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by

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ABSTRACT

The interest in transit time magnetic pumping as a method to heat the plasma in a Tokamak or in a Stellarator beyond the limits of Ohmic heating, has greatly increased in recent years. The existing theories, based on the motion of the guiding centre or on the properties of the dielectric tensor, gives however an incomplete and sometimes contradictory view of the process involved. It is shown here that a flux of parallel kinetic energy exists within the plasma. This energy flux is not accompanied by any transport of mass and accounts completely for the apparent disagreement between the existing theories.

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1. INTRODUCTION

We shall consider here the modified version of transit time magnetic pumping (a strong modulation of the magnetic field over a short distance [1]) consisting in the excitation into the plasma of an electromagnetic wave by an array of external coils. The frequency of the wave is much below the ion gyrofrequency and the phase velocity v_φ is close to the ion thermal velocity. At low β, the ratio of ionic kinetic pressure to magnetic pressure, v_φ is much less than the Alfven speed v_A so that the wave cannot propagate across the magnetic field and is essentially evanescent. Under these conditions the collisionless heating of the ions in the plasma has been evaluated [2,3], by using the concept of the motion of the guiding centre. The ions move under the force $F_z = -\mu \frac{\partial \tilde{B}_z}{\partial z}$, $\mu = mv_\perp^2/2B_o$ being the equivalent magnetic moment and $\tilde{B}_z$ the component of the oscillating magnetic field parallel to the static magnetic field B_o, and the electrons are assumed to be a fluid which is in mechanical and thermal equilibrium. Vacuum RF fields are considered and an electric field parallel to the static magnetic field is introduced consistent with charge neutrality and the relative motion of ions and electrons along the magnetic field lines. The main result is that the power absorbed by the plasma per unit volume is proportional to the plasma density, i.e. the rate of increase of ion parallel energy is independent of the local plasma density.

Using the properties of the dielectric tensor of a uniform plasma in a static magnetic field, the power lost by an external current flowing through the plasma has been evaluated [4], and the result is in agreement with the one which can be obtained from [2,3]. It can be shown however that the whole RF power is absorbed by the plasma in a very narrow region near the external coil, while the remainder of the plasma gives up, rather than absorbs, energy to the RF field. Notice incidentally that the boundary conditions used in reference [4] are incorrect, a discontinuity in the tangential component of the electric field at the coil radius has not been accounted for. In fact it is not possible to satisfy all the boundary conditions by considering the presence of one plasma wave only, a second strongly damped wave must be excited in the plasma and the power lost by this wave in a very narrow region near the external coil, accounts both for the power lost by the remainder of the plasma and by the external system.

In a subsequent paper [5] the RF power absorbed by a column of non-uniform plasma with the external currents generating the RF fields flowing outside the plasma column has been evaluated. It is found that, although the total

RF power absorbed by the plasma is still positive and in agreement with the previous estimates, the whole RF power is absorbed in a region of the plasma where a sufficiently high density gradient is present, while the remainder of the plasma rather supplies energy to the RF field. One would therefore expect that only a few ions at the plasma boundary or in the region where the external current flows can be heated at a very high rate, while the energy transfer from the external system to the plasma is accompanied by an anomalous 'heat transfer' from regions of nearly constant plasma density to the plasma boundary. As this would cast severe doubts on the feasibility of transit time magnetic pumping as a method to heat the plasma both in present day experiments and in a thermonuclear reactor, it seems worthwhile to reconsider here the whole problem and cast some light on the points which still remain obscure.

2. NON UNIFORM PLASMA WITH PLANE GEOMETRY

We consider here a non uniform plasma such that the plasma density at the coil radius has dropped much below its value in the region of maximum density. We choose for simplicity plane geometry with the density gradient in the $\hat{x}$ direction, the static magnetic field B_o in the $\hat{z}$ direction and the external current, which we assume to be of the travelling wave type $J_{ext} = J^{\neq} \delta(x) \exp\{i(kz-\omega t)\}$ in the $\hat{y}$ direction.

