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IGNITION CONDITION IN TOKAMAK EXPERIMENTS AND ROLE OF NEUTRAL INJECTION HEATING

D R SWEETMAN

CULHAM LABORATORY
Abingdon Berkshire

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IGNITION CONDITION IN TOKAMAK EXPERIMENTS AND ROLE OF NEUTRAL INJECTION HEATING

by

D. R. Sweetman

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ABSTRACT

The condition for toroidal experiments to become energetically self-sustaining is examined and a simple relationship is given between the self-ignition temperature and the plasma current. The temperature is limited at low density by synchrotron radiation and at high density by β_θ limitations. Tokamaks could, in principle, reach ignition conditions on ohmic heating alone but will probably be limited by synchrotron radiation. A quite modest additional heating source allows ignition at higher density and the consequent examination of β_θ limitations. The power and energy requirements for neutral beam heating are examined and shown to be compatible with existing technology.

U.K.A.E.A.
Culham Laboratory
Abingdon
Berks.

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1. INTRODUCTION

Several papers have dealt with the ignition of Tokamak and other toroidal reactors [1,2,3]. There has, however, been little discussion of the ignition condition in laboratory scale apparatus where the thermal conduction power loss always exceeds the hydrogenic bremsstrahlung loss. Golovin [1] discusses laboratory ignition experiments briefly but assumes a Bohm-like scaling of the heat loss which may be rather pessimistic.

In order to throw light on the ignition conditions for a laboratory scale apparatus we have used a simple model in which the ion and electron temperatures are decided by a balance of ohmic heating, neutral injection heating, α-heating, ion and electron thermal diffusion, bremsstrahlung and synchrotron radiation. Some conditions which limit the size of an ignition demonstration experiment using an auxiliary source of heating such as fast neutral injection are derived.

Factors influencing the choice of energy for a fast neutral injector are discussed, in particular the requirements of beam penetration and energy drain to the ions. In this paper there is no attempt to deal with the physical effects associated with injection (e.g. electric fields, induced currents and instabilities).

2. CONDITIONS FOR IGNITION OF A TOKAMAK EXPERIMENT

We first examine the conditions for ignition of a Tokamak experiment and note how they differ from those in a reactor. The model on which the numerical estimates are based is described in the Appendix. The thermal diffusion loss is taken to be neoclassical multiplied by an adjustable factor.

With parameters corresponding to a D-T reactor and with neoclassical conditions the plasma becomes thermally self-sustaining at a temperature for which the α-heating power equals the bremsstrahlung loss ($T \sim 4$ keV). At ignition the ohmic heating power is rather smaller than the α-heating power and the neo-classical ion thermal loss is rather small compared with the bremsstrahlung. Synchrotron radiation is not dominant provided the density is not too low. The power levels as a function of temperature ($T_i \simeq T_e = T$ for this purpose) are shown in Fig.1 for some typical reactor scale parameters.

With parameters corresponding to an ignition experiment the power balance is somewhat different. Fig.2 shows the power levels for a typical marginal ignition case. Even with neo-classical coefficients the ion thermal transport exceeds the bremsstrahlung loss by a substantial factor and the ohmic heating contribution is now significant for temperatures as high as 15 keV.

In order to establish the ignition condition and the effect of additional heating (for example by neutral injection), it is useful to plot the net power required to maintain a given temperature against the temperature [4]. Such a plot is shown in Fig.3. True ignition, with the plasma self-sustaining energetically without additional heating, is obtained at the temperature T_c in the figure. A useful relationship for this critical temperature is obtained by equating the α-heating power to the thermal conduction loss. For convenience this latter may be equated to the ion thermal conduction multiplied by a factor (χ_i) which takes into account the electron thermal conduction and departures from strict neo-classical behaviour. Combining equations (11) and (14) given in the Appendix, we arrive at the relationship:

$$I_p = 40 \left(A \frac{a}{R} \right)^{\frac{1}{4}} \left(\frac{\varepsilon_i \chi_i}{\varepsilon_\alpha} \right)^{\frac{1}{2}} T_c^{7/12} \exp \left(\frac{100}{T_c^{1/3}} \right) \text{ amps. } \quad \ldots (1)$$

Thus, for a given anomaly factor (χ_i) the critical temperature for the plasma to become self-sustaining is almost independent of all parameters except the plasma current. It should be noted that this condition is general for any toroidal system provided some equivalent toroidal current may be defined.

A plot of T_c against I_p is shown in Fig.4 for three assumed values of χ_i (χ_i = 1.2 corresponds approximately to the strictly neo-classical coefficients).

An additional limitation is set to the allowed temperature by constraints on β_θ. Using the definition

$$\beta_\theta = \frac{4}{I_p^2} \int_0^a n\, k(T_e + T_i)\, r\, dr \quad ,$$

for our distributions, $T_e = T_i$, and I_p in amps, we find

$$T = 9{\cdot}2.10^8 \, \frac{\beta_\theta \, I_p^2}{n\, a^2} \text{ eV} \quad . \qquad \ldots (2)$$

For a given β_θ limitation (for example $\approx$ R/a) this sets density dependent limitations to the obtainable temperature. Thus the combination of the relationship in equation (1) and the β_θ limitation set a minimum plasma current at which ignition can be obtained which is of order 1-2 megamps for the strictly neo-classical case.

To illustrate the behaviour as the density is varied it is instructive to plot the additional power required to maintain a given temperature as a function of temperature and density. An example experiment with marginal ignition assuming neo-classical losses (I_p = 1.3 megamps) is shown in Fig.5. The contours show the additional power required in kilowatts, where this is positive; the line labelled 0 gives the equilibrium temperature in the absence of additional heating and the line marked T_c is that given by equation (1). Also shown are lines of constant β_θ. Similar plots for χ_i = 5 (i.e. 4 x neo-classical heat loss) are shown in Figs.6 and 7. In Fig.6 the value of I_p has been raised by decreasing q so as to keep a 20 keV ignition temperature. In Fig.7 the magnetic field has been raised, whilst keeping q constant, to achieve the same result.

