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ANISOTROPY OF TURBULENCE IN A COLLISIONLESS SHOCK

by

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ABSTRACT

We have measured the level of density fluctuations $\langle \delta n_e^2(\vec{k}) \rangle$ for $|\vec{k}| \sim 1/\lambda_D$ as a function of the angle between $\vec{k}$ and the current within a collisionless shock. The cut-off angle is in good agreement with that predicted for current-driven ion wave turbulence and one-dimensional heating of the ions. This agreement provides further evidence that ion-wave instability is responsible for the observed turbulence within the shock.

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C O N T E N T S

1. INTRODUCTION

Collisionless shock waves propagating perpendicular to a magnetic field have been produced by the compression of an initial hydrogen plasma within a linear z-pinch, (PAUL et al. 1965 and 1967). The electron heating within the shock implies a resistivity which is two orders of magnitude larger than the Spitzer value and this is attributed to the presence of turbulent fluctuations.

The frequency and wave number spectra of the turbulence have been measured previously (DAUGHNEY et al. 1970) by scattering of ruby laser light from the shock. This yields the Fourier transform of the density fluctuations in the form

$$S(\omega,\vec{k}) = \langle \delta\ n_e^2(\omega,\vec{k}) \rangle / n_e \ . \qquad \ldots (1)$$

These measurements were made on shock waves with sub-critical Mach number ($M_A = 2.5$), for $\vec{k}$ collinear with the azimuthal current (J_θ) within the shock and $|\vec{k}| \sim 1/\lambda_D$. The spectra of turbulence are consistent with current driven ion-wave turbulence.

For an isotropic plasma, theory (KADOMTSEV, 1965) predicts that the turbulence extends within a half cone in k-space, centred on the direction of the electron current. The extreme or cut-off cone has an angle corresponding to that for zero growth on linear stability theory.

This angle depends only on the two ratios (STRINGER, 1964), (i) electron to ion temperature (T_e/T_i) and (ii) electron drift to thermal velocity (u_o/c_e).

Within such a 'collisionless' shock the electrons experience effective collisions and we assume that this maintains an isotropic electron temperature. The ions, however, are collisionless and not affected by the turbulence. We therefore expected the ions to be adiabatically heated only in the radial direction (i.e. normal to the shock front) in which they are slowed down and compressed by the radial electric field. The ion temperature is therefore anisotropic $T_r > T_\theta = T_z$.

We take this into account by assuming that the turbulence extends in k-space within a region defined by the contour of zero linear growth and this is no longer a cone. A linear stability calculation for $T_r \neq T_\theta = T_z$ yields the zero growth rates and corresponding cut-off angles:

(i) φ_c in the plane (r,θ) perpendicular to the shock front and B_z;

(ii) ψ_c in the plane (θ,z) containing the shock front and B_z.

The directions and angles are shown in Fig.1.

We then present the measured variation of $S(\varphi)$ in the plane (r,θ) obtained by the above scattering technique and averaged over the shock. These results have previously been reported briefly, (PAUL et al, 1971). The measured cut-off angle φ_c agrees well with the theory presented.

2. CALCULATION OF THE CUT-OFF ANGLES

The velocity distribution functions of electrons and ions averaged within the shock front can be expressed as follows in the shock frame shown in Fig.1:

$$f_e = \left(\frac{m_e}{2\pi\kappa T_e}\right)^{\frac{3}{2}} \exp\left[-\frac{m_e}{2\kappa T_e}\left\{v_r^2 + (v_\theta - u_o)^2 + v_z^2\right\}\right] \quad \ldots (2)$$

$$f_i = \left(\frac{m_i}{2\pi\kappa T_{ir}}\right)^{\frac{1}{2}} \left(\frac{m_i}{2\pi\kappa T_{i\theta}}\right) \exp\left[-\frac{m_i}{2k}\left\{\frac{v_r^2}{T_{ir}} + \frac{v_\theta^2 + v_z^2}{T_{i\theta}}\right\}\right] \quad \ldots (3)$$

where the notations have the obvious meanings. In equations (2) and (3), electron temperature is taken to be isotropic due to the rapid relaxation by the turbulence, whereas ion temperature is assumed anisotropic.