The zero-order distribution function can be constructed from the constant of the motion and we choose it to be of the form:

$$f_o(x,v) = \left(\frac{m}{2\pi KT}\right)^{\frac{1}{2}} n_o \; \psi\left(x + \frac{\varepsilon v_y}{\Omega}\right) e^{-\frac{m}{\partial KT} v^2} \qquad \dots (1)$$

with Ω , ε being respectively the cyclotron frequency and the sign of the electric charge of the particle considered (here: singly ionized ions of one sort and electrons).

We do not specify the function ψ but we suppose it to be continuous with all its derivatives and to change very slowly over a distance comparable with a Larmour radius so that:

$$n(x) = \iiint_{-\infty}^{\infty} f_o(x,v)\; d^3v \approx n_o \; \psi(x) \qquad \dots (2)$$

where $0 < \psi(x) \leq 1$ and n_o is the maximum plasma density. We linearise the Vlasov equation

$$\frac{\partial f_1}{\partial t} + v \cdot \nabla f_1 + \frac{\varepsilon e}{m} \frac{v \times B_o}{c} \cdot \nabla_v f_1 = \frac{-\varepsilon e}{m}\left(\tilde{E} + \frac{v \times \tilde{B}}{c}\right) \cdot \nabla_v f_o \qquad \dots (3)$$

and assume that all the perturbed quantities, f_1 , $\tilde{E}$, $\tilde{B}$ have a time space dependence of the form: $F(x)\exp\{i(kz-\omega t)\}$ and that ω has a small positive imaginary part such that at $t = -\infty$ f_1 vanishes. We can formally solve Eq. (5):

$$f_1(x,v,t) = -\frac{\varepsilon e}{m} \int_{-\infty}^{t} \left[E(x',t') + \frac{v' \times B(x',t')}{c}\right] \cdot \nabla_v \; f_o \; dt' \qquad \dots (4)$$

where the integral must be evaluated along the zero order trajectory

$$\dot{v}' = \epsilon \, \Omega \, \vec{v}' \times \vec{z} \qquad \ldots (5)$$

with $v'(t') = v$, $x'(t') = x$ for $t = t'$.

We assume now that all the perturbed quantities, as well as $\psi(x)$, vary very slowly over a distance comparable with a Larmor radius and expand them around $x = x'$.

We can then evaluate the first moments of the velocity distribution $f_1(x,v,t)$:

$$\langle v \rangle = \iiint_{-\infty}^{\infty} \vec{v} \, f_1(x,v,t) \, d^3v \qquad \ldots (6)$$

and combine the result obtained for ions and electrons to obtain the equivalent dielectric tensor in the form:

$$\bar{\bar{K}} = \bar{\bar{I}} + \frac{4\pi i e}{\omega} \left[\bar{\bar{M}}_i - \bar{\bar{M}}_e \right] \qquad \ldots (7)$$

where M_i, M_e are defined from

$$\langle \vec{v}_{i,e} \rangle = \bar{\bar{M}}_{i,e} \cdot \vec{E} \, . \qquad \ldots (8)$$

Neglecting terms of higher order in the small quantity $(\omega/\Omega_i)^2$ we obtain for the elements of $\bar{\bar{K}}$:

$$\epsilon_{11} = 1 + n_\parallel^2 \, \alpha_{11} \, \psi(x)$$

$$\epsilon_{22} = 1 + n_\parallel^2 \, \alpha_{11} \psi(x) - \frac{c^2}{\omega^2} \alpha_{22} \left\{ \psi(x) \frac{\partial^2}{\partial x^2} + \frac{d\psi}{dx} \frac{\partial}{\partial x} \right\}$$

$$\epsilon_{23} = \frac{kc^2}{\omega^2} \alpha_{23} \left[\psi(x) \frac{\partial}{\partial x} + \frac{d\psi}{dx} \right]$$

$$\epsilon_{32} = -\frac{kc^2}{\omega^2} \alpha_{23} \, \psi(x) \frac{\partial}{\partial x}$$

