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STRUCTURE OF ALMOST COLLISIONLESS SHOCKS IN A MAGNETO-PLASMA AND THE ION-ACOUSTIC INSTABILITY

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STRUCTURE OF ALMOST COLLISIONLESS SHOCKS IN A MAGNETO-PLASMA AND THE ION-ACOUSTIC INSTABILITY

by

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ABSTRACT

A phenomenological treatment of the anomalous resistivity resulting from the ion-acoustic instability is described. The effective "collision" frequency occurring in the resisivity is taken to be the maximum growth rate predicted by linear theory for the ion-acoustic instability. This resistivity is used to calculate shock structures for shocks propagating perpendicular to magnetic fields, and it is found that the shock thickness and downstream temperature ratio T_i/T_e agree well with some experimental results. It is concluded that the ion-acoustic instability does play an important role in determining collisionless shock wave structure.

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1. Introduction

Of the several instabilities proposed as the source of the turbulence, and hence of the anomalous resistivity required to account for the thickness of shock waves observed in collisionless plasmas, the ion-acoustic instability (Sagdeev 1967) is generally favoured. This streaming instability has maximum growth rates given by $\gamma = \alpha\omega_{pi}$ (Stringer 1964) where in the absence of a magnetic field α is a function of $r = T_i/T_e$ and $f = V_D/V_\theta$, the ratio of the drift and thermal velocities of the electrons. The marginal stability curve, $\alpha = 0$, passes through $f \approx 1$ at $r = 1$ and $f \approx 0.05$ at $r = 0.1$ and below this curve in the (r, f) - plane $\alpha = 0$. As f has rather small values at the leading edge of shock waves, some selective preheating of the electrons by another mechanism is required if this instability is to play a dominant role*. Classical resistivity, while small, is still sufficient to produce this preheating, albeit over lengths considerable greater than the shock "thickness" λ_s, which we take to be the sub-tangent of the steepest magnetic field gradient in the shock profile.

One common feature of the various calculations of anomalous resistivity from quasi-linear theory is that the effective collision frequency ν^* is found to be proportional to the maximum growth rate, i.e. for the ion-acoustic instability

$$\nu^* = K\,\alpha\,\omega_{pi}\,, \qquad (1)$$

where K is to be determined. In general at each point in the shock structure both K and α will depend on the strength of the magnetic field $|B|$, as well as f and r, but there are some indications

*But such preheating is not always essential, for it is easily shown that at the steepest gradient of a solitary wave $f \sim \beta_e^{-\frac{1}{2}}$ where $\beta_e = 2\mu nK\,T_e/B^2$.

that dependence on $|B|$ is not very important ($\nu^* \approx \omega_{ce}$, see (11)), so we shall ignore this.

For weak turbulence the product $K\alpha$ can be determined by quasi-linear theory (Sagdeev 1967, Tsytovich 1970), but there is disagreement on the values found and more important these calculations do not yield the $\alpha = 0$ curve in the (r, f) - plane, which means that the upstream and downstream regions of low gradients (and small f) in which classical resistivity is operative, are not accounted for. In view of these difficulties, for practical shock wave calculations, it is worth while to try a phenomonological approach, using Stringer's curves to determine $\alpha(r, f)$ and assigning K as a parameter. The shock thickness λ_s and the downstream temperature ratio r_2 can then be determined as functions of K, and the value of K then fixed from experimental values of λ_s and r_2. If very small or very large values of K are required to give agreement with experiment, the theory would be suspect, for heuristic arguments suggest values of about unity for K. Of course, this approach is no substitute for a basic theory but should $K \approx 1$ fit the experimental evidence, it would at least confirm the status of the ion-acoustic instability in collisionless shock wave theory.

2. The Structure Equations for Perpendicular Shock Waves

Because the Debye length is very much smaller than the shock thickness λ_s, we can neglect charge separation. The shortest scale length arising in the two-fluid equations adopted here is the collision-free skin depth $\delta_e = c/\omega_{pe} = (m_e/\mu n e^2)^{\frac{1}{2}}$, where n is the electron number density, and this will be used to non-dimensionalize the distance x perpendicular to the shock. The subscript "1" is used to denote values upstream of the shock at

$x = -\infty$, and the fluid velocity u is along the positive direction of the Ox-axis. The most important parameter is the Alfvén-Mach number, $M_A = u_1 (\mu\rho_1)^{\frac{1}{2}}/B_1$.

