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ON THE DAMPING OF FINITE AMPLITUDE ELECTRON PLASMA WAVES IN COLLISIONLESS PLASMA

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ABSTRACT

Experimental measurements of the collisionless damping of a finite amplitude electron wave confirm recent self-consistent numerical solutions of the non-linear Vlasov equation.

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It has been shown theoretically, by a number of authors[1-14], that the propagation of an electron plasma wave in collisionless plasma depends on the initial amplitude of the wave. This behaviour occurs because electrons which have velocities close to the wave phase velocity (resonant electrons) become trapped in, and oscillate in, the potential wells of the wave; the number of trapped electrons and their frequency of oscillation are determined by the depths of the wells. This can lead to a decrease, or even change of sign, in the instantaneous damping rate[1-14] and to a change in the phase velocity[14] with increasing initial amplitude, or to the growth of other waves in the system[15], the so-called side-band instability.

The theoretical approaches used to investigate the time development of a finite initial amplitude electron wave may be broadly divided into two groups. The first method, used by Knorr[1], Armstrong[2] and Nührenberg[3], is to solve simultaneously Vlasov's and Poisson's equations using numerical techniques. Their results show that the damping is exponential and that the rate γ_L is that predicted by the linear Landau theory[16] if the initial amplitude ϕ_o is very small ($e\phi_o/k_BT \ll 1$) or if the observation time t is very short ($\omega_B t \ll 1$); that the damping rate decreases (and may change sign, indicating wave re-growth) when ϕ_o and t are increased, but that at very large amplitudes ($e\phi_o/k_BT \sim 1$) even the initial damping rate is not exponential, the instantaneous rate exceeding γ_L. [k_B is Boltzmann's constant, T is the electron temperature, $\omega_B = k_o(e\phi_o/m)^{\frac{1}{2}}$ is the frequency of oscillation of the trapped electrons, e and m are the electronic charge and mass respectively, k_o is the wave number of the wave with angular frequency ω_o].

The second approach separates the electron distribution function into a resonant part and a non-resonant part, and solves exactly the equations of motion for the trapped electrons. This method was used by O'Neil[4] who assumed the trapped electrons to move in the potential wells of a constant amplitude wave (i.e. $\gamma_L \rightarrow 0$). His results show that after some initial Landau damping the wave amplitude oscillates and reaches a constant amplitude after a long observation time due to phase mixing. This is in contrast to the behaviour implicit in the work of Al'tshul and Karpman[5] where the amplitude oscillations persist for all times. Bailey and Denavit[6] extended the work of O'Neil to allow for a slow variation in the wave amplitude ($\gamma_L \neq 0$) for small values of the parameter $q \equiv \gamma_L/\omega_B$ ($q \ll 1$), and obtained results qualitatively similar to O'Neil's. Gary[7], using a perturbation method to solve the non-linear Vlasov equation, obtained results qualitatively similar to those of Knorr and Armstrong. All of this work, as was pointed out by Dawson and Shanny[8], is restricted by the assumption that the slope of the initial distribution function is constant over the resonant region, and the recovery of linear Landau damping at short observation times is a consequence of this restriction. Further, none of it is completely self-consistent insofar as it does not include the full effect of the varying wave amplitude on the electron distribution function and vice versa.

A third approach, which avoids these limitations, is computer simulation, in which the self-consistent motions of a large number of electrons (but much smaller than the number in a laboratory experiment) are followed. Dawson and Shanny[8] find, in agreement with earlier work[1-3], that when $e\phi_o/k_BT \sim 1$ the initial damping of the wave is not exponential, and is more rapid than that predicted by Landau[16]. Denavit and Kruer[9], using this method, show that for this condition the side-band instability occurs.

Recently Sugihara and Kamimura[10], extending the work of co-workers[11], computed self-consistent equilibrium solutions to the initial value problem which effectively cover the range $0 < q < \infty$ and which recover the result of O'Neil as $q \to 0$ and linear Landau damping as $q \to \infty$ (it is, however, still assumed that the slope of the initial electron distribution function in the resonant region is constant). Their solutions extend over much longer times than in earlier treatments and show that only for $q \geqslant 3$ does the wave damp at a constant rate γ_L; for $q > 0.77$ the damping rate decreases monotonically with time from its initial value γ_L, while for $q < 0.77$ the amplitude, after several oscillations, becomes constant with time, its actual value depending on the precise value of q. Oei and Swanson[12] have also very recently obtained self-consistent equilibrium solutions which appear to be similar to those of Sugihara and Kamimura. However, their results are presented in a way which makes direct, precise comparison with our experimental results difficult.