$$\epsilon_{33} = 1 + n_\parallel^2 \, \alpha_o \psi(x) + \frac{c^2}{\omega^2} \alpha_{33} \frac{\partial}{\partial x} \left\{ \psi(x) \frac{\partial}{\partial x} \right\}$$

$$\epsilon_{12} \approx \epsilon_{21} \approx \epsilon_{13} \approx \epsilon_{31} \approx 0 \, , \qquad \ldots (9)$$

with:

$$\alpha_{11} = \eta\ , \quad \alpha_{22} = \eta\ Z_o/\zeta_o\ , \quad \alpha_{23} = \eta\ \frac{\Omega_i}{\omega}\ \zeta_o Z_o$$

$$\alpha_{33} = \eta(1 + \zeta_o Z_o)\ , \quad \alpha_o = 2\zeta_o^2 \eta \left(\frac{\Omega_i}{\omega}\right)^2 \left(1 + \frac{T_i}{T_e} + \zeta_o Z_o\right)$$

$$\eta = \frac{\Pi_i^2}{k^2 c^2} \frac{\omega^2}{\Omega_i^2}\ , \quad \Pi_i = \left(\frac{4\pi n_o e^2}{m_i}\right)^{\frac{1}{2}}\ , \quad n_{\parallel} = \frac{kc}{\omega}$$

$$\zeta_o = \omega/k\ (m/2\kappa T)^{\frac{1}{2}}\ , \quad \rho_i = (\kappa T/m_i \Omega_i^2)^{\frac{1}{2}}\ . \qquad \ldots (10)$$

Here

$$Z_o = \exp(-\zeta_o^2)\left\{i\sqrt{\pi}\ \frac{k}{|k|} - 2\int_0^{\zeta_o} e^{t^2}\ dt\right\}$$

is the plasma dispersion function.

Combining Eq. (9) with Maxwell equation we obtain a 'dispersion relation' in the form:

$$\left[1 - \alpha_{11}\ \psi(x)\right] E_x + \frac{i}{k} \frac{\partial E_z}{\partial x} = 0$$

$$\left[1 - \alpha_{22}\ \psi(x)\right] \frac{\partial^2 E_y}{\partial x^2} - \alpha_{22} \frac{d\psi}{dx} \frac{\partial E_y}{\partial x} - k^2\left[1 - \alpha_{11}\ \psi(x)\right] E_y$$
$$+ k\ \alpha_{23} \frac{\partial}{\partial x}\left[\psi(x)\ \frac{\partial E_z}{\partial x}\right] = 0$$

$$\left[1 + \alpha_{33}\ \psi(x)\right] \frac{\partial^2 E_z}{\partial x^2} + \alpha_{33} \frac{d\psi}{dx} \frac{\partial E_z}{\partial x} + k^2 \alpha_o \psi(x) E_z$$
$$- k\alpha_{23} \psi(x) \frac{\partial E_y}{\partial x} - ik \frac{\partial E_x}{\partial x} = 0\ . \qquad \ldots (11)$$

For $v_\varphi = v_{th}$ we have $\eta \approx \beta \ll 1$ so that neglecting terms of the order η or $\rho_i \left|\frac{1}{\psi} \frac{d\psi}{dx}\right|$ with respect to unity, an approximate solution of Eq. (10) is found:

$$E_y = E_y^v\ , \qquad E_z = \frac{1}{K} \frac{\alpha_{23}}{\alpha_o} \frac{\partial E_y^v}{\partial v} \qquad \ldots (12)$$

where E_y^v is the electric field produced by the external currents in absence of plasma.

