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PENETRATION OF A SOLID LAYER BY A LIQUID JET

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ABSTRACT

The resistance of a solid layer to penetration by a liquid jet has been calculated for a variety of target geometries and in both the high and low jet-velocity regimes. The theory is applied to the cavitation of a liquid adjacent to a solid layer and results are presented for the water/solid aluminium and liquid sodium/solid uranium dioxide systems. It is shown that the thickness of the solid layer to be expected on the fuel during a thermal interaction is not an insurmountable barrier to jet penetration of the liquid fuel and the subsequent mixing of fuel and coolant.

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1. INTRODUCTION

It is well known that when a hot liquid comes into contact with a cold vaporizable liquid a violent explosion (thermal or fuel-coolant interaction) may occur. For example, interactions have been observed between a variety of molten metals (aluminium, tin, steel, lead, etc.) and water [Long (1957), Witte et al (1970)], liquefied natural gas and water [Enger (1972)], and in the nuclear field when hot molten core material, uranium dioxide, is dispersed into the coolant, liquid sodium [Brauer et al (1968)]. Although the mechanisms governing these interactions are not yet fully understood it is thought that the penetration of the fuel by jets of coolant is important since this will lead to rapid turbulent mixing and thus to an enhancement of the heat transfer rate. However, it must be recognized that a solid layer of fuel may surround the hot molten material, thus if mixing is to take place the jet of liquid coolant must be capable of penetrating this solid layer. The resistance of the solid layer to penetration by a liquid jet is calculated in this paper.

In section 2 we calculate the thickness of the solid layer as a function of time whilst in section 3 the resistance of the solid layer to penetration by a jet of coolant is considered. The latter calculation depends on whether or not the jet has a high velocity and also on the geometry of the solid surface. In section 4 the theory is applied to the cavitation problem in which jets are formed in collapsing bubbles. Finally, in section 5 we quote results for the H_2O/Al and Na/UO_2 cases. For the H_2O/Al system it is concluded that at atmospheric pressure it is the waterhammer pressure of the jet that causes penetration and it is shown that bubbles with an initial radius of about

3 times the thickness of the solid layer are sufficient to cause jet penetration. However, for the Na/UO_2 system it is the stagnant pressure that is the destructive agent.

Although the investigation concerns the penetration of liquid jets we do not consider specifically how the jets are formed. Experiments by Benjamin and Ellis (1966) have shown that jets form on bubbles collapsing adjacent to solid walls. In addition, Plesset and Chapman (1971) have done a numerical simulation of a bubble collapsing near a solid wall and they also observe jet formation. Although we have used such jets in the application, the theory applies equally well to the high-speed jets investigated by Birkhoff et al (1948), Walsh et al (1953) and Harlow and Pracht (1966).

2. THE SOLID LAYER THICKNESS

In this section we calculate the thickness of the solid layer as a function of time. Fig.1 displays the situation at time t. A solid layer of fuel (thickness X(t)) is sandwiched between the coolant and liquid fuel. Heat conduction is restricted to one dimension and the columns of fuel and coolant are assumed to have infinite length. We display two solutions to this problem, one exact and the other approximate. The advantage of the second method is that it is much easier to obtain a 'number' for any given substance and set of initial conditions.

The exact method is described by Carslaw and Jaeger (1959). By writing

$$X(t) = 2\lambda\sigma_2 t^{\frac{1}{2}} \qquad (1)$$

these authors are able to show that λ satisfies the equation

$$\frac{k_1\sigma_2(T_m-\tau_1)e^{-\lambda^2}}{k_2\sigma_1+k_1\sigma_2 \operatorname{erf}\lambda} - \frac{k_3\sigma_2}{k_2\sigma_3}\,\frac{(\tau_3-T_m)\exp(-\lambda^2\sigma_2^2/\sigma_3^2)}{\operatorname{erfc}(\lambda\sigma_2/\sigma_3)} = \frac{L\lambda\sqrt{\pi}}{c_2} \qquad (2)$$

where $\sigma_1^2 = k_1/\rho_1 c_1$, etc., τ_1 and τ_3 are the initial coolant and fuel temperatures, T_m and L are the melting temperature and latent heat of fusion of the fuel. The solid and liquid fuel densities are assumed to be equal.

