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# CONVECTION IN THE PRESENCE OF MAGNETIC FIELDS

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# CONVECTION IN THE PRESENCE OF MAGNETIC FIELDS

by

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## A B S T R A C T

Convection driven by horizontal temperature gradients in an electrically conducting, viscous, Boussinesq fluid has been studied on a computer. Results of numerical experiments with a Grashof number of $10^4$ are presented and compared with linear theory. Weak magnetic fields are distorted and concentrated without significantly affecting the motion, while strong fields virtually suppress convection. For moderate fields there is a non-linear regime in which the global magnetic and kinetic energies are comparable though the peak magnetic energy density may be an order of magnitude greater than the maximum kinetic energy density. The astrophysical relevance of these results is discussed.

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## 1. Introduction

The motion of the gas in stellar atmospheres, and of the material in the interior of the earth distorts the magnetic field which permeates it creating Lorentz forces which react back to modify the flow pattern.

The equations concerned are, in the notation of TABLE I

$$\left.\begin{aligned} &\frac{\partial \underline{B}}{\partial t} - \text{curl }(\underline{u} \wedge \underline{B}) = \eta \nabla^2 \underline{B} \\ &\rho \left\{ \frac{\partial \underline{u}}{\partial t} + (\underline{u}.\nabla)\, \underline{u} \right\} - \underline{j} \wedge \underline{B} = \rho\nu\nabla^2 \underline{u} - \nabla p + \rho g \end{aligned}\right\} . \quad (1)$$

If the right hand sides of the equations are ignored, the magnetic field is carried round by the fluid motions. The magnetic field and its associated current distribution result in a Lorentz force $\underline{j} \wedge \underline{B}$ which modifies the fluid motions. When the dissipative processes, viscosity and electrical resistivity, are included, the flow will die away unless some driving source of energy is present. In convection this is provided by thermal buoyancy forces, and non-trivial steady solutions result in certain regions of parameter space. The non-linear nature of these coupled vector partial differential equations makes computer simulation a necessary tool for the understanding of the processes.

## 2. Astrophysical Interest in Magnetic Convection

In a star, a strong magnetic field may reduce the efficiency of convection and in some cases suppress it altogether. The original treatment of convection in the presence of a magnetic field was by W B Thompson [1] in 1951 who showed that there is a critical "magnetic" Rayleigh number below which convection does not occur.

Chandrasekhar [2] describes fully the agreement which has been obtained

## Table 1

### NOTATION

| | | |
|---|---|---|
| $\underline{B}$ | magnetic field | |
| $\underline{u}$ | velocity vector | |
| $\underline{j}$ | electric current | |
| $\underline{g}$ | gravity downwards | |
| $\nu$ | kinetic viscosity | |
| $\eta$ | electrical resistivity | |
| $\kappa$ | thermometric conductivity | |
| $\beta$ | coefficient of thermal expansion | |
| $B_{max}$ | peak magnetic field strength | |
| $u_{max}$ | maximum velocity | |
| $B_o$ | undisturbed magnetic field strength | |
| $u_o$ | maximum velocity when $B_o = 0$ | |
| Gr | Grashof number | $\beta\ g\ T\ L^3/\nu^2$ |
| M | Hartman number | $Q^{\frac{1}{2}}$ |
| Pm | Magnetic Prandtl number | $\nu/\eta$ |
| Pr | Prandtl number | $\nu/\kappa$ |
| Rm | Magnetic Reynolds number | $u_{max}\ L/\eta$ |
| Q | Chandrasekhar number | $B^2L^2/\rho\ \nu\ \eta$ |

| | |
|---|---|
| t | time |
| $\rho$ | density |
| p | pressure |
| T | temperature |
| $\gamma$ | ratio of specific heats |
| $\nabla$ | d (log T)/d (log p) |
| L | distance between the plates |
| $\mu$ | constant of order unity |

| | |
|---|---|
| $E_{KIN}$ | normalised kinetic energy |
| $E_{MAG}$ | normalised magnetic energy |
| $E^0_{MAG}$ | undisturbed magnetic field energy |
| $E^{FREE}_{MAG}$ | $E_{MAG} - E^0_{MAG}$ |