2.1 Cut-off Angle ψ_c $(\theta, z$ plane$)$

The distribution functions in the ψ direction are obtained by resolving the velocity component in the shock plane into the direction ψ (velocity component of which is denoted by v_ψ) and perpendicular to this (velocity component $v_{\psi\perp}$) and integrating equations (2) and (3) with respect to v_r and $v_{\psi\perp}$. The results are

$$f_{e\psi} = \left(\frac{m_e}{2\pi\kappa T_e}\right)^{\frac{1}{2}} \exp\left\{-\frac{m_e(v_\psi - u_o\cos\psi)^2}{2\kappa T_e}\right\} \quad \ldots (4)$$

$$f_{i\psi} = \left(\frac{m_i}{2\pi\kappa T_{i\theta}}\right)^{\frac{1}{2}} \exp\left\{-\frac{m_i v_\psi^2}{2\kappa T_{i\theta}}\right\} \; . \quad \ldots (5)$$

STRINGER (1964) has presented a linear theory of the current-driven ion-wave instability. This can be used to get the cut-off angle ψ_c for the zero growth rate from equations of the form (4) and (5). The effective ion temperature is $T_{i\theta}$ and the effective drift $u_o \cos\psi$.

2.2 Cut-off angle φ_c (r, θ plane)

By the same procedure as above, we get the distribution function in the direction φ as follows:

$$f_{e\varphi} = \left(\frac{m_e}{2\pi\kappa T_e}\right)^{\frac{1}{2}} \exp\left\{-\frac{m_e(v_\varphi - u_o\cos\varphi)^2}{2\kappa T_e}\right\} \quad \ldots (6)$$

$$f_{i\varphi} = \left(\frac{m_i}{2\pi\kappa T_{i\varphi}}\right)^{\frac{1}{2}} \exp\left\{-\frac{m_i v_\varphi^2}{2\kappa T_{i\varphi}}\right\} \quad \ldots (7)$$

where

$$T_{i\varphi} = T_{ir}\sin^2\varphi + T_{i\theta}\cos^2\varphi \quad \ldots (8)$$

and v_φ is the velocity component in the φ direction. In this case, the effective ion temperature $T_{i\varphi}$ depends on φ and so the cut-off angle φ_c has to be obtained by iteration.

2.3 Cut-off Angles for the Experimental Condition

The initial parameters, (PAUL et al, 1965) for the shock wave with $M_A = 2.5$ are $n_{e1} = 6.4 \times 10^{20}\ m^{-3}$, $T_{e1} = T_{i1} = 1.2$ eV and $B_{z1} = 0.12$ T. From the measured electron temperature behind the shock, PAUL et al. (1967), $T_{e2} = 44$ eV, the shock thickness $L_S = 1.4$ mm, jump ratio $F = 2.5$ we obtain mean shock conditions

$$u_o/c_e = 0.25 \,,\ \bar{T}_e = \tfrac{1}{2}(T_{e1} + T_{e2}) = 23 \text{ eV} . \qquad \ldots (9)$$

Earlier work (PAUL et al, 1971) contains evidence that the turbulence arises from plasma with approximately this mean temperature. If the ions were adiabatically heated in three dimension (i.e. isotropic T_i), then $T_i = 2.7$ eV, $\bar{T}_i = 1.7$ eV and the theory of STRINGER (1964) yields

$$\varphi_c = \psi_c = 81^\circ . \qquad \ldots (10)$$

For adiabatic heating of the ions in one dimension, as is expected $T_{ir2} = 7.5$ eV and $\bar{T}_{ir} = 4.4$ eV. For this mean and the above conditions the cut-off angles calculated by the above method are

$$\left.\begin{aligned} \psi_c &= 84^\circ \\ \varphi_c &= 55^\circ \end{aligned}\right\} . \qquad \ldots (11)$$

In the experiment there is no resolution within the shock but φ_c is not critically dependent on T_{ir}, for an extreme example $T_{ir} = T_{ir2} = 7.5$ eV yields $\varphi_c = 41^\circ$.