Although this method is exact we have to solve numerically the transcendental equation (2) to obtain a number for λ. As an alternative we use Cho's et al (1971) method of equivalent temperature difference. The latent heat L is equivalent to the sensible heat $c\Delta T$. Thus the effect of latent heat may be taken into account by increasing the initial temperature of the fuel from τ_3 to $\tau_3 + \Delta T$. The effect of increasing τ_3 is to increase temperature gradients; however, Cho et al have found that for the Na/UO_2 system, if $(\tau_3 - \tau_1) > 1000K$ the approximate method gives the correct heat conduction to within 10%. This condition is normally satisfied in a fuel-coolant interaction. The heat conduction equations that must be solved now are

$$\frac{\partial T_1}{\partial t} = \sigma_1^2 \frac{\partial^2 T_1}{\partial x^2} \qquad x < 0$$

$$\frac{\partial T_3}{\partial t} = \sigma_3^2 \frac{\partial^2 T_3}{\partial x^2} \qquad x > 0$$

with the initial conditions $T_1 = \tau_1$, $T_3 = \tau_3 + \Delta T$. We also require that T and $k\partial T/\partial x$ be continuous at k = 0. We find

$$T_3 = \frac{1}{k_1\sigma_3 + k_3\sigma_1} \left\{ k_3\sigma_1(\tau_3 + \Delta T) + k_1\sigma_3\tau_1 + k_1\sigma_3(\tau_3 + \Delta T - \tau_1) \operatorname{erf} \frac{x}{2\sigma_3 t^{\frac{1}{2}}} \right\} . \tag{3}$$

The thickness of the solid layer at time t is determined by solving this equation for x with $T_3 = T_m$. Hence

$$X(t) = 2\sigma_3\sqrt{t} \operatorname{erf}^{-1} \left\{ \frac{T_m(k_3\sigma_1 + k_1\sigma_3) - k_3\sigma_1(\tau_3 + \Delta T) - k_1\sigma_3\tau_1}{k_1\sigma_3(\tau_3 + \Delta T - \tau_1)} \right\} \tag{4}$$

where erf^{-1} is the inverse error function.

Thus the thickness of the solid layer is given by

$$X(t) = X_o \, t^{\frac{1}{2}} \qquad (5)$$

where X_o is determined via Eqs.(2) or (4).

The following examples show that the approximate method must not be used indiscriminately. For the Na/UO_2 system we find (τ_1=1100K, $\tau_3=T_m$=3000K)

$$X_{exact} = 1.165 \; t^{\frac{1}{2}} mm$$

$$X_{approx.} = 0.942 \; t^{\frac{1}{2}} mm$$

Thus it may be seen that in this case the approximate method underestimates the thickness by some 17%. This result may seem surprising in view of the fact that the approximate method steepens temperature gradients; it is a result of the fuel being at an effective temperature greater than T_m. To solidify, the fuel has to cool to the melting temperature and this process takes a certain time. In the exact analysis the release of latent heat is instantaneous. For the H_2O/Al system we find (τ_1=293K, $\tau_3=T_m$=933K)

$$X_{exact} = 1.097 \; t^{\frac{1}{2}} mm$$

but the approximate analysis gives no real solution. This is because Eq.(3) gives $T(t \rightarrow \infty) > T_m$. The difference between the two examples is that on the one hand the conductivity of the coolant is much greater than that of the fuel (Na/UO_2) while the reverse is true for the H_2O/Al case. Thus we conclude that the approximate method may be used provided the conductivity of the coolant is much greater than that of the fuel.

Although it is possible to derive formulae for the case of finite lengths of fuel and coolant columns it is not necessary for the purpose of investigating jet penetration applied to fuel-coolant interactions. The maximum value of t that is of relevance is 2 or 3 ms and at this time $X < 0.1$mm. As the initial dimensions of the fuel are orders of magnitude larger than this the assumption of infinite length is justified

3. JET PENETRATION

The resistance of a solid to penetration by a liquid jet is both a function of the jet velocity and the geometry of the target. If the jet has a high enough velocity the depth of penetration is controlled by the inertia of the target but for lower velocities the mechanical strength of the solid is the dominating factor. We consider first the penetration of high-velocity jets and then go on to deal with the mechanically dominated situation for both spherical and plane target geometries.