The RF power $P = \left\{- \frac{i\omega}{16\pi} \vec{E}^{*} \cdot (\bar{\bar{K}} - \bar{\bar{I}}) \cdot \vec{E} + cc\right\}$ absorbed by the plasma per unit volume is then given by

$$P = - \frac{\kappa T c^2}{2B_o^2 \omega} \left[n(x) E_y^v \frac{\partial^2 E_y^v}{\partial x^2} + \frac{dn}{dx} E_y^v \frac{\partial E_y^v}{\partial x} \right]$$

$$\times \; \mathrm{Im}\left\{ \zeta_o Z_o \left(2 + \frac{\zeta_o Z_o}{1 + T_i/T_e + \zeta_o Z_o} \right) \right\} . \qquad \ldots (13)$$

It is evident from (12) that positive power absorption is only possible if $\frac{dn}{dx}$ and $\frac{dE_y}{dx}$ have opposite sign and

$$\left| \frac{1}{n} \frac{dn}{dx} \right| > \left| \frac{1}{E_y^v} \frac{dE_y^v}{dx} \right| . \qquad \ldots (14)$$

If these conditions are not satisfied the plasma supplies energy to the RF field. A uniform plasma therefore gives up, rather than absorbs, energy to the RF field.

3. A PHYSICAL PICTURE OF MAGNETIC PUMPING

In order to give an insight in the physical process of transit time magnetic pumping which accounts for the rather surprising result of the preceding section, we consider now the motion of a charged particle drifting through an electromagnetic field of the form given in (12). We simplify the mathematical calculations by neglecting the component of the electric field parallel to the static magnetic field. This is physically compatible with the self consistent electromagnetic field (12) if we assume $T_e \ll T_i$, T_e , T_i being electron and ion temperature respectively.

The components of the electromagnetic field are then given by:

$$E_y = \varepsilon(x) \cos(kz - \omega t)$$

$$B_x = -\frac{kc}{\omega} \varepsilon(x) \cos(kz - \omega t) \qquad \ldots (15)$$

$$B_z = \frac{c}{\omega} \frac{\partial \varepsilon}{\partial x} \sin(kz - \omega t) \quad .$$

We do not specify the function $\varepsilon(x)$ but we assume it to change very slowly over a distance comparable with a Larmor radius so that $\rho_i \left| \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial x} \right| \ll 1$.

The equation of motion of a single particle is then given by

$$\dot{v}_x = \Omega\, v_y + \frac{e}{m} \frac{v_y}{\omega} \frac{\partial \varepsilon}{\partial x} \sin(kz - \omega t)$$

$$\dot{v}_y = -\, \Omega\, v_x - \frac{e}{m} \frac{v_x}{\omega} \frac{\partial \varepsilon}{\partial x} \sin(kz - \omega t) - \frac{e}{m}\left(\frac{kv_z}{\omega} - 1\right) \varepsilon(x) \cos(kz - \omega t)$$

$$\dot{v}_z = \frac{e}{m} \frac{k}{\omega} v_y\, \varepsilon(x) \cos(kz - \omega t) \quad . \qquad \ldots (16)$$

The zero-order solution of Eq.(16) is the solution for zero electric field:

$$\dot{v}_{xo} = \Omega v_{yo} \quad , \quad \dot{v}_{yo} = -\, \Omega v_{xo} \quad , \quad \dot{v}_{zo} = 0 \; . \qquad \ldots (17)$$

After solving Eq.(17) we linearise Eq.(16) to obtain the equation of motion for the first order velocity v_1 :

$$\dot{v}_{x1} = \Omega v_{y1} + \frac{e}{m} \frac{v_{yo}}{\omega} \frac{\partial \varepsilon}{\partial x} \sin\beta$$

$$\dot{v}_{y1} = -\, \Omega v_{x1} - \frac{e}{m} \frac{v_{xo}}{\omega} \frac{\partial \varepsilon}{\partial x} \sin\beta - \frac{e}{m} \frac{\alpha_o}{\omega} \varepsilon(x) \cos\beta$$

$$\dot{v}_{z1} = \frac{e}{m} \frac{k}{\omega} v_{yo}\, \varepsilon(x) \cos\beta \qquad \ldots (18)$$

where $\beta = kz_o + \alpha_o t$, $\alpha_o = kv_{zo} - \omega$ and $\varepsilon(x)$ is to be evaluated along the zero order trajectory $\dot{x}(t) = v_{xo}(t)$ with $x = x_o$, $z = z_o$, for $t = 0$.