3. EXPERIMENT

We have measured the intensity of turbulence as a function of the angle φ from the direction of azimuthal current in the plane perpendicular to the magnetic field, which is obtained from equation (1) as

$$S(\varphi) = \int_{-\infty}^{\infty} \int_{\Delta k} \int_{\varphi-(\Delta\varphi/2)}^{\varphi+(\Delta\varphi/2)} S(\omega, k, \varphi, \psi = 0)\, d\omega dk d\varphi \quad . \quad \ldots \quad (12)$$

The technique employed here is essentially the same as reported previously (DAUGHNEY et al, 1970). Light is scattered from a 50 MW ruby laser beam during the transit of the shock through the beam. The signal is detected by a photomultiplier with a 35 Å pass band optical filter. The pulse of scattered light is collected through a window which is covered by different masks in order to vary the scattering plane. These masks accept scattered light with mean $k = 7.7 \times 10^5$ m^{-1} ($\sim 1/\lambda_D$) and spread $\Delta k = 3.0 \times 10^5$ m^{-1} and with $\Delta\varphi$ of either 4^o or 8^o with midpoint in the range between $\varphi = 0^o$ and 90^o. The results for $M_A = 2.5$ are shown in Fig.2 and correspond to $\varphi_c = 50^o$.

Although the light is detected in directions corresponding to $0^o < \varphi < 90^o$, there is ambiguity about the sign of $\vec{k}$ because the scattering waves have $\vec{k} = \pm (\vec{k}_i - \vec{k}_s)$ and corresponding frequency $\omega = \pm (\omega_i - \omega_s)$ where i and s are incident and scattered (Fig.1(b)). Previous experiments, DAUGHNEY et al. (1970), showed that the positive sign must be taken and consequently $\vec{k}$ makes an angle $\varphi + 180^o$ ($\equiv \varphi^*$) and the observed turbulence covers $0^o < \varphi^* < 50^o$. We are thus observing scattered light with $\vec{k}$ directed in the sector between $-\theta$ and r. In terms of shock parameters this sector is between $\vec{J}_e = -\vec{J}_\theta (\varphi^* = 0^o)$ and $-J_\theta \times B_z (\varphi^* = 90^o)$.

The possibility of an effect on $S(\varphi)$ due to a variation of the form of $S(k)$ with φ has been excluded by checking $S(k)$ for two ranges of φ, $\varphi = 0$ to 8^o and 32^o to 40^o. The ratios of the signal from full aperture ($\Delta k = 6.2 \times 10^5$ to 9.2×10^5 m^{-1}) to that from smaller one ($\Delta k = 6.2 \times 10^5$ to 6.75×10^5 m^{-1}) are the same within experimental error for the two ranges of φ.

4. DISCUSSION

The observed cut-off angle $\varphi_c = 50^o$ and extreme angle of 60^o (Fig.2) are in good agreement with the value of 55^o calculated on the above model of ion-wave turbulence with $T_{ir} > T_{i\theta}$ and not with the value of 81^o for $T_{ir} = T_{i\theta}$.

We must consider the possibility that the observed cut-off does not correspond to zero growth but to sufficient linear growth within the shock ($t \sim 6 \times 10^{-9}$ s) for the level of turbulence to be observed. The observed enhancement over the thermal level for $\varphi = 0$ is $\epsilon_{ob} \sim 10^2$

(DAUGHNEY et al, 1970). (As the theoretical linear growth for $\varphi = 0$ would yield $\epsilon = e^{2\gamma t} \sim 10^8$ we assume non-linear limitation.) If we assume the large cut-off angle $\varphi_C = 81^o$ (isotropic heating), linear growth should give $\epsilon \sim 10^2$ for $\varphi = 67^o$ and $\epsilon \sim 10^5$ by $\varphi = 50^o$. The rapid increase in γ as φ decreases from φ_C means that, if the cut-off is related to the linear growth, it must be near $\gamma = 0$.

If we were to assume isotropic ion heating, the above mean shock conditions and the observed cut-off $\varphi_C = 50^o$, then Stringer's theory would yield $T_{i2} = 6.2\ eV$. This is about three times the adiabatic value and is therefore counter to the normal assumption that ion heating is adiabatic below the critical Alfven Mach number $M_A = 2.7$. However the experimental evidence (PAUL et al, 1967; PAUL et al, 1971), cannot preclude the possibility of the small amount of irreversible ion heating required here.

Other linear instabilities have been proposed (GARY, 1970; GARY and SANDERSON, 1970; GARY and BISKAMP, 1971; LASHMORE-DAVIES, 1970) which could drive the turbulence within the perpendicular shock wave, namely

(a) Growth of negative energy (Doppler shifted) Bernstein wave when Landau damped, and

(b) Unstable crossing of Doppler shifted Bernstein modes with the ion-wave mode.