The only published experimental data relevant to all this theoretical work is that by Malmberg and Wharton[17]: they observed spatial amplitude oscillations when ϕ_o was increased, in qualitative agreement with O'Neil's theory modified to the spatial situation, as by Lee and Schmidt[13].

In this letter we report measurements of the spatial damping of an electron plasma wave which show, in detail, the transition from linear Landau damping to oscillatory behaviour, and which are well described by the results of Sugihara and Kamimura[10] for the range of amplitudes below that at which side-bands appear and begin to extract significant energy from the original wave.

Our experiments were performed in a quiescent, sodium plasma column, radially confined by a strong (2kG), uniform, axial magnetic field. The electrons were produced by thermionic emission and the Na^+ ions by contact ionization at a single tungsten surface, diameter 2.5 cm, uniformly heated ($\pm$ 30°K) to a temperature $\sim$2,500°K. The column was terminated 80 cm from the hot plate by a cold, plane, tantalum disk which, because it was at floating potential, reflected all but the fastest electrons; thus the unperturbed electron distribution can be regarded as a full one-dimensional Maxwellian with the same temperature as the hot plate. At the low densities used, $n \sim (1 - 3) \times 10^7\,\mathrm{cm}^{-3}$ ($\omega_{pe}/2\pi \sim$ 30-50 MHz) the axial density was uniform to better than 1% and the mean free path for an electron was several times the column length. The neutral background gas pressure was $\leq 5 \times 10^{-7}$ torr. The propagation of small amplitude waves in this plasma is well understood and, as has been shown already[18], is well described by linear Landau theory modified for finite radial effects.

Waves, propagating in their lowest radial eigenmode, were launched at $x = 0$ from a short, fine wire probe antenna connected to the end of a coaxial transmission line matched, close to its end, to a signal generator. Resulting plasma fluctuations were detected at positions x between the transmitter and the cold-end-plate by a similar probe which was matched to the plasma with a high-input-impedance amplifier. Because of uncertainty in the probe-plasma impedance absolute values of ϕ_o could be estimated to an accuracy ± 20% although its relative values were known more accurately (± 2%). The spatial variation in the amplitude $\phi(x)$ of the plasma fluctuations was recorded logarithmically using a very narrow band ($\Delta\omega$ = 300 Hz) r.f. receiver and an x-y recorder. Phase velocities, determined from the interferogram obtained by comparing the phase of the plasma signal with that of a reference signal from the signal generator in a balanced mixer, were chosen so that there were between three and five Landau damping lengths included in the length of plasma used (50 cm). This allowed the non-linear behaviour to develop sufficiently, in the distance available, for it to be clearly distinguishable from the linear. Changes in the phase velocity due to non-linear effects[14] were undetectable (< 1% for our conditions).

Fig.1 shows experimental points, taken from traces similar to those shown in Fig.3, for the relative amplitudes of waves of the same frequency but different initial amplitudes ϕ_o, analysed and plotted in terms of the dimensionless quantities used in reference 10, i.e. $\log(\phi/\phi_o)$ versus $\gamma_L t \equiv k_i x$ (for weakly damped waves $\gamma_L = k_i \partial\omega/\partial k$ where k_i is the inverse damping length[13]) with parameter q. The experimental uncertainty in

k_i was ±2% and that in $\partial\omega/\partial k$, determined from the measured dispersion, was ±5%, so that with the previously mentioned uncertainty in ϕ_o, q was known absolutely to within ±17%. However, for a fixed frequency, the relative values of q were known to ~1%. The results show very good agreement with the theoretically predicted curves[10] for $q \geqslant 0.45$; in particular they demonstrate the monotonic but non-exponential decrease in amplitude for $q > 0.8$ and a transition to periodic behaviour for $q < 0.6$. (The theory predicts an asymptotic stationary amplitude $\phi^* \simeq 0.04\ \phi_o$ for $q = 0.77$). For $q \leqslant 0.4$ side-bands[15] could be detected above the background noise level; this presumably explains the greater damping suffered by the larger amplitude waves than that predicted by the theory (which does not consider the stability of the system).