between experiment and linear theory for the influence of a magnetic field on convection in a liquid. However, the compressibility of stellar material and the complicated geometry of a star make the study of convection in a real star much more difficult. Sunspots provide the most obvious location in astrophysics where magnetic fields affect convection. There much stronger magnetic fields are found than in the surrounding material. Biermann [3] suggested that these fields inhibited convection inside sunspots, and that the observed cooling was a consequence of reduced heat flux. This view is given support by the analysis for incompressible fluids and by work on the onset of convection in a compressible gas. In practice convection is only partially inhibited and not totally suppressed. Crudely, to suppress convection the magnetic field pressure must exceed the buoyancy force. However the direction of the magnetic field is important. In plane parallel geometry in a compressible gas, a sufficient condition for the suppression of convection can be shown to be (Gough & Tayler [4])

$$\frac{B_v^{\,2}}{B_v^{\,2} + \gamma P} > \nabla - \nabla_{ad} \qquad (2)$$

where $\nabla \equiv d(\log T)/d(\log P)$ and $\nabla_{ad}$ is the adiabatic value of $\nabla$. $B_v$ is the vertical component of the magnetic field; the horizontal component does not enter. If convection is suppressed totally in the interior of a star, energy is carried by radiation, ie $\nabla = \nabla_{rad}$. Hence equation (2) indicates that very large magnetic fields are required to suppress convection throughout the core of a star. In the outer convective envelope, Moss & Tayler [5] indicate that it is probably not possible for convection to be completely suppressed by a magnetic field: the inevitable regions where the field is horizontal cannot be stabilized.

In a satisfactory theory of the large-scale stellar or solar magnetic field which is effectively frozen into the ionized gaseous fluid, it is necessary to describe the motion in the convective zone. Danielson [6] investigated the effect of a magnetic field on the solar convection zone. He applied Chandrasekhar's analysis [2] for a conducting fluid permeated by a magnetic field to photospheric conditions. Chandrasekhar had previously shown that for magnetic fields greater than a minimum value ($\sim 10^{-2}$ gauss for the sun) that instability first occurs as overstable oscillations when $\kappa > \eta$, that is Pr < Pm.[7] Danielson found that this inequality is satisfied by nearly six orders of magnitude in the solar photosphere when the radiative conductivity is used for $\kappa$. Overstable oscillations can be viewed in a Bénard cell as occurring when the magnetic field is strong enough to stop and reverse the motion. The cell then has oscillatory rather than circulatory motion. Alternatively the configuration can be viewed as being composed of standing Alfven waves, and in our calculations for high Hartmann number we have found these, being damped away by resistivity, as one would expect.

The scale of solar active regions (comparable with the solar radius) can be taken to indicate that magnetic fields penetrate deeply into the convective zone, which can be divided into three layers each with its preferred scale of motion [8]. The largest would be giant cells with diameters of about 300,000 km and velocities around 0.1 km/sec as suggested by Bumba [9]. The intermediate convective circulatory field in the sun, the supergranulation, was discovered by Leighton [10] in 1960 and has been shown to be responsible for the structure of the chromospheric network. A feature of that relationship is that the cellular circulatory motions concentrate the weak, large-scale field of a magnetic region into narrow lanes at the boundaries of the convection cells. There is little evidence that the ordinary granulation concentrates the magnetic field significantly at its cell boundaries.

At the solar photospheric level, the average magnetic energy density is small compared with the energy of motion, despite the presence of strong local fields. Hence one may consider smoothed magnetic fields and treat them kinematically. This provided the motivation for early computer studies of the interaction of convection and magnetic fields [12, 13, 14] .

Smoothed fields and some form of eddy diffusion form a basic ingredient of most current solar models. The effect of a meridional circulation on a poloidal field has been computed by Maheswaran [15]. The main features of the solar dynamo (Babcock [16]) are now generally accepted, and have been reviewed elsewhere ([8], [17] ). Many details of these dynamos remain to be investigated. Further we must now aim to comprehend more fully the effect of individual convection cells in producing the flux ropes that emerge into the photosphere.

## 3. Convective Concentration of Magnetic Fields - Numerical Studies

An understanding of the interaction of convection and a magnetic field requires answers to questions such as "In what circumstances will convection wind up a magnetic field to such an extent that the field is annihilated by the enhanced dissipation?"; "When will the convection concentrate the field lines into localized regions with no substantial reduction in flux?" ; "When are fluid motions and magnetic fields in rough equipartition energetically?" At the present time such questions can only be answered using computer simulations.