The equilibrium temperature in the absence of additional heating is given essentially by the balance of ohmic heating and thermal conduction loss which, as is well known, results in a limit to β_θ of order 0.5. For our distributions and $\ell n\ \Lambda = 15$ we get for this limit

$$\beta_\theta = 0.22 \left(\frac{\chi_\Omega}{\chi_i} \right)^{\frac{1}{2}} \left(\frac{R}{a} \right)^{\frac{1}{4}} . \qquad \ldots (3)$$

In the absence of synchrotron radiation, ignition could occur without additional heating when the density is sufficiently low. Limits to the density are however provided by the runaway condition and synchrotron radiation.

Neglecting the variation of plasma current with the minor radius, the density limitation provided by the runaway condition is approximately given by

$$n \gtrsim 4 \cdot 2.10^{11} \frac{I_p}{\pi a^2 T_e^{\frac{1}{2}}} \ cm^{-3} \qquad \ldots (4)$$

where again I_p is in amps and T_e in eV. For most experimental parameters of interest here this density limit is not serious ($< 10^{12}\ cm^{-3}$). It might be important in some small scale high-field experiments however.

Synchrotron radiation provides a major temperature limitation at low densities. Also it is worth noting that because of the limitations of the present synchrotron radiation calculation the relatively high temperatures at low densities shown in Figs.5-7 should be viewed with caution. It is possible that more realistic calculations will show a lower temperature barrier. In any case the balance at these low densities is essentially between ohmic heating input and synchrotron radiation with α-heating contributing only a fraction of the total heat input. It is doubtful if this could be considered a test of ignition in any very real sense especially as β_θ is not required to exceed that of present experiments.

An equally serious restriction to an ignition demonstration at low densities might be the implied energy containment time (τ_E). Considering for this purpose only ion thermal conduction heat loss

$$\tau_E = \frac{3.2}{A^{\frac{1}{2}} \varepsilon_i \chi_i} \left(\frac{R}{a} \right)^{\frac{1}{2}} \frac{I_p^2 T^{\frac{1}{2}}}{n} \text{ sec} \qquad \ldots (5)$$

where I_p is in amps. Near the maximum self-ignition density of Fig.5 the containment time is $\sim$ 20 seconds (or some 10^5 Bohm times!). Depending on the nature of any anomalous losses present it might be preferable to work at larger density where the implied loss time is correspondingly smaller.

An important feature of Figs.5-7 is the comparatively modest level of additional power required to surmount the barrier between the ohmic heating limit and true ignition. The availability of such an additional source of heating also adds flexibility and enables a wider range of β_θ to be investigated.

3. α-PARTICLE DRIFT ORBITS

These have been discussed by Morosov and Solovyev [5] and other workers. Two applications are of interest: to the orbits of the injected particles and to the thermonuclear α-particles. The former has been discussed in some detail by Stix [6] and will not repeated here. For the scale of apparatus considered here drift loss of the injected particles does not present serious problems for $E_0 << 1$ MeV. However the loss of α-particles sets a limit to the size of an ignition experiment.

Particles not trapped by the variation of B along the field lines (passing particles) have approximately circular drift motion. In the uniform current distribution case, the displacement of the centre from the minor axis is given by (provided $v_{\parallel}/v$ is not too small)

$$\delta = \frac{q\, r_L}{2} \frac{(1 + \cos^2 \psi)}{\cos \psi}$$

where $r_L = mcv/eB$ and $\cos \psi = v_{\parallel}/v$.

Trapped particles may have somewhat larger displacements. Thus if $\delta = a$, essentially all the α-particles are lost: for our case this gives a limiting plasma current below which there is essentially no α containment

$$I_p = \frac{mcv}{2e} \frac{a}{R} .$$

For an inverse aspect ratio of 0.33 this corresponds to a current of 0.45 megamperes.

A proper assessment of the fraction of α-particles lost at a given plasma current requires a numerical integration of the fraction which drift into the wall over the spatial and angular distribution of the created α-particles. This work is in progress but in the absence of a definitive answer we merely indicate on Fig.4 the current at which a substantial loss of α-particles occurs.

4. HEATING BY NEUTRAL INJECTION

The neutral injection power is simply $P_{NI} = I.E_0$ watts where I is the injected beam current (in amps) after losses due to charge exchange and drifts have been subtracted and E_0 is the injected energy (in eV). This power (and the power introduced by the α-particles) is distributed between the ions and electrons. The instantaneous rate of transfer to the ions is given by [7]

$$\frac{1}{E_i} \frac{dE_i}{dt} = - 1 \cdot 8.10^{-7} \frac{A_i^{\frac{1}{2}}\, z^2\, \ell n\Lambda\, n}{A\, E_i^{\frac{1}{2}}} \;\; \sec^{-1}$$

where E_i, A_i, z are the energy (in eV), the atomic weight and charge of the trapped ions respectively. Assuming $v_e >> v_i$, the instantaneous transfer rate to the electrons is given by [10]

$$\frac{1}{E_i}\frac{dE_i}{dt} = - 3\cdot 3.10^{-9} \frac{z^2 \ \ell n\Lambda \ n}{A_i \ T_e^{3/2}} \quad \sec^{-1} ,$$

hence the ratio

$$\frac{\text{transfer rate to ions}}{\text{transfer rate to electrons}} = \frac{56}{A}\left(\frac{A_i \ T_e}{E_i}\right)^{3/2} .$$

To calculate the fraction of the total energy that is conveyed to the ions we must integrate the fraction transferred from $E_i = E_o$ to $E_i = 0$. Thus the fraction going to the ions is given by

$$f_i = \int_{k_o}^{o} \left(1 + \frac{1}{k^{3/2}}\right) dk$$

$$= \frac{2}{k}\left[-\frac{1}{6}\log_e\left\{\frac{(1+k^{\frac{1}{2}})^2}{k - k^{\frac{1}{2}} + 1}\right\} + \frac{1}{3^{\frac{1}{2}}}\arctan\left(\frac{3^{\frac{1}{2}} \ k^{\frac{1}{2}}}{2 - k^{\frac{1}{2}}}\right)\right] \quad \ldots(6)$$

where

$$k = \frac{A^{2/3}}{14.6 \ A_i} \cdot \frac{E_o}{T_e}$$

is the ratio of the fast particle energy to that at which there is equal energy transfer to ions and electrons. A plot of this expression is shown in Fig.8, which is similar to that given in reference 6. The ions and electrons are heated equally when $k = 2.4$.

5. NEUTRAL INJECTION POWER AND ENERGY REQUIREMENTS

We might consider two limiting cases depending on whether neutral injection power or total injected energy are the important factors.