We can now expand $\varepsilon(x)$ around $x = x_o$:
$\varepsilon(x) = \varepsilon(x_o) + \partial\varepsilon/\partial x \, (x - x_o) + \ldots$ and solve Eq. (18) as an initial value problem: $\vec{v}_1 = 0$ at $t = 0$.

The second order velocity $\vec{v}_2$, again imposing $\vec{v}_2 = 0$ at $t = 0$, can now be obtained from:

$$\dot{v}_{x1} = \Omega v_{y2} + \frac{e}{m}\frac{v_{y1}}{\omega}\frac{\partial\varepsilon}{\partial x}\sin\beta + \frac{e}{m}\frac{v_{yo}}{\omega}\left(\frac{\partial^2\varepsilon}{\partial x^2} x_1 \sin\beta + \frac{\partial\varepsilon}{\partial x} kz_1 \cos\beta\right)$$

$$\dot{v}_{y2} = -\Omega v_{x2} - \frac{e}{m}\frac{v_{x1}}{\omega}\frac{\partial\varepsilon}{\partial x}\sin\beta - \frac{e}{m}\frac{v_{xo}}{\omega}\left(\frac{\partial^2\varepsilon}{\partial x^2} x_1 \sin\beta + \frac{\partial\varepsilon}{\partial x} kz_1 \cos\beta\right)$$

$$- \frac{e}{m}\frac{k}{\omega} v_{z1}\,\varepsilon(x)\cos\beta - \frac{e}{m}\frac{\alpha_o}{\omega}\left(\frac{\partial\varepsilon}{\partial x} x_1 \cos\beta - \varepsilon(x) kz_1 \sin\beta\right)$$

$$\dot{v}_{z2} = \frac{e}{m}\frac{k}{\omega} v_{y1}\,\varepsilon(x)\cos\beta + \frac{e}{m}\frac{kv_{yo}}{\omega}\left(\frac{\partial\varepsilon}{\varepsilon x} x_1 \cos\beta - \varepsilon(x) kz_1 \sin\beta\right) \quad \ldots (19)$$

where $x_1 = \int_0^t v_{x1}(t)\,dt$, $z_1 = \int_0^t v_{z1}(t)\,dt$ and $\varepsilon(x)$ is again to be evaluated along the zero order trajectory.

After solving Eq. (17) to Eq. (19) the zero, first and second order ion velocities can be obtained as a function of initial velocity and initial position. We can now evaluate the rate of change of particle energy:

$$\frac{d}{dt}\frac{mv^2}{2} = \vec{v}_1 \cdot \frac{d}{dt} m\vec{v}_1 + \vec{v}_2 \cdot \frac{d}{dt} m\vec{v}_o + \vec{v}_o \cdot \frac{d}{dt} m\vec{v}_2 \ldots \quad (20)$$

where on the right side of Eq. (20) we shall consider only terms which are non periodic in z_o. After averaging over the initial position z_o and over a Larmor period, the mathematical details are long and tedious and shall not be reported here, we obtain :

$$\frac{d}{dt}\left(\frac{mv_\perp^2}{2}\right) = 0$$

$$\frac{d}{dt}\left(\frac{mv_\parallel^2}{2}\right) = -\frac{e^2}{m}\frac{k^2 v_{\perp o}^4}{8\,\omega^2\,\Omega^2}\left(\frac{\partial \varepsilon}{\partial x}\right)^2\left[\omega\,\frac{\sin\alpha_o t}{\alpha_o^2} - k\,v_{zo}t\,\frac{\cos\alpha_o t}{\alpha_o}\right] \qquad \ldots (21)$$

where $v_{\perp o}$ is the initial perpendicular velocity and only terms to the second order in the small quantities $(^{\omega}/\Omega_i)^n$ or $\rho_i^n\left|\frac{1}{\varepsilon}\frac{\partial^n \varepsilon}{\partial x^n}\right|$ have been retained.