The depth of penetration of a high-velocity fluid jet striking a solid has been the subject of a number of authors. By 'high-velocity' we mean that the pressure exerted by the jet is very much greater than the yield stress, σ_y, of the solid target,

$$\text{i.e.} \quad \tfrac{1}{2}\rho_1 V^2 \gg \sigma_y$$

where ρ_1 and V are the jet density and velocity. If this condition is satisfied it is the inertia of the target that determines the depth of penetration. Birkhoff et al (1948) and, independently, Pack and Evans (1951) have shown that the velocity of penetration, U, and the depth of penetration, d, are given by

$$U = V/[1+(\rho_2/\gamma_e \rho_1)^{\frac{1}{2}}] \tag{6}$$

$$d = L(\gamma_e \rho_1/\rho_2)^{\frac{1}{2}} \tag{7}$$

where ρ_2 is the target density and L is the length of the jet. γ_e is a dimensionless number which Eichelberger (1956) calls the 'break-up factor' since it supposedly takes account of the jet breaking into particles. In fact, Eichelberger also states that γ_e accounts for "all the factors that produce changes" from the simple derivation of Eqs.(6) and (7) from Bernoulli's law. For our purposes it is sufficient to note that γ_e is of order unity.

The time taken for penetration is

$$t = \frac{L}{V}\left\{1 + (\gamma_e \rho_1/\rho_2)^{\frac{1}{2}}\right\} \quad . \tag{8}$$

If the jet starts penetrating at time t_o then the thickness of the solid layer after the end of penetration is

$$X = X_o(t_o+t)^{\frac{1}{2}} \quad . \tag{9}$$

Eqs.(7) and (9) may be compared to see if the jet completely penetrates the solid layer,

$$\text{i.e.} \quad \frac{d}{X} = \frac{\bar{L}\alpha}{\{1+(1+\alpha)\bar{t}\}^{\frac{1}{2}}} \tag{10}$$

where $\bar{L} = L/X_o t_o^{\frac{1}{2}}$, $\bar{t} = L/Vt_o$ and $\alpha = (\gamma_e \rho_1/\rho_2)^{\frac{1}{2}}$ and if $d/X > 1$ complete penetration occurs.

For a low velocity jet it is the elastic properties of the target that determine whether jet penetration takes place. We assume that the jet subjects the target to a uniform pressure q over an area equal to the cross-section of the jet. Initially we shall also assume that q is constant in time and examine later how the dynamic aspect of q affects the theory. The method of calculation is to determine the stress produced by q in the solid layer and then to compare this stress with the yielding stress of the material in order to determine whether or not penetration takes place. The stress produced by q is a function of the target geometry. It should be noted that the possibility of the jet penetrating only part of the solid layer does not exist in this calculation. If the stress produced by q is greater than the yielding stress complete penetration takes place, otherwise the target remains intact. We consider first the case of a spherical target.

If the solid layer forms a spherical surface of thickness X and of radius $r (r \gg X)$ then the maximum stress due to q is [Timoshenko (1956)]

$$\sigma = qr/2X \quad . \tag{11}$$

Eq.(11) is a result of pure membrane theory and consequently bending is not taken into account.

We consider now the case of a flat target. In Eq.(11) if r becomes infinite we see that the slightest pressure is sufficient to cause penetration. This is contrary to experimental fact; for the flat target we must take account of bending. We assume that the target forms a disc of radius a and thickness X and that the jet is centred on the disc. The maximum stress on the disc due to q depends on whether or not the disc is clamped at the edges or simply supported. Timoshenko (1956) has calculated the maximum bending stresses at the centre of the disc for the case of clamped edges. The calculation for the simply supported case is similar. Thus

$$\sigma_c = \frac{3}{2} \frac{(1+\mu)qb^2}{X^2} \left[\ln \frac{a}{b} + \frac{b^2}{4a^2} \right] \tag{12}$$

$$\sigma_s = \frac{3}{2} \frac{(1+\mu)qb^2}{X^2} \left[\ln \frac{a}{b} + \frac{1}{1+\mu} - \frac{(1-\mu)}{(1+\mu)} \frac{b^2}{4a^2} \right] \tag{13}$$

where b is the radius of the jet and μ is Poisson's ratio for the solid. σ_c and σ_s are the maximum bending stresses at the centre for the clamped and supported cases.