(i) Fast heating

In this case we consider heating to ignition in a time (τ_h) short compared with some characteristic energy loss time τ_E. Considering for this purpose only thermal conduction heat loss equation (5) gives times of a few seconds for typical marginal ignition cases.

When $\tau_h \ll \tau_E$ then the total energy required is simply that required to warm up the plasma. This is most conveniently expressed in terms of the initial and final β_θ

$$E = 4{\cdot}6.10^{-9}\ R\ I_p^2\ (\beta_\theta^F - \beta_\theta^I)\ \text{joules} \qquad \ldots(7)$$

This energy goes to zero as the density is lowered to the point where, neglecting synchrotron radiation, ignition occurs on ohmic heating alone (i.e. $\beta_\theta^I = \beta_\theta^F$).

If, as an example, we take the experimental parameters corresponding to Figs. 5-7 then the total energy required to heat to ignition at a density of 5.10^{13} cm^{-3} is 4.10^6 joules. For the strictly neoclassical case τ_E would be 3.4 seconds (at $\beta_\theta = 1$). The power implied by $\tau_h < \tau_E$ is therefore $> 1{\cdot}2.10^6$ watts.

(ii) Slow heating

The minimum heating power is used if sufficient is applied to just overcome the losses given in Figs.5-7. Under these conditions $\tau_h \gg \tau_E$ and, because of the losses during heating, the total energy requirement may exceed that given by equation (7) by a substantial factor. Fig.9 shows the total energy input required to reach ignition, based on time dependent calculation of the ion and electron temperatures; the total time involved is also shown. The starting plasma conditions were assumed to be the equilibrium values in the absence of additional heating.

6. NEUTRAL BEAM TRAPPING

A beam of fast neutral atoms (velocity v_o) becomes trapped by ionizing collisions with the plasma ions (v_i) and electrons (v_e). The beam attentuation is governed by an equation of the form

$$I = I_o\, e^{-n\sigma x} = I_o\, e^{-\frac{nx}{D}}$$

where D is some characteristic plasma "thickness" for 1/e beam attenuation: for the case $v_e > v_o > v_i$ then

$$D = \frac{1}{\sigma} = \frac{1}{\sigma_x + \sigma_i + \frac{\overline{\sigma_e v_e}}{v_o}} \text{ cm}^{-2} , \quad \ldots (8)$$

where σ_x is the cross section for charge exchange and σ_i is the cross section for ionization on a plasma ion; $\overline{\sigma_e v_e}$ is the ionization rate averaged over the electron distribution.

Fig.10 shows the separate cross sections appropriate to hydrogen or deuterium beams as a function of the fast neutral energy for a range of values of T_e and Fig.11 shows the attenuation thickness D. The basic data is that published by Riviere [8]; for $v_e < v_o$ the electron contribution is taken to be the cross section for a δ-function distribution with relative velocity v_o. For $E_o < 40$ keV the attenuation is primarily by charge exchange. For higher energies the predominant process is ionization by the plasma ions with, if the electron temperature is near the $\overline{\sigma_e v_e}$ maximum (∿ 100 eV), electron ionization also playing a role.

For many Tokamak experiments the practical energies of interest lie in the range 20-100 keV. Over this range a reasonable approximation to the penetration thickness is given by

$$(H^o \text{ atoms}) \qquad D = 5{\cdot}5.10^{10}\, E_o \text{ cm}^{-2} ,$$
$$(D^o \text{ atoms}) \qquad D = 2{\cdot}7.10^{10}\, E_o \text{ cm}^{-2} , \quad \ldots (9)$$

where E_o is the atom energy in eV.

In order to avoid excessive heating of the vacuum wall and to maximise the heating power it is desirable to trap most of the beam. For the beam to attenuate by say $(1/e)^2$ this sets a maximum thickness ($\int n d\ell$ along the beam) equal to 2D. For injection tangential to the major circumference this may be set equal to some average density ($\overline{n}$) times the mean chord length, $2(2Ra)^{\frac{1}{2}}$.

The minimum energy is set by the need to deposit energy near the minor axis. For injection perpendicular to the minor axis

$2D \gtrsim \bar{n}a$. Therefore, using (9) the range of energies of interest is given by

$$(H^o \text{ atoms}) \qquad a \lesssim 5\cdot 5.10^{10}\, E_o/\bar{n} \lesssim \left(\frac{2R}{a}\right)^{\frac{1}{2}} a \quad ,$$

$$(D^o \text{ atoms}) \qquad a \lesssim 2\cdot 7.10^{10}\, E_o/\bar{n} \lesssim \left(\frac{2R}{a}\right)^{\frac{1}{2}} a \quad . \qquad \ldots (10)$$

In practice other physical considerations may decide between tangential and radial injection and the range of energy may therefore be more restricted.

7. BEAM SPECIES

In Section 6 the cross sections appropriate for hydrogen or deuterium beams were considered: other beam species might have advantages from the beam production point of view. For beams of other than the primary plasma constituents however, it is important to consider the implied impurity concentration. For purposes of this discussion it is simplest to consider the case $\tau_h < \tau_E$ and that none of the injected ions are lost during the heating phase.

In that case we may equate the heat required to raise the temperature to ignition to that brought in by the beam

i.e
$$\int \Delta(3\ nkT)\ dV = I_o\ E_o \tau_h \quad .$$

The relative contamination level (ξ) at the end of the heating phase is given by

$$\xi = \frac{I_o\ \tau_h}{\int ndV}$$

$\therefore$ combining these

$$\xi = \frac{\int \Delta\ (3\ nkT)\ dV}{E_o \int ndV}$$

$$= \frac{4}{5} \cdot \frac{\Delta(3\ kT)}{E_o} \qquad\qquad (\xi \ll 1)$$

where the factor 4/5 comes from the integration over the radial distributions we have assumed and the density change is taken to be not too large.

Thus for a hydrogen beam where, from penetration considerations, E_o = 40-100 keV and ΔkT to ignition = 16 keV the hydrogen level at the end of the heating phase is between 1.0 and 0.4 times the D-T level. This might have serious consequences if β_θ is limited.

An upper bound to the atomic mass of the injected species is found by comparing the impurity level to that required to say double the bremsstrahlung emission. If we assume that the trapping cross section is dependent only on the velocity of the injected particle, then we may for general comparative purposes take the injected energy required **for** penetration as proportional to the particle mass

i.e.
$$\xi \approx \frac{4}{5} \cdot \frac{\Delta(3\ kT)}{A_i\ E_o'}$$

where E_o' is the hydrogen penetration energy given by equation (10).