Now observationally magnetic flux is mainly concentrated around the perimeter of solar convection cells and particularly at corners where several cells meet. For weak fields the concentration is purely kinematic and the field is limited by the magnetic Reynolds number Rm. Weiss [14] investigated the effects of an inexorable, non-divergent velocity field simulating convective cells on a seed magnetic field by following the time evolution of the system. He obtained a steady state solution of the induction equation with a balance between diffusion and advection, which is achieved after the initial seed field $\underline{B}_o$ has been amplified locally to a magnitude

$$B \simeq {R_m}^{\alpha} B_o \tag{3}$$

where $\alpha = \frac{1}{2}$ or 1 for two or three dimensions respectively (Parker [18]; Clark [19]; Weiss [14]; Clark & Johnson [20]). Moss [21] developed a kinematic numerical model of two-dimensional hydromagnetic turbulence. He found $\alpha = 0.35$, and further that some flux may always remain in a region even though most of it has been turbulently expelled. His method was one in which the magnetic field was stochastically displaced under the influence of a pseudo-random velocity field of quite a coarse scale. In a recent study (Peckover [22]), a two-dimensional dynamic simulation has been carried out. The important parameters are Q, the Chandrasekhar number Gr, the Grashof number, and Pm the magnetic Prandtl number. The ordinary Prandtl number Pr was zero. At the kinematic end of parameter space the law, $B_{max} = Rm^{\frac{1}{2}} B_o$, was substantiated and the gradual deviation from it as Q increased could be observed. At the magnetically dominated end of parameter space the simulation agreed well with linear theory. In the non-linear range computations were able to provide further data for the non-equipartition of magnetic and kinetic energies under some circumstances.

The particular configuration considered in this simulation is as follows:- the fluid is confined between two rigid horizontal plates. In the absence of motion the magnetic field would be vertical. The y-direction is vertical and $\partial/\partial z \equiv 0$ for all physical quantities. The thermal conductivity is assumed to be so high that the Prandtl number is zero. The temperature field is then Laplacian, and T is taken proportional to cos kx sinh ky. Periodicity is assumed in the x-direction, and the horizontal plates are assumed to be rigid and impervious such that no tangential viscous or magnetic stress occurs there.

The changing pattern of magnetic field lines as Q varies can be seen for steady state conditions for $Gr = 10^4$, $Pm = 1$ in figures 1 & 2. If the magnetic field is strong enough ($Q \to \infty$) the field is undisturbed. For Q = 1 the Lorentz force is weak compared with the temperature driving field, and flux is concentrated into ropes, and is almost completely expelled from the central region. The regime where non-linear induction effects are important lies between $Q = 25$ and $Q = 2500$. In these simulations, the stream line pattern remains little changed as a "cell in a box". (see figure 3). The Reynolds number is less than 200 and the flow is laminar. Although the eddy structure is maintained its strength falls as the graph of $u(= 2\psi_{max})$ as a function of $Q$ indicates (figure 4).

Figure 5 shows the global magnetic and kinetic energies as functions of the Chandrasekhar number Q. The kinetic energy falls with Q from the flat plateau through the non-linear régime to the $Q^{-2}$ asymptotic value. There is a region, around $Q \sim 100$, where $E_{KIN} \sim E_{MAG}$, but this is a factor of ten down from the value of $E_{KIN}$ when $Q \sim 0$. Statements of the form

$$E_{KIN} + E_{MAG} = \text{const}$$

or

$$E_{KIN} + E_{MAG}^{FREE} = \text{const}$$

independent of field strength, are not found to hold.

Observations of the photosphere should be able to provide us not only with mean values but also with peak values. In figure 6 the peak value of the velocity $u_{max}$ is plotted as a function of $B_o$. After a flat start it tails off as $Q^{-1}$ as $Q \to \infty$. The peak magnetic field goes as $Rm^{\frac{1}{2}} B_o$ for weak magnetic field and simulation agrees with kinematic theory (Weiss [14]). For strong fields, $B_{max}$ approaches $B_o$ as $Q^{-1}$, in agreement with linear theory. Crudely the field is concentrated until $B_{max} \sim u_{max}$ in the flux ropes. The magnetic field is strong enough to balance the buoyancy force and $u_{max}$ falls. Note $B_{max} \lesssim (u_{max})_o$ for all values of $Q$ except for the magnetically dominated limit.