It is convenient to use the following non-dimensional dependent variables

$$b = B/B_1 \ , \quad \omega = u/u_1 \ , \quad \theta = \mu n_1 k\, T_e/B_1^2 \ , \quad \phi = \mu n_1 k\, T_i/B_1^2 \ ,$$

and the independent variable $X = x/\delta_{e1}$; the derivative d/dX will be denoted by a dot. Notice that $2(\theta_1 + \phi_1)$ is the plasma "beta" at $x = -\infty$. By continuity $\omega = u/u_1 = n_1/n$. The four equations expressing momentum and energy balance for the two fluids may be expressed (Woods 1969)

$$a\,\mathring{b} = b\omega - 1 \ , \tag{2}$$

$$c\,\omega\,\mathring{\omega} = M_A^2\,\omega^2 - \omega\,(M_A^2 + \theta_1 + \phi_1 + \tfrac{1}{2} - \tfrac{1}{2} b^2) + \theta + \phi \ , \tag{3}$$

$$0 = -\tfrac{1}{2} M_A^2\,(\omega - 1)^2 + \tfrac{3}{2}\,(\theta + \phi - \theta_1 - \phi_1) + (\omega - 1)(\theta_1 + \phi_1 + \tfrac{1}{2}) + b - \tfrac{1}{2} - \tfrac{1}{2}\,\omega\, b^2 \ , \tag{4}$$

$$\tfrac{3}{2}\,\omega\,\mathring{\phi} + \phi\mathring{\omega} = c_i\,\omega\,(\mathring{\omega})^2 \ , \tag{5}$$

where a, c and c_i are transport coefficients to be discussed shortly. Several approximations have been introduced here. First the electron inertia term has been dropped from Ohm's law (2) which amounts to the assumption that the ratio $(\delta_e/\lambda_s)^2$ is very much less than unity (values of 0.01 are typical for laboratory shocks). Secondly the thermal conductivity is neglected on the expectation that anomalous resistivity is the dominant dissipative process, and finally thermoelectric effects are neglected.

Now $f = V_D/V_\theta$ where $V_D \approx -j/en = dB/dx/(en\mu)$ and $V_\theta = (k\,T_e/m_e)^{\frac{1}{2}}$ may be expressed in the non-dimensional form

$$f = \mathring{b}\,\omega\,\theta^{-\frac{1}{2}} \ , \tag{6}$$

so that

$$\alpha = \alpha(r, f) , \tag{7}$$

may be evaluated from Stringer's curves at each point in the shock structure.

It is assumed here that the non-dimensional resistivity a in (2) can be written

$$a = (\eta_c + \eta^*)/(\mu\, u_1\, \delta_{e1}) , \tag{8}$$

where η_c, η^* are the collisional and anomalous resistivities respectively. Now

$$\eta_c = \mu\, \delta_e^2\, \nu_c = \mu\, \delta_{e1}^2\, \nu_{c1}\, (\theta_1/\theta)^{\frac{3}{2}} ,$$

ν_c being the collision frequency and the usual $T_e^{-\frac{3}{2}}$ dependence for ν_c/n has been adopted. By (1)

$$\eta^* = \mu\, \delta_e^2\, K\, \alpha\, \omega_{pi} = \mu c \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \omega^{\frac{1}{2}}\, K\, \alpha\, \delta_{e1} ,$$

whence

$$a = a_c + a^*, \quad a_c = \left(\frac{\delta_e\, \nu_c}{u}\right)_1 \left(\frac{\theta_1}{\theta}\right)^{\frac{3}{2}}, \quad a^* = \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \frac{c}{u_1}\, \omega^{\frac{1}{2}}\, K\, \alpha . \tag{9}$$

When the anomalous resistivity is dominant, it follows from (2) and (8) that $\lambda_s \approx \eta^*/\mu u = \delta_e^2 \nu^*/u$. Now the instability will have time to develop within the shock front provided $1/\gamma$ is much less than the transit time λ_s/u for a fluid element. The assumption $\nu^* \approx \gamma$ thus meets this requirement provided $(\delta_e/\lambda_s)^2 \ll 1$, which inequality has already been adopted.