To restrict the bremsstrahlung emission we take $\xi Z_e^2 < 1$ where Z_e is an appropriate effective atomic charge. This gives

$$\frac{Z_e^2}{A_i} < \frac{5}{4} \cdot \frac{E_o'}{\Delta(3\ kT)} \ .$$

If $A_i = 2 Z_e$ then

$$A_i < \frac{5\ E_o'}{\Delta(3\ kT)}$$

which for the range of E_o' given by equation (10) and $\Delta kT \sim 16$ keV gives $A_i <$ 4-10.

If $\tau_h > \tau_E$ similar considerations apply except that the ratio of the injected ion containment time to the plasma energy containment time must be taken into account.

Thus the range of possible injected species, other than deuterium or tritium, is rather restricted. It is possible however that a more specific consideration of He or Li might reveal significant advantages in their use.

8. CONCLUSIONS

We have seen from the above discussion that if thermal losses scale in an essentially neo-classical way then the plasma current required to achieve an energetically self-sustaining experiment is a unique function of the plasma temperature. At high plasma density an upper limit is set to the temperature by β_θ limitations and at low density by the dominance of synchrotron radiation. Thus the critical current for ignition is rather clearly defined and is about 1-1.5 megamperes if the losses are strictly neo-classical. At this low current α-particle drift losses are probably serious and need more detailed evaluation. If the thermal losses are, say, five times the neo-classical ion thermal conductivity the critical plasma current required is 2-3 megamperes.

At the lower densities very high temperatures can, in principle, be achieved without any additional heating power. Under these conditions the energy balance is mainly between ohmic heating and synchrotron radiation and it is doubtful if this could be regarded as a true test of ignition since the ohmic heating cannot be switched off.

The main role of additional heating is to enable a genuinely self-sustaining condition to be achieved at higher densities where the synchrotron radiation is not a limit and where the all-important β_θ limitations can be tested. The additional power required to give this flexibility is not very dependent on the neo-classical loss factor provided the plasma current is scaled so as to make the self-sustaining condition possible. The power levels required are typically a few hundred kilowatts, though to give reasonable heating times a power level of 1-2 megawatts is desirable. For the reactor case, power levels of $\sim$ 30 megawatts are required to give reasonable heating times [9].

It is significant that, because of β_θ limitations, marginal experiments must work at a comparatively low density and hence a long containment time. This makes them particularly susceptible to losses which do not scale with density (e.g. Bohm-like). It would appear that if ignition experiments with significantly less than reactor dimensions are to be constructed then $\sim 10^4$-10^5 Bohm times must be achieved. This gives an additional reason for igniting at the maximum possible density.

If fast neutral injection is used as the source of additional heating then the beam energy is fairly well defined by the penetration considerations discussed in Section 6. For next generation experiments ($a = 70$ cm, $R/a \sim 3$, $n = 3.10^{13}$) equation (10) gives a range of D^0 energy from 80-200 keV depending on the angle of injection. At this energy, assuming classical friction processes, most of the power goes to the ions. It is a very convenient energy range for ion source technology [10] and the implied beam currents (say 10 A at 100 keV) are close to being achieved with a single unit. In the case of reactors the D^0 energy required is in the range 2-5 MeV which represents a significant extension of existing technology though the beam current required (say 15 A at 2 MeV) is similar to that available now.

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APPENDIX

Model used in Time Dependent Calculations

The radial density and temperature distributions are assumed to be of the form $n(r) = n(1 - r^4/a^4)$ and $T_{i,e}(r) = T_{i,e}(1 - r^4/a^4)$ respectively. These are reasonably close to equilibrium distributions obtained from 1D code calculations [11,12]. Following Artsimovitch [13] the local plasma current density is taken to be proportional to $T_e^{3/2}$. The ion and electron thermal conductivities are derived from the work of Rosenbluth, Hazeltine and Hinton [14] for the banana regime, though multiplication factors have been included to allow for non neo-classical effects. In particular, the total power loss due to ion thermal conductivity is taken to be

$$P_i = 1{\cdot}6.10^{-18} \,.\, A^{\frac{1}{2}}(a/R)^{5/2} R^3 \frac{n^2 T_i^{\frac{1}{2}}}{I_p^2} \,.\, \varepsilon_i \chi_i \text{ watts} \qquad \ldots (11)$$

where I_p is the plasma current in amps, R is the major radius in cms, the temperatures are in eV and a/R is the plasma inverse aspect ratio. ε_i essentially describes the temperature gradient ($\approx \frac{a}{T}\frac{dT}{dr}$): a calculation of the power flux as a function of radius for the above distributions leads to a mean value of about 2.5 for ε_i. χ_i is included as an adjustable factor to simulate non-classical effects. A is the average atomic mass of the plasma ions.

The electron thermal loss is taken to be

$$P_e = 0.77{\cdot}10^{-19} (a/R)^{5/2} R^3 \frac{n^2 T_e^{\frac{1}{2}}}{I_p^2} \left(\delta_i + \delta_2 \frac{T_i}{T_e}\right) \varepsilon_e \chi_e \text{ watts} \qquad \ldots (12)$$

where for our distributions $\varepsilon_e \approx 2.5$ and $\delta_1 = 2.0$, $\delta_2 = 1.07$.

The ohmic heating is taken to be

$$P_\Omega = 1\cdot 0.10^{-2}\ \ell n\Lambda\ \frac{R\ I_p^2}{a^2\ T_e^{3/2}} \cdot \varepsilon_\Omega \cdot \chi_\Omega \ \text{watts} \qquad \ldots (13)$$

$\varepsilon_\Omega = 1$ is appropriate for temperature and current density uniform over the minor radius. For our case $\varepsilon_\Omega = 1.7$. χ_Ω is again included to simulate non-classical effects which include in this case the effect of trapped particles. For present purposes χ_Ω is taken as 1.5.