For values of Pm other than unity, computations showed that $B_{max} \sim (R_m)^{\frac{1}{2}} B_o$ in the kinematic regime, and $u_{max} \sim \frac{1}{Pm} B_o^{-2}$ in the magnetically dominated regime. The expected behaviour for Pm << 1 and Pm >> 1 is shown in figures 7 & 8.

For small Pm, where the fluid is highly resistive, the magnetic field will differ little from its ambient configuration and $B_{max} \sim (1 + \mu R_m^{\frac{1}{2}}) B_o$ for $\mu \sim 1$ and $B_o < 1$. For $B_o > 1$, then $B_{max} \sim B_o$. The peak velocity $u_{max}$ will be unabated until $B_{max} \sim U_{max}$. For $B_o > 1$, $u_{max} \sim \frac{1}{Pm} B_o^{-2}$. As functions of $B_o$, $u_{max}$ and $B_{max}$ are almost piecewise linear.

For large Pm, when the viscosity is the dominant dissipative process, $B_{max} \sim (Rm)^{\frac{1}{2}} B_o$ until $B_{max} \sim u_o$ at $P'$. Thereafter there is a range of $B_o$ in which $B_{max}$ is constant $\sim u_o$, the non-magnetic peak velocity (note $B_{max}$ is measured in the units of Alfven velocity). $B_{max}$ falls as the convective velocity falls until $B_{max} \sim B_o$ for $B_o > 1$. The peak velocity $u_{max}$ is unabated until $P'$ when $B_{max} \sim u_{max}$. Thereafter, after a transitional knee $\left(\text{in which } \frac{d\, u_{max}}{d\, B_o} > -\infty\right)$, the peak velocity asymptotically has the form

$$\left(\frac{\pi}{\pi-1}\right)\left(\frac{1}{\nu\, Pm}\right) B_o^{-2} \quad \text{as } B_o \to \infty.$$

## 4. Conclusion

Thus, for magnetic convection caused by a particular persistent driving buoyancy force which is a continuous function of position, equipartition of magnetic and kinetic energy only occurs in a small region of (Q - Pm) parameter space. This is true whether one is considering global or peak values. It indicates that Beckers' suggestion [23] that theoretical equipartition can be used with the observed peak velocities to provide a method for calculating the magnetic fields needs to be treated with caution.

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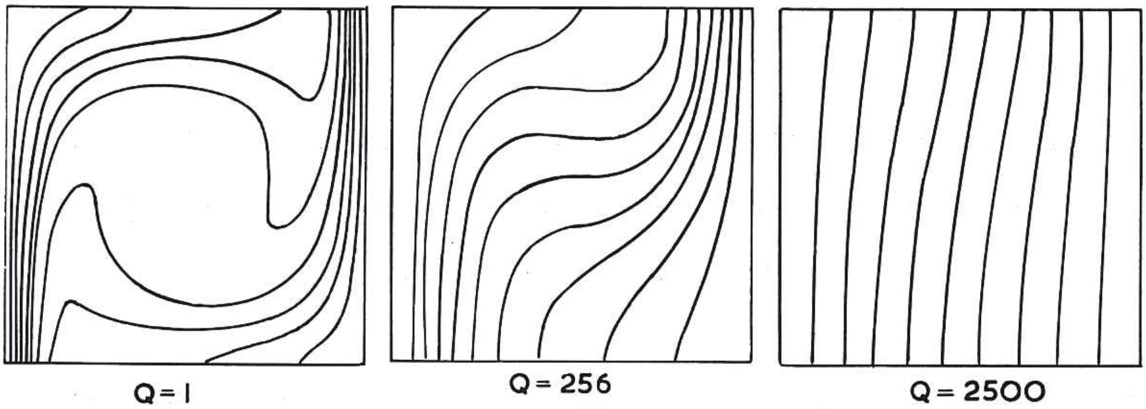


Fig.1 The changing pattern of magnetic field lines as Q varies.