We average now Eq. (21) over the distribution of initial velocity which we assume to be of the form :

$$f_i(v_o) = \left(\frac{m}{2\pi K T_\perp}\right) e^{-mv_{\perp o}^2/2KT_\perp}\, g(v_{zo})$$

$$g(v_{zo}) = \left(\frac{m}{2\pi K T_\parallel}\right)^{\frac{1}{2}} e^{-mv_{zo}^2/2KT_\parallel} \,. \qquad \ldots (22)$$

We have

$$\left\langle\frac{d}{dt}\frac{mv_\parallel^2}{2}\right\rangle = -\frac{m}{4B_o^2}\frac{k^2c^2}{\omega^2}\left(\frac{2KT_\perp}{m}\right)^2\left(\frac{\partial\varepsilon}{\partial x}\right)^2\int_{-\infty}^{\infty} g(v_{zo})\left[\omega\,\frac{\sin\alpha_o t}{\alpha_o^2} - kv_{zo}t\,\frac{\cos\alpha_o t}{\alpha_o}\right] dv_{zo} \,. \qquad \ldots (23)$$

As α_o is real and the integrand well behaved at $\alpha_o = 0$ we may take the principal part of the sum and obtain, for large values of t :

$$\left\langle \frac{d}{dt} \frac{mv_{\parallel}^2}{2} \right\rangle = - \frac{m}{4B_o^2} \frac{k^2c^2}{\omega^2} \left(\frac{2\kappa T_\perp}{m}\right)^2 \left(\frac{\partial \varepsilon}{\partial x}\right)^2 \frac{\pi\omega}{k|k|} \left[\frac{\partial g(v_{zo})}{\partial v_{zo}}\right]_{v_{zo} = \omega/k} \quad \ldots (24)$$

and substituting for $g(v_{zo})$ its expression (22):

$$\left\langle \frac{d}{dt} \frac{mv_{\parallel}^2}{2} \right\rangle = \frac{\kappa T c^2}{B_o^2 \omega} \frac{T_\perp}{T_\parallel} \left(\frac{\partial \varepsilon}{\partial x}\right)^2 \cdot \sqrt{\pi} \frac{k}{|k|} \zeta_o e^{-\zeta_o^2} \quad \ldots (25)$$

so that, as we have found, Eq. (12), that, at low β, $\varepsilon(x)$ is in first approximation unaffected by the presence of the plasma, the rate of increase of ion kinetic energy is always positve and does not depend on the local value of plasma density.

We can now evaluate the RF power P absorbed by the plasma per unit volume:

$$P = n(x) \frac{d}{dt} \frac{mv^2}{2} + \nabla \cdot \vec{\Phi} \left(\frac{mv^2}{2}\right) \quad \ldots (26)$$

where $\vec{\Phi}\left(\frac{mv^2}{2}\right) = n(x) \frac{mv^2}{2} \vec{v}$ is the flux of ion kinetic energy and $n(x)$ the local plasma density. As we have assumed no dependence on the y coordinate and the dependence on the z coordinate vanishes after averaging even initial position z_o only the x component of the energy flux needs to be considered here :

$$\varphi_x\left(\frac{mv_\perp^2}{2}\right) = n(x) \frac{m}{2} \left\{\left(v_{x1}^2 + v_{y1}^2 + 2v_{xo}v_{x2} + 2v_{yo}v_{y2}\right) v_{xo} + 2\left(v_{xo}v_{x1} + v_{yo}v_{y1}\right) v_{x1} + \left(v_{xo}^2 + v_{yo}^2\right) v_{x2}\right\} \quad \ldots (27$$

$$\varphi_x\left(\frac{mv_\parallel^2}{2}\right) = n(x) \frac{m}{2} \left\{\left(v_{z1}^2 + 2v_{zo}v_{z2}\right)v_{xo} + 2\left(v_{zo}v_{z1}\right)v_{x1} + v_{zo}^2 v_{x2}\right\} \quad \ldots (28)$$

where again only terms non periodic in z_o shall be retained in the right side of Eq. (27), Eq. (28).