The α-heating is derived from a Gamov fit to the D-T cross section curve (a 50:50 mixture is assumed):

$$P_\alpha = 1\cdot 0.10^{-21}\ n^2\ (a/R)^2\ R^3\ \frac{\exp(-200/T_i^{1/3})}{T_i^{2/3}} \cdot \varepsilon_\alpha \ \text{watts} \ldots (14)$$

Again $\varepsilon_\alpha = 1$ is appropriate for uniform density and temperature and for our case $\varepsilon_\alpha = 0.4$. This expression somewhat underestimates the α-heating in the energy range 10-30 keV (by about 30%) but has been retained in this form as a very approximate allowance for α-particle loss due to drifts (see Section 3).

The bremsstrahlung power loss is given by

$$P_{br} = 3\cdot 3.10^{-31}\ n^2\ (a/R)^2\ R^3\ T_e^{\frac{1}{2}}\ \varepsilon_{br}\ \text{watts} \qquad \ldots (15)$$

In our case $\varepsilon_{br} = 0.49$.

The transfer between electrons and ions is given by

$$P_{ei} = 1\cdot 5.10^{-26}\ \frac{\ell n\Lambda\ n^2\ (a/R)^2\ R^3}{A} \cdot \frac{(T_e - T_i)}{T_e^{3/2}} \cdot \varepsilon_{ei}\ \text{watts} \qquad \ldots (16)$$

where $\varepsilon_{ei} = 0.59$.

Synchrotron radiation is somewhat more difficult to estimate accurately since the temperatures of interest tend to be higher than those for which the weakly relativistic analysis of Rosenbluth [15] ($\lesssim$ 8 keV) is valid but lower than that for the continuum treatment to be valid [16] ($\gtrsim$ 20 keV). For our present purposes, to calculate the net radiated power (P_s), we have taken the expression given by Trubnikov [16] but multiplied by a factor of two to allow for radiation beyond the black body limit [17]. A reflection coefficient of 0.8 is assumed.

The approach to equilibrium temperatures is followed by solving the differential equations

$$\frac{dT_e}{dt} = \frac{P_\Omega + P^e_{NI} + P^e_\alpha - P_e - P_{ei} - P_{br} - P_s}{\frac{3}{2} \cdot 1{\cdot}6.10^{-19} \; n \; \bar{V}} \quad \text{eV/sec} \quad ,$$

and

$$\frac{dT_i}{dt} = \frac{P_{ei} + P^i + P^i_{NI} - P_i}{\frac{3}{2} \cdot 1{\cdot}6.10^{-19} \; n \; \bar{V}} \quad \text{eV/sec} \quad ,$$

where, for our distributions,

$$\bar{V} = \frac{8}{15} \cdot 2\pi^2 \, Ra^2 \quad .$$

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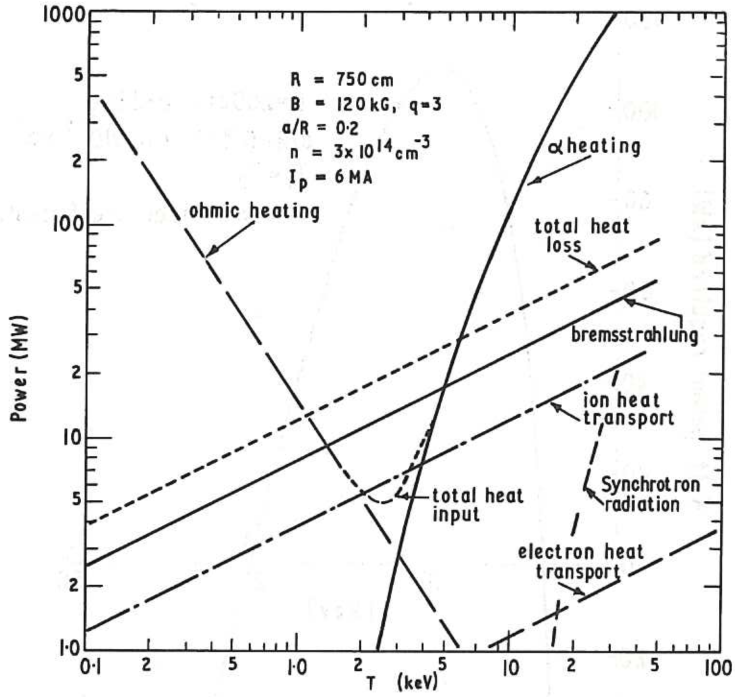

Fig.1 Power balance - reactor case. For this case parameters typical of those used for reactor design studies at Culham have been used. For this graph neo-classical heat losses are assumed.

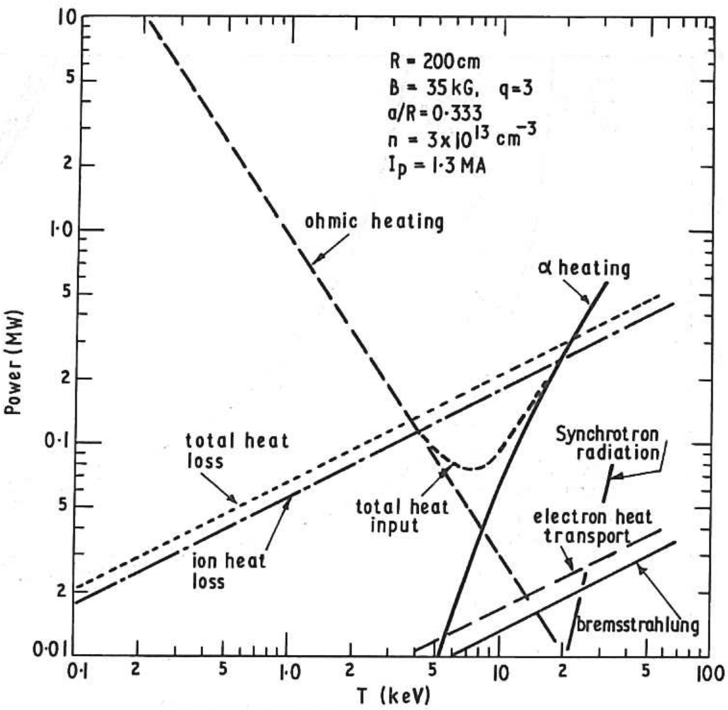

Fig.2 Power balance - ignition experiment. This case corresponds to the minimum plasma current that could give ignition at a reasonable temperature and assumes neo-classical losses.