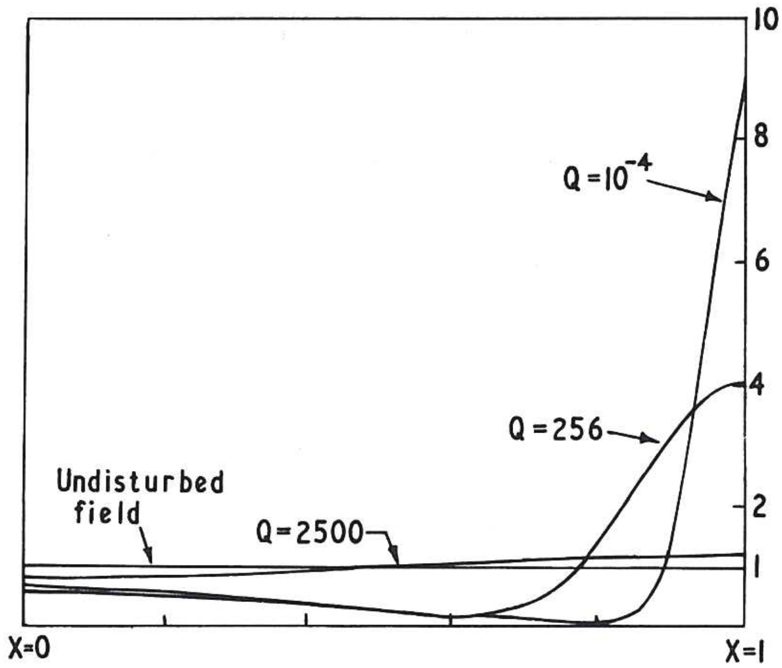


Fig.2 The amplitude of the vertical component of magnetic field on the horizontal boundary $y = 1$, for magnetic Prandtl number $Pm = 1$.

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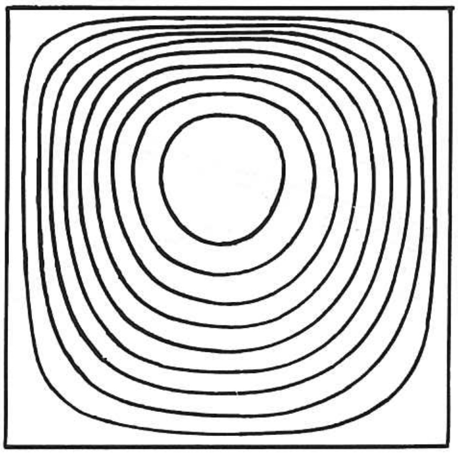


Fig.3 The streamline pattern of a convective eddy in a box when Q = 0 for a Laplacian temperature field.

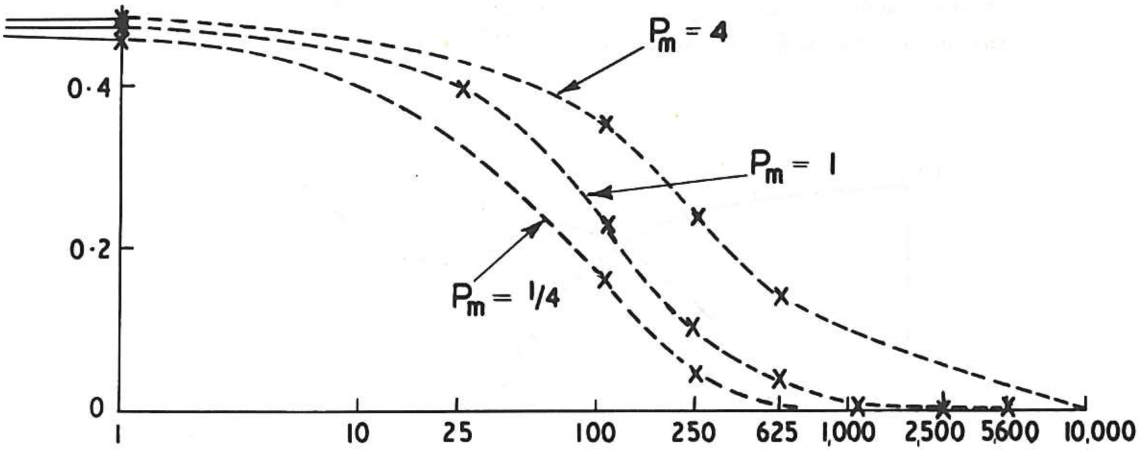


Fig.4 The variation of eddy strength with Q for Pm = ¼, 1 and 4.