As δ_e is known to be the thickness of a laminar collision-free large amplitude wave, λ_s/δ_e must be somewhat larger than unity. In fact more experimental results show this ratio to be between about 6 and 20, and it follows that the number a will have a similar value. From equation (9) we see that a shock can be classified as being collisionless in regions where

$$\delta_e < u/\nu_c \,. \tag{10}$$

From $\delta_e \, \omega_{ce} = (m_i/m_e)^{\frac{1}{2}} \; u/M_A$ and the estimate $\lambda_s \approx \delta_e^2 \, \nu^*/u$, we find that

$$\frac{\omega_{ce}}{\nu^*} = \frac{\delta_e}{\lambda_s} \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} \frac{1}{M_A} \,, \tag{11}$$

which is of order unity for many laboratory shock waves.

As α is strongly dependent on the temperature ratio r, it is important to allow properly for the heating of the ions (or electrons). In equations (3) and (5) c_i and $(c - c_i)$ are non-dimensionalized ion and electron viscosities as given by kinetic theory (Braginskii 1965), and for the details of their complicated dependence on $(\omega_c\tau)_i$ and $(\omega_c\tau)_e$ the reader is referred to Woods (1969). In our model the ions are heated by compression and viscous dissipation, and the balance of the energy is given to the electrons. It appears from the calculations that about half the ion energy downstream of the shock is due to compression and about half is due to viscosity. On the other hand the electrons are heated mainly by the anomalous resistivity.

3 Results and Conclusions

At each point in the shock structure (6) was used to find f, (7) then gave the value of α to be used in (2) and (9). The results were obtained as functions of K. It is useful to plot the shock profile in the (r, f) - plane; figure 1 shows a typical curve for a transverse shock propagating at an Alfvén-Mach number of 2.5 into a 1 eV hydrogen plasma, with $n_1 = 6.4 \times 10^{14}/\mathrm{cm}^3$, and $u_1 = 2.5 \times 10^7$ cm/sec. For this calculation $K = 1.75$. The shock starts at point 1 where $T_i = T_e$, i.e. $r_1 = 1$, and terminates downstream at point 2 where $R_2 = 0.075$. In figure 2 the magnetic field B/B_1 and voltage profiles V are shown, the latter being obtained by integrating the normal component of the electron momentum equation. The shock length is $\lambda_s \approx 7.8\ \delta_{e1}$. Notice the (collisional) resistive heating of the electrons in the stable region, which occupies a distance of about $20\ \delta_{e1}$, giving the shock a long non-turbulent "foot". The experimental values (Paul 1967) for λ_s and r_2 are $7\ \delta_{e1}$ and 0.1. The quantities appearing in (10) are (in mm) $\delta_{e1} = 0.21$; $\delta_{e2} = 0.14$ and $(u/\nu_c)_1 = 0.02$, $(u/\nu_c)_2 = 1.3$, showing that the shock becomes collisionless somewhere through the structure. This point is illustrated in figure 3, which shows curves of a^* and a_c, the classical and anomalous non-dimensional resistivities.

As K is increased, the average value of α becomes smaller, for by (2) and (6) larger values of a yield smaller values of f; in the limit $K = \infty$ the profile follows the marginal stability curve $\alpha = 0$. The other extreme, $K = 0$, gives a purely collisional shock. We find that λ_s varies from about $3\ \delta_{e1}$ at $K = 0$ to about $20\ \delta_{e1}$ at $K = 10$ and that r_2 varies from 0.2 at $K = 0$ to 0.05 at $K = 10$. As values of K between 1 and 2 give fair agreement with the experimental results just mentioned, as well as with the shock profile, we conclude that the ion-acoustic instability can explain at least some of the observed collisionless shock structures.