These effects can also be seen clearly in Fig. 2, which shows, for the same data as Fig. 1, the attenuation suffered by a wave in traversing a distance $x = 44$ cm, ($k_i x = 4$) as a function of ϕ_o compared with the theory. Only for very small initial amplitudes ($\phi_o \leqslant 0.1$ mV) would the measurements agree with the linear theory, while for $\phi_o > 2$ mV the attenuation exceeds that of the non-linear theory. The amplitude at which side-bands were observed is indicated with an arrow.

To demonstrate that the observed departure from linear Landau damping is caused by electrons trapped in the potential wells of the wave (ω_o, k_o), the damping of the wave was measured in the presence of a second perturbing wave (ω_1, k_1) whose frequency was well removed from that of the first. When the amplitudes of the two waves were comparable, the only time invariant potential well in which the electrons could be trapped travelled with the phase velocity of the beat wave[19] $(\omega_o - \omega_1)/(\underline{k}_o - \underline{k}_1)$, i.e. well removed from $v_\phi = \omega_o/k_o$. For such conditions the amplitude variations due to trapped electrons are expected to disappear. Experimental data illustrating this are given in Fig. 3: it shows (i) a very small amplitude wave ($e\phi_o/k_BT \simeq 2\times10^{-3}$) which exhibits linear Landau damping; (ii) a larger amplitude wave ($e\phi_o/k_BT \simeq 10^{-2}$) whose damping departs, at a distance $x \geqslant 15$ cm, from a pure exponential, and (iii) the same wave as shown in (ii) but propagating in the presence of a second perturbing wave, demonstrating that the damping is the same as that of (i) (i.e. essentially the linear value).

For a one-dimensional Maxwellian, the condition that the slope of the initial electron distribution function over the resonant region can be regarded as constant can be expressed as $2e\phi_o/k_BT < [v_\phi v_T/(2v_\phi^2 - v_T^2)]^2$ where $v_T = (2k_BT/m)^{\frac{1}{2}}$ is the electron thermal velocity[8]. For the conditions of Figs. 1 and 2 this inequality, which is satisfied for $\phi_o \leq 4$ mV($q \geq 0.33$), was never seriously violated: thus the observed initial Landau damping is to be expected. Rapid, non-exponential, initial damping predicted by some of the theoretical work[1-3,8] was not observed and it would appear from our experiments that under most laboratory conditions increased damping of this sort would be obscured by other non-linear effects, for example the side-band instability[15]. Further, at sufficiently large amplitudes the decay of an electron wave into either a second electron wave and an ion wave[20] or into two other electron wave modes[21] would also lead to a damping exceeding the linear Landau rate. The spectrum was carefully checked during these measurements to ensure that none of these processes occurred.

We feel that a comment is in order on the plasma conditions needed for such measurements. It is essential that the plasma column have a very uniform axial density. A variation in density of only 1% along the column would cause the average linear damping rate to change by 20% for the conditions of Fig. 1; this <u>linear</u> effect of inhomogeneity would obscure any change in the damping caused by the <u>non-linear</u> electron trapping process. The method used to measure damping is reliable only for linear values of k_i in the range $5 \times 10^{-3} \leqslant k_i \leqslant 0.5\text{cm}^{-1}$; it must therefore be ensured that the effective collision frequency for the electrons ν is sufficiently small for Landau damping, and not collisional damping to be the dominant damping mechanism somewhere within this range. In our experiment $\nu/\omega_o \sim 2 \times 10^{-3}$, and thus measured values of $k_i/k_o \geqslant 10^{-2}$ may be unambiguously attributed to collisionless Landau damping[18]. Further, it is important that $\nu/\omega_B << 1$; in our case this ratio was always < 0.2 even at the smallest amplitudes used ($\phi_o = 0.5$ mV). At higher plasma densities both ν/ω_o and ν/ω_B would have increased sufficiently that the plasma could no longer have been considered collisionless in the required sense.