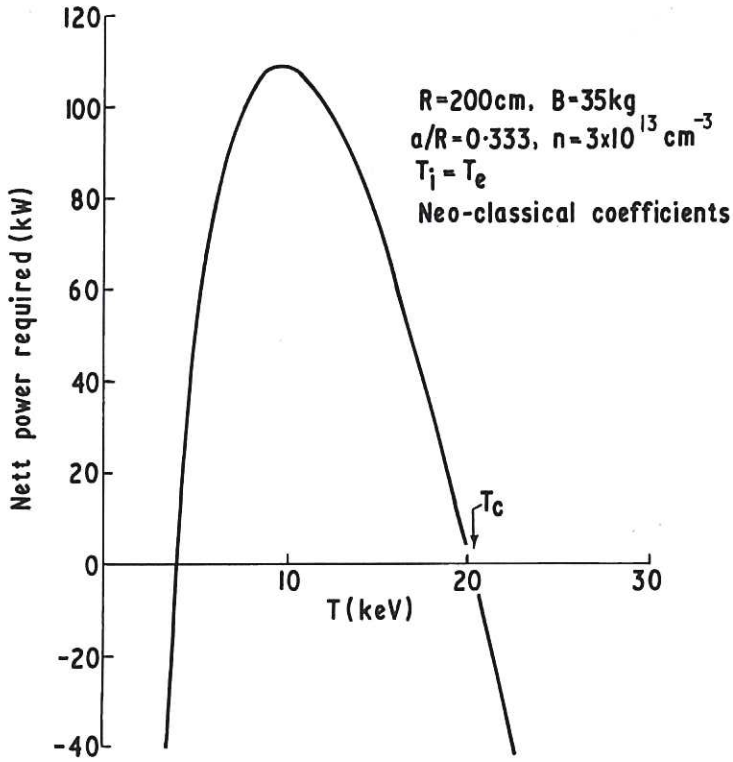

Fig.3 Nett power required to maintain the plasma at a temperature T for our minimum ignition case. The critical temperature at which the plasma becomes self-sustaining is shown.

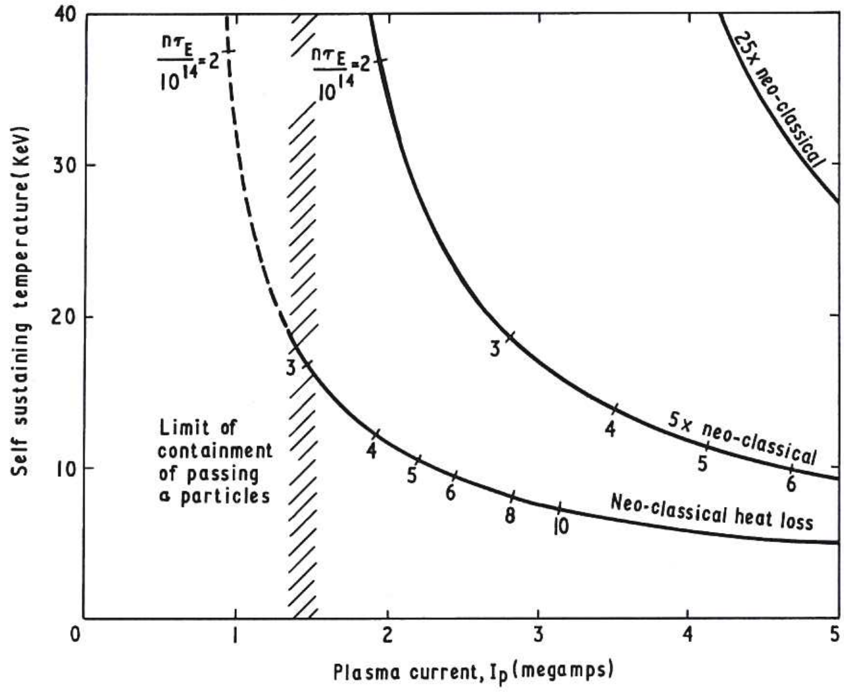

Fig.4 The critical temperature for the plasma to become self-sustaining energetically v. plasma current for different assumed neo-classical coefficients. The values of $n\,\tau_E$ are indicated on the curves.

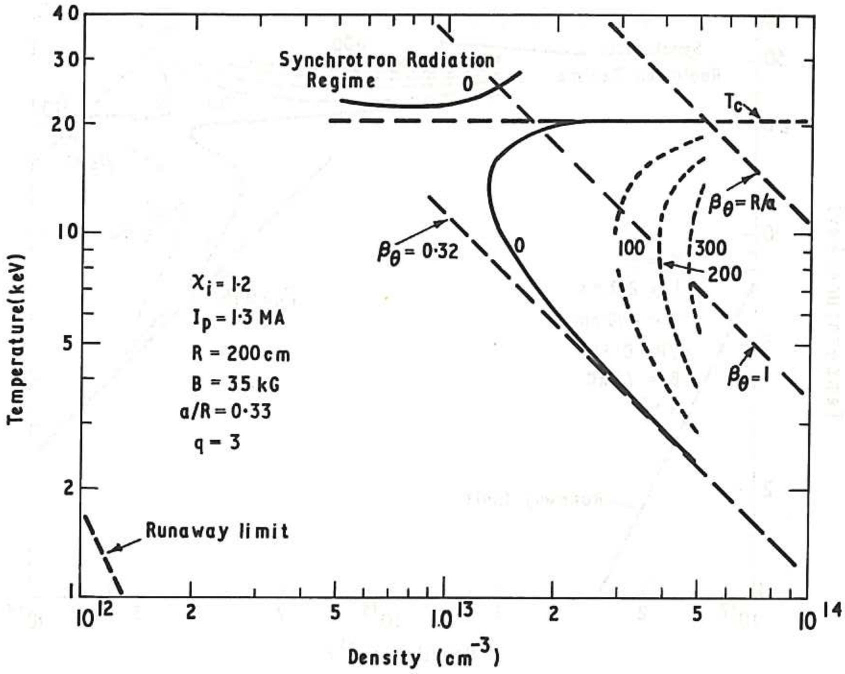

Fig.5 Contours of power required to sustain a given temperature at a given density for a strictly neo-classical case ($\chi_i = 1.2$). The line marked 0 assumes no additional heating, the dotted curves show the equilibrium temperature for 100, 200, 300 kilowatts heating respectively. The ohmic heating limit given by equation (3) and the self-sustaining temperature given by equation (1) are also shown.