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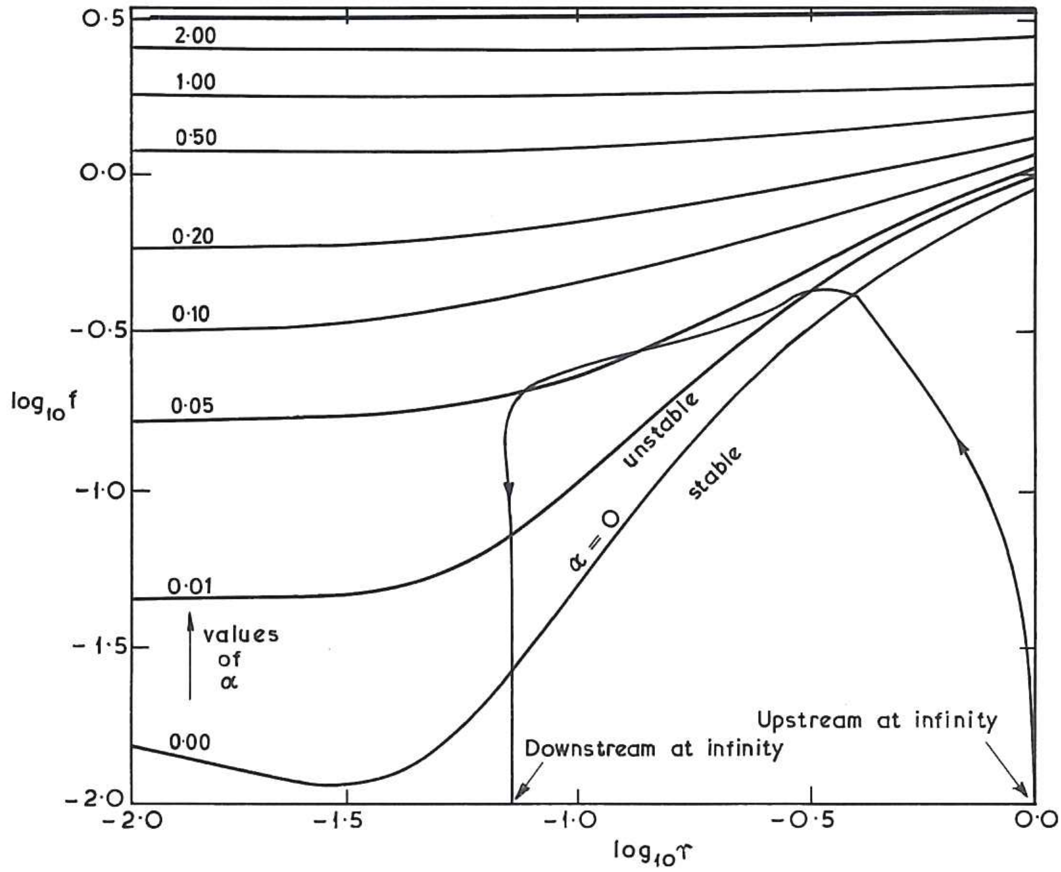