While the scaling of frequency spectrum with ω_{pi} for different plasma condition (DAUGHNEY et al. 1970; PAUL et al., 1971) excludes the possibility of the former, the latter has not been completely ruled out.

The instability due to Bernstein waves are restricted to a fan about $\vec{J}_e$ within a few degrees of perpendicular to B, i.e. $\psi_c \sim 2^\circ$. Consequently the cut-off angle ψ_c in the plane containing the magnetic field is crucial in deciding which instability is dominant. The measurements have been limited, so far, to the plane perpendicular to the magnetic field. If the cut-off angle ψ_c is larger than φ_c as suggested by equation (11), this will not only further support the model of ion-wave turbulence, but also the presence of ion temperature anisotropy within the shock front.

The peak in $S(\varphi)$ at $\varphi^* = 20^\circ$ (i.e. between $-\vec{J}_\theta$ and $-\vec{J}_\theta \times \vec{B}_z$) appears to be genuine and has been confirmed recently in another experiment (MACHALEK, 1972). Also a recent computation (CHODURA, 1972), has produced a maximum in $S(\varphi)$ shifted by 25° in the same direction as is observed. Unfortunately, at present there is no physical interpretation of this angular shift. These two-dimensional computations also confirm our assumption (PAUL et al., 1971) that the magnetic field has little effect on the basic ion wave turbulence produced by current across a magnetic field as in the shock described.

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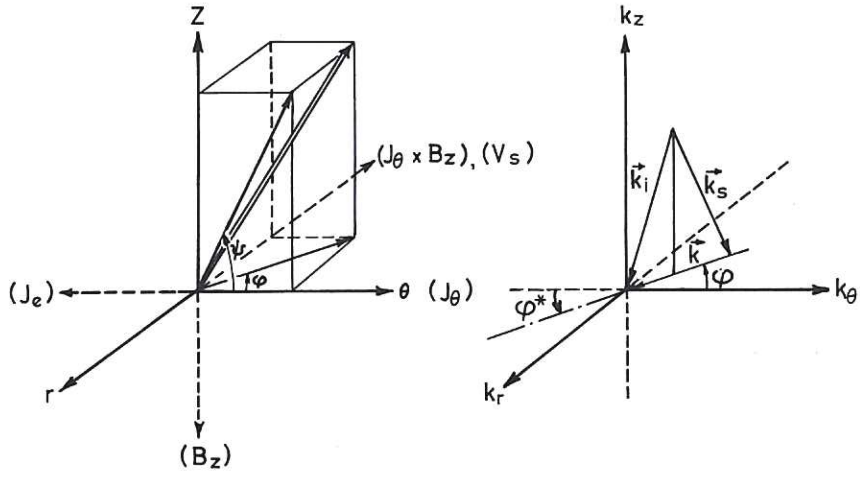

Fig. 1 Coordinates. (a) Physical space. The directions of shock propagation, current, and magnetic field are expressed by $-$ r, θ, and $-$z, respectively. (b) k-space. $\vec{k} = \vec{k}_i - \vec{k}_s$.

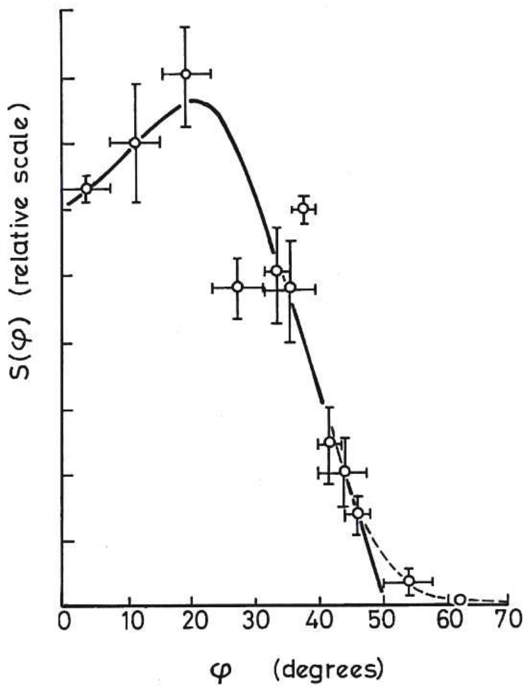

Fig. 2 Dependence of the intensity of turbulence on the angle ϕ from the direction of azimuthal current in the plane perpendicular to the magnetic field.