Clearly the solid layer in the fuel coolant interaction is neither clamped nor freely supported and so the stress due to q will be somewhere between the two extremes given by Eqs.(12) and (13). Nevertheless, for a large number of cases Eqs.(12) and (13) can be used to determine whether penetration takes place. If $\sigma_c > \sigma_y$ then penetration definitely takes place; if $\sigma_s < \sigma_y$ then penetration definitely does not take place. For intermediate cases further calculation is required and this will be the subject of a future paper.

The pressure, q, exerted by the jet is either the 'waterhammer pressure', q_{WH}, or the stagnant pressure, q_s, where

$$q_{WH} = \rho_1 s_1 V\left(\frac{\rho_2 s_2}{\rho_1 s_1 + \rho_2 s_2}\right) \tag{14}$$

$$q_s = \tfrac{1}{2} \rho_1 V^2 \ . \tag{15}$$

s_1 and s_2 are the sound velocities in the fluid and solid respectively. For a low-velocity jet $q_{WH} > q_s$, however, the duration of waterhammer pressure is of order b/s_1 whilst that of the stagnant pressure is L/V. The pressure-time history is illustrated schematically in Figure 2. Thus we must take into account the dynamic nature of q and this is done by allowing the influence of q to be felt only at points within a radius $s_2 L/V$ or $s_2 b/s_1$ depending on whether q_s or q_{WH} is important. We assume that for q_{WH} to cause penetration a necessary condition is $s_2 b/s_1 \geqslant X$ so that the effect of q_{WH} can be felt simultaneously throughout the thickness of the solid. If this condition is not satisfied then q_{WH} has no effect. Of course, even if this condition is satisfied q_{WH} must be large enough to overcome the yield stress of the solid before penetration will occur. If q_{WH} has no effect we can then examine in a similar manner the effect of q_s. Hence the values of a and χ to be inserted in Eqs.(12) or (13) are simply $s_2 L/V$ and $X_o(t_o+L/V)^{\frac{1}{2}}$ or $s_2 b/s_1$ and $X_o(t_o+b/s_1)^{\frac{1}{2}}$ corresponding to q_s and q_{WH} respectively. For many applications L/V (or b/s_1) will be negligible compared with t_o. To take account of the fact that q_{WH} and q_s are applied suddenly and not gradually we use the fact that a load applied suddenly is equivalent to twice the same load applied gradually [Duggan (1964)].

4. APPLICATION TO CAVITATION

In this section we apply the foregoing theory to the problem of an initially spherical vapour cavity in the neighbourhood of the solid layer. It has been known for a long time that collapsing bubbles in the neighbourhood of a solid surface can lead to pitting of the surface. That the damage is caused by liquid jets on the bubbles was first suggested by Kornfeld and Suvorov (1944) and experiments by Benjamin and Ellis (1966) have confirmed that jets do indeed form on bubbles collapsing near a solid wall. Plesset and Chapman (1971) have done a numerical simulation of bubble collapse near a wall. They find that the jet velocity scales as $(\Delta P/\rho_1)^{\frac{1}{2}}$ and the jet dimensions scale as R. ΔP is the pressure difference causing the collapse and R is the initial bubble radius. Thus

$$V = V_c\left(\frac{\Delta P}{\rho_1}\right)^{\frac{1}{2}}$$

$$L = L_c R$$

$$b = b_c R$$

where V_c, L_c and b_c are constants and can be found from Plesset and Chapman's results

$$V_c = 13.0$$

$$L_c = 0.4929$$

$$b_c = 0.1186 \quad .$$

For complete penetration by a high-velocity jet we require $F_{HV} > 1$ where

$$F_{HV} = \frac{\alpha L_c Z}{\left\{1 + \frac{(1+\alpha)L_c}{V_c}\tau\right\}^{\frac{1}{2}}} \quad . \tag{16}$$

$Z = R/X_o t_o^{\frac{1}{2}}$ and $\tau = R\left(\frac{\rho_1}{\Delta P}\right)^{\frac{1}{2}}/t_o$. For small τ, i.e. large driving pressure, ΔP, or thick solid layer, we see that the initial bubble radius required to produce penetration is proportional to the thickness of the solid layer. In order that Eq.(16) be applicable we require

$$\Delta P \gg 2\sigma_y/V_c^2$$

and for aluminium this condition implies $\Delta P \gg 12$ atmos.