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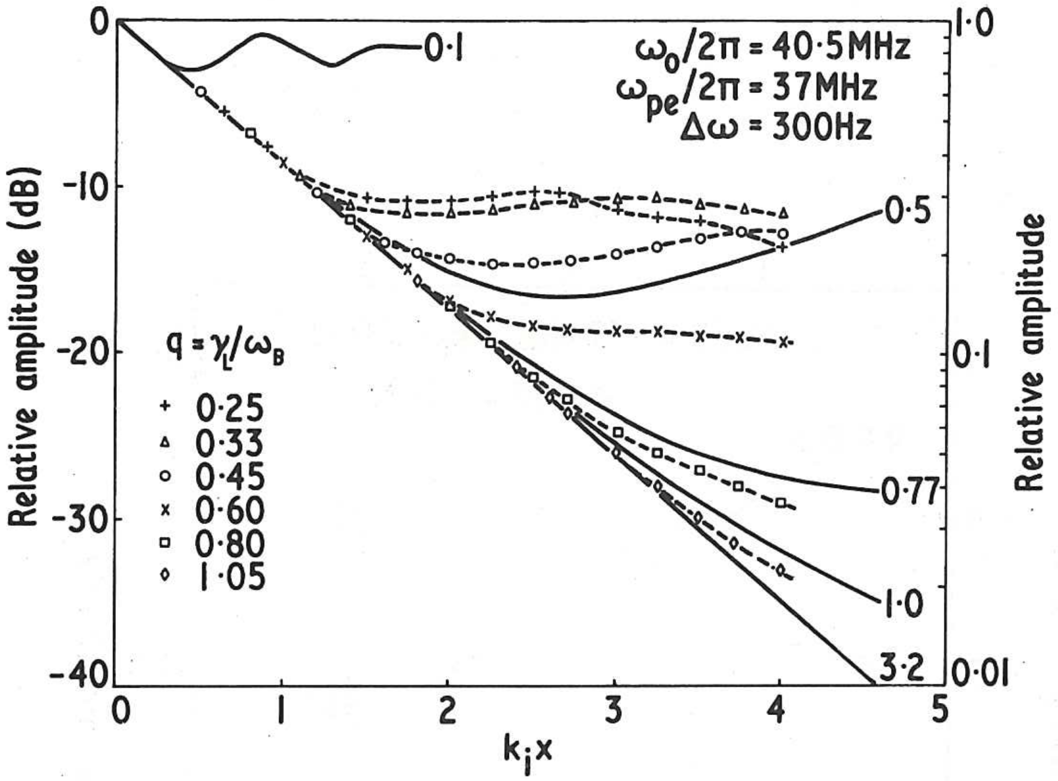

Figure 1 Experimental data and theoretical curves[10] showing relative spatial amplitude variations for different initial amplitudes. $k_o = 3.64\ \mathrm{cm}^{-1}$, $k_i = 0.09\ \mathrm{cm}^{-1}$.

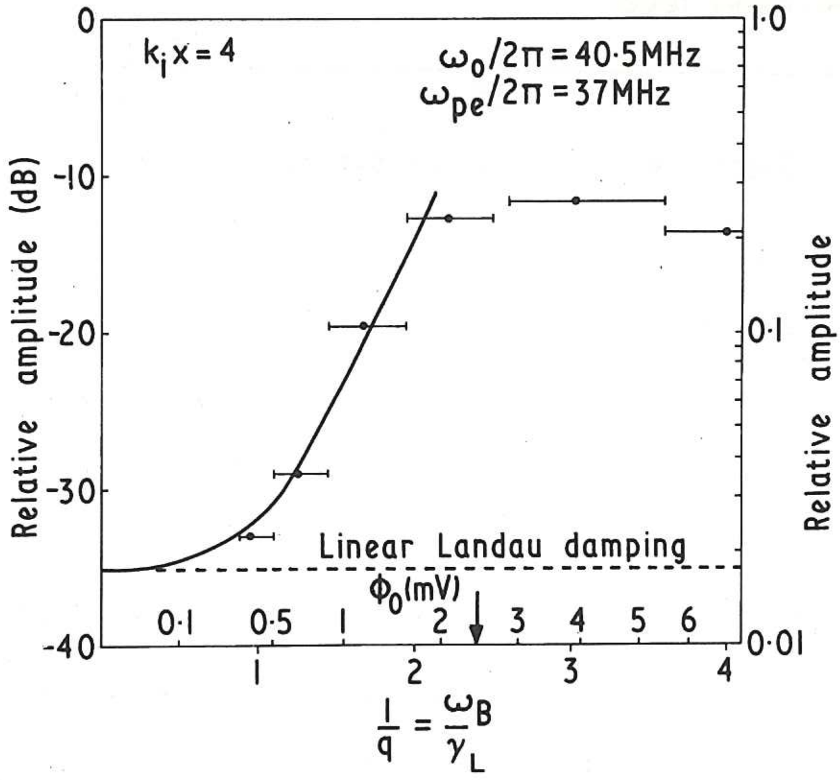

Figure 2 Effect of different initial wave amplitudes on attenuation.