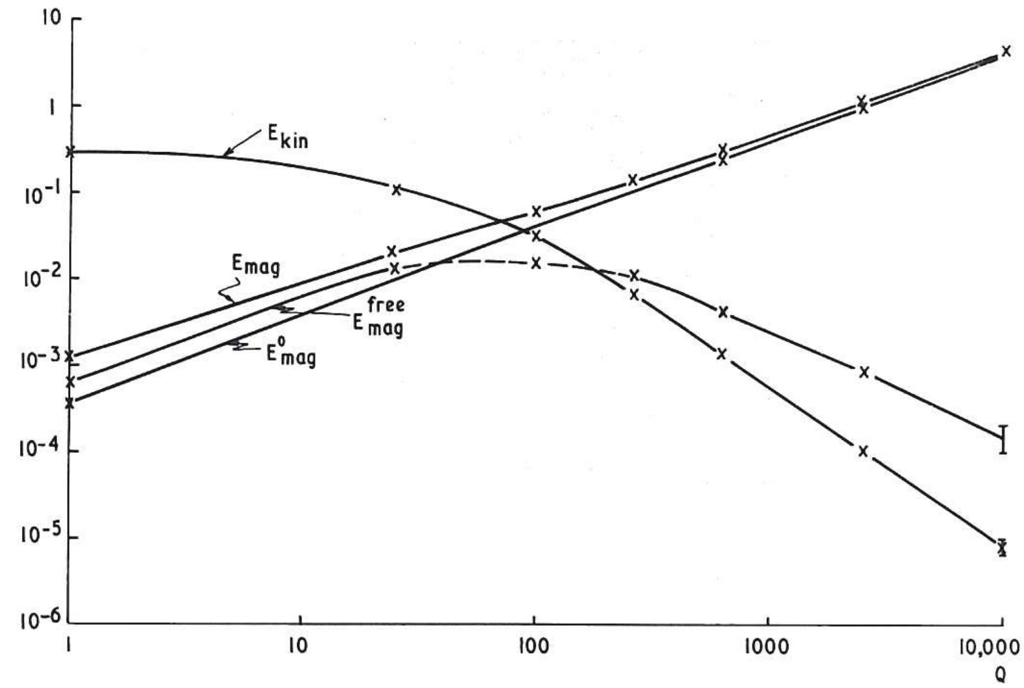


Fig.5 The global energies as a function of Q when Pm = 1 $E_{KIN}$ is the kinetic energy; $E_{MAG}$ is the magnetic energy; $E^{o}_{MAG}$ is the undistorted magnetic field energy; $E^{FREE}_{MAG} = E_{MAG} - E^{o}_{MAG}$.

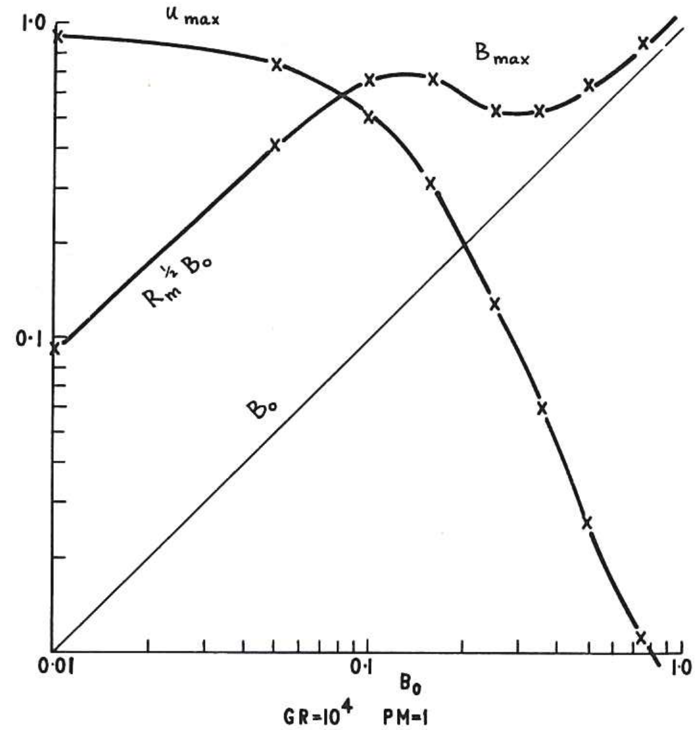


Fig.6 The steady state peak velocity $U_{MAX}$, and the peak magnetic field $B_{MAX}$ (measured as an Alfven velocity) plotted as functions of the ambient field strength $B_0$ when Pm = 1.