For penetration by a low velocity jet we require $F_{LV} > 1$ where F_{LV} is a function of the target geometry and the pressure of the jet. For example, for a plane target that is clamped and subjected to the stagnant pressure

$$F_{LV} = \frac{3(1+\mu)V_c^2 b_c^2 \rho_1}{2\sigma_y} Q^2 Z^2 \left\{\ln \frac{s_2 L_c}{b_c V_c} \frac{1}{Q} + \frac{b_c^2 V_c^2}{4 s_2^2 L_c^2} Q^2\right\} \tag{17}$$

where $Q = (\Delta P/\rho_1)^{\frac{1}{2}}$. We also need $Z/Q > V_c/s_2 L_c$. In this case we see that the penetrative power of the jet goes up as the square of the initial bubble radius. If q_{WH} is the important pressure then

$$F_{LV} = \frac{3(1+\mu)b_c^2 \rho_1 s_1 \rho_2 s_2 V_c}{\sigma_y(\rho_1 s_1 + \rho_2 s_2)} QZ^2 \left\{\ln \frac{s_2}{s_1} + \frac{s_1^2}{4 s_2^2}\right\} \tag{18}$$

and we also require $Z > s_1/b_c s_2$. In this case the penetrative power is again proportional to the initial radius squared but the dependence on ΔP is much less marked than in the stagnant pressure case.

In the case of a spherical target F_{LV} is given by

$$F_{LV} = \frac{V_c^{\,2} \rho_1}{2\sigma_y} Q^2 Y^2$$

for q_s , and

$$F_{LV} = \frac{\rho_1 s_1 \rho_2 s_2}{\sigma_y(\rho_1 s_1 + \rho_2 s_2)} Q\, Y$$

for q_{WH} where $Y = r/X_o t_o^{\frac{1}{2}}$.

5. APPLICATIONS TO H_2O/Al AND Na/UO_2 SYSTEMS

Tables 1 and 2 show the values of Z required for various Q values in order that $F_{LV} = 1$ for both Eqs.(17) and (18). The liquid jets are water and sodium respectively and the solid layers are aluminium and uranium dioxide.

ΔP (atmos)	Q	Z Eq(17)	Z Eq(18)
0.01	1	51.79	8.818
1	10	6.241	2.790
100	100	0.843	0.882

TABLE 1

Z values as a function of Q that give $F_{LV} = 1$. The values of the constants for water are the normal room temperature values whilst for the aluminium layer we assume that σ_y is to be identified with the ultimate compressive strength ($\sim 10^8\ n/m^2$). Clearly if the break is by shearing then the ultimate shear strength should be used ($\sim 2x10^8\ n/m^2$); however, the Z values will only be altered by a factor of $\sqrt{2}$. Similarly the value of s_2 depends on the propagation mechanism in the solid. We have used $s_2 = 5102$ m/s. In addition it is the values of the properties of the solid just below the melting point that should be used and not the room temperature values. Using the correct values will make penetration easier. The conditions $Z > V_c Q/s_2 L_c$ and $Z > s_1/b_c s_2$ are satisfied by all the values of Z in this table except $Z = 0.882$ corresponding to Eq.(18) with $Q = 100$.

From Table 1 we see that for $Q = 1$ and $Q = 10$ it is the waterhammer pressure that is the destructive mechanism. However, for $Q = 100$ the Z value predicted via Eq.(18) that makes $F_{LV} = 1$ results in too brief a pressure pulse. Using Eq.(18) and the condition $Z > s_1/s_2 b_c$ we find that if $Q > 12.65$ we must have $Z > 2.48$ for penetration to occur. For smaller Z values the duration of the waterhammer pressure is insufficient to cause penetration and it is the stagnant pressure that is important.

For water bubbles collapsing under one atmosphere of pressure, $Q = 10$, we see that bubbles of radius 2.79 times the thickness of the solid will cause penetration. Using Eq.(5) with the exact value of X_o we find that the thickness of the solid layer after 1 ms is 3.47×10^{-2} mm and thus a bubble of initial radius 9.68×10^{-2} mm will cause penetration of this layer.