Fig.1 Shock profile

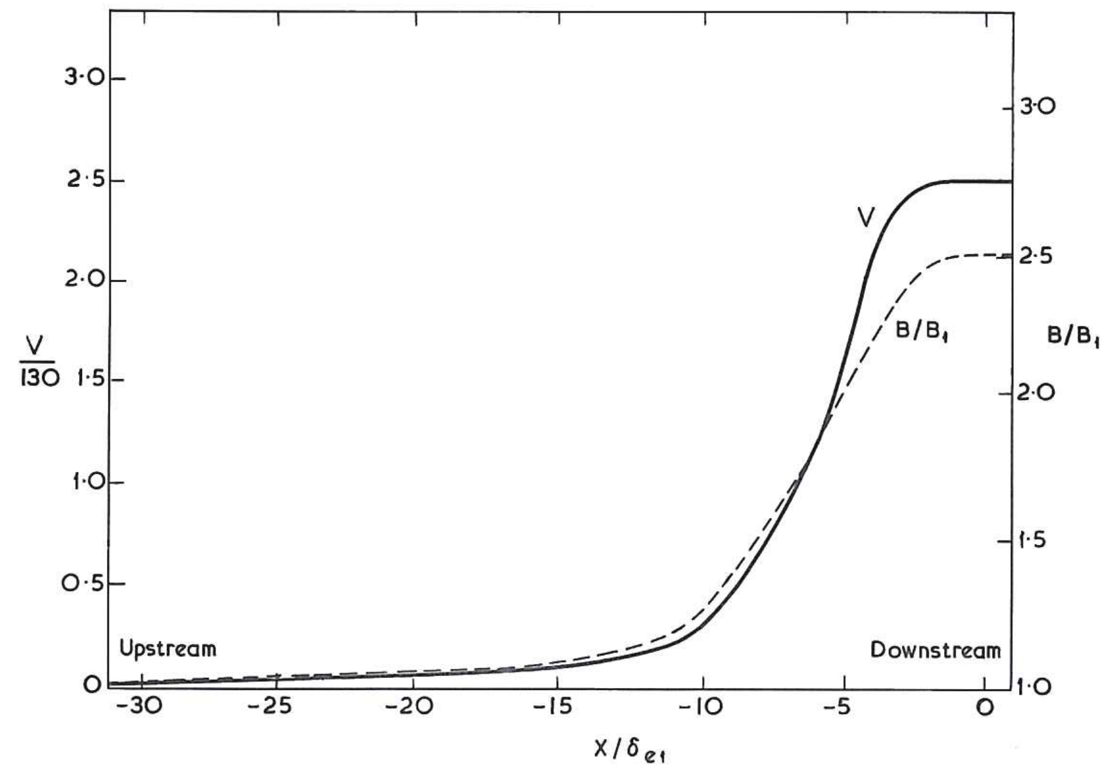

Fig.2 Shock profiles with distance

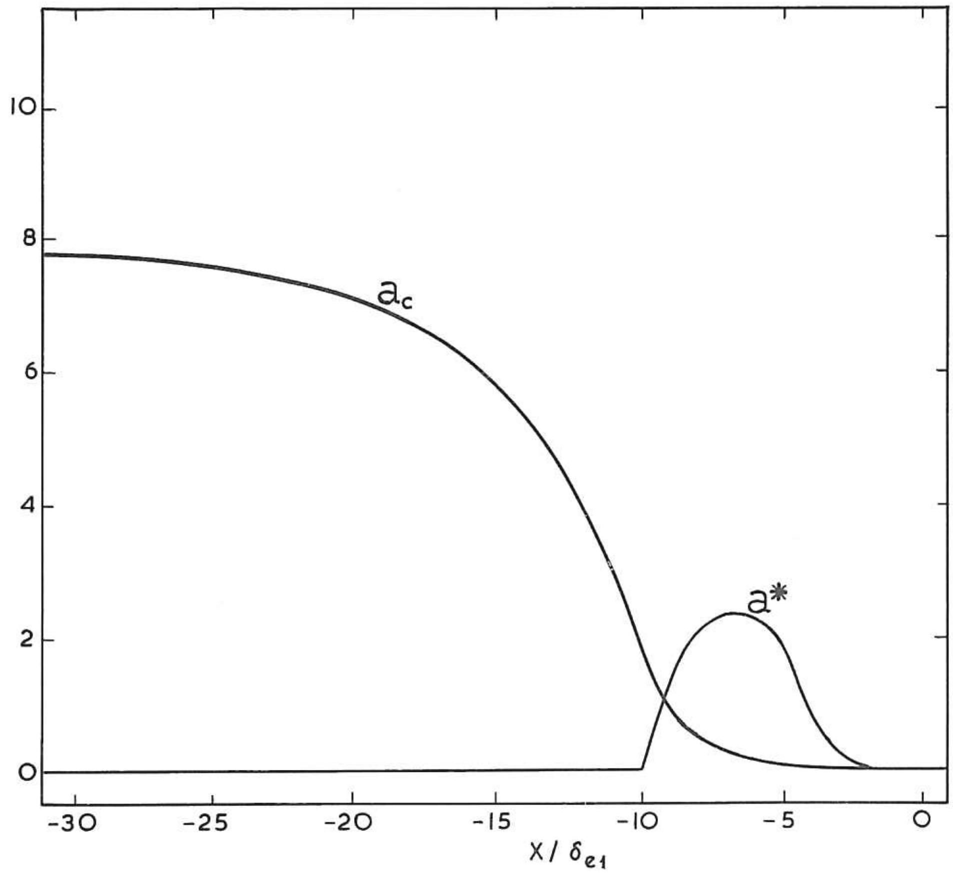

Fig.3 Profiles of classical and anomalous resistivity.

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