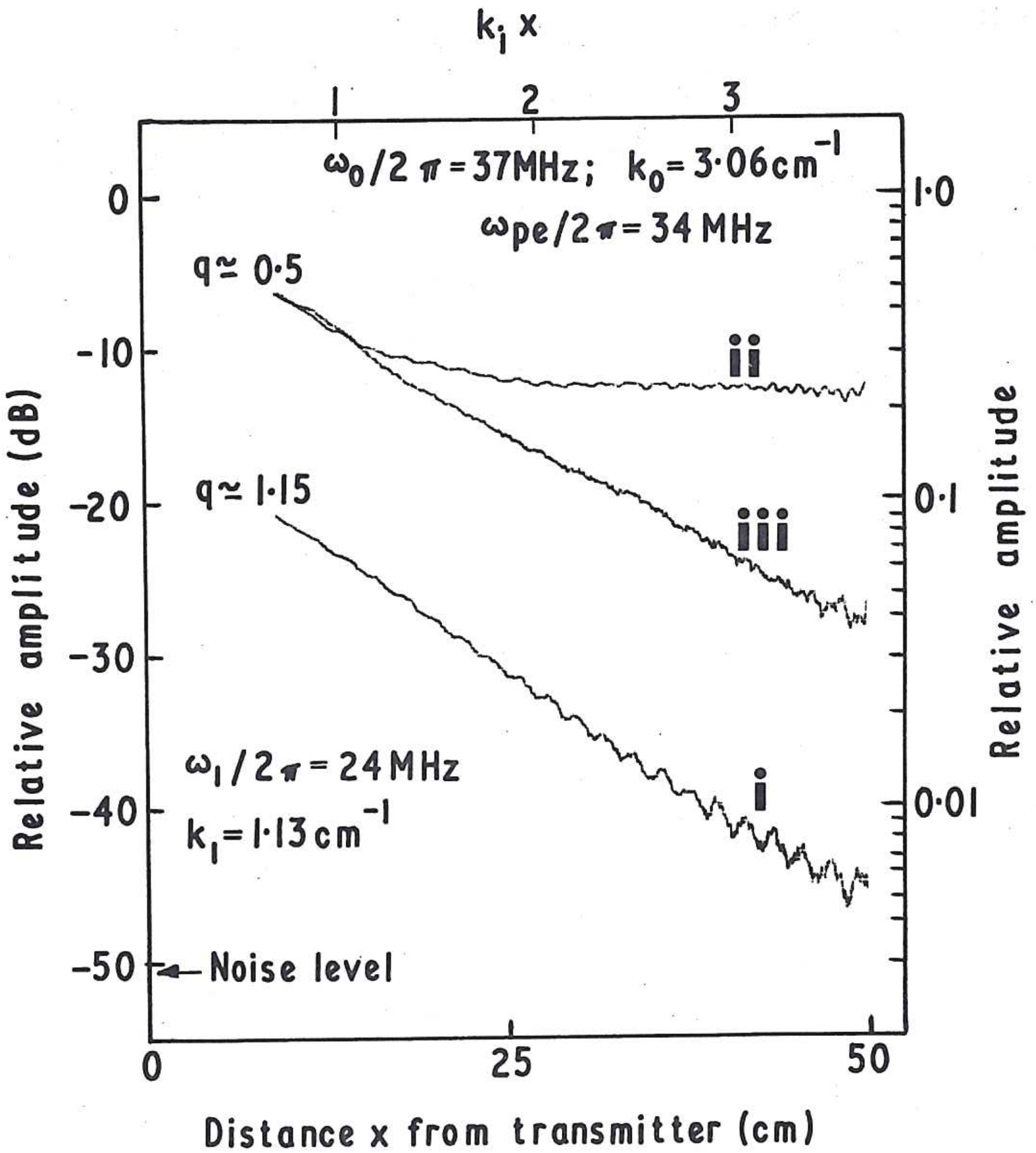

Figure 3 Raw experimental data showing spatial amplitude variation of (i) wave (ω_o, k_o) with $\phi_o \simeq 0.4$ mV; (ii) wave (ω_o, k_o) with $\phi_o \simeq 2.2$ mV, and (iii) same wave as (ii) but in presence of undamped wave (ω_1, k_1) launched at x = 67 cm with amplitude $\simeq$ 4 mV and propagating in opposite direction to (ω_o, k_o). $k_i/k_o = 2.4 \times 10^{-2}$.