ΔP (atmos)	Q	Z Eq(17)	Z Eq(18)
0.0083	1	287.9	185.5
0.83	10	35.63	58.67
83.	100	5.203	18.55

TABLE 2

Z values as a function of Q that give $F_{LV} = 1$. (Na/UO_2 system). The values of the constants are (MKS units)

$\rho_1 = 830$ $\quad s_1 = 2595$ $\quad \mu = 0.23$

$\rho_2 = 9800$ $\quad s_2 = 2370$ $\quad \sigma_y = 2x10^9$

The value of μ is that appropriate to U (not UO_2) whilst the value of σ_y is that for a high grade steel.

The conditions $Z > V_c Q/s_2 L_c$ and $Z > s_1/b_c s_2$ are satisfied by all the Z values. From these results we see that for small Q values it is the sodiumhammer pressure that is the destructive agent. As Q increases we find that small bubbles are capable of penetrating via q_s when q_{WH} is resisted. For example, with Q = 10 (0.83 atmos.) a bubble of initial radius 35.63 times the solid layer thickness is capable of penetrating via q_s , but one of 58.67 times the solid layer thickness is required if penetration is via q_{WH}. For Q = 10 and $t_o = 1$ ms we find that a bubble of initial radius 1.31 mm will penetrate a solid layer of thickness 3.68×10^{-2} mm.

It should be noted that in these calculations we have neglected L/V and $s_2 b/s_1$ in comparison with t_o the time at which the jet starts to exert its pressure. Thus the thickness of the solid layer us $X_o t_o^{\frac{1}{2}}$. For $t_o = 1$ ms this approximation is highly accurate.

6. CONCLUSIONS

We have calculated the thickness of a hot solid layer adjacent to a cool liquid as a function of time. The resistance of the layer to penetration by a jet of liquid has been estimated for a variety of geometries and also in both the high and low jet-velocity regimes. The theory has been applied to the cavitation problem and, more specifically, to the H_2O/Al and Na/UO_2 systems. It has been shown that the solid layer to be expected on the fuel during a thermal interaction can be penetrated by jets produced by the collapse of very small bubbles.

Although we have not considered what causes the pressure difference under which the bubbles collapse and jet, this may lead to another condition of the minimum size of bubble. For example, if the pressure difference is due to rapid condensation of the vapour within the bubble then we may require that the bubble be large enough initially for part of it to be in the unheated part of the liquid. (The bubble is adjacent to a hot solid surface). If this condition is necessary then the bubble diameter at time t, when it begins to collapse, must be greater than the thermal diffusion length $2\sigma\sqrt{t}$.

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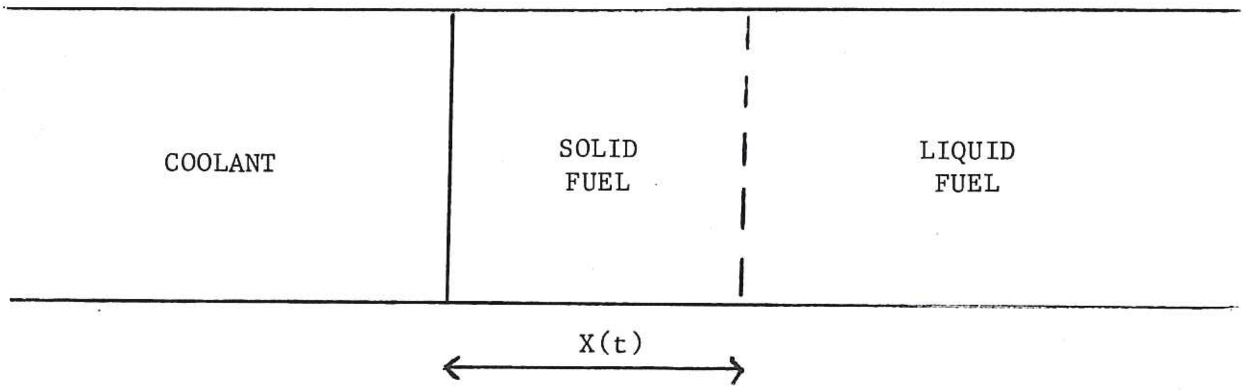

Figure 1. The physical situation at time t. k_1, ρ_1, c_1 are the thermal conductivity, density and specific heat of the coolant; k_2 k_3, etc. are defined similarly for the solid and liquid fuel respectively.