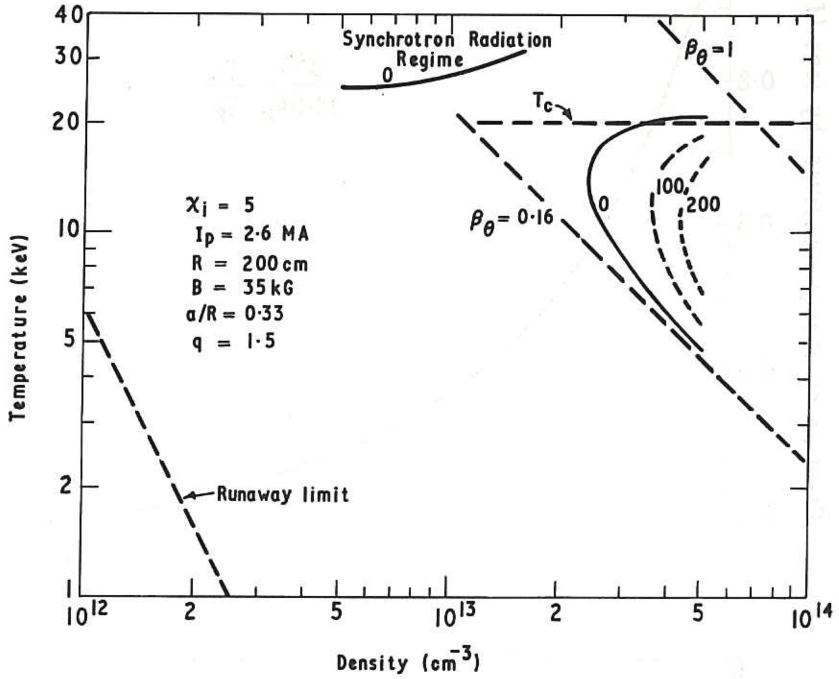

Fig.6 Contours of power required to sustain a given temperature for 4 × neo-classical heat loss ($\chi_i = 5$). q has been reduced to 1.5 to raise I_p and keep the ignition temperature at 20 keV.

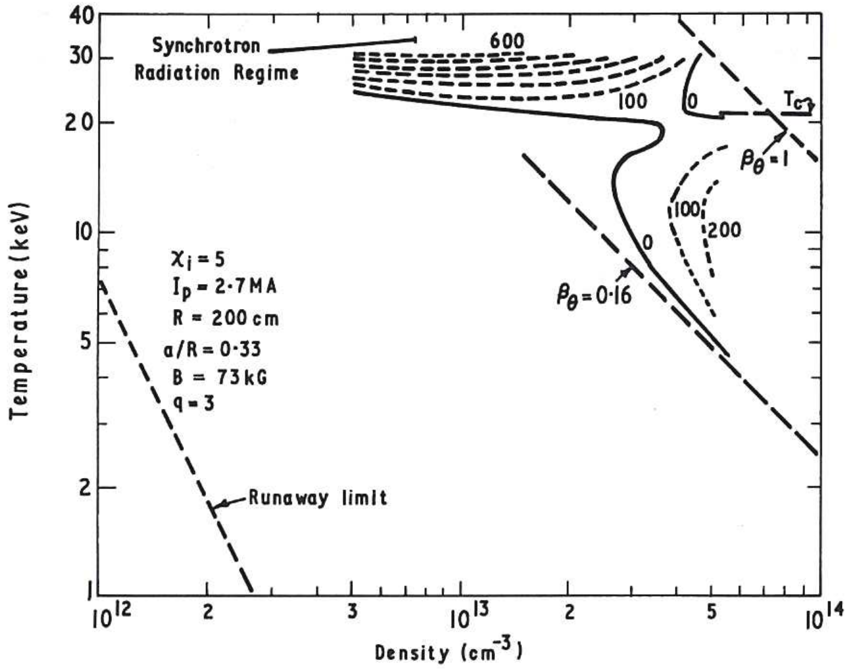

Fig.7 Same conditions as for Fig.6 except that, keeping q = 3, the magnetic field has been raised to keep the ignition temperature at 20 keV.

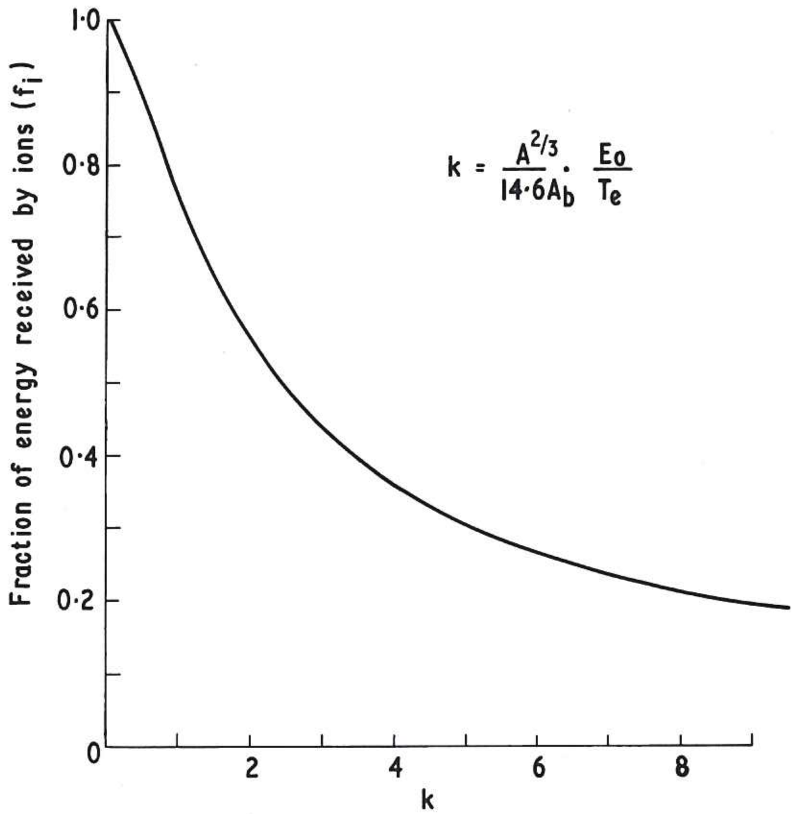

Fig.8 Fraction of injected particle energy conveyed to the plasma ions v. the injected particle energy expressed in normalised units.