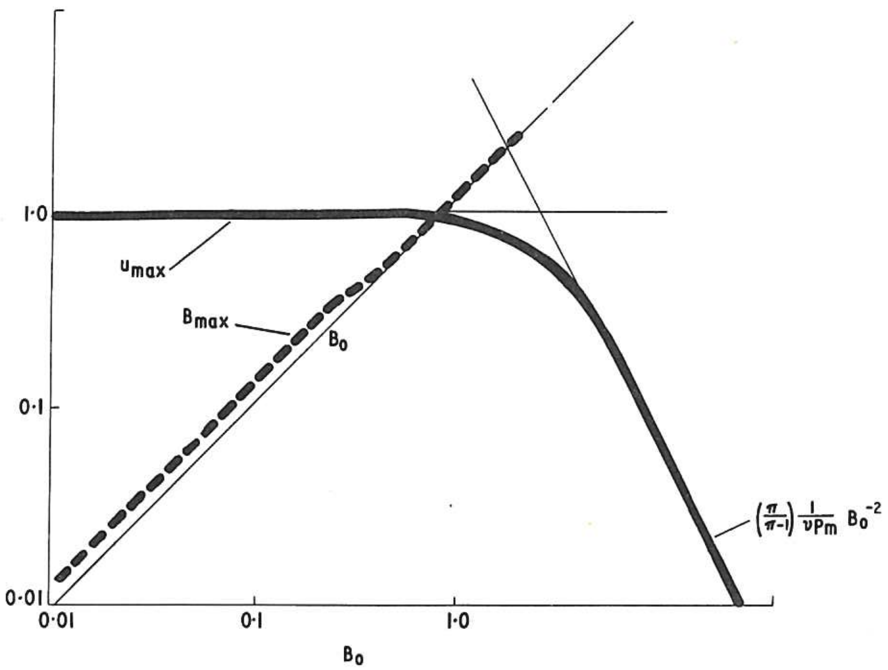


Fig.7 The conjectured form for the steady state peak local values $U_{MAX}$ and $B_{MAX}$ as functions of the ambient field strength $B_0$ when Pm is small.

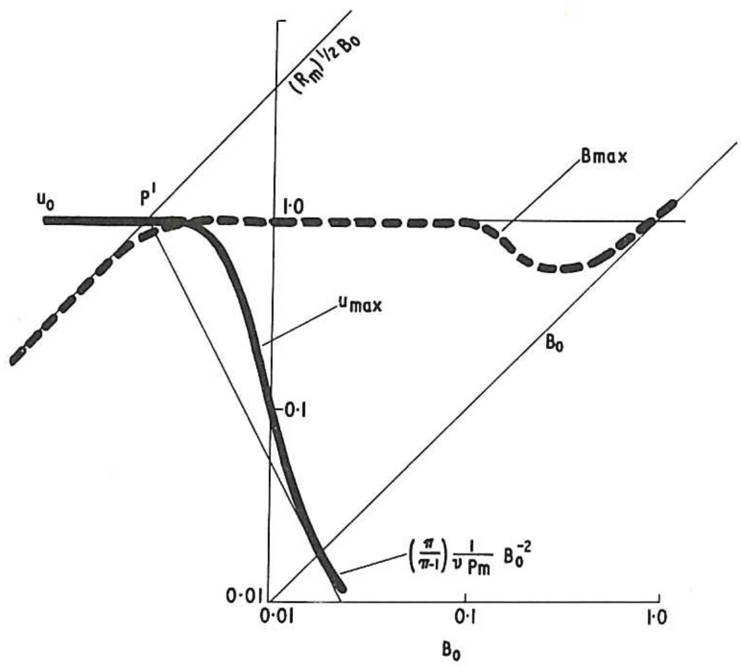


Fig.8 The conjectured form for the steady state peak local values $U_{MAX}$ and $B_{MAX}$ as functions of the ambient field strength $B_0$ when Pm is large.