After averaging over z_o and over a Larmor period we find that only the first term on the right side of Eq. (28) does not vanish :

$$\Phi_x\left(\frac{mv_\perp^2}{2}\right) = 0$$

$$\Phi_x\left(\frac{mv_\parallel^2}{2}\right) = n(x)\,\frac{e^2}{m}\,\frac{k^2\ v_{\perp o}^4}{8\,\omega^2\,\Omega^2}\,\varepsilon(x)\,\frac{\partial\varepsilon}{\partial x}\left[\omega\,\frac{\sin\alpha_o t}{\alpha_o^2} - kv_{zo}t\,\frac{\cos\alpha_o t}{\alpha_o}\right]. \quad \ldots (29)$$

So that only parallel energy contributes to energy flux and, as no contribution arises from terms proportional to v_{x2}, the flux of kinetic energy is not accompanied by any transport of mass. The origin of this energy flux can be made physically more intuitive: as the ions move along the magnetic field lines under the slowly varying average force, $F_z = -\mu\,\Delta B$ they are subject to a rapidly varying force, $F_z = -v_y B_x$, and, in turn, acquire and lose energy according to the sign of v_y. An ion whose gyro centre lies close enough to a plane $x = \cos t$ will therefore cross this plane back and forth with different energies according to the sign of v_x.

We average now over the distribution of initial velocity (22) and obtain

$$\left\langle\varphi_x\left(\frac{mv_\parallel^2}{2}\right)\right\rangle = n(x)\,\frac{m}{4B_o^2}\,\frac{k^2 c^2}{\omega^2}\left(\frac{2\kappa T_\perp}{m}\right)^2\varepsilon(x)\left(\frac{\partial\varepsilon}{\partial x}\right)\frac{\pi\omega}{k|k|}\left[\frac{\partial g(v_{zo})}{\partial v_{zo}}\right]_{v_{zo}=\omega/k} \quad \ldots (30)$$

or

$$\langle \varphi_x \left(\frac{mv_\parallel^2}{2}\right) \rangle = - n(x) \frac{\kappa T_\perp c^2}{B_o^2 \omega} \frac{T_\perp}{T_\parallel} \varepsilon(x) \frac{\partial \varepsilon}{\partial x} \left[\sqrt{\pi} \frac{k}{|k|} \zeta_o e^{-\zeta_o^2}\right] . \quad \ldots (31)$$

Substituting now (25), (26) in Eq.(26) we obtain for the RF power absorbed by the plasma per unit volume:

$$P = \frac{\kappa T_\perp c^2}{B_o^2 \omega} \frac{T_\perp}{T_\parallel} \left\{ n(x) \left(\frac{\partial \varepsilon}{\partial x}\right)^2 - \frac{\partial}{\partial x}\left[n(x)\, \varepsilon(x) \frac{\partial \varepsilon}{\partial x}\right]\right\} \left[\sqrt{\pi} \frac{k}{|k|} \zeta_o e^{-\zeta_o^2}\right] . \quad \ldots (32)$$

For $T_\parallel = T_\perp$ Eq.(32) is identical to Eq.(31) if the contribution of the parallel electric field is neglected there and we find again that RF power can be absorbed by the plasma only if conditions (14) are satisfied.

4. CONCLUSIONS

From a detailed analysis of the ion motion the rate of increase of ion kinetic energy and the RF power absorbed by the plasma per unit volume in transit time magnetic pumping has been evaluated. It is found that, at low β, the rate of change of ion kinetic energy is positive through the whole plasma volume and independent of the local plasma density while RF power can be absorbed by the plasma only in a region where a sufficiently high density gradient is present. It is shown that a flux of ion kinetic energy, which is not accompanied by any transport of mass, exists within the plasma and that it accounts for the apparent contradiction.

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