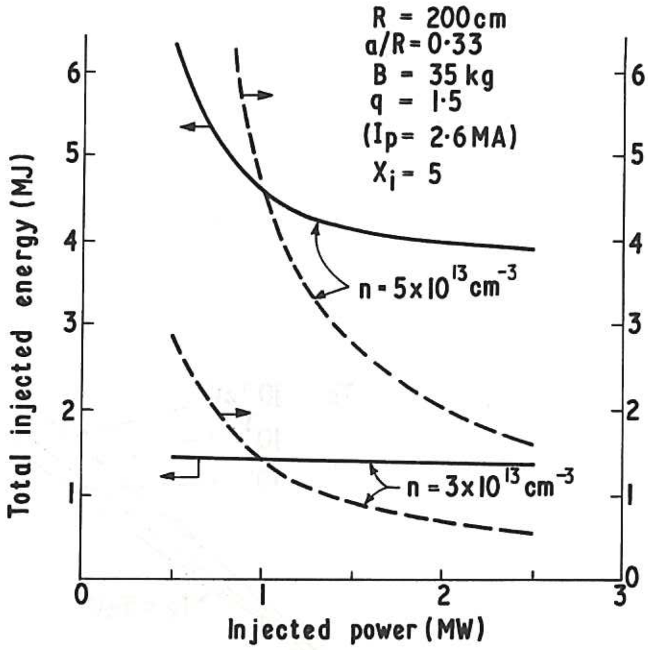

Fig.9 The total injected energy and the time to reach the self-sustaining condition v. injected power for two assumed densities. 4 × neo-classical loss is assumed with the same conditions as for Fig.6.

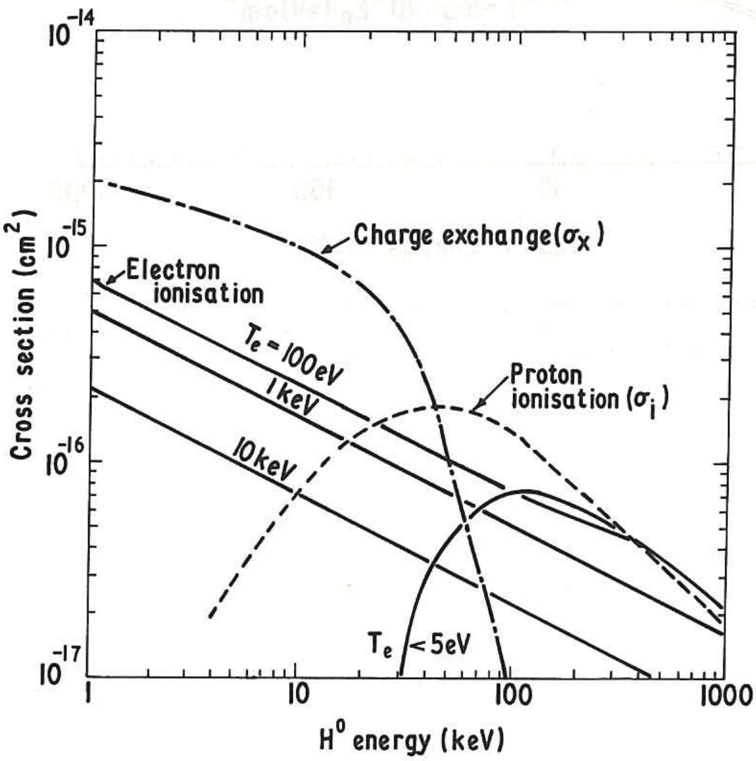

Fig.10 The separated cross sections for ionization of the injected neutral beam v. energy. In the case of electron ionization, when $v_e > v_o$, $\overline{\sigma_e v_e}/v_o$ is shown for a maxwellian electron velocity distribution at temperature T_e; for $v_e < v_o$ the cross section σ_e is shown for a velocity v_o.

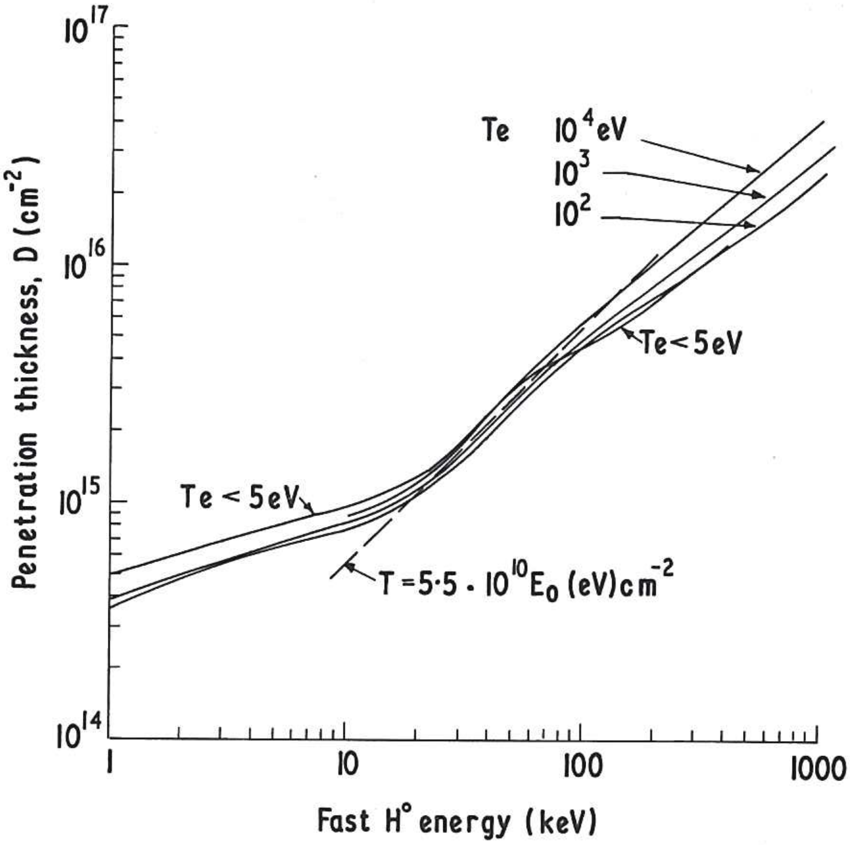

Fig.11 The penetration thickness $\int n d\ell$ for a 1/e attenuation of an H^o injected beam v. injected energy for various values of T_e. For injection of D^o atoms the energy scale should be multiplied by 2.