 |
|---|---|---|---|---|---|
| Lt $\eta(c) \to 0$ | $8.7^{12}\,\eta(c)$ | $9.8^{3}\,\eta(c)$ | $2.2^{-1}\,\eta(c)$ | $7.3^{-4}\,\eta(c)$ | $2.9^{-5}\,\eta(c)$ |
| $10^8$ | $8.4^{20}$ | $9.6^{11}$ | $2.1^{7}$ | $7.3^{4}$ | $2.9^{3}$ |
| $10^9$ | $7.7^{21}$ | $9.1^{12}$ | $2.1^{8}$ | $7.3^{5}$ | $2.9^{4}$ |
| $10^{10}$ | $7.4^{22}$ | $9.0^{18}$ | $2.1^{9}$ | $7.2^{6}$ | $2.8^{5}$ |
| $10^{11}$ | $5.5^{24}$ | $8.5^{14}$ | $1.9^{10}$ | $6.7^{7}$ | $2.7^{6}$ |
| $10^{12}$ | $5.2^{24}$ | $7.1^{15}$ | $1.8^{11}$ | $6.3^{8}$ | $2.5^{7}$ |
| $10^{13}$ | $3.3^{25}$ | $6.1^{16}$ | $1.5^{12}$ | $5.5^{9}$ | $2.3^{8}$ |
| $10^{14}$ | $2.4^{26}$ | $3.5^{17}$ | $1.0^{13}$ | $3.9^{10}$ | $1.6^{9}$ |
| $10^{15}$ | $2.3^{27}$ | $2.3^{18}$ | $7.0^{13}$ | $2.5^{11}$ | $1.1^{10}$ |
| $10^{16}$ | $4.0^{28}$ | $4.5^{19}$ | $1.1^{15}$ | $3.3^{12}$ | $1.2^{11}$ |
| $10^{17}$ | $2.7^{30}$ | $2.5^{20}$ | $4.5^{16}$ | $1.2^{14}$ | $3.8^{12}$ |
| $10^{18}$ | $2.2^{32}$ | $2.2^{23}$ | $4.0^{18}$ | $1.0^{16}$ | $3.0^{14}$ |
| $X\,\eta_s(1)/Z^{11}$ in Lt $\eta(c) \to \infty$ } $X\,\eta_E(1)/Z^{11}$ for all $\eta(c)$ } | $2.3^{-4}\eta(c)^2$ | $2.2^{-13}\eta(c)^2$ | $4.0^{-18}\eta(c)^2$ | $1.0^{-20}\eta(c)^2$ | $3.0^{-22}\eta(c)^2$ |




**CLM – P3 FIGURE I.**

Variation with log T of $\log\left[\underset{n(c)\to 0}{\mathrm{Lt}}\ \alpha\right]$ and of $\log\left[\underset{n(c)\to\infty}{\mathrm{Lt}}\ \frac{10^{10}\alpha}{n(c)}\right]$

where T is the temperature ($^{0}$K), n(c) is the number density of free electrons ($cm^{-3}$) and $\alpha$ is the collisional-radiative recombination coefficient ($cm^3\ sec^{-1}$) in an optically thin hydrogen ion plasma.




CLM–P3 FIGURE 2.

Relative importance of the processes populating the ground level when the reduced temperature $T/Z^2$ is $4000^0$K plotted against the logarithm of the reduced number density $n(c)/Z^7$ in $cm^3\ sec^{-1}$.
Curve a, radiative transitions from excited levels;
curve b, radiative recombination from the continuum;
curve c, collisional transitions from excited levels;
curve d, (not shown) three body recombination from the continuum (everywhere less than 0.5 percent).