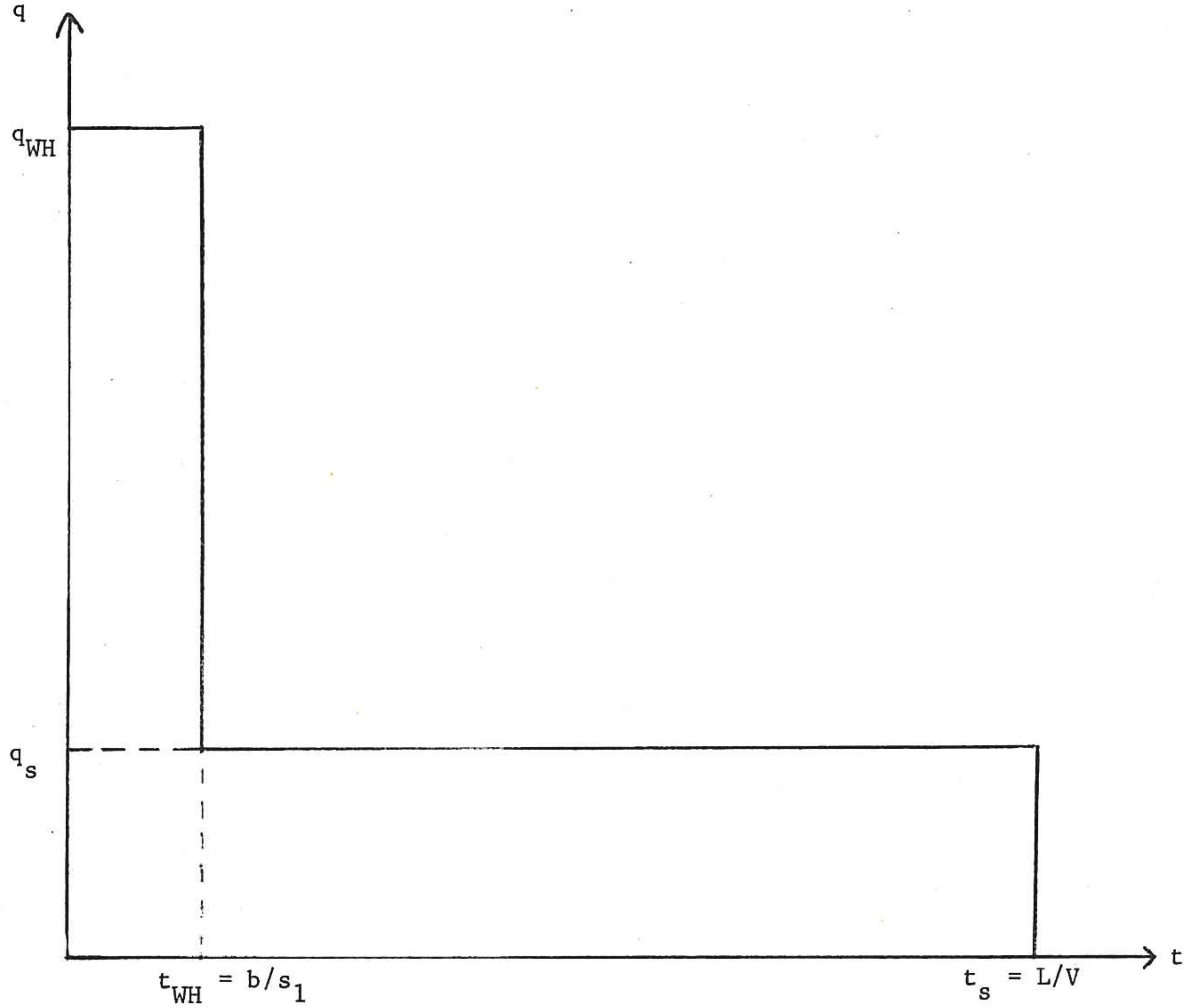

Figure 2. The pressure-time history produced by a jet impinging on a solid layer.